https://doi.org/10.1016/j.omega.2019.01.005

Designing time-of-use tariffs in electricity retail markets using a bi-level model – Estimating bounds when the lower level problem cannot be exactly solved

Inês Soares[1], Maria João Alves[1,2], Carlos Henggeler Antunes[1,3]

[1] Institute for Systems Engineering and Computers at Coimbra (INESC Coimbra), Coimbra, Portugal

[2] CeBER and Faculty of Economics of the University of Coimbra, Coimbra, Portugal

[3] Department of Electrical Engineering and Computers, University of Coimbra, Coimbra, Portugal

Contacts:

Inês Soares (corresponding author): inesgsoares@gmail.com

Maria João Alves: mjalves@fe.uc.pt

Carlos Henggeler Antunes: ch@deec.uc.pt

ABSTRACT

Time-of-use tariffs are a pricing strategy for a product or service in which the supplier establishes time-differentiated prices. Dynamic (e.g., day-ahead) time-differentiated electricity prices can contribute to increase the retailer's profit, allow end-users to reduce the consumption costs and enhance grid efficiency. The electricity retailer and the consumer are hierarchically related. The interaction between them can be modeled by a bi-level (BL) optimization model - the retailer is the upper level decision maker and the consumer is the lower level decision maker. The retailer and the consumer have different and conflicting goals: the retailer establishes the pricing scheme to sell electricity to consumers to maximize his profit; the consumer reacts to these prices by determining the operation of the controllable loads in order to minimize the discomfort and the electricity bill.

In this work, a BL optimization model incorporating shiftable, interruptible and thermostatic loads is proposed. The upper level problem is tackled by a particle swarm optimization algorithm while the lower level problem is solved by an exact mixed-integer programming solver. The inclusion of the thermostatic load in the lower level problem imposes a much higher computational burden. Therefore, it may not be possible to find the optimal lower level solution, and a sub-optimal lower level solution is infeasible to the BL problem. Considering a computational budget, this work proposes an approach to compute good quality estimates of bounds for the upper level objective function, providing the leader further information and allowing him to make sounder decisions in an adequate time frame.

Keywords: Bi-level optimization; Hybrid algorithms; Dynamic tariffs; Demand response; Pricing problem; Electricity retail market.

1. Introduction

Time-of-use (ToU) tariffs are a pricing strategy for a product or service in which the supplier establishes time-differentiated prices, with higher rates being charged during peak demand periods. Dynamic ToU tariffs are expected to become a common scheme in smart grids, in which prices are announced with short antecedence (e.g. one day-ahead). This type of tariff structure can bring benefits for grid operators (contributing to alleviate congestion in distribution networks and enhancing the utilization of renewable sources), retailers (enabling to manage wholesale buying and retail selling prices) and consumers (engaging in demand response actions to reduce the bill without jeopardizing comfort). The retailer (upper level decision maker) defines commercial offers consisting of dynamic ToU tariffs to maximize profits in face of variable wholesale and network access prices. The consumer (lower level decision maker) reacts to the time differentiated retail prices by resetting/rescheduling (shiftable, interruptible and thermostatic) appliance operation to lower priced periods to minimize the electricity bill. Some bi-level (BL) approaches for modelling the interaction between electricity retailers and consumers have been developed over the past few years. BL optimization approaches have also been widely used in several research areas to address other pricing problems (for instance, see the works by (Labbé and Violin 2016) and (Lunday and Robbins 2018)).

In the context of electricity retail markets, (Zugno et al. 2013) developed a BL model to compute the dynamic price signal to maximize the retailer's profit subject to optimal appliance usage patterns by consumers, consisting of the maximization of a utility function minus the electricity procurement costs, considering stochastic prices, weather data and must-serve load. The BL model is transformed into a single level mixed-integer programming (MIP) problem.

Some approaches considering BL models have been proposed by Meng and Zeng to determine ToU tariffs that maximize the retailer's profit while minimizing the consumers' costs at the lower level. (Meng and Zeng 2013) reformulated the BL model as a single-level problem by applying the Karush-Kuhn-Tucker conditions, which is then solved using a branch and bound algorithm to deal with complementarity constraints. These authors then extended this model to distinguish (interruptible and non-interruptible) shiftable, non-shiftable and curtailable appliances leading to independent linear sub-problems at the lower level. A cost for the waiting time of appliance operation was also included (Meng and Zeng 2016). These models are solved using a genetic algorithm (GA) for the upper level problem and a linear programming solver for the lower level problems. These models consider that loads should be supplied with a given amount of energy for the completion of the service, thus not accounting for the appliance operation cycles (e.g., in a washing machine: heating the water, centrifuging, etc.).

In the work developed by (Sekizaki et al. 2016), a GA is proposed to deal with a BL model that considers different types of consumers (residential and not residential). Each type of consumer has an objective function that results from a weighted sum of the cost of purchasing power from the retailer and the disutility caused by the reduction of the load served.

In (Alves et al. 2016), a BL programming model was proposed to study the interaction between an electricity retailer and consumers, which was dealt with a hybrid approach including a GA for the upper level problem and an exact MIP solver for the lower level problem. This work was then extended to a semi-vectorial BL optimization model (Alves and Antunes 2018), by considering two lower level objective functions: minimize the consumption costs and minimize the dissatisfaction caused by rescheduling the operation of controllable appliances. To address the same BL problem as the one presented in (Alves et al. 2016), (Carrasqueira et al. 2017) proposed and compared two novel BL population-based algorithms, one based on an evolutionary algorithm and the other on a particle swarm optimization algorithm applied to both upper and lower optimization levels. The advantage of these approaches is that they can address higher dimension problems because the lower level problem is also dealt with a metaheuristic method. However, the optimality of the lower level solution cannot be guaranteed and it is not known how far this solution is from the optimal one, which raises difficulties in ensuring the feasibility of the BL solution.

The BL model proposed in (Alves et al. 2016, Carrasqueira et al. 2017) was pioneering in the context of electricity retail market by including detailed appliance data, although just considering shiftable loads. In the BL model proposed in the current paper, the whole range of typical household appliances is considered. These appliances are categorized as shiftable, interruptible and thermostatic loads, according to the type of control that can be exerted on their operation (Soares et al. 2014). The physical characterization of the operation cycles of shiftable appliances (dishwasher, laundry machine, clothes dryer), interruptible loads (electric water heater and electric vehicle) and thermostatic loads (an air conditioning system) is embedded in the model. The incorporation into the model of the physical information associated with the operation and control of all categories of appliances allows a more realistic characterization of the loads and enables a more accurate representation of the residential consumer's energy management problem.

Since the lower level optimization problem arises as a constraint of the upper level problem, it is necessary to guarantee obtaining the optimal solution to the lower level problem to ensure BL feasibility. However, if the lower level problem is difficult to solve, namely due to its combinatorial nature and/or whenever a limited computational budget is available, it may not be possible to obtain its optimal solution, and therefore a feasible solution to the BL problem. The modeling of the thermostatically-controlled appliance strongly increases the computational difficulty and the time required to solve the consumer's problem of the MIP model presented in section 3. In fact, even after a significant computational time with a state-of-the-art solver, the MIP gap (relative MIP gap yielded by the *cplex* solver) is still positive for a problem considering a planning period of 24 hours divided into intervals of 15 minutes. In some instances, more than 42 hours to obtain a MIP gap just below 5% are necessary and more than 5 days to obtain a MIP gap below 2%. This renders the exact approach impractical with a reasonable computational budget, which derives from the requisites of the problem. Without being able to guarantee the lower level optimal solution, the value obtained for the upper level objective function may be misleading, in the sense that it may be better than the optimal solution. In face of this difficulty, the aim is to offer the leader good quality estimates of bounds for his objective function, so he can hedge against the risk involved by overestimating his outcome and make a sound decision. Sub-optimal solutions to the lower level problem (i.e. for which the MIP gap is still positive) are used to determine upper estimates for the leader's optimal objective function value. Lower estimates for the upper level objective function are obtained by exploiting the characteristics of the problem, particularly the existence of symmetrical terms in the consumer's cost objective function and in the retailer's profit objective function. Using an incremental strategy, a set of solutions is selected for further analysis, each one being represented by a [lower estimate, upper estimate] range for the leader's objective function. Techniques for comparing interval numbers are then used to offer meaningful information of practical interest to the leader regarding risk vs. opportunity associated with different ToU prices (upper level solutions). This methodology can be replicated with the necessary adaptations in other BL models concerning pricing problems.

In summary, the main contributions of the paper are the following: a new comprehensive lower level MIP model integrating different types of appliances (shiftable, interruptible and thermostatic loads) considering their physical characteristics in a detailed manner; a hybrid algorithm combing a particle swarm optimization (PSO) algorithm, which guides the upper level search, with a MIP solver, which provides solutions to the lower level model for each instantiation of the upper level variables; a novel approach, which makes the most of the structure of the problem, to compute good estimates of bounds for the upper level objective function value whenever a limited computational budget exists. This latter issue is of utmost importance in face of the computational effort to solve the MIP model to optimality due to the detailed physical modelling of the thermostatic load.

