Inflation, Economic Growth, and Education Expenditure

Tiago Neves Sequeira§

Univ Coimbra, CeBER, Faculdade de Economia

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§ Av Dias da Silva 165, 3004-512 Coimbra. The author acknowledges the contributions of two anonymous referees and the editor, which improved the article. The usual disclaimer applies.

Abstract

Recent evidence shows that different inflation rates have effects on long-run economic growth. We follow the increasing interest in the issue offering a new explanation for the influence of monetary policy on economic growth: cash requirements for household expenditures in education. We devise an endogenous growth model with cash-in-advance (CIA) constraints in several sectors (education, horizontal R&D, vertical R&D, and manufacturing and consumption) and study its steady-state and transitional dynamics. In particular, the CIA constraint in education expenditures of households is essential to obtain a negative relationship between inflation and economic growth in the long-run. Quantitatively, this monetary endogenous growth model replicates both the small influence of monetary policy on growth, while highlighting the effects it can have on welfare and the allocation of resources in different sectors of the economy. *Keywords:* endogenous economic growth, inflation, interest rate, monetary policy, cash-in-advance (CIA). *JEL:* O30, O40, E13, E17, E61.

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1 Introduction

Until very recently, endogenous growth theory ignored the mechanics under which monetary policy can influence endogenous growth (examples of early studies are Jones and Manuelli (1995) and Chari et al. (1995)). As is recognized by Klump and LaGrandville (2011) and Klump and Jurkat (2018), empirical research has pointed out the negative effects of inflation on growth while economic growth theory has been dominated by the *superneutrality* result. On the one hand, most central banks are constitutionally committed to price stability. On the other hand, price instability seems to have important short-run welfare effects (Lucas, 2000),[1] which explains that the inflation-growth nexus is paramount. Our novel contribution to that explanation considers a human capital accumulation sector in which households need cash to cover expenditures related to education. This proves to be crucial in explaining the long-term relationship between inflation and economic growth. We generally refer to such costs as *tuition fees,* although they include all non-opportunity costs linked with education.[2]

More recent empirical results reported that the effects of inflation on economic growth are only quantitatively relevant above a given inflation rate threshold (see Bick, 2010; Arawatari et al., 2018). Also, Lopez-Villavicencio and Mignon (2011) present empirical evidence according to which there is a threshold above which inflation exerts a negative effect on growth and below which it enhances growth for advanced countries. This means that we should observe very small (positive or even negative) effects of inflation on growth for low inflation rates and somewhat higher (negative) effects for high inflation rates.

Recently, several contributions have shed light on the mechanisms that link inflation to the real side of the economy. First, there was a concern to explain the negative influence of inflation on growth (Frunk and Kromer, 2010; Chu and Lai, 2013; Chu and Cozzi, 2014). Some of these studies also quantify the welfare effects of a drop in inflation in the context of endogenous growth models (Chu and Lai, 2013; Chu and Cozzi, 2014). Heterogeneity in firms (Chu et al., 2019a) and households (Arawatari et al., 2018) have been introduced in the models in order to study the effect of inflation on income distribution, and simultaneously, to allow for the empirically validated inverted U-shaped relationship between economic growth and inflation to be replicated. The non-monotonous (U-shaped

[1]Fischer (1979) and Cohen (1985) were able to show analytically that inflation can hasten convergence toward the steady state.

[2] e.g., books and studying material, rents, or transportation costs.

or inverted-U-shaped) relationship between inflation and economic growth has been one of the most debated issues in these contributions. However, some papers have been unsuccessful in replicating the inverted U-shaped pattern. Most articles just suggest a monotonous negative relationship, while others obtain a convex curve (e.g., Gil and Iglesias, 2020). The effects of inflation on macroeconomic variables other than economic growth were also analyzed (e.g., Chu and Ji, 2016;Gil and Iglesias, 2020).[3] Most of these studies present comparative steady-state analyses and qualitative assessments of the transitional dynamics and do not assess the transition paths quantitatively. Exceptions are the most recent studies by Chu et al. (2019b), Zheng et al. (2019), and Klump and Jurkat (2018). Also, most of these studies justify the money demand through the cash-in-advance (CIA) channel (and not through the money-in-utility -MIU - channel). An exception using the MIU approach is Chu and Lai (2013).

Among those, the papers most related to ours are Chu et al. (2019b) and He (2018), because they, like ours, are the only studies to-date that consider human capital accumulation as a source of growth in a monetary endogenous growth model. We differ from them in four fundamental features.

First, we incorporate cash requirements in several expenditures in the model, including all sources of long-run growth (human capital accumulation, and R&D through increasing varieties and improving qualities). Crucially, this differs from Chu et al. (2019b) as we also introduce a cash requirement for household expenditures in education. In fact, the existence of education costs (or the *tuition fee*) is a very realistic feature in most developed countries that have been overlooked in endogenous growth literature. These education costs are a proportion of the wage the agent (or his household) earns in the labor market. This can be thought of as the *fee* to pay for depending on household labor income (as is the case in the USA through the expected family contribution dating back to the 1965 Higher Education Act) or a wage-contingent payment of a loan the agent takes out to earn a college degree.

[3]We refer to Chu (2020) for a recent survey of this literature.

As shown in Brossard et al. (2015) and Iori et al. (2016), a non-negligible percentage of education costs are supported by the households, even for primary schooling, as is the case in most developed countries.[4] Several policy-oriented documents (e.g., Patrinos, 2007) support that at least some families need cash for education. From this comes some of the support for demand-side policies for education (e.g., vouchers). It is natural to think of cash-in-advance (CIA) requirements for education household expenditures, since book and school material costs are naturally incurred before the learning process is completed. Fees, transportation, and rentals for housing are also paid before degrees are earned. Available data on household cash holdings seems to support this claim. In fact, US Census data showed that only 6% and 12% of total wealth of households with no children is allocated to education savings accounts and cash, respectively. However, those percentages rise to nearly 23% each for households with one or more child under 18 years old. Additionally, younger (below 35) married couples - who typically invest more in their own and their children's education hold much more cash (nearly 42%) than the average household (nearly 9%). In the model, this cash requirement to pay the *fee* is essential to obtaining a negative long-run effect of inflation on economic growth.

Furthermore, our contribution also differs from He (2018) who *only* considered a CIA constraint in human capital because we consider (more realistically) that other sectors in the economy also face CIA constraints, while also addressing the allocation of resources among different sectors.[5]

[4]For example, in Spain and the United Kingdom the percentage of the households supporting costs with primary education is around 10%, while in South Korea it reaches almost 20%. This cost proportion rises to 25% for secondary education in Japan and South Korea. In Spain, New Zealand, Italy, and Lithuania, families support 20% to 30% of tertiary education (colleges and universities), while this value rises to nearly 75% in the USA, United Kingdom, and Japan.

[5] He (2018) does not present results on transitional dynamics and only includes material inputs in education. Contrary to what Chu et al. (2019b) assumed, we consider that skilled labor (human capital) is used in all sectors in the economy — in the tradition of Arnold (1998), which implies that in our case, the CIA in human capital accumulation is in fact crucial to obtain a (non)neutral effect of monetary policy on economic growth at the steady-state. Moreover, for the first time, we devise a model in which both the education and the manufacturing sector are subject to CIA constraints.

Second, in an extension of our model, incorporating a nonlinear tuition fee, we feature a theoretical concave and nonlinear relationship between inflation and economic growth that conforms to the one that is empirically validated.

Third, our model incorporates both horizontal and vertical R&D. In fact, this is the first time that in this literature, both vertical and horizontal R&D are considered together with human capital accumulation.[6] This contribution is most important for quantitative reasons. Finally, we also consider extensions never studied before in the context of monetary endogenous growth models: a depreciation of human capital, population growth, and the possibility of altruistic agents.

