Group Decision and Negotiation **15:** 171–184, 2006.
DOI: 10.1007/s10726-006-9027-9

An Approach to Support Negotiation Processes with Imprecise Information Multicriteria Additive Models

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Abstract

This paper discusses the possible uses of the VIP (Variable Interdependent Parameters) Analysis software and methodology in negotiation support. VIP Analysis is a decision support tool that incorporates complementary approaches to deal with the aggregation of multi-criteria performances under imprecise information. Its purpose is to support the evaluation of a discrete set of alternatives according to multi-attribute additive value functions. We propose extensions of the methodology of VIP Analysis to address explicitly the differences among the actors in terms of the weights space.

Key words: multicriteria aggregation, additive utility/value models, imprecise information, VIP analysis

1. Introduction

Group decision and negotiation processes constitute very complex human activities. Improving their understanding justifies multidisciplinary studies based on cognitive psychology, organizational science, sociology, political science, etc. Furthermore, the development of computer based support tools for these activities is rooted not only in communication/information technologies and OR/management science models, but also in human and organizational behavior studies.

VIP Analysis (Dias and Clímaco 2000) is a decision support tool to evaluate a discrete set of alternatives according to a multiattribute additive value function. Its main characteristic is that it does not require precise values for the scaling constants (scaling weights). Rather, it can accept imprecise information (i.e., intervals and linear constraints) on these values. VIP Analysis may be used to discover robust conclusions, i.e. those that hold for every admissible combination of the parameters, and to identify which results are more affected by the imprecision in the parameter values, i.e. the variability of the results.

The standalone version of VIP Analysis can be a useful tool to support a group of Decision Makers (DMs) meeting face to face around a computer, but it excludes settings where the DMs are spatially distributed. It does not allow DMs to analyze the problem individually and it implicitly requires an a priori aggregation of the different models that the DMs may have. Recent work (Dias and Clímaco 2005) discussed how the stand alone version can be extended to become a distributed Group Decision Support System (GDSS) – VIP – G – suitable to support groups of DMs. Other related methodologies for multicriteria group decision support with imprecise information are, for instance, (Contreras and Mármol 2004), (Kim et al. 1999), (Lahdelma and Salminen 2001), (Salo 1995), and (Tavares 2004).

This paper discusses the way in which VIP-G may evolve to situations where, instead of cooperative group decision making, strong conflicting interests are present and voting is not an option – the process must end with an agreement (even if the agreement is that negotiations failed). Examples of this type of situation occur, for instance, when managers in a company negotiate the selection of the company's R&D projects which should be considered of top priority; when performing community-based assessment of alternatives for managing a river, involving different stakeholders and interest groups; or when it is necessary to prepare a call for tenders for a new information system for a service of the Public Administration by actors representing different departments and user perspectives. Sometimes aggregation studies considering sub-groups of the actors may be adequate, namely to support the members of coalitions that may emerge. In these circumstances, besides the conflict, interdependence among the DMs/actors is crucial. Negotiation Support Tools may help the identification of the critical constraints imposed to the evaluation model's parameters, trying to help in narrowing the differences and/or identifying the characteristics of new virtual alternatives which should be acceptable by the parties, overcoming the degree of opposition among them. A negotiation process is, in any case, an evolutionary process and the system must provide the tools to increase the chances of opinions' convergence. The core of this paper consists in the discussion of some possible extensions of the VIP-G Analysis methodology to negotiation support.

This paper is structured as follows. We first review the basic aspects of VIP Analysis (Section 2) and the idea of using it to support groups using implicit preference aggregation. In Section 3, we review the explicit aggregation methods proposed in (Dias and Clímaco 2005) concerning the results space, and suggest extensions addressing the weights space. A further extension introducing the concept of convergence paths is proposed in Section 4. An illustrative example is presented in Section 5, followed by the conclusions section.

2. Basic VIP Analysis Concepts

We will first overview the current VIP Analysis methodology and software (Dias and Clímaco 2000). The purpose of VIP Analysis is the support of the evaluation of a discrete set of alternatives, in order to choose the most preferred one, according to a multiattribute additive value function (Keeney and Raiffa 1976). The global value of an alternative a_i is a sum of its values for the n criteria ($v_1(a_i), \ldots, v_n(a_i)$), weighted by n scaling weights $w = (w_1, \ldots, w_n)$ that indirectly reflect the importance of the criteria:

$$V(a_i, w) = \sum_{j=1}^{n} w_j v_j(a_i), \quad \text{with} \quad \sum_{j=1}^{n} w_j = 1 \quad \text{and} \quad w \geq 0. \tag{1}$$

One of the most difficult steps of the decision aid process is setting the values of the scaling weights, since these parameters will reflect the DMs values and trade-offs (e.g., how much would you be willing to lose in attribute "cost" to gain one unit in attribute "safety"?).