The manuscript is organized as follows. In section 2 the main concepts of BL optimization are summarized, including a brief review of the main methodological approaches. A new BL model for the interaction between a retailer and a cluster of consumers with similar consumption patterns and demand responses in the retail electricity market is presented in section 3. Section 4 describes the methodological approach to compute lower/upper estimates for the leader's objective function considering a computational budget. The results of the study are presented and discussed in section 5. The conclusions are drawn in section 6.

2. Bi-level optimization

BL optimization involves two optimization problems which are hierarchically associated. The upper level refers to the leader's interests while the lower level problem, which appears in the constraints of the upper level problem, concerns the aims of the follower. The leader decides first, and the follower then optimizes its objective function within the feasible region set by the instantiations of the leader's decision variables. However, the follower's decision also affects the leader's objective value and therefore the leader should anticipate the follower's reaction.

A BL general optimization problem can be defined as follows:

$$\max_{x \in X} \quad F(x, y)$$

$$s.t. \quad G(x, y) \leq 0$$

$$y \in \arg\min_{y^* \in Y} \{f(x, y^*) : g(x, y^*) \leq 0\}$$

where $X \subset \mathbb{R}^n$ and $Y \subset \mathbb{R}^m$ are closed sets, *n* is the number of upper level variables and *m* is the number of lower level variables. The decision variables *x* are controlled by the leader, while the follower controls the decision variables *y*. $F(x, y)$ is the leader's objective function and $f(x, y)$ is the follower's objective function. Variables x are kept constant when the lower level objective function is optimized, since the follower optimizes his objective function $f(x, y)$ after decision variables x are set by the leader.

Let $x \in X$ be a vector of decision variables set by the leader. The *feasible* and the *rational reaction* sets of the follower are $Y(x) = \{y \in Y: g(x, y) \leq 0\}$ and $\Psi(x) = \left\{y \in Y: y \in \underset{y^* \in Y(x)}{\arg\min} f(x, y^*)\right\}$, respectively. The feasible set of the BL problem described above, also called *inducible region*, is $IR = \{(x, y): x \in X, G(x, y) \leq 0, y \in \Psi(x)\}$. Solving a BL optimization problem is methodologically and computationally challenging, since the problem is inherently non-convex. Even the linear BL problem is NP-hard (Dempe 2002).

Classical approaches and several metaheuristics have been developed to address BL problems. Due to the difficulties of BL optimization, most of the classical solution methods consider problems where functions have convenient properties, such as linearity or convexity. Classical approaches include using the Karush-Kuhn-Tucker (KKT) conditions, penalty methods and techniques exploring vertices in linear problems (among others). The first approach involves replacing the lower level problem by its KKT conditions, thus transforming the problem into a single-level problem. The resulting problem with complementary constraints is then solved, e.g., by a branch-and-bound method. The second approach consists of formulating a nonlinear programming problem approximating the original one, which is solved iteratively by means of a penalty function method applied to the lower level problem; this leads, under certain conditions, to a sequence of approximated solutions converging to the optimal solution. The third approach is based on the property that only vertices of the constraint region need to be considered for the computation of the optimal solution to a linear BL problem. Less attention has been given to BL problems with integer variables, for which few classical approaches were developed. Surveys on classical methods can be found in (Vicente and Calamai 1994, Bard 1998, Dempe 2002, Colson et al. 2005, Colson et al. 2007, Sinha et al. 2018).

To cope with the difficulties of solving BL problems, which are further aggravated if the problem has nonlinear functions/constraints or integer variables, several metaheuristic approaches have been developed. Typical metaheuristics that have been used to deal with BL problems are population-based algorithms, including evolutionary algorithms, particle swarm optimization and differential evolution. (Sinha et al. 2018) present a recent review on classical and evolutionary approaches to (single and multiobjective) BL optimization.

The BL model we propose in this paper for the interaction of an electricity retailer and consumers can be regarded as a price setting problem (Labbé and Violin 2016). The upper level variables (x) are the prices and, as in general price setting problems, these variables only appear in the lower level objective function and do not figure in the lower level constraints. The lower level is a mixed-integer problem with many binary variables; the objective function is nonlinear because prices x multiply by the lower level variables associated with load control. Therefore, for each instantiation of x, a MIP problem results at the lower level. Taking profit from this structure, a hybrid approach is developed in this work, in which a PSO metaheuristic is used for guiding the upper level search and an exact method is called to solve the lower level problem for each instantiation of x.

3. A bi-level model for pricing problems in the residential electricity retail market

In the electricity market for residential consumers in smart grids, it is expected that retailers define commercial offers with dynamic time differentiated electricity prices with the goal of maximizing profits (i.e., making the most of buying in wholesale markets and selling to end-users). Consumers respond to these ToU prices by resetting controls or rescheduling the operation of appliances in order to minimize cost taking into account comfort requirements. This interaction can be modeled as a BL problem with the retailer as the upper level decision maker (who sets the prices) and the consumer as the lower level decision maker (who optimizes load operation in face of those prices).

In the current work, a comprehensive BL optimization model is developed encompassing shiftable, interruptible and thermostatic loads, with different physical characteristics and type of control. In addition to shiftable loads, i.e. appliances whose operation cycle cannot be interrupted once initiated, as in the model presented by Alves et al. (Alves et al. 2016), the model herein proposed considers new types of controllable loads: interruptible and thermostatic appliances. Interruptible appliances are loads for which the energy supply can be interrupted as long as the required amount of energy is supplied by a certain point in time (as a proxy for quality of service). Thermostatic loads are controlled by a thermostat device switched by the indoor temperature, which is determined by a thermal model of the space being conditioned, exhibiting a hysteresis behavior within a deadband around a reference setpoint established by the consumer. The consumer's cost objective includes energy costs associated with (shiftable, interruptible and thermostatic) load operation, costs due to the power peak demand, and a term associated with the monetization of indoor temperature discomfort (i.e. deviations from the range of comfort temperature).

The inclusion of the thermostatic load in the lower level problem imposes a much higher computational burden, namely regarding modelling the hysteresis operation of the thermostat (which prevents excessive switching when the indoor temperature is around the setpoint).

This BL model of the residential electricity retail market problem is described below in detail.

The planning period $\mathrm{T} = \{1, \ldots, T\}$, with T being the number of time intervals, is divided into I sub-periods $P_i = \left[P_{1_i}, P_{2_i}\right] \subset \mathrm{T},\ i \in \{1, \ldots, I\}$, such that $\bigcup_{i=1}^{I} P_i = \mathrm{T}$, $P_{1_i} = P_{2_{i-1}} + 1, i \in \{2, \ldots, I\}$, with $P_{1_1} = 1$, and $\overline{P_i} = P_{2_i} - P_{1_i} + 1$ is the amplitude of P_i. Each x_i (in €/kWh), $i \in \{1, \ldots, I\}$, is the upper level decision variable corresponding to the price of electricity to be charged to the consumers by the retailer in each pre-defined sub-period P_i, with h being the unit of time the planning period is discretized into (hour, minute, quarter of hour, etc.). For each instantiation of prices $x = (x_1, \dots, x_I)$ defined by the retailer, a lower level problem is solved (consumer's problem).

In the upper level problem, the electricity prices set by the retailer are limited to minimum and maximum values, $\underline{x_i}$ and $\overline{x_i}$ respectively, for each sub-period P_i, i.e. $\underline{x_i} \leq x_i \leq \overline{x_i},\ i \in \{1, \dots, I\}$, (constraints (C1) and (C2)). Additionally, an average electricity price for the whole planning period T is imposed, $x^{AVG} = \frac{1}{T}\sum_{i=1}^{I} \overline{P_i}\, x_i$ (constraint (C3) in the model, similar to the one considered by (Zugno et al. 2013)), as a proxy to model competition in the electricity retail market.

UPPER LEVEL CONSTRAINTS

$$x_i \leq \overline{x_i}\,, \quad i = 1, \dots, I \tag{C1}$$

$$x_i \geq \underline{x_i}\,, \quad i = 1, \dots, I \tag{C2}$$

$$\frac{1}{T}\sum_{i=1}^{I} \overline{P_i} x_i = x^{AVG} \tag{C3}$$

The BL model considers a cluster of consumers with similar consumption and demand response profiles, which can be defined using smart meter data. In addition to the controllable load, the model considers a base load, which is not deemed for control due to its characteristics (e.g. entertainment, oven, …). The base load is denoted by b_t (in W) in each time t of the planning period T. Regarding the controllable loads, J shiftable appliances (whose operation cycles cannot be interrupted once initiated), K interruptible appliances and a thermostatic load (air conditioner) are considered. The retailer also defines levels of power demand and the consumer pays for the power level corresponding to the peak, i.e. attained by the maximum power requested to the grid at any time of the whole planning period T.

For the shiftable loads, the consumer should specify the comfort time slot $T_j = \left[T_{1_j}, T_{2_j}\right] \subseteq$ T in which each load $j \in \{1, \dots, J\}$ should operate, according to his preferences and routines. The power requested by load j at stage $r \in \{1, \dots, d_j\}$ of its operation cycle is f_{jr} (in W), being d_j the duration of the working cycle of load j. The lower level decision variables associated with shiftable load j are w_{jrt} and p_{jt}. Variables w_{jrt} specify whether load j is "on" or "off" at time $t \in T_j$ and stage r of its operation cycle; p_{jt} is the power (in W) requested to the grid by load j at time $t \in \{1, \dots, T\}$. Variables p_{jt} are auxiliary variables depending on w_{jrt}. The set of constraints (C4)-(C11) models the operation of shiftable loads.