We offer alternative explanations for small and nonlinear (inverted U-shaped) effects of inflation on economic growth. Moreover, we calculate the welfare effects of those policies. While the simple costs associated with CIA constraints may account for small growth effects (which are potentially different for different growth sources), a nonlinear cost for the *tuition fee* is needed to account for the empirically plausible nonlinear inflation-growth nexus.

The paper is organized as follows. After this introduction, in Section 2, we present a monetary endogenous growth model with human capital accumulation and horizontal and vertical R&D. In Section 3, we present the steady-state analysis and the most important theoretical results. In Section 4, we derive the dynamic system describing the model economy. In Section 5, we simulate the economic evolution following a monetary policy shock on the nominal interest rate and study the influence of several monetary features on the growth rate and welfare (which takes into account transitional dynamics), and we offer results from a number of extensions of the baseline model. In Section 6, we conclude the paper.

[6] Thus, our model builds on Strulik (2005) — which equilibrium stability and transitional dynamics properties were studied in Sequeira et al. (2014) — to devise a monetary endogenous growth model.

2. The Model

In this section we devise the model. The model includes human capital accumulation and both horizontal and vertical R&D. Additionally, we introduce the possibility of monetary policy and the study of its effects on growth in both the steady-state and during the transitional dynamics. The subsections include the description of households, monetary authority, and firm behavior as well the aggregation of the innovation process, money, and skilled labor demand.

2.1. Households

The representative agent maximizes intertemporal utility U depending on

consumption C_t

$$U = \int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \qquad (1)$$

where $\rho > 0$ denotes the time preference rate, and $1/\theta$ (with $\theta > 1$) is the elasticity of intertemporal substitution. They also earn returns, r, per unit of aggregate wealth, A_t, and retain real money holdings, $m_t = \frac{M_t}{P_t}$, with M_t being the nominal money supply and P_t the price level in the economy. They invest in their own education whose stock is denoted by H_t, which is devoted to human resources H_{Ht}. This education effort is subject to an opportunity cost because of being out of the labor market (which is a common assumption). It is also subject to a non-opportunity cost of education, $p > 0$, which we call a *tuition fee* or just fee. All this leads to the following budget constraint:[7]

$$\dot{A} + \dot{m} = rA_t + w_t(H_t - (1+p)H_{H_t}) - \Pi_t m_t - C_t + i_t b_t + T_t \qquad (2)$$

where w_t is the real wage from labor, Π_t is inflation rate that determines the cost of holding money, i_t is the nominal interest rate, which is the return for the amount of money borrowed from households to firms (b_t), and T_t represents lump-sum transfers (or taxes, if negative) from the monetary authority

[7]Variable x variation is denoted by $\dot{x}$. Also, let g_x denote x's growth rate, $g_x = \frac{\dot{x}}{x}$.

to households (seigniorage). Human capital is accumulated according to the following Lucas (1988) - type law of motion:

$$\dot{H} = \xi H_{H_t}, \qquad (3)$$

where £ > 0 represents productivity in the accumulation of human capital. Households (and firms) are subject to CIA constraints, ensuring that some of the costs they incur must be paid in liquid money, thus:

$$m_t \geq \Theta_c C_t + \Theta_p p w_t H_{H_t} + b_t, \qquad (4)$$

And $b_t = \Theta_x w_t H_{x_t} + \Theta_n w_t H_{n_t} + \Theta_Q w_t H_Q$, where parameters $\Theta j (j = \text{p}, H, x, n, Q)$ measure the strength of each CIA constraint for each sector (education/human capital employing H_{Ht}, manufacturing employing H_{Xt}, horizontal R&D, employing H_{nt}, and vertical R&D, employing HQ_t, respectively) in the model. The first-order conditions for an interior solution of the maximization problem of utility (1), with restrictions (2), (3), and (4) yield:[8]

$$g_{C_t} = \frac{r_t - \rho}{\theta}, \qquad (5)$$

$$g_{w_t} = r_t - \frac{\xi}{1 + p(1 + i_t \Theta_p)}. \qquad (6)$$

$$i_t = r_t + \Pi_t. \qquad (7)$$

Equation (5) comes from the standard Ramsey rule. Equation (6) indicates that investment in human capital is ensured when the overall return on human capital $g_w + \frac{\xi}{1+p(1+i_t\Theta_p)}$ is equal to the return on financial assets r_t. It is worth noting that the *tuition fee* introduced in this paper is relevant just because we are in a monetary endogenous growth model, i.e., because $i_t > 0$ and $\Theta_P > 0$. Otherwise, the *tuition fee p,* although empirically relevant, would be equivalent to a discount of the productivity of learning (ξ) in the wages growth rate. Equation (7) is a no-arbitrage condition between real money balances and real financial assets (which is the well-known Fisher equation).

[8]Control variables are C_t, b_t and H_{H_t} and state variables are a_t, m_t, and H_t. More details on the solution of the optimal control problem are given in a (not for publication) Appendix.

2.2. Monetary Authority

The monetary sector in each country is considered using the same setup that is now usual in monetary endogenous (or Schumpeterian) growth literature (e.g., Chu and Cozzi, 2014; Chu et al., 2019a, b). The growth rate of nominal money supply, M_t, is $\mu_t = \frac{\dot{M}_t}{M_t}$. Since the growth rate of P_t is the inflation rate, $\Pi_t = \frac{\dot{P}_t}{P_t}$, the growth rate of m_t is $\frac{\dot{m}_t}{m_t} = \mu_t - \Pi_t$. We consider that the monetary authority adopts an inflation targeting framework in which the monetary policy instrument is the nominal interest rate. In this context, we follow the literature and assume that the nominal interest rate is exogenously chosen by the monetary authority (e.g., Chu and Cozzi, 2014; Chu and Ji, 2016; Chu et al., 2017; Chu et al., 2019a,b), such that i_t = i, and thus Π_t is endogenously determined according to the Fisher equation (7), for each r_t: $\Pi_t = i - r_t$. Then, given Π_t, the growth rate of the nominal money supply will be endogenously determined according to $\frac{\dot{m}_t}{m_t} + \Pi_t = \mu_t$. That is, the monetary authority will endogenously adjust the money growth rate to whatever level is needed for the interest rate i to prevail. As is usual in the literature, we consider that to balance its budget, the monetary authority returns the seigniorage revenues to households as a lump-sum transfer, i.e., $T_t = \frac{\dot{M}_t}{P_t} = \frac{(m_t P_t)}{P_t} = \frac{\dot{m}_t P_t + \dot{P}_t m_t}{P_t} = \dot{m}_t + \Pi_t m_t$.[9]

2.3. Firms and Markets

2.3.1. Goods Markets

Intermediate goods are used to produce a final good Y that is sold on a competitive market. Technology has constant returns to scale with elasticity of substitution $\alpha > 1$. Aggregate output of Y can be written as

$$Y = [\int_0^n (\sum_{kj=0}^{kj} q_{kj} x_{kj})^{1-1/\sigma} dj]^{1/(1-1/\sigma)} \qquad (8)$$

After k innovations in quality (vertical) research, a variety/good $j = 1, \ldots, n$ is available at qualities q_{kj}, with $kj = 0.1, \ldots. Kj$, which is used in quantities x_{kj}.

[9]The time subscript t is eliminated from now on for simplification purposes.

Profit maximization in the final goods sector yields demand for intermediates

$$x_{k_j} = \frac{q_{k_j}^{\sigma-1} p_{k_j}^{-\sigma} Y}{\int_0^n \left(\frac{q_{k_j}^{\sigma-1}}{p_{k_j}^{\sigma-1}} \right) dj}, \tag{9}$$

where p_{kj} denotes the price of a unit of x_{kj} denominated in units of final goods.