Indeed, not only DMs may find it hard to provide precise figures about their preferences, taking into account how these figures match their intuitive notion of importance, but also these preferences may change as the decision aid process evolves. Moreover, the questioning techniques that can be used to elicit the values of the importance parameters may require more time and patience than the DMs can spare and, in group decision situations, the opinions and preferences of the DMs diverge frequently.

To overcome these difficulties, VIP Analysis proposes to advance in the decision process with Variable Interdependent Parameters. This means that instead of requiring precise values for the scaling weights, it can accept intervals or any other linear constraints on these values. For instance a group of DMs may be doubtful about setting $w_1 = 0.2$ and $w_2 = 0.1$ (these precise values) but may find it easy to agree that $w_1 > w_2$. This kind of information is often designated as poor, imprecise, incomplete, or partial information (e.g., see (Dias and Clímaco 2000), (Weber 1987)). The constraints usually stem from imprecise answers from the DM (e.g. providing an interval for the trade-off rate between two criteria) or from holistic judgments about alternatives that the DM is able to compare (e.g. a_1 is preferred to a_2).

Let W denote the set of all combinations (vectors) of parameter values ($w_1, \ldots, w_n$) that satisfy all the established constraints. Once W is defined, VIP Analysis may be used to discover robust conclusions (those that hold for every combination in W) and to identify which results are more affected by the imprecision in the parameter values (the results that vary more). The results produced by VIP Analysis from a set W of acceptable combinations of values for the importance parameters and a set $A = \{ a_1, \ldots, a_m\}$ of alternatives include the following:

(a) Computation of a range of value for each alternative $a_i \in A$: the minimum value of a_i given W can be computed by solving a linear program (LP) with the scaling weights $w = (w_1, \ldots, w_n)$ as variables

$$\min\{V(a_i, w) \colon w \in W\}, \tag{2}$$

and similarly the maximum value of a_i given W can be computed by solving another LP

$$\max\{V(a_i, w) \colon w \in W\}. \tag{3}$$

If the maximum value for an alternative a_x is less than the minimum value for an alternative a_y, then the first alternative could be discarded as the second one is clearly superior. The minimum value may be used as a ranking rule – the "maximin" rule (e.g., Salo and Hämäläinen 2001).

(b) Computation of the highest difference of value for each ordered pair of alternatives: given an ordered pair of alternatives $(a_i, a_j) \in A^2$ and W, an LP may be solved to find the maximum possible advantage of the first alternative over the second one

$$m_{ij} = \max\{V(a_i, w) - V(a_j, w) \colon w \in W\}. \tag{4}$$

If the maximum difference is negative or null then $V(a_j, w) \geq V(a_i, w)\ \forall w \in W$, which we denote as $a_j \Delta a_i$ (a_j "dominates" a_i). If the maximum difference does not exceed a tolerance parameter ε, then $V(a_j, w) \geq V(a_i, w) - \varepsilon \ \forall w \in W$, and we denote this as $a_j \Delta_\epsilon a_i (a_j$ "quasi-dominates" a_i with tolerance ε).

(c) Computation of the "maximum regret" associated with choosing each alternative: given an alternative $a_i \in A$, the set $A \backslash \{a_i\}$, and W, this amounts to find the maximum difference of value by which a_i can lose to another alternative in $A \backslash \{a_i\}$. The scaling weights $w = (w_1, \ldots, w_n)$ are considered as variables (rather than being fixed) to allow the regret to be as high as possible given A and W

$$\text{Regret}_{\max}(a_i) = \max_{w \in W} \{ \max_{j=1,\ldots,m} \{V(a_j, w)\} - V(a_i, w) \} . \tag{5}$$

Rather than directly computing (5), after finding the maximal differences of value (4) for all pairs of alternatives, the maximum regret associated with choosing each one can be found by noting that (Dias and Clímaco 2000)

$$\text{Regret}_{\max}(a_i) = \max_{j=1,\ldots,m} \{m_{ji}\} \tag{6}$$

If $\text{Regret}_{\max}\ (a_i) = 0$ then we can say that a_i is "optimal"; if $\text{Regret}_{\max}(a_i) \leq \varepsilon$ we can say that a_i is "quasi-optimal" with tolerance ε. The "minimax regret" rule can also be used to rank alternatives (e.g., Salo and Hämäläinen 2001), although it is well known for not respecting Arrow's independence condition (adding or removing an alternative to/from A may result in rank reversals among the other alternatives).