Constraints (C4) and (C5) set the lower level auxiliary variables p_{jt} according to f_{jr} and w_{jrt}. Equations (C4) set p_{jt} for $t \in T_j$, i.e., when load $j \in \{1, \dots, J\}$ is allowed to operate. Equations (C5) set $p_{jt} = 0$ for intervals in which load j is not allowed to operate due to the consumer's preferences.

Constraints (C6) ensure that, at time t of the planning period, each shiftable load $j \in \{1, \dots, J\}$ is either "off" or "on" at only one stage r of its operation cycle. Constraints (C7) ensure that, if load j is "on" at time t and at stage $r < d_j$ of its operation cycle, then it must also be "on" at time $t + 1$ and stage $r + 1$. Constraints (C8) guarantee that each shiftable load $j \in \{1, \dots, J\}$ operates exactly once at stage r and this should occur within its comfort time slot. Constraints (C9) guarantee that load j starts its working cycle within its allowed comfort time slot, i.e. at most at time $T_{2_j} - d_j + 1$, thus assuring that it never finishes later than T_{2_j}. Thus, constraints (C7-C9) ensure that each shiftable load j operates exactly d_j consecutive time intervals, thus forcing the lower level decision variables w_{jrt} to be zero whenever load j is "off". Constraints (C10) define w_{jrt} as binary variables, where 1 means that load j is "on" and 0 means that it is "off" at time $t \in T_j$ and at stage r, and (C11) define p_{jt} as non-negative for the whole planning period.

SHIFTABLE LOADS

$$p_{jt} = \sum_{r=1}^{d_j} f_{jr} w_{jrt} \ , \ j = 1, \dots, J \ , t = T_{1_j}, \dots, T_{2_j} \tag{C4}$$

$$p_{jt} = 0 \ , \ j = 1, \dots, J \ , t < T_{1_j} \vee t > T_{2_j} \tag{C5}$$

$$\sum_{r=1}^{d_j} w_{jrt} \leq 1 \ , \ j = 1, \dots, J \ , t = T_{1_j}, \dots, T_{2_j} \tag{C6}$$

$$w_{j(r+1)(t+1)} \geq w_{jrt} \ , j = 1, \dots, J \ , \ r = 1, \dots, (d_j - 1) \ , t = T_{1_j}, \dots, \left(T_{2_j} - 1\right) \tag{C7}$$

$$\sum_{t=T_{1_j}}^{T_{2_j}} w_{jrt} = 1 \ , \ j = 1, \dots, J \ , r = 1, \dots, d_j \tag{C8}$$

$$\sum_{t=T_{1_j}}^{T_{2_j} - d_j + 1} w_{j1t} = 1 \ , \ j = 1, \dots, J \tag{C9}$$

$$w_{jrt} \in \{0,1\} \ , j = 1, \dots, J \ , r = 1, \dots, d_j \ , \ t = T_{1_j}, \dots, T_{2_j} \tag{C10}$$

$$p_{jt} \geq 0 \ , \ j = 1, \dots, J \ , t = 1, \dots, T \tag{C11}$$

Figure 1 displays the control capability of a shiftable load j.

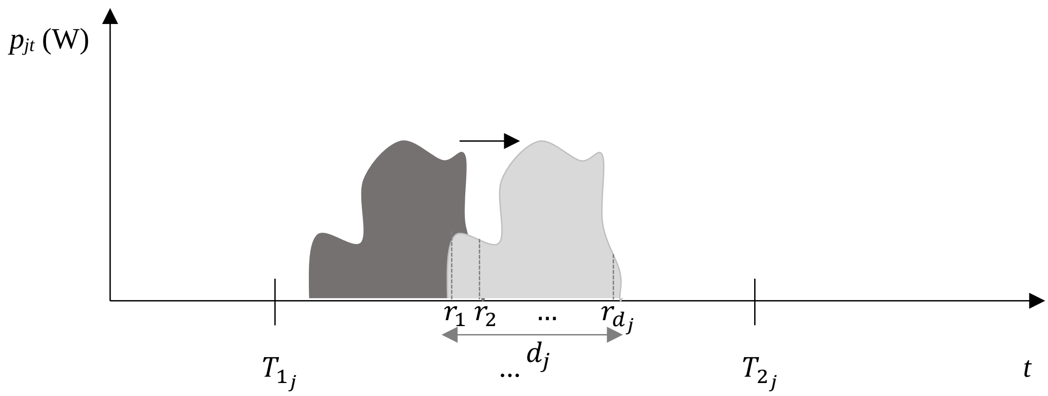

Figure 1. Control of shiftable (non-interruptible) load $j \in \{1, \dots, J\}$.

For interruptible loads, the consumer should also specify the comfort time slot $T_k = [T_{1_k}, T_{2_k}] \subseteq$ T in which each load $k \in \{1, \dots, K\}$ should operate. The power requested by each load k is Q_k (in W) and the total energy required is $E_k = d_k \times Q_k$, where d_k is the duration of the operation of load k. The lower level decision variables for each interruptible load k are v_{kt} and q_{kt}. Binary variables v_{kt} specify whether load k is "on" or "off" at time $t \in T_k$; each auxiliary variable q_{kt} is the power (in W) requested to the grid by interruptible load k at time $t \in \{1, \dots, T\}$, which is either 0 or Q_k. The set of constraints (C12)-(C16) models the operation of interruptible load.

Constraints (C12) set the variables q_{kt} for $t \in T_k$ as function of v_{kt}, when the load $k \in \{1, \dots, K\}$ is allowed to operate, and (C13) impose that the power requested to the grid is 0 outside the comfort time slot. Constraints (C14) guarantee that the total amount of energy consumed by load k within the comfort time slot is E_k.

Constraints (C15) define v_{kt} as binary variables, where 1 means that interruptible load k is "on" and 0 means that it is "off" at time $t \in T_k$,and (C16) define q_{kt} as non-negative for the whole planning period T.

INTERRUPTIBLE LOADS

$$q_{kt} = v_{kt} Q_k \ , \ k = 1, \dots, K \ , t = T_{1_k}, \dots, T_{2_k} \quad \text{(C12)}$$

$$q_{kt} = 0 \ , \ k = 1, \dots, K \ , t < T_{1_k} \vee t > T_{2_k} \quad \text{(C13)}$$

$$\sum_{t=T_{1_k}}^{T_{2_k}} q_{kt} = E_k \ , \ k = 1, \dots, K \quad \text{(C14)}$$

$$v_{kt} \in \{0,1\} \ , \quad k = 1, \dots, K, t = T_{1_k}, \dots, T_{2_k} \quad \text{(C15)}$$

$$q_{kt} \geq 0 \ , \ k = 1, \dots, K \ , t = 1, \dots, T \quad \text{(C16)}$$

Figure 2 displays the control capability of an interruptible load k to which a quantity E_k of energy should be supplied.

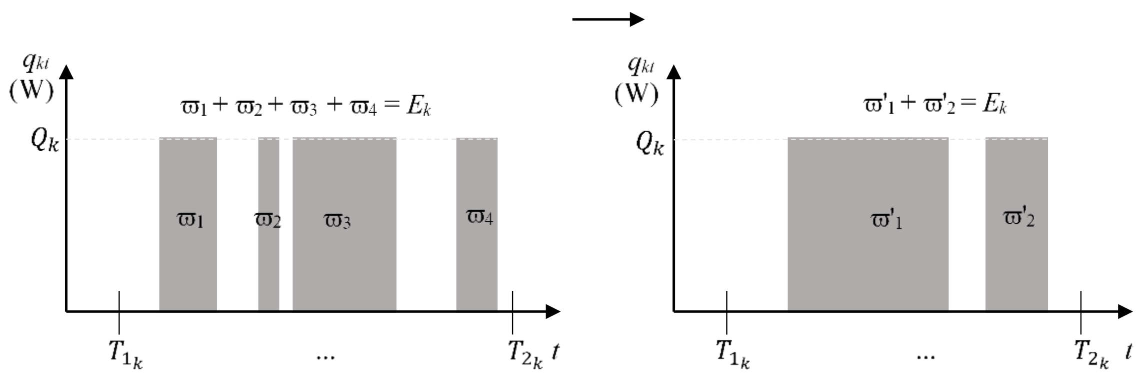

Figure 2. Control of interruptible load $k \in \{1, \dots, K\}$.