Each intermediate good is produced using human capital such that $x_{kj} = H_{Xj}$. Intermediate goods are produced by monopolistically competitive firms that take wages w, and nominal interest rate i as given and maximize profits

$$\pi_{k_j} = p_{k_j} x_{k_j} - wH_{x_j} - i\Theta_x wH_{x_j} = p_{k_j} x_{k_j} - (1 - i\Theta_x) wH_{x_j}. \tag{10}$$

In equation (10), costs are composed by the usual real labor (human capital) costs and the costs associated with the CIA constraint. After substituting (9) in (10), profit maximization yields prices

$$p_{k_j} = w\frac{\sigma(1+i\Theta_x)}{\sigma - 1} \tag{11}$$

Inserting (11) into (9) yields supply of good j:

$$x_{k_j} = \frac{\sigma - 1}{\sigma(1+i\Theta_x)} \frac{q_{k_j}^{\sigma-1} Y}{wnQ}. \tag{12}$$

Therefore, instantaneous profits of the firm supplying good j are:

$$\pi_j = \frac{1}{\sigma} \frac{q_{k_j}^{\sigma-1} Y}{nQ}. \tag{13}$$

This means that intermediate firm profits are not affected by monetary policy. This is because the CIA costs are repassed by intermediated goods firms to the prices, thus not affecting profits.

2.3.2. Horizontal R&D

It is worth noting that research occurs both because of the increasing number of technologies (increasing varieties - horizontal R&D) and the increase in their quality (vertical R&D). First, we will characterize the horizontal R&D sector.

New technologies appear in the economy as a result of the activity of an

increasing varieties sector such that

$$\dot{n} = A_n H_n, \qquad A_n = \frac{n^{\beta_1} Q^{\beta_2} H_n^{-\chi}}{Q}, \tag{14}$$

where n is the number of varieties in the economy, Q is the aggregate level of quality, which is defined below, (recall that) H_n is the allocation of human capital to the horizontal R&D sector, β_1 measures the typical spillover effect within the horizontal R&D sector *(standing-on-the-shoulders* effect), β_2 is the cross-sector spillover from the quality (vertical R&D) sector to the horizontal R&D sector, and x measures the duplication effect *(stepping-on-toes*). In fact, A_n is the aggregate productivity *per quality level* of each individual human capital unit hired to increasing variety R&D.

Firms in this sector maximize their expected value (the number of varieties produced times its value V_n) minus research costs

$$\pi_n = \dot{n}V_n - wH_n - i\Theta_n wH_n. \qquad (15)$$

The free-entry condition is thus: $V_n n = \mathrm{w}(1 + iQ_n)H_n$, using (14), implying that

$$\frac{\dot{V_n}}{V_n} = \frac{\dot{w}}{w} - \beta_1\frac{\dot{n}}{n} - (\beta_2 - 1)\frac{\dot{Q}}{Q} + \chi\frac{\dot{H_n}}{H_n}. \qquad (16)$$

This means that the value of an innovation times the number of innovations must be equal to the cost of producing those innovations.

The non-arbitrage condition for the increasing varieties firm is $r = \pi_{0j} + \frac{\dot{V_n}}{V_n} - \varsigma\frac{V_{0j}}{V_n}$, meaning that the return on investing in assets of this sector (rV_n) plus the innovators' expected losses through the first quality improvement (q) times the value of that innovation (V_{0i}) must be equal to the instant profits from selling the respective intermediate good (produced with the state-of-the-art technology) (n_{0j}) plus the valorization of the patent (Vi). Note that ςV_{0j} denotes the (horizontal) innovators' expected loss through the first quality improvement. Then $V_{0_j} = V_{k_j}\frac{Q}{q_{k_j}^{\sigma-1}}$.

Inserting the free-entry condition yield by (15) into the no-arbitrage condition yields

$$r = \frac{1}{\sigma}\frac{An^{\beta_1-1}Q^{\beta_2-1}H_n^{-\chi}Y}{w(1+i\Theta_n)} + \frac{\dot{w}}{w} - \beta_1\frac{\dot{n}}{n} - (\beta_2-1)\frac{\dot{Q}}{Q} + \chi\frac{\dot{H_n}}{H_n} - \varsigma Q\frac{V_{k_j}}{V_n}\frac{1}{q_{k_j}^{\sigma-1}}. \quad (17)$$

2.3.3. Vertical R&D

As mentioned before, there is also a vertical R&D sector. At each point in time an improvement from quality level k to $k+1$ occurs with probability

$$\varsigma_{k_j} = \frac{A_Q}{q_{k_j}^{\sigma-1}} H_Q, \quad A_Q = A n^{\alpha_1} Q^{\alpha_2} H_Q^{-\chi}. \tag{18}$$

where $Q = \frac{1}{n}\int_0^n \left(q_{k_j}^{\sigma-1}\right) dj$ measures the cross-sector spillovers effects from horizontal to vertical R&D, and α_2 measures the own sector spillover effects in the vertical R&D sector. When invented at time T, a good j has initial quality $Q_\tau^{\frac{1}{\sigma-1}}$ and after K quality innovations, it has quality $q_{k_j} = \gamma^{k_j} Q_\tau^{\frac{1}{\sigma-1}}$, where seven governs the increase in quality from one innovation to another, such that each good is equivalent to seven units of the next best good. If an innovation occurs in a sector j, quality grows at $\left(\gamma^{(k_j+1)(\sigma-1)} - \gamma^{k_j(\sigma-1)}\right)/\gamma^{k_j(\sigma-1)} = \gamma^{\sigma-1} - 1$ rate. As the parameter γ measures the increase in quality within each vertical R&D sector, the $\gamma^{\sigma-1} - 1$ term measures the creative destruction effect. In any sector at any time, an innovation occurs with probability q per unit of time. Hence, expected growth of the quality index is $\frac{\dot{Q}}{Q} = \varsigma(\gamma^{\sigma-1} - 1) = (\gamma^{\sigma-1} - 1)\frac{A_Q}{Q}\frac{H_Q}{n}$.

Thus, vertical R&D process follows the accumulation function

$$\dot{Q} = (\gamma^{\sigma-1} - 1) A n^{\alpha_1 - 1} Q^{\alpha_2 - 1} H_Q^{1-\chi}, \tag{19}$$

Recall that the allocation of human capital to the vertical R&D sector is H_Q. Firms in this sector maximize expected revenue minus research cost

$$\pi_Q = \varsigma_{k_j} V_{k_j} - w H_{Q_j} - i\Theta_Q w H_Q, \tag{20}$$

The free-entry condition is, $\varsigma_{k_j} V_{k_j} = w(1 + i\Theta_Q) H_{Q_j}$ which implies

$$\frac{\dot{V}_{k_j}}{V_{k_j}} = \frac{\dot{w}}{w} - \alpha_1 \frac{\dot{n}}{n} - \alpha_2 \frac{\dot{Q}}{Q} + \chi \frac{\dot{H}_Q}{H_Q}. \tag{21}$$

This means that the value of the innovation times the number of quality improvements made on the available technologies (varieties) must be equal to the cost of producing these innovations.