VIP Analysis can be seen as a toolbox offering complementary approaches to analyze a decision situation with imprecise information. This can be useful to support a group of DMs meeting face-to-face around a computer, sharing an imprecisely defined model, therefore agreeing on the robust conclusions. The outputs of VIP Analysis may allow the decision process to progress, as the DMs learn about the model and the problem, postponing the elicitation questions they find difficult to answer, or prone to generate conflict. For instance, they may not agree on precise values for the scaling weights, but may agree on a ranking of those weights. Or they may merely agree on a partial ranking (e.g. $w_1 \geq w_2; w_1 \geq w_3$ and $w_3 \geq w_4$). In any case, VIP Analysis provides results that may allow to eliminate some alternatives or to highlight the most promising ones. Furthermore, the results can be used to direct the elicitation of further information with the purpose of progressively reducing the imprecision (e.g. "can alternative a_x really attain such a high value?", "can all DMs agree that a_x is worse than a_y?"). Very often, only a few constraints, for instance a ranking of the scaling weights (Salo and Hämäläinen 2001), (Dias and Clímaco 2000) suffice to clearly indicating one or two alternatives as being potentially the best ones. However, note there is little connection between the number of constraints and the reduction of the volume of W: in some cases a single constraint may drastically reduce W, and in other cases two or more constraints may hardly reduce its size. At the individual level, VIP Analysis may be a useful tool also in negotiation contexts, allowing each actor to study the problem and learn about his/her own preferences, both before and during negotiations.

3. VIP Analysis Extensions Addressing the Weights Space

The VIP Analysis software excludes settings where the DMs are spatially distributed. It requires group members to meet together around a computer and it implicitly requires an *a priori* aggregation of the different models that the DMs may have. It does not allow DMs to analyze the problem in private. For this reason, we have proposed an extension that we designated as VIP-G (Dias and Clímaco 2005), offering explicit aggregation tools for the K elements of the group, and allowing DMs to work privately and spatially distributed, while keeping the advantages mentioned above. Each group member can have his/her set of constraints on W defining his/her model in an imprecise manner. Nevertheless, along the interaction process, group members can agree to incorporate in their private models common constraints acceptable by all of them. Each DM can perform private analyses, which may be confronted with the aggregated results and/or conclusions of the group or part of it, considering or not tolerance parameters.

VIP-G maintains the "exploration" philosophy of VIP Analysis, not demanding precise information, and providing robust conclusions with wide support from the group (possibly with a tolerance). Moreover, the feedback each member receives concerning what can be accepted by the other members can facilitate issuing proposals (e.g. "can we drop a_7 from consideration?", "can we agree that $w_1 \geq w_2$?") and can encourage the convergence of opinions. So, the focus has been directed to the aggregation of results, emphasizing the sort of duality between shared results and shared conclusions, and the compromise between the extension of majorities in the group and the tolerance.

In VIP-G (Dias and Climaco 2005) introduced several possibilities for explicitly aggregating the preferences of the group. Performing the aggregation at the output level, i.e. of the results, is simpler and, since there is a many-to-one correspondence between parameter values and results (e.g., there are many combinations of parameter values yielding the same alternative as the best one), it is more likely to facilitate consensus. The "α-majority" aggregation operator for the results has been defined as:

$$R_{(\alpha)} = h_\alpha(R_1, \ldots, R_K) = \left\{ r \in \bigcup_{k=1}^{K} R_k : \frac{\#\{k \in \{1, \ldots, K\} : r \in R_k\}}{k} \geq \alpha \right\} (\alpha \in [1/K, 1]). \quad (7)$$