The thermostatic load is an air conditioner system (AC) that requires from the grid a nominal power P_{AC}^{nom} (in W) when it is "on". The operation of the system depends on outdoor and indoor temperatures, θ_t^{ext} and θ_t^{in} respectively, for each unit of time $t \in$ T. The BL model optimizes the operation of the thermostat by determining the minimum indoor temperature for which the system should turn on (θ_t^{min}). The consumer should specify the reference temperature, θ^{ref}, the maximum allowed indoor temperature, θ^{max}, and the absolute minimum allowed indoor temperature, θ_{Abs}^{min}. The control temperatures θ^{max} and θ_{Abs}^{min} can also be set automatically with respect to the (setpoint) reference temperature θ^{ref}. The lower level variables for the AC are s_t, θ_t^{in}, θ_t^{min}, z_t , y_t , Δ_t^+ and Δ_t^-. Decision variables s_t establish whether the AC is "on" ($s_t = 1$) or "off" ($s_t = 0$) at time $t \in$ T; θ_t^{in} and θ_t^{min} specify the indoor temperature and the minimum indoor temperature at each time $t \in$ T, respectively, which derive from the optimization of the thermostat operation; auxiliary variables z_t and y_t are used to ensure the consistency between the thermostat status and the indoor temperature, at each time *t* of the planning period; Δ_t^+ and Δ_t^- are the positive and negative deviations, respectively, of the minimum temperature, θ_t^{min}, from the reference temperature, θ^{ref}. The discomfort caused by the operation of the AC is measured by these deviations, which are included in the consumer's overall cost objective function. The set of constraints (C17)-(C31) models the operation of the AC.

Figure 3 displays the behavior of thermostat controlling the AC in heating mode, at each time *t* of T, by determining θ_t^{min}. The BL model is developed considering heating mode, but it can be easily adapted for cooling mode.

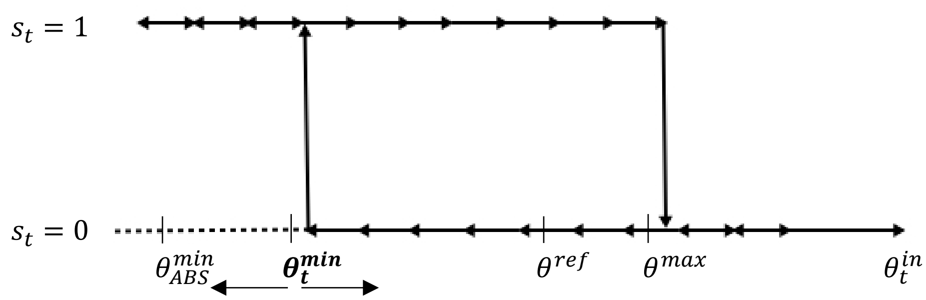

Figure 3. Behavior of a thermostat in heating mode at each time *t* of T.

For each time *t* of the planning period T, equations (C17) represent the thermal model defining the indoor temperature at time *t* as a function of indoor temperature at time *t*-1, the outdoor temperature and the operation of the AC. Constants α, β, and γ are derived from the building characteristics (area, envelope, etc.) and the coefficient of performance of the AC (Antunes et al. 2018).

Constraints (C18) ensure that the AC is "on" when the indoor temperature is below the minimum temperature, by forcing decision variable s_t to be 1. Constraints (C19) – (C22) guarantee that the system keeps the state "on" or "off" when the indoor temperature lies between the minimum temperature and the maximum temperature. Constraints (C23) force $s_t = 0$ when the indoor temperature is above the maximum allowed temperature, thus ensuring that the AC is "off".

Equations (C24) define the discomfort variables Δ_t^+ and Δ_t^- at each time *t* of T. Constraints (C25) and (C26) ensure that the minimum temperature is always within the bounds defined by the absolute minimum temperature and the maximum temperature allowed, respectively.

Constraints (C27) define the auxiliary lower level variables z_t and y_t and the decision variables s_t as binary variables. Constraints (C28) define temperatures and temperature deviations as non-negative variables.

The computational complexity associated with the thermostatic load results from allowing θ_t^{min} to vary in each *t* of T in order to make the most of time differentiated electricity prices. If θ_t^{min} was fixed, thus defining a thermostat deadband independent of the optimization process, the model could be solved very fast. But with θ_t^{min} being an outcome of the optimization process, the combinatorial nature of the model makes it computationally very demanding. The optimization process can set θ_t^{min} according to what is the best for the overall lower level cost function considering the dynamic ToU tariffs set by the retailer (only imposing that it is not lower than the minimum allowed θ_{Abs}^{min}). Therefore, it may be advantageous to increase θ_t^{min} to heat the room in periods of low electricity prices to account for comfort in periods of high prices. In this way, the model displays an increased flexibility thus becoming more realistic vis-à-vis the implementation of demand response actions to price signals reflecting wholesale prices (e.g. renewable generation availability) and grid status (e.g., network congestion).

Although θ_t^{min} variables are established as continuous variables in (C28), in practice we have considered variables with only one decimal place. The main reason is that they are used together with the reference temperature θ^{ref} to define the discomfort values and temperature differences lower than 0.1 ºC should not be accounted for because they do not correspond to actual consumer's discomfort. This change is operationalized in the model by defining the θ_t^{min} variables as integer and replacing θ_t^{min} with $\theta_t^{min} \times 0.1$ in (C24)-(C26). We have implemented the model with this change, which turns it more realistic, but further increases the computational effort required to solve the lower level problem, since continuous variables become integer variables.

THERMOSTATIC LOAD

$$\theta_t^{in} = \alpha\theta_{t-1}^{in} + \beta\theta_{t-1}^{ext} + \gamma s_{t-1} P_{AC}^{nom}\ , \quad t = 1, \ldots, T \qquad \text{(C17)}$$

$$\theta_t^{in} \geq \theta_t^{min} - M s_t\ , \quad t = 1, \ldots, T \qquad \text{(C18)}$$

$$\theta_t^{in} \leq \theta_t^{min} + Mz_t , \quad t = 1, \dots, T \tag{C19}$$

$$\theta_t^{in} \geq \theta^{max} - My_t , \quad t = 1, \dots, T \tag{C20}$$

$$z_t + y_t - s_{t-1} + s_t \leq 2 , \quad t = 1, \dots, T \tag{C21}$$

$$z_t + y_t + s_{t-1} - s_t \leq 2 , \quad t = 1, \dots, T \tag{C22}$$

$$\theta_t^{in} \leq \theta^{max} + M(1 - s_t) , \quad t = 1, \dots, T \tag{C23}$$

$$\theta_t^{min} - \theta^{ref} = \Delta_t^+ - \Delta_t^- , \quad t = 1, \dots, T \tag{C24}$$

$$\theta_t^{min} \geq \theta_{Abs}^{min} , \quad t = 1, \dots, T \tag{C25}$$

$$\theta_t^{min} \leq \theta^{max} , \quad t = 1, \dots, T \tag{C26}$$

$$s_t, z_t, y_t \in \{0,1\} , \quad t = 1, \dots, T \tag{C27}$$

$$\theta_t^{in}, \theta_t^{min}, \Delta_t^-, \Delta_t^+ \geq 0 , \quad t = 1, \dots, T \tag{C28}$$

For the power cost component, the retailer defines L levels of power (in W), P_l^{Cont}, $l \in \{1, \dots, L\}$, and the corresponding prices to be charged to the consumers, e_l (in €). The lower level decision variables for the power component are u_l, which specify the peak power level $l \in \{1, \dots, L\}$ the consumers should pay. Constraint (C29) ensures that a single power price should be charged to the consumer in the whole planning period. Constraints (C30) guarantee that the total power required from the grid at each time $t \in \{1, \dots T\}$ should satisfy the operation of all types of loads.

Constraints (C31) define the lower level decision variables u_l, $l \in \{1, \dots, L\}$, as binary variables, where 1 means that the consumer should pay for the power price corresponding to the *l* power level.

POWER COMPONENT

$$\sum_{l=1}^{L} u_l = 1 \tag{C29}$$

$$b_t + \sum_{j=1}^{J} p_{jt} + \sum_{k=1}^{k} q_{kt} + s_t P_{AC}^{nom} \leq \sum_{l=1}^{L} P_l^{Cont} u_l , \quad t = 1, \dots, T \tag{C30}$$

$$u_l \in \{0,1\} , \quad l = 1, \dots, L \tag{C31}$$

In the lower level problem, the consumer's objective is to minimize the sum of the electricity bill and the monetization of the discomfort caused by the thermostatic load (equation (F-LL)), i.e. the sum of the cost of the energy consumed by uncontrollable, shiftable, interruptible and thermostatic loads (term (A) in equation (F-LL)), the power cost (term (B)) and the costs resulting from the monetization of the positive and negative deviations (Δ_t^+ and Δ_t^-, respectively) of the minimum temperature from the reference temperature (term (C)). Constants c^+ and c^- in term (C) are the costs (in €/ºCh) incurred by the positive and negative temperature deviations, respectively, with $c^- > c^+$ since we are considering heating mode. The factor $\frac{1}{1000}$ converts the power in W to kW.

LOWER LEVEL PROBLEM

$$\min_{\substack{p,w,q,v,s,z,y,u,\\ \Delta^+,\Delta^-,\theta^{in},\theta^{min}}} f = \underbrace{\frac{1}{1000}\sum_{i=1}^{I}\sum_{t\in P_i} x_i\left(b_t + \sum_{j=1}^{J} p_{jt} + \sum_{k=1}^{K} q_{kt} + s_t P_{AC}^{nom}\right)}_{(A)} + \underbrace{\sum_{l=1}^{L} e_l u_l}_{(B)} + \underbrace{\sum_{t\in T}(c^+\Delta_t^+ + c^-\Delta_t^-)}_{(C)} \qquad \text{(F-LL)}$$

s.t.