The non-arbitrage condition for the quality-ladder firm states that $r = \pi_{k_j}/V_{k_j} + \frac{\dot{V}_{k_j}}{V_{k_j}} - \varsigma_{k_j}$., meaning that the return from investing in assets (rV_k.) must be equal to the instant profits from selling the respective state-of-the-art quality intermediate good (π_k.) plus the increase in the value of the patent ($\dot{V}_k$.) minus the probability that a competitor succeeds in research and drives the incumbent out of business (ς_k). Inserting (13), (18), and the free-entry condition given by (20) into the no-arbitrage condition provides

$$r = \frac{1}{\sigma}\frac{An^{\alpha_1-1}Q^{\alpha_2-1}H_Q^{-\chi}Y}{w(1+i\Theta_Q)} + \frac{\dot{w}}{w} - \alpha_1\frac{\dot{n}}{n} - \alpha_2\frac{\dot{Q}}{Q} + \chi\frac{\dot{H}_Q}{H_Q} - \varsigma_{k_j}, \qquad (22)$$

2.4. Aggregate Innovation, money, and skilled labor demand

Demand for research in the vertical R&D is given by transforming (18) in

$$H_Q = \int_0^n H_{Q_j}\,dj = \frac{\varsigma\int_0^n q_{k_j}^{(\sigma-1)}dj}{n^{\alpha_1}Q^{\alpha_2}H_Q} = \frac{\varsigma nQ}{n^{\alpha_1}Q^{\alpha_2}H_Q}. \qquad (23)$$

Integrating (12), we obtain the demand for human capital in the intermediate goods sector H_x

$$H_x = \frac{\sigma-1}{\sigma(1+i\Theta_x)}\frac{Y}{w}, \qquad (24)$$

which implies that $\frac{\dot{H}_x}{H_x} = g_Y - g_w$.

Solving (8) and taking into account that $Q = \int_0^n q_j^{\sigma-1}dj$, the final good output can be written as follows:

$$Y = (nQ)^{\frac{1}{\sigma-1}}H_x. \qquad (25)$$

Combining growth rates resulting from (24) and (25), the growth rate of wages is

$$g_w = \frac{1}{\sigma-1}(g_n + g_Q). \qquad (26)$$

This equation tells us that the growth rate of real wages is proportional to the technological growth rate in the economy.

2.5. Characterization of the Equilibrium

The general equilibrium is a time path of prices $\{p(j), r, w, i, V_n, V_k(j)\}$ and allocations $\{C, m, b, Y, x(j), H_H, H_X, H_Q, H_n\}$, which satisfy the following conditions at each instance of time:

- households maximize utility taking prices $\{r, w, i\}$ as given
- competitive final goods firms maximize profit taking p(j) as given
- monopolistic intermediate goods firms choose $\{p(j), H_X\}$ to maximize profit taking w as given
- the skilled labor (or human capital) market clears (that is, $H_H + H_X + H_Q + H_n = H$)
- the final goods market clears (that is, $Y = C$)
- the (total) value of monopolistic firms $(V_n, V_k(j))$ adds up to the value of household assets (A_t)
- the CIA constraint $\Theta_c C_t + \Theta_p p w_t H_{H_t} + b_t = m_t$ binds
- the real money balance borrowed by R&D entrepreneurs from the household is $b_t = \Theta_x w_t H_{x_t} + \Theta_n w_t H_{n_t} + \Theta_Q w_t H_{Q_t}$

3. Steady-State Analysis

First, we obtain a modified version of the no-arbitrage conditions after consideration of the free-entry conditions in R&D, i.e., modified versions of equations (17) and (22), respectively. Using (6), (14), and (24) and the fact that the expected growth of the quality index is $\frac{\dot{Q}}{Q} = \varsigma(\gamma^{\sigma-1} - 1)$, we obtain the first of the following equations. Then, using (6), (19), and (24), we obtain the second of the following equations.[10]

$$\frac{\xi}{1+p(1+i\Theta_p)} = \left(\frac{H_x}{H_Q}\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_Q)} - 1\right)\frac{\dot{Q}}{Q}\frac{1}{\gamma^{\sigma-1}-1} - \alpha_1\frac{\dot{n}}{n} - \alpha_2\frac{\dot{Q}}{Q} + \chi\frac{\dot{H}_Q}{H_Q} \tag{27}$$

$$\frac{\xi}{1+p(1+i\Theta_p)} = \left(\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_n)}\frac{H_x}{H_n} - \frac{1+i\Theta_Q}{1+i\Theta_n}\frac{H_Q}{H_n}\right)\frac{\dot{n}}{n} - \beta_1\frac{\dot{n}}{n} - (\beta_2-1)\frac{\dot{Q}}{Q} + \chi\frac{\dot{H}_n}{H_n}. \tag{28}$$

Second, as usual in growth theory, we define a steady-state in which both the growth rates and allocation of human capital to different sectors in the economy are constant. Using (6), we obtain a value for r that can be substituted in (5). Using (26), we obtain the economic growth rate at the steady-state as

$$g_Y = \frac{1}{\theta}\left(\frac{\xi}{1+p(1+i\Theta_p)} + \frac{1}{\sigma-1}(g_n+g_Q)\right). \tag{29}$$

Furthermore, using R&D technologies in (14) and (19) and assuring that growth rates are constant yields the growth rates of technological varieties (from horizontal R&D) and quality (from vertical R&D)

$$g_n = \frac{(\alpha_2-\beta_2)(1-\chi)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)}g_H, \tag{30}$$

and

$$g_Q = \frac{(\beta_1-\alpha_1)(1-\chi)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)}g_H. \tag{31}$$

For positive growth, one of the following pairs of conditions has to be fulfilled: $\alpha_2 > \beta_2$ and $\beta_1 > \alpha_1$ or $\alpha_2 < \beta_2$ and $\beta_1 < \alpha_1$.

Inserting (30) and (31) into (29) and using the growth rate version of (25), we obtain the steady-state growth of human capital:

$$g_H = \frac{\left(\frac{\xi}{1+p(1+i\Theta_p)} - \rho\right)/(\sigma-1)}{\frac{(\theta-1)(1-\chi)(\beta_1-\alpha_1+\alpha_2-\beta_2)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)} + \theta}. \tag{32}$$

[10]For the second parcel in the term in parentheses in equation (28), we need to use free-entry conditions $V_n\dot{n} = w(1+i\Theta_n)H_n$, $\varsigma_{k_j}V_{k_j} = w(1+i\Theta_Q)H_Q$ and the following already mentioned definitions $V_{0_j} = V_{k_j}\frac{Q}{q_{k_j}^{\sigma-1}}$ and $Q = \int_0^n q_j^{\sigma-1}dj$.

Inserting (32) into (29) after using (30) and (31), taking into account that money holdings are constant at the steady-state (which is implied by equation 4) yields the steady-state growth of consumption, money holdings, and output *per capita* as determined solely by the model's parameters of technology and preferences: [10]

$$g_c = g_m = g_Y = \frac{\frac{\xi}{1+p(1+i\Theta_p)} - \rho}{\theta - 1}\left(1 - \frac{1}{\frac{(\theta-1)(1-\chi)(\beta_1-\alpha_1+\alpha_2-\beta_2)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)} + \theta}\right). \quad (33)$$

Regarding the relationship between inflation and growth, we arrive at the following proposition:

Theorem 1. *The effect of inflation in the steady-state growth rate is negative, monotonous, and nonlinear.*

Proof. From (33), use the Fisher equation, and obtain $\frac{\partial g_Y}{\partial \Pi}$. This yields: $-(\Upsilon/(\theta-1))\frac{\xi p \Theta_p}{(1+p(1+i\Theta_p))^2} < 0$, where $\Upsilon = 1 - \frac{1}{\frac{(\theta-1)(1-\chi)(\beta_1-\alpha_1+\alpha_2-\beta_2)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)}+\theta}$. This result holds for all values of parameters that ensure positive economic growth ($\alpha_2 > \beta_2$ and $\beta_1 > \alpha_1$ or $\alpha_2 < \beta_2$ and $\beta_1 < \alpha_1$ and $\frac{\xi}{1+p(1+i\Theta_p)} > \rho$), and in particular, for all values of i, ensuring a monotonous relationship. Finally, inspection of the expression of $\frac{\partial g_Y}{\partial \Pi}$ easily yield the conclusion that the derivative is not constant (depends on i) and thus the relationship is nonlinear. In particular $\frac{\partial^2 g_Y}{\partial \Pi^2} > 0$. □

The proposition makes it clear that the negative effect of inflation on growth is obtained due to the *tuition fee* p.[12] It is worth noting that the growth rate is always decreasing for positive economic growth rates. Higher the negative effect of inflation on growth, higher is the productivity of education (£). Both the *tuition fee* and the CIA parameter ©p have a non-linear relationship with economic growth. Exact quantitative effects are highlighted through calibration in the next subsections.