This means that a result r (a number) is considered acceptable (i.e. as belonging to $R_{(\alpha)}$) if at least $\alpha.\,K$ DMs include it in their result sets. As particular cases, we have $\alpha = 1$ (all DMs include it) and $\alpha = 1/K$ (at least one DM includes it):

$$h_1(R_1, \ldots, R_K) = \bigcap_{k=1}^{K} R_k \quad \text{and} \quad h_{1/K}(R_1, \ldots, R_K) = \bigcup_{k=1}^{K} R_k. \quad (8)$$

It may happen that $R_{(\alpha)}$ becomes void, which implies that $W_{(\alpha)}$ is also void ($W_{(\alpha)}$ denotes the set of weight vectors w that are considered acceptable by at least $K\alpha$ DMs). However, given the many-to-one correspondence between weights and results, there may exist situations where $W_{(\alpha)}$ is void but not $R_{((\alpha)}$. The purpose of computing the set $R_{(\alpha)}$ is not to impose a "consensus" set of results, but to provide some feedback to the individual DMs, who may confront their private results with the ones accepted by the group (all the members if $\alpha = 1$, at least one member if $\alpha = 1/K$, or some required majority level in between this extremes).

The conclusions that are drawn from $R_{(\alpha)}$ need some careful interpretation. For instance, consider a group composed of three DMs evaluating an alternative a_i. The first DM accepts a convex set of possible weight values (W_1), which (let us suppose) yield $\min\{V(a_i, w)$: $w \in W\} = 0,4$ and $\max\{V(a_i, w)$: $w \in W\} = 0,65$. From the convexity of W_1 and the linearity of $V(.)$, the interval of possible results corresponding to W_1 is $V(a_i) \in [0.4, 0.65]$. Let us further suppose that the set W_2 of possible input values accepted by the second DM yielded the interval $V(a_i) \in [0.5, 0.7]$, whereas the third DM reached the interval $V(a_i) \in [0.6, 0.75]$. In this case, $R_{(1)}$ would yield that $V(a_i) \in [0.6, 0.65]$ (i.e. values acceptable by all DMs), $R_{(2/3)}$ would yield that $V(a_i) \in [0.5, 0.7]$ (i.e. values acceptable by at least 2 out of 3 DMs), and $R_{(1/3)}$ would yield that $V(a_i) \in [0.4, 0.75]$ (i.e. values acceptable by at least one DM). Now, it is not reasonable to say that the group unanimously considers that $V(a_i)$ is not less than 0.6, just because the intersection $R_{(1)}$ yields that $V(a_i) \in [0.6, 0.65]$. Indeed, only the third DM supports that conclusion. What we could say (from $R_{(1/3)}$) is that the group unanimously agrees that $V(a_i)$ cannot be less than 0.4.

This approach to aggregating the outputs may hence be seen from another perspective while keeping the same idea. Let us consider C_k $(k = 1, \ldots K)$ as sets of individual conclusions (propositions) of the following types: $V(a_i) \geq x$; $V(a_i) \leq x$; $\text{Regret}_{\max}(a_i) \leq x$; and $V(a_i) - V(a_j) \leq x$.

The "α-majority" aggregation operator ($\alpha \in [1/K, 1]$) yields a set of conclusions acceptable by the group as follows:

$$C_{(\alpha)} = q_\alpha(C_1, \ldots, C_K) = \left\{ c \in \bigcup_{k=1}^{K} C_k : \frac{\#\{k \in \{1, \ldots, K\} : c \in C_k \vee (c' \Rightarrow c \wedge c' \in C_k)\}}{K} \geq \alpha \right\} \tag{9}$$

This means that a conclusion c (a proposition) is considered acceptable (i.e. as belonging to $C_{(\alpha)}$) if at least $\alpha \cdot K$ DMs reach the same conclusion (or a conclusion that implies it; e.g., the conclusion $V(a_i) \geq 0.8$ implies the conclusion $V(a_i) \geq 0.7$). As particular cases, we have $\alpha = 1$ (all DMs reach a conclusion) and $\alpha = 1/K$ (at least one DM reaches it).

There is a direct correspondence (a sort of duality) between sets $C_{(\alpha)}$ and $R_{(1-\alpha+1/K)}$: the consequences in the first are drawn from the results in the latter, and vice-versa. In particular, if we know from $R_{(1/K)}$ that for a given variable $v(.)$ all values in an interval [x, y] are acceptable by at least one DM, then we will know that the conclusions "$v(.)$ cannot be lower than x" and "$v(.)$ cannot be higher than y" are robust conclusions accepted by all the group (i.e. belong to $C_{(1)}$).