Constraints (C4) to (C31)

The objective function of the upper level problem (equation (F-UL)) consists of the maximization of the retailer's profit, being defined as the difference between the revenue with the sale of energy to consumers (term (A)+(B)) and the cost of buying energy in the wholesale market (term (D)). Coefficients π_t in equation (F-UL) are the prices of energy incurred by retailer at time $t \in \{1, \dots, T\}$.

BL MODEL

$$\max_{x} F = \underbrace{\frac{1}{1000}\sum_{i=1}^{I}\sum_{t\in P_i} x_i\left(b_t + \sum_{j=1}^{J} p_{jt} + \sum_{k=1}^{K} q_{kt} + s_t P_{AC}^{nom}\right)}_{(A)} + \underbrace{\sum_{l=1}^{L} e_l u_l}_{(B)} -$$

$$\underbrace{\frac{1}{1000}\sum_{t=1}^{T} \pi_t\left(b_t + \sum_{j=1}^{J} p_{jt} + \sum_{k=1}^{K} q_{kt} + s_t P_{AC}^{nom}\right)}_{(D)} \qquad \text{(F-UL)}$$

s.t.

Constraints (C1) to (C3)

$$\min_{\substack{p,w,q,v,s,z,y,u,\\ \Delta^+,\Delta^-,\theta^{in},\theta^{min}}} f = \underbrace{\frac{1}{1000}\sum_{i=1}^{I}\sum_{t\in P_i} x_i\left(b_t + \sum_{j=1}^{J} p_{jt} + \sum_{k=1}^{K} q_{kt} + s_t P_{AC}^{nom}\right)}_{(A)} + \underbrace{\sum_{l=1}^{L} e_l u_l}_{(B)} + \underbrace{\sum_{t\in T}(c^+\Delta_t^+ + c^-\Delta_t^-)}_{(C)} \qquad \text{(F-LL)}$$

s.t.

Constraints (C4) to (C31)

4. Methodological Approach

In this section, a hybrid approach for solving the BL model described in the previous section is presented. The approach combines a population-based algorithm to solve the upper level problem ((F-UL), (C1) – (C3)) and an external MIP solver (Cplex) for the lower level problem ((F-LL), (C4) – (C34)).

4.1. Global framework

The proposed algorithm starts by creating an initial population of N individuals $x^h = (x_1^h, \dots, x_I^h)$, $h = 1, \dots, N$, representing the electricity prices set by the retailer in the I sub-periods of the planning period. For each x^h, a lower level solution y^h is computed, where y denotes the vector of all decision variables of the lower level model (see (F-LL), (C4)-(C34)).The following pseudocode summarizes the steps of the proposed hybrid approach.

PSEUDOCODE OF THE HYBRID APPROACH

1. Create the initial population Pop of N individuals $x^h = (x_1^h, \dots, x_I^h), h = 1, \dots, N$
2. Repeat
   2.1. Repair each solution $x^h, h \in \{1, \dots, N\}$, of *Pop* to satisfy the upper level constrains (C1)-(C3).;
   2.2. Solve the lower level problem ((F-LL), (C4)-(C31)) using Cplex for each x^h, $h \in \{1, \dots, N\}$, in *Pop* to obtain the corresponding y^h;
   2.3. Compute $F(x^h, y^h)$ for all (x^h, y^h), $h \in \{1, \dots, N\}$, and retain the best solution (x, y);
   2.4. Modify each solution x^h, $h \in \{1, \dots, N\}$, of *Pop* according to the PSO algorithm (described in section 4.2);
3. Until G iterations are performed;
4. Output: (x, y) with the best $F(x, y)$.

In step 2.1, if a solution x^h, $h \in \{1, \dots, N\}$, does not satisfy the upper level constraints (C1)-(C3), then it is repaired using a routine based on the one described in Alves et al. (2016). This routine has been improved to cope with a fixed number of decimals since electricity prices are generally given in this way (e.g., 5 decimals). Therefore, although the x variables have been defined as continuous in the model, after creating or modifying a solution, the x values are truncated and then adjusted by the Repairing routine to satisfy the upper-level constraints, while still maintaining the number of decimal places.

Firstly, if any component $x_i^h, i \in \{1, \dots, I\}$, is out of the bounds imposed by constraints (C1) and (C2), then it is pushed to the closest bound (minimum $\underline{x_i}$ or maximum $\overline{x_i}$). Due to the fixed number of decimals, it may be not possible to satisfy the constraint (C3) as an equality; therefore, the Repairing routine adjusts the solution to satisfy $\frac{1}{T}\sum_{i=1}^{I} \overline{P_i} x_i \le x^{AVG} \;\; \wedge \;\; \frac{1}{T}\sum_{i=1}^{I} \overline{P_i} x_i \ge x^{AVG}$-$\varepsilon$, i.e. a downward tolerance (ε) with respect to the average price x^{AVG} is allowed but x^{AVG} can never be surpassed (the aim is that the electricity prices contracted by the consumer are strictly respected). The Repairing routine consists of an iterative process, in which the deviation of the violated inequality is divided by the variables that can still move - *free* variables - defining an amount a_i that must be added to each x_i^h: a variable is *free* if it can be increased (x_i^h is below its maximum) and a_i is positive, or it can be decreased (x_i^h is above its minimum) and a_i is negative. Whenever one or more a_i cannot be fully added to the respective variable, because it would lead x_i^h to be outside its bounds, another iteration of the process is done in which this variable is no longer *free*. It may happen that the Repairing routine stops with an infeasible solution and no *free* variables, although this situation has been rarely observed in practice. In that case the solution is discarded.

The output of the hybrid approach is the solution (x, y) found throughout the G iterations of the algorithm that gives the highest retailer's profit $F(x, y)$.

4.2. Upper level search based on PSO

In previous experiments we have done with other BL models, the PSO algorithm revealed a better performance in comparison with a GA. Therefore, in the current work, the upper level search of the BL model described in section 3 is performed by a PSO algorithm.

After randomly creating an initial population of N solutions $x^h = (x_1^h, \dots, x_I^h)$, $h = 1, \dots, N$, commonly known as a swarm of particles in PSO, and evaluating their fitness $F(x^h, y^h)$, the PSO algorithm iteratively moves the particles towards best positions. The movements are influenced by the best position visited by each particle x^h (personal best – $x^{h^{best}}$) and guided by the best known position of the entire swarm (global best – g^{best}).

In each iteration q, the velocity component ${v_i}^h$, $i \in \{1, \dots, I\}$, of solution x^h is determined according to the following expression (Eberhart and Yuhui 2001):

$$ {v_i}^{h^q} = \eta {v_i}^{h^{q-1}} + r_1 C_1 \left(x_i^{h^{best}} - {x_i}^{h^{q-1}} \right) + r_2 C_2 \left(g_i^{best} - {x_i}^{h^{q-1}} \right) $$

where η is the inertia weight, C_1 and C_2 are the cognitive and social parameters, r_1 and r_2 are uniform random numbers in the interval [0,1]. The new position of particle x^h is then determined by the following equation:

$$ x^{h^q} = x^{h^{q-1}} + v^{h^q} $$

If this solution does not satisfy the upper level constraints (C1)-(C3) then it is repaired. After a feasible x^{h^q} is obtained, the lower level problem is solved for x^{h^q} and the corresponding optimal lower level y^{h^q} is computed. The personal best, $x^{h^{best}}$, and the global best, g^{best}, are updated whenever better solutions according to $F(x^{h^q}, y^{h^q})$ are found.

When the value of F^{best} (objective function value for g^{best}) does not improve over a predefined number G' of consecutive iterations, then the exploration capability is enhanced by inducing some turbulence in the population with a probability p_m. Therefore, for each solution x^{h^q}, after its movement and before entering the Repairing routine, each coordinate ${x_i}^{h^q}, i \in [1, I]$ is subject to a perturbation ζ randomly generated in the range $\left[-\delta \left(\overline{x_i} - \underline{x_i} \right), \delta \left(\overline{x_i} - \underline{x_i} \right) \right]$, i.e., ${x_i}^{h^q} \leftarrow {x_i}^{h^q} + \zeta$.

In the current work, it is considered that the value of F^{best} does not improve from one iteration to the next when the relative change in its value is below a given threshold τ, i.e. the following inequality is satisfied:

$$\frac{F^{best^{q}} - F^{best^{q-1}}}{F^{best^{q}}} < \tau$$

In previous experiments, this adaptive mutation scheme led to significant improvements in the solutions obtained.

In our implementation, $\tau = 0.001$ and δ = 0.2. $G' = 5$ is the number of consecutive iterations without improvement of F^{best} that triggers turbulence with a probability $p_m = 0.1$. The inertia parameter η was set to 0.4 and the learning parameters in both cognitive and social components were kept equal, $C_1 = C_2 = 1$.

4.3. What to do if the computational effort is too high to guarantee lower level optimality?

Assuming a realistic h for the discretization of the planning period T, the lower level problem may not be amenable to exact resolution because it may require a high computational effort due to the combinatorial nature of the mathematical model. We have observed, in particular, that the inclusion of the thermostatic load strongly increases the computational difficulty and the time required to solve the problem.