[10]This is a common feature of a non-scale endogenous growth model. [12]For $p = 0$, $\frac{\partial g_Y}{\partial \Pi} = 0$. .

3.1. Comparative Steady-State Analysis

Following Strulik (2005) and Sequeira et al. (2014), we consider $\beta_1 = 0.25$, $\beta_2 = 0.25$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\chi = 0.5$, $\sigma = 6.00$, $\xi = 0.0675$, $\rho = 0.02$, $\theta = 2$, and $\gamma = 1.05$. As in Arawatari et al. (2018), we consider a baseline situation in which all expenditures are made in cash such that $\Theta_p = 1$, and we consider a fee that is 25% of the wage, thus $p = 0.25$. Figure 1 shows the relationship between economic growth and the inflation rate in the steady-state for a series of inflation ranging from 1% to 70%. Despite the theoretical result in Proposition 1, which points to a nonlinear (convex) relationship, Figure 1 highlights a quasi-linear (negative) relationship between inflation rates and economic growth. Note also the very small effect of rising inflation in the long-run.[13] An increase in inflation between 1% and 60% implies a decrease of just 0.1 percentage point in the economic growth rate.

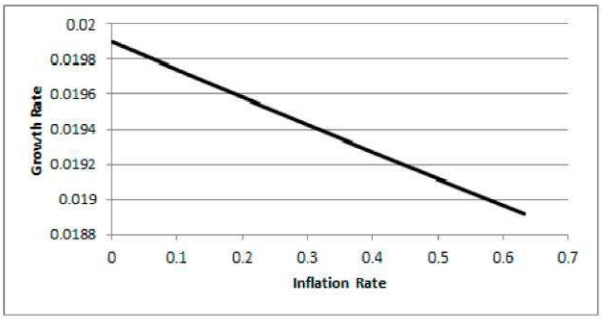


Figure 1: Relationship between steady-state growth rate and inflation rate

As the center of the non-neutrality result on long-run growth is the *tuition fee,* we propose that a nonlinear structure for such a cost may be an explanation for the non-linearity that resembles the small effects for low inflation levels and higher effects for quite high levels of the inflation rate. Suppose that the *fee* has a relationship with the inflation rate. Thus, assume that the fee, *p,* has a base value *p* and a variable value (i—i)^2 that is related to a given threshold for inflation, *i.* Intuitively, this means that the *tuition fee* highly increases in times of high inflation.[14] Specifically, assume the following *fee* function:

$$\hat{p} = p + (i - \bar{i})^2 \qquad (34)$$

If we set *i* = 0. 25, we can plot the long-run relationship between inflation and economic growth in Figure 2. This figure highlights three important results. First, it allows for slightly positive effects of inflation on growth for very low levels of inflation, a result that resembles the one pointed out by Fischer (1979) and Cohen (1985), but for the steady-state in our case. Second, it also replicates the stronger (negative) effects of rising inflation on steady-state economic growth also highlighted by Arawatari et al. (2018). Third, the concavity of the relationship crucially depends on the percentage of cash requirements for education, parameter ©$_p$. It is also worth noting that the relationship that best replicates the empirical relationship that has been reported in empirical studies (see also Arawatari et al., 2018) is the one that assumes the highest money requirement for schooling expenditures (blue line, with Θ_p=1).

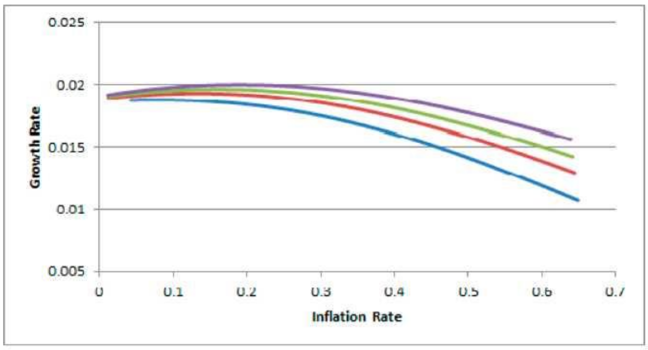


Figure 2: Relationship between steady-state growth rate and inflation rate under the alternative nonlinear *fee*.

Notes: blue line is for $\Theta_p = 1$, red line is for $\Theta_p = 0.75$, green line is for $\Theta_p = 0.25$, and purpleline is for $\Theta_p = 0$.

4. Stationary Variables and Transitional Dynamics

We will solve the dynamics of the model taking into account the following five stationary variables:

$$\upsilon_Q = \frac{Q}{H^{\frac{\beta_1-\alpha_1}{D}(1-\chi)}}; \tag{35}$$

$$\upsilon_n = \frac{n}{H^{\frac{\alpha_2-\beta_2}{D}(1-\chi)}}; \tag{36}$$

$$u_X = \frac{H_X}{H}, u_Q = \frac{H_Q}{H}, u_n = \frac{H_n}{H}. \tag{37}$$

where $D = (1-\alpha_1)(1-\beta_2) - (1-\alpha_2)(1-\beta_1)$. The first two variables are state variables and the last three are control variables. Based on these variables, we will derive a system of five differential equations. Saddle-path stability is thus guaranteed when the system features two negative eigenvalues, which happen in all our simulations. [15] By log-differentiation of (35), using the equations for the growth rate of qualities and the growth rate of human capital, given above, we obtain

$$\frac{\dot{\upsilon}_Q}{\upsilon_Q} = (\gamma^{\sigma-1}-1)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi} - \frac{\beta_1-\alpha_1}{D}(1-\chi)\left[\xi\left(1-u_X-u_Q-u_n\right)\right] \tag{38}$$

By log-differentiating (36), using the equations for the growth rate of varieties and the growth rate of human capital given above, we obtain

$$\frac{\dot{\upsilon}_n}{\upsilon_n} = A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi} - \frac{\alpha_2-\beta_2}{D}(1-\chi)\left[\xi\left(1-u_X-u_Q-u_n\right)\right] \tag{39}$$

By the log-differentiation of u_X, u_Q, and u_n, we obtain the following dynamic equations. For the equation of u_X, we use the log-differentiated version of (25) and (26) and the above law of motion for human capital (see equations 3 and the fact that $u_H + u_X + u_Q + u_n = 1$). For the equation for u_Q we use (27) and the law of motion for human capital. Finally, for the equation for u_n, we resort to (28) and to the law of motion for human capital.