Concerning the tolerance, the idea proposed in VIP-G is finding out how much of a tolerance would each actor have to concede to have an agreement on each pairwise comparison. This tolerance was defined with respect to differences of global value. For instance, if the actors agree to a tolerance of 0.05 units of value, then they may accept that one alternative a_x is not worse than another alternative a_y, provided that $\min\{V(a_x) - V(a_y)$: $w \in W\} \geq -0.05$. In group decision situations, there is a trade-off between the accepted tolerance and the number of group members accepting a quasi-dominance conclusion. If we denote by $a_i \Delta_{\varepsilon(\alpha)} a_j$ the assertion "a_i quasi-dominates a_j with tolerance ε for a majority of α", then one may need a higher tolerance to obtain a wider majority supporting the conclusion. For instance, one may find all members of a group with 4 DMs agree that $a_i\ \Delta_{0.03(4/4)}\ a_j$, half of them agree that $a_i\ \Delta_{0.02(2/4)}\ a_j$; and one of them finds that $a_i\ \Delta_{(1/4)}\ a_j$, i.e. only one of them supports the conclusion that a_i dominates a_j without considering any tolerance (for an example see (Dias and Clímaco 2005)).

Although the same strategy may be followed in negotiation settings, where negotiating parties may seek to know the conclusions that are easier to reach an agreement on, we now propose to follow a similar strategy, but considering an analysis of the weights space rather than the results space. In earlier work (Tavares 2004), a model was proposed to support the process of search for a consensus when the actors have different additive value functions and so prefer different rankings of the alternatives. In the proposed model the weights space is used to provide useful insights to help the DMs adopting a consensus ranking, by computing a central vector of weights. A central vector of weights is also computed in (Contreras and Mármol 2004), based on a different approach (the lexicographical minimization of the maximum disagreement between the values assigned to the alternatives by the group members). Even earlier (e.g., (Kersten 1985)), distance minimization formulations were proposed, but in the offers space.

The idea of this paper is to consider tolerances defined with respect to the right-hand-sides (RHS) of the linear constraints that bound the acceptable weights defined by a polyhedron $W_k = \{w : A_k\, w \leq b_k\}$ for each actor ($k = 1,\ldots,K$), i.e. a relaxation of those constraints. To find the minimum relaxation τ on the RHS defining $W_1, \ldots, W_K$ that allows an alternative a_x to be potentially considered not worse than another alternative a_y, the following linear program (LP1) would have to be solved:

$$
\begin{aligned}
&\text{Min } \tau \\
&\text{s.t.} \\
&A_k\, w - \tau \leq b_k \ (k = 1, \ldots, K) \\
&V(a_x, w) - V(a_y, w) \geq 0
\end{aligned}
\qquad \text{(LP1)}
$$

The solution of (LP1) will indicate the critical constraints as regards the pair (a_x, a_y). These will be the constraints that are binding at the optimal solution. More important, if τ^* (the optimum value of (LP1)) is considered low, then the actors may agree that a_x can be better than a_y, when comparing this value with the optimum τ^* obtained for the reversed pair (a_y, a_x).

This strategy is analogous to the analysis proposed in (Dias and Clímaco 2005), but now considering the weights space. It requires, however, special attention to circumvent a problem related with the way constraints are coded: writing a constraint as $2w_1 - w_2 \geq 0$ will not yield the same results as writing the same constraint as $w_1 - 0.5\, w_2 \geq 0$. Hence, this strategy requires rules about the coding of constraints defining each $W_k (k = 1, \ldots, \mathrm{K})$, e.g., only accept constraints of type $w_s/w_t \in [L, U]$, or only accept constraints of type $V(a_y) \geq V(a_z)$.

We may also note that this LP amounts to minimize a Chebyshev distance (L_∞ norm). Of course, other norms could be used as a distance to minimize. The chosen distance, however, fits better the use of tolerances in VIP Analysis.