If the lower level problem cannot be solved to optimality (namely due to practical computational budget requirements) then it is not possible to guarantee the feasibility of the solution to the BL model. Indeed, solutions obtained for the BL model with non-optimal lower level solutions may be better for the upper level objective function than the real optimal solution, but these solutions may be misleading. Therefore, the aim should be offering the leader good quality estimates of bounds for his objective function in order to support informed decisions.

In the present model, obtaining sub-optimal solutions to the lower level problem enables to approximate upper bounds (*upper estimates*) for the optimal value of the leader's objective function (F^*). Recall that the leader's objective is to maximize the profit resulting from the revenue with selling electricity to consumers minus the cost of buying electricity in the wholesale market; on the other hand, consumers want to minimize electricity cost, which corresponds to the leader's revenue. Hence, sub-optimal solutions to consumers, i.e. solutions with higher cost, lead to retailer's revenues higher than the real ones. Note that this is an upper bound for F in solutions with the same x vector and, in theory, this may be not an upper bound for F^* because the BL optimal solution could be given by another x. However, the incremental strategy (described below) to obtain this estimate aims at overcoming this issue in practice, leading to relevant information for decision support in face of the rational follower's reaction. Guaranteed upper bounds could be computed using relaxations of the BL problem, which were also tested, but the information provided would have considerably less practical value.

In the current work, we assume that a computational budget exists; our main goal is taking advantage of the information that can be obtained within that budget and use it as support to the leader's decision. A reasonable computational time limit of ξ_1 is defined to solve each instantiation of the lower level problem and, rather than keeping just the best solution, the algorithm stores the $\mathcal{G}$ solutions with the highest value of F.

When the lower level problem is a MIP, as in the model presented in section 3, the solver provides the (relative) MIP gap when the computational budget is attained. If this gap is strictly positive, a non-optimal solution may be delivered. In this case, after completing the pre-specified number of iterations G (see pseudocode in section 4.1), the lower level problem is solved again for each final individual solution $x^{g}, g \in \{1, \dots, \mathcal{G}\}$, with an increased time limit of ξ_2. The solutions obtained, $(x^{g}, y^{g}), g \in \{1, \dots, \mathcal{G}\}$, are then evaluated according to F.

The output of the algorithm is the best solution (x, y) among the $\mathcal{G}$ solutions, i.e. the one that presents the highest retailer's profit $F(x, y)$.

Due to the stochastic nature of the PSO procedure, the algorithm is run more than once even if the exact optimal solution to the lower level problem can be obtained for each instantiation of the upper level variables. Therefore, the algorithm is performed r independent runs (each with G iterations and N individuals), generating r *best* solutions. Then, a total of $\frac{r}{2}$ solutions are selected from these r *best* solutions for further analysis when the MIP gap is still positive: half of them $(\frac{r}{4})$ are the ones with the highest F values and the other half are randomly selected from the remaining solutions. This further analysis consists of solving the lower level problem for each upper level vector $x^{\omega}, \omega \in \left\{1, \dots, \frac{r}{2}\right\}$ considering a longer time limit, $\xi_3 > \xi_2$. This analysis aims at decreasing, for each x^{ω}, the MIP gap of the lower level solution to the optimal one, so that the upper estimate yielded for the leader's objective function value can be tightened still within a reasonable computational time limit, in an incremental strategy. Thus, at the end of this process, $\frac{r}{2}$ solutions $(x^{\omega}, \hat{y}^{\omega})$ to the BL model are obtained with $F(x^{\omega}, \hat{y}^{\omega})$ values expected to be good upper estimates for F^{*}.

This procedure to derive upper estimates for F^{*} can be adapted for other problems in which the lower level problem is defined by a mathematical model, but it is not possible, or practical, to obtain its optimal solution due to the need of an excessive computational effort. In particular, this procedure can be replicated in pricing problems in which the lower level cost objective function is a part of the upper level revenue objective function.

The next step is to derive good quality *lower estimates* for the upper level objective function. This should be accomplished by profiting from the characteristics of the problem to narrow as much as possible the interval defined by lower/upper estimates for the leader's objective function. Thereafter, these intervals will be assessed through standard techniques for comparison of interval numbers with the aim of providing meaningful information to the leader.

In order to compute lower estimates for F^*, the following characteristics of the problem are considered: the lower level objective function includes a term associated with the monetized discomfort of the indoor temperature controlled by the air conditioner, which does not constitute a revenue of the retailer (term (C) in F-LL); the revenue terms of the upper level objective function are terms (A) and (B), which are the energy costs and the power costs for the operation of all types of loads incurred by the consumer; therefore, if the discomfort component (C) is removed from the lower level objective function, by defining null temperature deviation costs, and only the energy and power components are considered, then this leads to minimum cost solutions at lower level and consequently to minimum retailer's revenues. Note that this option also induces, in general, the air conditioner to work just to guarantee the minimum indoor temperature allowed, because the temperature deviation is discarded from the analysis, which further leads to lower revenues for the leader. This information is used to compute realistic good quality lower estimates for the leader's objective function. So, the discomfort component is removed from the lower level objective function and the lower level problem is solved for each upper level vector $x^{\omega}, \omega \in \left\{1, \dots, \frac{r}{2}\right\}$ considering the time limit ξ_3 (the same used before for computing the upper estimates for F^*). At the end of this process, $\frac{r}{2}$ solutions $(x^{\omega}, \breve{y}^{\omega})$ are obtained with $F(x^{\omega}, \breve{y}^{\omega})$ values expected to be good *lower estimates* for F^*. As a result, the retailer's profit estimates obtained with an acceptable computational effort can be used as decision support information to establish the electricity prices (i.e., a commercial offer in the retail market).

The comparison between the $\frac{r}{2}$ objective function values represented by interval numbers, i.e. uncertain but limited, $[F(x^{\omega}, \breve{y}^{\omega}), F(x^{\omega}, \hat{y}^{\omega})\ ], \omega \in \left\{1, \dots, \frac{r}{2}\right\}$, is made using the relation proposed by (Jiang et al. 2012). Being A and B two interval numbers, the *reliability-based possibility* of $P(A \leq B)$ is determined as

$$P(A \leq B) = \min\left\{\max\left\{\frac{B^R - A^L}{2A^w + 2B^w}, 0\right\}, 1\right\}$$

where the subscripts *R* and *L* denote the right and the left limits and *w* denotes the amplitude of the interval number. Other equivalent expressions for this purpose exist.

The computation of $P(A \leq B)$ for all pairs of lower/upper estimates for the *F* values enables to assess the strength of this relation as well as ranking the interval numbers [lower estimate, upper estimate] ([LE, UE]) and eventually recognizing a solution, i.e. a vector of electricity prices x^{ω}, as an interesting one (in some way balancing risk vs. opportunity to obtain a higher profit) in face of the impossibility of guaranteeing the optimality of lower level solutions.

At this point we would like to emphasize that, even if the optimal solution to lower level problem can be obtained for each *x*-vector, the global optimal solution and the *F** value may not be obtained because a meta-heuristic is used to explore the upper level solution space. However, the main difference between obtaining optimal solutions and sub-optimal solutions to the lower level is that, in the first case, the value obtained for the upper level (maximizing) function, say F', certainly satisfies $F' \leq F^*$ because the algorithm only works with feasible solutions; in the latter case, the final value of F may be less or greater than F^* because sub-optimal solutions to the lower level are infeasible to the BL problem. Therefore, obtaining good upper/lower estimates for F^* is of utmost importance in the latter case.

5. Experimental results and discussion

The computational budget to solve the lower level optimization model presented in section 3 renders impossible, in general, to obtain its optimal solution, namely due to the modelling of the thermostatic load. Therefore, the aim is to offer the retailer insights to assist him in making a decision regarding the prices to be imposed to consumers employing the methodology described in section 4. In the following, the problem data and the parameters of the algorithm are described; then the results obtained are presented and analyzed.

5.1. Data and Parameters

In the experiments carried out, a planning period of 24 hours was considered, split into intervals of 15 minutes, leading to a planning period of 96 units of time. Then, $\mathrm{T} = \{1, \ldots, 96\}$ where each time unit $t \in \mathrm{T}$ represents a quarter-hour (i.e. h = $^1/_4$ h).

In this study, dynamic tariffs with $I = 6$ sub-periods of energy prices charged to the consumers by the retailer, $P_i = [P_{1_i}, P_{2_i}] \subset \mathrm{T},\ i \in \{1, \ldots, 6\}$, were considered. Also, $L = 9$ power levels were considered for the power cost component. Detailed data, regarding the prices the retailer buys energy in the spot market (€/kWh), the minimum/maximum and average energy prices the retailer can sell energy in the retail market (€/kWh; e.g., to comply with competition of commercial offers), and the power level prices (€) are displayed in Tables SM-1, SM-2 and SM-3 (Supplementary Material).

Six appliances were considered in the lower level model: three shiftable non-interruptible loads ($J = 3$) – dishwasher (DW), laundry machine (LM) and clothes dryer (CD); two interruptible loads ($K = 2$) – electric vehicle (EV) and electric water heater (EWH); one thermostatic load – air conditioner system (AC). The information related with the operation cycles of controllable appliances, the thermostatic load, as well as non-controllable (base) load is displayed in Table SM-4 to Table SM-8.