$$\begin{aligned}\frac{\dot{u}_X}{u_X} &= \frac{1-\theta}{\theta}\frac{1}{\sigma-1}\left[A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi} + (\gamma^{\sigma-1}-1)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi}\right] \\ &\quad +\frac{1}{\theta}\left(\frac{\xi}{1+p(1+i\Theta_p)}-\rho\right) - \left[\xi\left(1-u_X-u_Q-u_n\right)\right]\end{aligned} \tag{40}$$

$$\begin{aligned}\frac{\dot{u}_Q}{u_Q} &= \frac{1}{\chi}\frac{\xi}{1+p(1+i\Theta_p)} - \frac{1}{\chi}\left(\frac{u_X}{u_Q}\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_Q)} - 1 - \alpha_2(\gamma^{\sigma-1}-1)\right)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi} + \\ &\quad +\frac{1}{\chi}\alpha_1 A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi} - \left[\xi\left(1-u_X-u_Q-u_n\right)\right]\end{aligned} \tag{41}$$

$$\begin{aligned}\frac{\dot{u}_n}{u_n} &= \frac{1}{\chi}\left[\frac{\xi}{1+p(1+i\Theta_p)} - \left(\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_n)}\frac{u_X}{u_n} - \frac{1+i\Theta_Q}{(1+i\Theta_n)}\frac{u_Q}{u_n} - \beta_1\right)A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi}\right. \\ &\quad \left. + (\beta_2-1)(\gamma^{\sigma-1}-1)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi}\right] - \left[\xi\left(1-u_X-u_Q-u_n\right)\right]\end{aligned} \tag{42}$$

5. Transitional Dynamics

In this section, we are particularly interested in detailing the transitional dynamics of variables of interest after an expansionary monetary policy that drops the nominal interest rate. In particular, we want to explore the allocation of human capital, growth, and the welfare effects of a monetary policy during the transitional dynamics. After that, in an analysis of robustness, we present the growth and welfare effects of a given monetary policy within different models, corresponding to changes to the underlying assumptions of the baseline model devised above.[16]

5.1. Baseline analysis

We consider the following set of initial parameters: $\beta_1 = 0.25$, $\beta_2 = 0.25$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\chi = 0.5$, $\sigma = 6.00$, $\xi = 0.0675$, $\rho = 0.02$, $\theta = 2$, and $\gamma = 1.05$, and $p = 0.25$.[17] As in Arawatari et al. (2018), we consider a baseline situation in which all education expenditures are made in cash such that $\Theta_c = \Theta_p = \Theta_x = \Theta_n = \Theta_Q = 1$. The initial nominal interest rate set by the monetary authority is $i_0 = 0.10$—corresponding to a high 10% inflation rate. This yields a (realistic) steady-state growth rate of $g_Y = 1.75\%$. Then, we introduce a monetary policy shock that fixed the new nominal interest rate to $i_{new} = 0.06$.[18] This will have a steady-state effect yielding a new $g_Y = 1.77\%$.

In an alternative exercise, we consider lower values for parameters associated with the CIA constraint, assuming that some of the expenditures do not need cash-in-advance. As argued in Arawatari et al. (2018) and in references therein (e.g., Schifer and Weder, 2001 ; Beck et al., 2005), smaller firms, especially in countries with high inflation, face more difficulties in doing business due to higher inflation. Thus, higher values for parameters © should be associated with higher inflation. In a situation in which we want to simulate a more developed, higher-growth economy, we may consider different values for different parameters such that $\Theta < 1$. In this case, we assume that firms can make proportionally fewer payments in money than families and that R&D firms need higher cash flow for payments than intermediate goods firms (as also assumed in Chu and Cozzi, 2014). This can be a situation such that $\Theta_c = \Theta_p = 0.9$, $\Theta_n = \Theta_Q = 0.7$, and $\Theta_x = 0.5$. Initial and final nominal interest rates set by the monetary authority are equal to the exercise described above. This yields an initial steady-state growth rate of $g_Y = 1.76\%$ and a final steady-state $g_Y = 1.78\%$. This exercise is done to evaluate the effects of having different CIA features. The immediate conclusion is that the lower the cash holding requirements, the higher the economic growth rate. However, a given change in the nominal interest rate will have almost the same quantitative effect in both scenarios (a higher economic growth rate in 0.02 percentage points).

5.2. Growth, allocation, and welfare effects of monetary policy

Figure 3 illustrates the transitional dynamics for the baseline exercise described above for the main variables in the model. Variables u_Q and u_n are the state variables in the model, so they cannot jump after a monetary shock (see Figures 3a and 3b). Intuitively, these variables measure the weight of technological knowledge when compared to embodied knowledge (human capital) as they are ratios between each of the varieties (from horizontal R&D) and quality (from vertical R&D) indexes and human capital. They decrease following a monetary policy shock indicating that the technological intensity in the economy decreases when compared to embodied knowledge intensity, resulting from the costs that R&D firms face due to inflation. Inflation decreases at once from nearly 5% to 4.6% and then continues to slowly decrease toward its new steady-state value (see Figure 3c). Allocation effects are interesting as they are mainly characterized by a reallocation of resources to human capital accumulation from the production of the final good, which is induced by the policy (see Figures 3d and 3f).[19] There are also transitional reallocations affecting R&D sectors but they are smaller, especially due to the smaller dimensions of these sectors in the economy (see Figures 3g and 3h). The monetary policy that decreases nominal interest rates and inflation is expansionary as it permanently increases the economic growth rate (see Figure 3e) both at the moment of the shock and through transition. However, as mentioned above and according to empirical evidence, the effects on economic growth are very small. This exercise uncovers that the mechanics that yield a positive (although small) inflation effect on growth are consistent with a reallocation of resources toward human capital accumulation.

Figure 4 shows the results of the alternative exercise with only partial cash holding requirements described above ($\Theta_c = \Theta_p = 0.9$, $\Theta_n = \Theta_Q = 0.7$, and $\Theta x = 0.5$). When compared to the previous case (depicted at Figure 3), qualitative effects are similar and quantitative effects are also close to the previous ones. The only remarkable difference is a slightly higher reallocation toward the R&D sectors that can be seen in Figures 4g and 4h. These results are seen essentially throughout transitional dynamics, while R&D benefits from the decreasing input (human capital) costs resulting from the high expansion in human capital.

Figure 5 depicts the time path of consumption with and without the monetary policy shock. The figure highlights small changes that have welfare gains, amounting to an increase in 0.63% (in the case of total cash requirements) and 0.58% (in the case of partial cash requirements), respectively. The welfare gains from an alternative drop in nominal interest rates from 10% to 0% were 1.58% (total cash requirements) and 1.45% (partial cash requirements). [20]

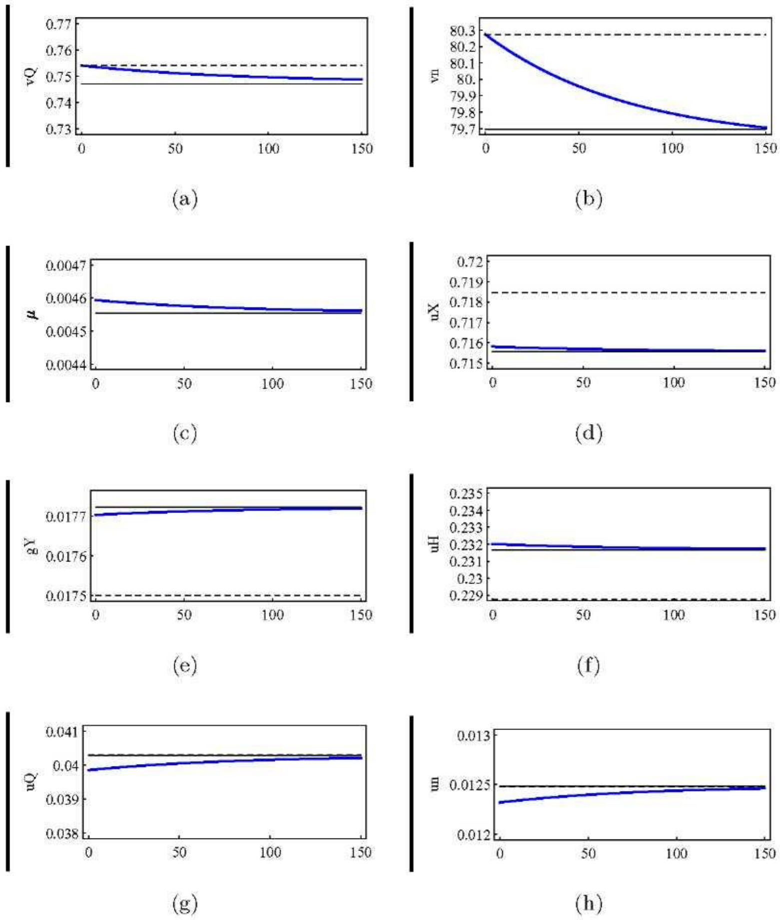


Figure 3: Effects of a shock on nominal interest rate i from 10% to 6%, with all expenses paid by cash.