4. A Further Extension: Paths of Convergence

In this section we present another strategy to extend the analysis based on the weights space, in a way that overcomes the problem found in the previous approach. Consider two actors, actor 1 and actor 2. The idea is to find a path (a line segment) linking W_1 to W_2, in case they are disjoint. After computing this path, the actors may analyze which conclusions about the comparison of alternatives hold at each point. Analyzing a sequence of points from W_1 to W_2 (or vice-versa), they may discover that some conclusions hold only for points closer to W_1, other conclusions hold only for points closer to W_2, whereas there may exist other conclusions that hold along all the points in the path. Furthermore, this path will indicate to each actor a direction he/she could follow (i.e., the constraints that could be relaxed) in order to get closer to the other actor.

If $W_1 = \{w : A_{(1)}w \le b_{(1)}\}$ and $W_2 = \{w : A_{(2)}w \le b_{(2)}\}$, then to obtain a path from W_1 to W_2, one may proceed according to Process 1 or Process 2:

Process 1: To define two extreme points according to RHS tolerances

1. Solve the following LP (variables are vector w and scalar τ_2) to find a first point in the path, a point in W_1's border

$$\begin{aligned}&\text{Min } \tau_2\\ &\text{s.t.}\\ &A_{(1)}w \le b_{(1)}\\ &A_{(2)}w - \tau_2 \le b_{(2)}\\ &w \ge 0, \quad \tau_2 \ge 0\end{aligned} \tag{LP2}$$

If $\tau_2^* = 0$, then W_1 and W_2 intersect and the algorithm stops, since there is no interest in finding a path linking W_1 to W_2 in this case. Otherwise, $\tau_2^* > 0$, meaning that the intersection of acceptable weights $W_1 \cap W_2$ is empty. However, there would exist a common vector of weights w^* (i.e., the sets would intersect) if actor 2 relaxed his/her critical constraints by a tolerance of τ_2^*. Let $p^{(1)} = w^*$.

2. Solve (LP3) to find the last point in the path, a point in W_2's border:

$$\begin{aligned}&\text{Min } \tau_1\\ &\text{s.t.}\\ &A_{(1)}w - \tau_1 \le b_{(1)}\\ &A_{(2)}w \le b_{(2)}\\ &w \ge 0, \quad \tau_1 \ge 0\end{aligned} \tag{LP3}$$

τ_1^* must now be strictly positive. The solution of this LP states that there would exist a common set of weights w* if actor 1 relaxed his critical constraints by a tolerance of τ_1^*. Let $p^{(2)} = w^*$.

One property of this process is that no comparisons among the actor's tolerances are made. If the actors agreed to accept the midpoint of $\overline{p^{(1)}p^{(2)}}$ as a compromise weights vector, this would mean a concession that is equivalent for both of them, although actor 1 would be accepting a relaxation of $\tau_1^*/2$, whereas actor 2 would be accepting a relaxation of $\tau_2^*/2$. The resulting midpoint does not depend on the way the constraints are coded (multiplying both sides of an inequality by a positive constant would make no difference), although the values τ_1^* and τ_2^* might change.

Process 2: To define two extreme points by minimizing a L_1 distance

In this case, the idea is to find $p^{(1)} \in W_1$ and $p^{(2)} \in W_2$ such that the distance between these two points is as small as possible according to the L_1 metric. This may be achieved by solving the following LP (variables are vectors $w_{(1)}$ and $w_{(2)}$, plus the vectors x_p and x_n):

$$
\begin{aligned}
&\text{Min} \sum_{i=1}^{n} [x_p]_i + [x_n]_i \\
&\text{s.t.} \\
&A_{(1)} w_{(1)} \leq b_{(1)} \\
&A_{(2)} w_{(2)} \leq b_{(2)} \\
&w_{(1)} - w_{(2)} = x_p - x_n \qquad\qquad \text{(LP4)} \\
&w_{(1)}, \quad w_{(2)}, \quad x_p, \quad x_n \geq 0
\end{aligned}
$$

The solution of this LP yields the two points: $p^{(1)} = w^*_{(1)}$, $p^{(2)} = w^*_{(2)}$. This second process does not inform about the tolerances involved.

Independently of the process chosen, $\overline{p^{(1)}p^{(2)}}$ is a line segment defining a path from W_1 to W_2. The analysis of quasi-dominance and quasi-optimality results for the points in this path allows the DMs to see if there are any conclusions that they might easily agree on. If they agree on a conclusion, e.g., $V(a_1) \geq V(a_2)$, then they may incorporate this constraint in their models (i.e., in the definition of weights they consider acceptable – those making $V(a_1) \geq V(a_2)$). These points also show each DM how much of a tolerance they have to concede in order to approximate to another actor, and which constraints are involved. The actors may even exchange concessions, e.g., actor 1 accepts one conclusion that does not hold for W_1, but holds for a point near W_1, while actor 2 accepts one conclusion that does not hold for W_2, but holds for a point near W_2.