Typical values for the parameters α, β and γ (defining constraint (C17) in the model) were considered: 0.8569, 0.1431 and 0.002775, respectively, for a time discretization of 15 minutes (Antunes et al. 2018). The temperature deviation costs in heating mode were defined as $c^+ = 0.005$ and $c^- = 0.010$ (€/ºCh).

Given the range of temperatures, the value of the big number M was defined as 100.

In each instantiation of the upper level variables, the number of binary and continuous variables as well as the number of constraints in the lower level problems are displayed in Table 1 (with and without the thermostatic load). The upper level problem has 6 decision variables and 13 constraints (lower/upper bounds for the prices plus the average price constraint).

Table 1. Dimension of the lower level problem

	Number of Variables		Number of Constraints	
	Binary	Continuous	Inequality	Equality
with all loads	839	535	1346	354
without the thermostatic load	551	151	578	162

In the computational experiments, the hybrid BL algorithm was run $r = 20$ independent times, each one consisting of $G = 70$ iterations with a population size of $N = 20$ individuals. These settings resulted from preliminary studies, which showed that no further improvement usually occurred for the upper level objective function by increasing the number of iterations or the population size.

In each run, a total of 1400 MIP problems, i.e. 20 individuals $\times$ 70 iterations, are solved (plus the corresponding upper level operations), with a computation time limit of $\xi_1 = 15$ s for the resolution of each instantiation of the lower level problem by the MIP solver. After completing the 70 iterations, the lower level problems are solved again for a final set of the $\mathcal{G} = 10$ best solutions of that run, with an increased time limit of the MIP solver of $\xi_2 = 60\ s$ aiming to further improve them. These lower level solutions are then evaluated according to the upper level objective function and the best one is the output of the run.

Therefore, after running the algorithm $r = 20$ independent times, then $\frac{r}{2} = 10$ solutions are selected from the set of 20 solutions for further analysis. For this purpose, the lower level problem for each best (upper level) solution is solved again allowing a longer time limit of $\xi_3 = 5$ min for the MIP solver. This incremental strategy aims to use the computational budget in more promising solutions to further improve the MIP gap of the lower level solutions and, consequently, better approximate the true value of the upper level objective function.

Several experiments were performed to fine tune the values of time limit parameters $\xi_k, k = 1, 2, 3.$ The values used in the experiments revealed a suitable compromise between running time and convergence for good quality results.

The algorithm was written in R language and all runs were carried out in a computer with an Intel Core i7-7700K CPU@3.6GHz and 64GB RAM. In summary, one iteration involves the upper level operations plus solving 20 MIP problems. Each complete run involves 70 iterations plus resolving 10 extra lower level problems, taking approximately 6 hours and 10 minutes.

Considering a similar BL model but without the thermostatic load, the algorithm runs fast and is able to obtain optimal solutions at the lower level; in this case, one complete run performs in approximately 13 minutes.

5.2. Results

The upper level vectors associated with the $\frac{r}{2} = 10$ solutions selected from the set obtained in all the runs (with a computational time limited to 60 seconds for the resolution of each lower level problem) are displayed in Table SM-9 (Supplementary Material). Statistics about these 10 solutions are presented in Table 2: the maximum, minimum, average and standard deviation of the retailer's profit, *F*, and the corresponding consumers' costs, *f* , as well as the relative MIP gaps.

Table 2. Statistics of the 10 selected solutions with a limited computational time to solve the lower level problem.

Computational time to solve the lower level problem		Maximum	Minimum	Average	Standard Deviation
60 seconds	*F*	4.36875	4.31026	4.33793	0.01810
	f	7.31917	7.23766	7.29493	0.02228
	MIP gap	5.43%	4.67%	5.02%	0.28%
5 minutes	*F*	4.36493	4.23427	4.27368	0.03677
	f	7.28510	7.19252	7.23547	0.02370
	MIP gap	4.33%	2.85%	3.70%	0.45%

The results displayed in Table 2 show that, for a computational time limited to 60 seconds, the lower level computations produce solutions with relatively high MIP gap values (average gaps of approximately 5%), which do not reduce significantly if the computational time is extended to 5 minutes (MIP gap average value of 3.7%).

According to the methodology described in section 4, the coefficients of the monetized discomfort component of the lower level objective function are then set to $c^{+} = c^{-} = 0$. The aim is to have a good estimate of the lower bound for the retailer's objective function. This lower estimate can be defined as the difference between the minimum retailer's revenue (i.e., the consumer's cost obtained with null temperature deviation costs) and the maximum retailer's acquisition cost (i.e., the maximum value between the cost of energy acquisition with $c^+ = c^- = 0$ and the cost of energy acquisition with $c^+ = 0.005$ and $c^- = 0.010$). The corresponding results are displayed in Table 3.

Table 3. Statistics of the 10 solutions corresponding to the lower estimates of *F*, with a computational time limited to 5 minutes to solve the lower level problem.

	Maximum	Minimum	Average	Standard Deviation
F	4.26037	4.14528	4.22618	0.03577
f	7.14236	7.01616	7.11241	0.03592
MIP gap	5.87%	4.72%	5.43%	0.38%

The values in Table 3 show lower consumer' costs, i.e. lower values of *f* in comparison with the ones obtained with positive temperature deviation costs. However, disregarding the discomfort cost from the consumer's objective function leads to a higher variability of *f* across runs, as measured by the standard deviation. Also, the relative MIP gap values corresponding to the best solutions obtained with the computational budget are higher than the ones obtained with $c^+ = 0.005$ and $c^- = 0.010$.
Therefore, the objective function values obtained with positive values of c^+ and c^- can be assumed as upper estimates for the retailer's profit; objective function values obtained with null values of c^+ and c^- are used to compute the lower estimates. Figure 4 and Table 4 display the [lower estimate, upper estimate] intervals ([LE,UE]) for the retailer's profit in each of the 10 selected final solutions.

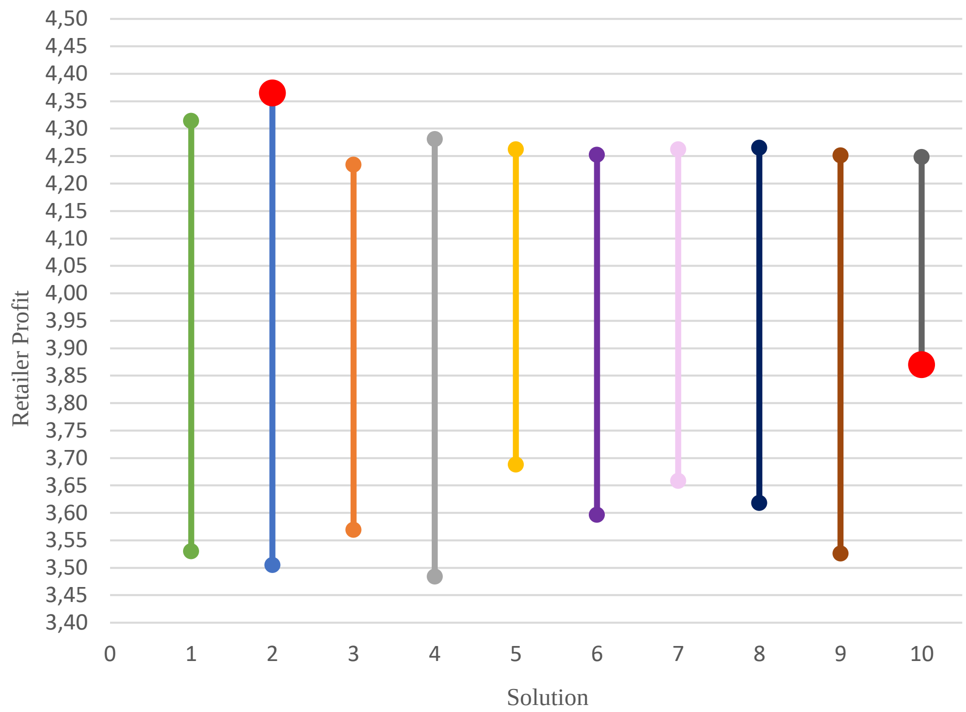

Fig. 4. Retailer's profit intervals, [LE,UE], in the 10 final solutions.

Table 4. Retailer's profit intervals, [LE,UE], in the 10 final solutions.

Solution	Lower Estimate	Upper Estimate	Average	$\frac{\text{UE}}{\text{max UE}}$	$\frac{\text{LE}}{\text{max UE}}$	$\frac{\text{LE}}{\text{max UE}}$
1	3.53006	4.31436	3.922208	98.84%	91.22%	80.87%
2	3.50489	**4.36493**	3.934908	100.00%	90.57%	80.30%
3	3.56940	4.23427	3.901835	97.01%	92.23%	81.77%
4	3.48438	4.28127	3.882823	98.08%	90.04%	79.83%
5	3.68842	4.26254	3.975478	97.65%	95.31%	84.50%
6	3.59667	4.25229	3.924480	97.42%	92.94%	82.40%
7	3.65836	4.26216	3.960260	97.65%	94.53%	83.81%
8	3.61796	4.26536	3.941655	97.72%	93.49%	82.89%
9	3.52595	4.25124	3.888593	97.40%	91.11%	80.78%
10	**3.86998**	4.24842	4.059195	97.33%	100.00%	88.66%

The maximum lower estimate and upper estimate for the retailer's profit obtained for these 10 solutions are marked with a large circle (in red) in Figure 4, which correspond to solutions 10 and 2, respectively (in bold in Table 4). Table 4 shows that all upper estimates are within a 3% range with respect to the maximum upper estimate. The lower estimates are distant of the maximum lower estimate at most 10%.