Note: Thin black line represents initial steady-state values. Dashed line represents final steady-state values. If they coincide, initial and final steady-states coincide, despite the transitional dynamics. Both initial and final steady-states are saddle-path stable as the eigenvalues of the Jacobian of the dynamic system (38)-(42) are 0.0907421, 0.0520588, 0.0440464, -0.0123153, -0.00146365 and 0.0916291, 0.0518626, 0.0443532, -0.0124619, and -0.00148179, respectively.

5.3. Robustness and effects on growth and welfare

In order to analyze the robustness of the main results of changes in functional forms of the model preferences and production functions, we perform a

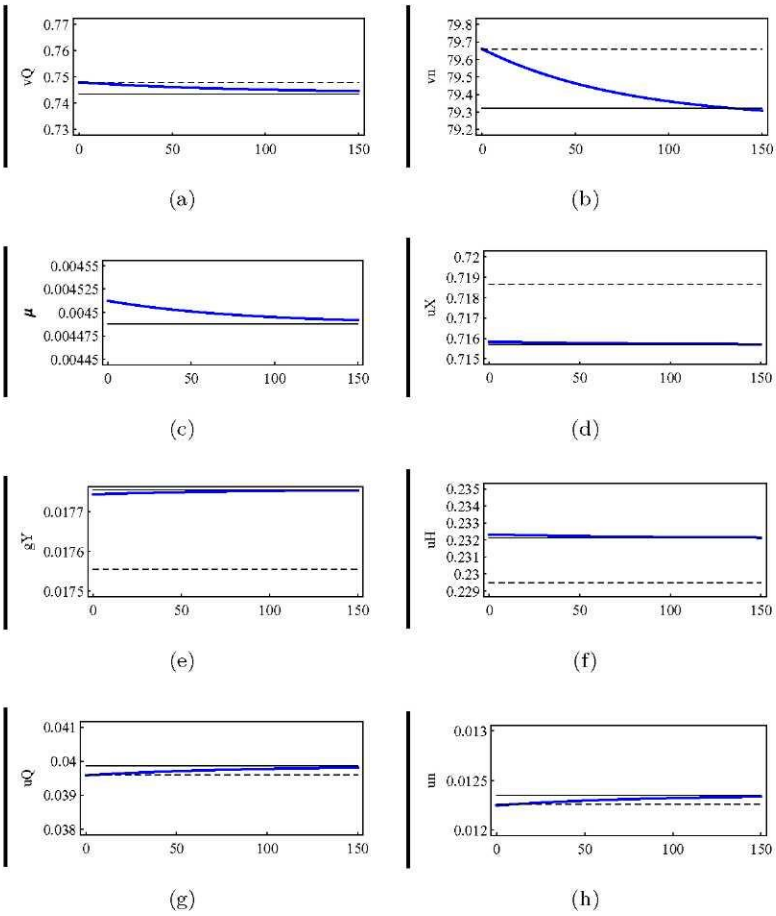
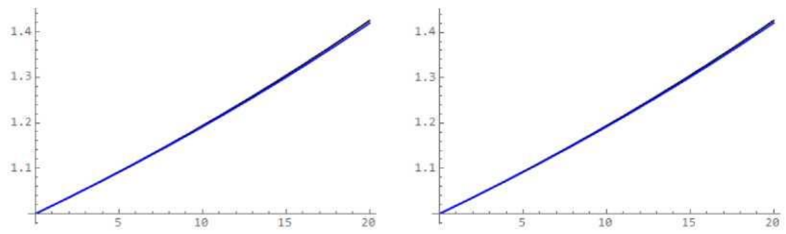


Figure 4: Effects of a shock on nominal interest rate i from 10% to 6%, with expenses paid only partially by cash.

Note: Thin black line represents initial steady-state values. Dashed line represents final steady-state values. If they coincide, initial and final steady-states coincide, despite the transitional dynamics. Both initial and final steady-states are saddle-path stable as the eigenvalues of the Jacobian of the dynamic system (38)-(42) are 0.0915953, 0.0520101, 0.0446733, -0.0123751, -0.00146911 and 0.0921589, 0.051833, 0.0447436, -0.0124987, -0.00148514, respectively.

number of different changes and evaluate the effects on steady-state growth and allocations of a given monetary policy, and on welfare (which takes into account the transitional dynamics after the policy change represented by a drop of 4 percentage points in the nominal interest rate (from 10% to 6%). Results are presented



(a) With total cash requirements (b) With partial cash requirements

Figure 5: Effects of a shock on nominal interest rate i from 10% to 6% in Consumption. Note: Black line represents consumption with the policy. Blue line represents consumption
without the policy.

in Table 1, where the baseline (described in Section 5.2) is included for ease of comparison. [21] First, we want to evaluate the changes that may stem from the alternative *tuition fee* function proposed in Section 3.1. As becomes clear from the first line in the table, we observe negative growth (-0.007%) and welfare (-0.22%) effects of decreasing inflation, resulting in a positive relationship between the two rates, an effect opposite what is obtained in the baseline analysis, but consistent with the first part of the nonlinear relationship presented in Figure 2. In this case, there is a reallocation of human capital from the education sector (and also from R&D sectors) to the final good sector, which implies a decrease in the *per capita* output growth rate. This is due to the nonlinear form of the *tuition fee* function (see again Figure 2). As expected from this nonlinear relationship, had we considered higher levels of the initial nominal interest rate (e.g., 30%), we would have obtained positive effects of decreasing inflation. For instance, had we considered a drop in the nominal interest rate from 30% to 26%, we would have obtained a 0.84% increase in welfare, which is even higher than in the baseline analysis. This means that the nonlinear effect of inflation on growth induced by the nonlinear *fee* can also be seen in transitional dynamics, explaining both positive and negative growth and the welfare effects of an expansionary monetary policy according to different initial inflation rates. In particular, an expansionary monetary policy that departs from a low inflation level may imply growth and welfare losses, while an expansionary monetary policy that departs from a high inflation level may imply relevant growth and welfare gains.

Secondly, we introduce a depreciation rate in human capital accumulation, which has also been considered in Strulik (2005) and in Sequeira and Reis (2007), though in our case, we consider a constant depreciation rate such that

$$\dot{H} \quad \xi H_{H_t} - \delta_H H, \qquad \xi > 0. \tag{43}$$

In this case, we obtain growth and allocation effects similar to those in the baseline analysis. The crucial difference is in welfare, as it increases nearly 10 times to 2.24%. This is not only due to the fact that initial welfare is much smaller (12.8 compared to 23.3 in the baseline analysis), but also due to transitional effects. In fact, welfare increases nearly 0.15 in the baseline analysis and 0.29 in the model with human capital depreciation. The higher welfare effect (in variations) obtained in this exercise is solely due to transitional dynamics.[5] First, the initial jump in consumption growth rates is higher, and they approach the new steady-state faster from below.

Finally, we consider two changes in preferences: (i) positive population growth affecting the discounting of future consumption, and (ii) altruism (measured by m) affecting the discounting factor. In this case, the utility function is changed to

$$U = \int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-(\rho - m g_L)t} dt, \tag{44}$$

We consider two alternatives regarding altruism: $m = 0$ where agents maximize *per capita* –utility, they do not care about future generations, although the population is growing ($g_L = 0.01$, i.e., population is growing at an average rate of 1%), and $m = 0.5$, where agents are partially altruistic, caring about the utility of future generations in the dynasty. In the first of these cases, we obtain

Table 1: Growth, allocation and Welfare Effects

Inflation drop from:			10% to 6%				10% to 0%
Δg_Y	Δu_H	Δu_Q	Δu_n	Δu_x	$\Delta \mu$	$g_{Welfare}$	$g_{Welfare}$
Baseline							
0.022%	0.29%	0.00%	0.00%	-0.30%	-4.04%	0.63%	1.58%
Alternative nonlinear education cost p function							
-0.007%	-0.10%	0.00%	0.00%	0.11%	-4%	-0.22%	-0.80%
Human Capital Depreciation ($\delta = 0.02$)							
0.022%	0.29%	0.02%	0.00%	-0.32%	-3.21%	2.24%	5.60%
Population Growth Rate ($g_L = 0.01$) & ($\delta = 0.02$)							
0.021%	0.29%	0.00%	0.00%	-0.29%	-4.06%	8.60%	21.29%
Population Growth and Altruism ($g_L = 0.01$; $m = 0.5$) & ($\delta = 0.02$)							
0.023%	0.29%	0.01%	0.00%	-0.27%	-4.04%	3.29%	8.08%

Note: Δ is variation in the variable and g the growth rate of Welfare from a monetary policy that decreases the nominal interest rate from 10% to 6%.

very similar growth and allocation effects, but a quite higher welfare effect than in the baseline analysis. Again, this welfare effect is not only seen in growth rates, but also in variations. In the second case, altruism keeps the growth and allocation effects quite unchanged but decreases welfare effects when compared to the case with population growth but not altruism. All in all, the fact that we notice small changes between steady-state values across different experiments (lines in Table 1) but substantial differences in welfare effects points out the great importance of the pattern of transitional dynamics in shaping the welfare changes due to monetary interventions.

As before, a higher nominal interest rate change (from 10% to 0%) yields significantly greater welfare effects. When those effects are compared with those that were previously obtained, e.g., by Lucas (2000), Chu et al. (2019b), and He (2018), our baseline scenario falls between the Lucas (2000) and Chu et al. (2019) estimations. This makes our estimations the closest to those of Lucas (2000) in the context of monetary endogenous growth models. Interestingly, the extensions considered (human capital depreciation, population growth, and altruistic agents) increase the welfare effects substantially.[22]

6. Conclusions

The effects of monetary policy on economic growth rates in the long run are present in empirical results, while theoretical approaches have been dominated by the *superneutrality* result until recently. This contradiction between empirical and theoretical results has garnered increasing interest from economists, especially due to the real-economy-oriented policy approaches some central banks have implemented following the subprime crisis and even before (e.g., in the Japanese case). Thus, the study of the inflation-growth nexus within the endogenous growth framework has seen an increasing number of contributions in the last decade. Those contributions tried to replicate a small but decreasing and nonlinear (i.e., inverted U-shaped) effect of inflation on economic growth relying on R&D as the only source of long-run growth. In fact, most contributions so far have ignored human capital accumulation as a source of growth as well as the role of transitional dynamics and of the reallocation of resources in the economy. We help fill this gap in the endogenous growth literature by devising a model with human capital accumulation and horizontal and vertical R&D. We devise a monetary endogenous growth model through a CIA constraint that affects both households and firms in different sectors. We show that the mechanics to obtain the decreasing, small, and nonlinear effect of inflation on the long-run growth must be based on household cash requirements for education, which we called a *tuition fee*.

Quantitatively, the model replicates the small influence of monetary policy on growth but also highlights the effects it can have on welfare as well as the allocation of resources throughout different sectors in the economy. We offer explanations for small (and also nonlinear) effects of inflation on economic growth following monetary policies both on transitional dynamics and in the long-run. Another interesting finding is that the empirical plausible effects of monetary policy on economic growth are consistent with the reallocation of resources from the goods production sector to the education sector. Interestingly, these (production and education) sectors were precisely the two sectors, which have not been studied jointly as facing CIA constraints.

It is worth noting that our results subsist for a number of robustness analyses regarding the crucial assumptions of the model. In fact, a depreciation in human capital and population growth would imply very similar effects on growth, but higher effects on welfare when compared to the baseline scenario, in which there is no depreciation in human capital or population growth. More importantly, the nonlinear *tuition fee* for education proposed in this paper can also explain both positive and negative growth as well as the welfare effects of an expansionary monetary policy according to different initial inflation rates. In particular, an expansionary monetary policy that departs from a low inflation level may imply negative growth and welfare effects, while an expansionary monetary policy that departs from a high inflation level may imply relevant growth and welfare losses.

From a more practical point of view, the more cash households (need to) hold to fund education, the higher the negative effects of inflation on economic growth. Additionally, an increase in education costs incurred by households in a context of high inflation may have more serious effects on economic growth.

Declaration of Interest

The author has not conflict of interest regarding the research in "Inflation, Economic Growth and Education Expenditure"

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Appendix A. Details on the households' maximization problem

The households' maximization problem is solved through the following current value Hamiltonian:

$$\Gamma = \frac{C_t^{1-\theta}-1}{1-\theta} + \lambda\left[rA_t + w_t(H_t-(1+p)H_{H_t}) - \Pi_t m_t - C_t + i_t b_t + T_t\right] + \\ +\iota(m_t - \Theta_c C_t - \Theta_p p w_t H_{H_t} - b_t) + \\ +\varpi\left[\xi H_{H_t}\right] \tag{A.1}$$

where co-states are λ, ι and ϖ for which we are not indicating time indexes. The first-order conditions for C_t, H_{H_t}, b_t, A_t, H_t, m_t are the following, respectively:

$$C_t^{-\theta} - \lambda - \iota\Theta_c = 0 \tag{A.2}$$

$$-(1+p)w_t\lambda - \gamma\Theta_p p w_t - \varpi\xi = 0 \tag{A.3}$$

$$\lambda i_t - \iota = 0 \tag{A.4}$$

$$r_t\lambda = \rho\lambda - \dot{\lambda} \tag{A.5}$$

$$w_t\lambda = \rho\varpi - \dot{\varpi} \tag{A.6}$$

$$-\Pi_t\lambda + \iota = \rho\lambda - \dot{\lambda} \tag{A.7}$$

Substituting i in equation (A.7) by the value given in (A.4) and using (A.5) yields the Fisher equation (7). Then, using (A.2), we see that (acknowledging that for the household, the nominal interest rate is taken as constant,see Section 2.2) $g_\lambda = -\theta g_c$. Then, we use equation (A.5) to obtain equation (5). From equation (A.3), we obtain the following

$$w\lambda = \frac{\varpi\xi}{1-p(1+i_t\Theta_p)} \tag{A.8}$$

and acknowledging that for the household, the nominal interest rate is taken as constant,see Section 2.2:

$$\frac{\varpi}{\varpi} = g_w + g_\lambda \tag{A.9}$$

Substituting $w\lambda$ from (A.8) in (A.6), we obtain:

$$\frac{\dot{\varpi}}{\varpi} = \rho - \frac{\varpi\xi}{1-p(1+i_t\Theta_p)} \tag{A.10}$$

Now use (A.5) to substitute for g_λ in (A.9) and solve the resulting equation for g_w to obtain equation (6).