Solution 10 has the higher lower estimate and the higher average retailer profit. Additionally, solution 10 presents good values of lower estimate and upper estimate with respect to the best ones, also displaying the best relation between the lower estimate and the upper estimate values. This suggests that solution 10 may be an adequate option for a risk-averse decision maker.

Table 5 displays the *reliability-based possibility* of $P(A \leq B)$, where A and B represent the F interval values [LE,UE] for the 10 solutions. The column "Count $P(A \leq B) \geq 0.5$" in Table 5 gives the frequency of $P(A \leq B) \geq 0.5$, thus stating the superiority of B. The column "Rank" gives the ranking of the interval solutions according to the frequency of $P(A \leq B) \geq 0.5$. This analysis corroborates that solution 10 seems to be superior to the other solutions. Solutions 5 and 7 are also good options for a decision maker not willing to engage in a high risk, while solution 2 may yield a very good objective function value but with a higher risk of obtaining a low value.

Table 5. $P(A \leq B)$, where A and B represent the F interval values [LE,UE] for the 10 solutions.

$P(A \leq B)$		A										Count	Rank
		1	2	3	4	5	6	7	8	9	10	$P(A \leq B) \geq 0.5$	
B	1		0.492	0.514	0.525	0.461	0.498	0.473	0.486	0.522	0.382	3	7
	2	0.508		0.522	0.531	0.472	0.507	0.483	0.496	0.529	0.400	5	5
	3	0.486	0.478		0.513	0.441	0.483	0.454	0.470	0.510	0.349	2	8
	4	0.475	0.469	0.487		0.432	0.471	0.445	0.459	0.496	0.350	0	10
	5	0.539	0.528	0.559	0.568		0.541	0.513	0.528	0.567	0.412	8	2
	6	0.502	0.493	0.517	0.529	0.459		0.472	0.487	0.526	0.370	4	6
	7	0.527	0.517	0.546	0.555	0.487	0.528		0.515	0.554	0.400	7	3
	8	0.514	0.504	0.530	0.541	0.472	0.513	0.485		0.539	0.385	6	4
	9	0.478	0.471	0.490	0.504	0.433	0.474	0.446	0.461		0.345	1	9
	10	0.618	0.600	0.650	0.650	0.588	0.630	0.600	0.615	0.655		9	1

The structure of prices of solution 10 is presented in Table 6, which leads to a retailer's profit in the interval [3.86998, 4.24842].

Table 6. Electricity prices (in €/kWh) of solution 10.

Solution	Prices (€/kWh)					
	P_1 [1,28]	P_2 [29,44]	P_3 [45,56]	P_4 [57,72]	P_5 [73,84]	P_6 [85,96]
10	0.0996	0.2739	0.2828	0.0817	0.1548	0.1438

6. Conclusions

In this work, a novel BL model of the interaction between a retailer and consumers in the electricity retail market was developed, including shiftable, interruptible and thermostatic loads, which can be controlled by an energy management system. The aim is to determine the optimal dynamic ToU electricity prices to be established by a retailer to maximize profits in face of consumers' demand response to minimize costs considering comfort requirements of time slots for shiftable load operation, energy supplied to interruptible loads and indoor temperature range for the thermostatic load. The BL model is dealt with a hybrid approach based on a PSO algorithm that calls a MIP solver to deal with the consumer's problem of appliance scheduling and thermostat setting for a given instantiation of electricity prices (upper level decision variables).

The consideration of the thermostatic load in the MIP model imposes a high computational burden, due to the combinatorial nature of the model, which impairs obtaining the lower level optimal solution in an acceptable computational time. Since only optimal solutions to the lower level problem are feasible to the BL problem, non-optimal solutions to the lower level problem may lead to misleading solutions to the BL problem, i.e. solutions that may display better upper level objective function values but are indeed infeasible. However, sound information that can be exploited in practice should be given to the (upper level) decision maker. For this purpose, we propose a novel approach to compute lower/upper estimates for the optimal solution of the BL problem when a computational budget should be considered to obtain solutions to the lower level problem. Solutions to the lower level problem are identified and successively refined, within a given computational budget compatible with the decision time frame, to obtain upper estimates for the upper level objective function. Good quality lower estimates are determined by making the most of the characteristics of the problem, in particular the relations between the upper level and the lower level objective functions. A set of solutions for the leader is obtained, each one giving an interval number for the upper level objective function. Additional information to support decision making is obtained by comparing interval numbers defined by the lower/upper estimates previously computed.

In this setting, we intend to further develop techniques to derive risk vs. opportunity indexes useful to inform the upper level decision making process whenever lower level optimal solutions cannot be guaranteed due to the computational difficulties of the lower level problem. Furthermore, we intend to consider the lower level problem as a multi-objective model by explicitly including cost and comfort objective functions.

Acknowledgments

This work was partially supported by projects UID/MULTI/00308/2013 and by the European Regional Development Fund through the COMPETE 2020 Programme, FCT – Portuguese Foundation for Science and Technology and Regional Operational Program of the Center Region (CENTRO2020) within projects ESGRIDS (POCI-01-0145-FEDER-016434), SUSPENSE (CENTRO-01-0145-FEDER-000006) and MAnAGER (POCI-01-0145-FEDER-028040).

References

Alves, M. J. & Antunes, C. H. (2018). "A semivectorial bilevel programming approach to optimize electricity dynamic time-of-use retail pricing." Computers & Operations Research. 92: 130-144.

Alves, M. J., Antunes, C. H. & Carrasqueira, P. (2016). A Hybrid Genetic Algorithm for the Interaction of Electricity Retailers with Demand Response. Applications of Evolutionary Computation. 459-474.

Antunes, C. H., Rasouli, V., Alves, M. J., Gomes, Á., Costa, J. J. & Gaspar, A. (2018). "A Discussion of Mixed Integer Linear Programming Models of Thermostatic Loads in Demand Response." 2nd International Symposium on Energy System Optimization. Karlsruhe Institute of Technology, Germany, October 10-11, 2018.

Bard, J. F. (1998). "Practical Bilevel Optimization: Algorithms and Applications." Springer Science + Business Media, Dordrecht.

Carrasqueira, P., Alves, M. J. & Antunes, C. H. (2017). "Bi-level particle swarm optimization and evolutionary algorithm approaches for residential demand response with different user profiles." Information Sciences. 418-419: 405-420.

Colson, B., Marcotte, P. & Savard, G. (2005). "Bilevel programming: A survey." 4or. 3(2): 87-107.

Colson, B., Marcotte, P. & Savard, G. (2007). "An overview of bilevel optimization." Annals of Operations Research. 153(1): 235-256.

Dempe, S. (2002). "Foundations of Bilevel Programming." Kluwer Academic Publishers, Dordrecht.

Eberhart & Yuhui, S. (2001). Particle swarm optimization: developments, applications and resources. Proceedings of the 2001 Congress on Evolutionary Computation (IEEE Cat. No.01TH8546)**:** 81-86.

Jiang, C., Han, X. & Li, D. (2012). "A New Interval Comparison Relation and Application in Interval Number Programming for Uncertain Problems." Computers Materials & Continua. 27(3): 275-303.

Labbé, M. & Violin, A. (2016). "Bilevel programming and price setting problems." Annals of Operations Research. 240(1): 141-169.

Lunday, B. J. & Robbins, M. J. (2018). "Collaboratively-developed vaccine pricing and stable profit sharing mechanisms." Omega. In Press.

Meng, F. L. & Zeng, X. J. (2013). "An Optimal Real-time Pricing Algorithm for the Smart Grid: A Bi-level Programming Approach." 2013 Imperial College Computing Student Workshop (ICCSW'13) - OpenAccess Series in Informatics: 81-88.

Meng, F. L. & Zeng, X. J. (2016). "A bilevel optimization approach to demand response management for the smart grid." 2016 IEEE Congress on Evolutionary Computation (CEC): 287-294.

Sekizaki, S., Nishizaki, I. & Hayashida, T. (2016). "Electricity retail market model with flexible price settings and elastic price-based demand responses by consumers in distribution network." International Journal of Electrical Power & Energy Systems. 81: 371-386.

Sinha, A., Malo, P. & Deb, K. (2018). "A Review on Bilevel Optimization: From Classical to Evolutionary Approaches and Applications." IEEE Transactions on Evolutionary Computation. 22(2): 276-295.

Soares, A., Gomes, Á. & Antunes, C. H. (2014). "Categorization of residential electricity consumption as a basis for the assessment of the impacts of demand response actions." Renewable and Sustainable Energy Reviews. 30: 490-503.

Vicente, L. s. N. & Calamai, P. H. (1994). "Bilevel and multilevel programming: A bibliography review." Journal of Global Optimization. 5(3): 291-306.

Zugno, M., Morales, J. M., Pinson, P. & Madsen, H. (2013). "A bilevel model for electricity retailers' participation in a demand response market environment." Energy Economics. 36: 182-197.