Another interesting analysis that can be made is that $\overline{p^{(1)}p^{(2)}}$ indicates the direction one actor should follow (i.e., the constraints that should be relaxed) in order to get closer to another actor. Actor 1, for instance, will know that he/she has to relax the constraints defining $p^{(1)}$ in order to reach $p^{(2)}$. He/she may also compare $p^{(1)}$ with $p^{(2)}$ (which represent weights vectors), to know which weights should increase or decrease in order to get closer to another actor. This may indicate which criteria are originating the conflict: what the actor 2 values that actor 1 does not value as much. This in turn may stimulate thinking about how to "enlarge the pie": actor 1 may try to devise new alternatives that improve current ones in the criteria both actors value most.

5. A Short Illustrative Example

Let us consider a small illustrative example, considering two actors and three evaluation criteria. The sets of weights acceptable by each actor are the following (Figure 1):

$$W_1 = \{(w_1, w_2, w_3) \in W : w_1/w_2 \in [2, 3], w_1/w_3 \in [4, 6]\} \quad \text{(Actor1)}$$
$$W_2 = \{(w_1, w_2, w_3) \in W : w_2/w_1 \in [1.2, 2], w_2/w_3 \in [2, 4]\} \quad \text{(Actor2)}$$
$$\text{with} \quad W = \{(w_1, w_2, w_3) \in \Re^3 : w_1, w_2, w_3 \geq 0; \quad w_1 + w_2 + w_3 = 1\}.$$

If Process 1 is followed to find $\overline{p^{(1)}p^{(2)}}$, then $p^{(1)} = (0.571, 0.286, 0.143)$ corresponds to a relaxation $\tau_2^* = 0.4$ by actor 2 (Figure 2), whereas $p^{(2)} = (0.357, 0.429, 0.214)$ corresponds to a relaxation $\tau_1^* = 0.5$ by actor 1 (Figure 3). The path $\overline{p^{(1)}p^{(2)}}$ is depicted in Figure 4. If Process 2 is followed instead, the path is the one depicted in Figure 5. The difference is that $p^{(2)} = (0.390, 0.467, 0.143)$. We may observe that the critical constraints for actor 1 are

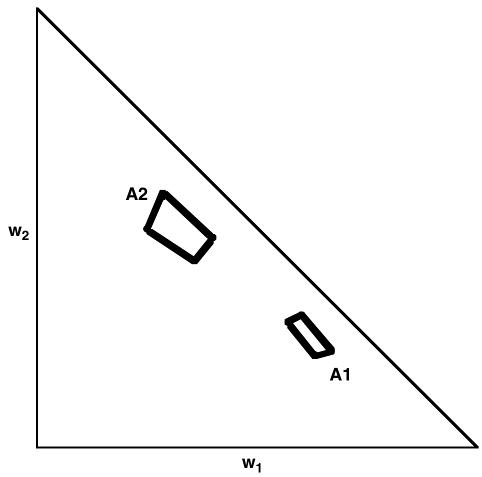

Figure 1. Acceptable weights for actor 1 (A1) and actor 2 (A2) viewed on a projection of the unit simplex *W* on axis w_1 and w_2.

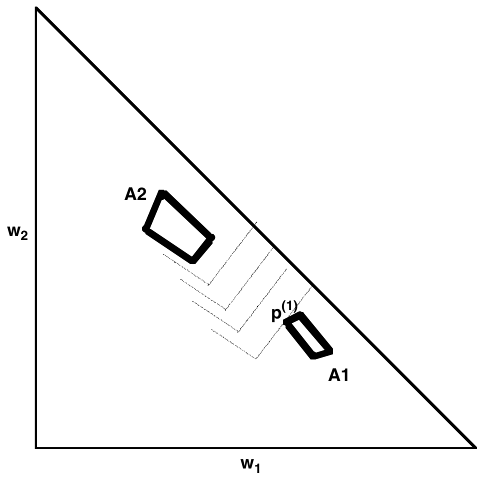

Figure 2. Computation of $p^{(1)}$.

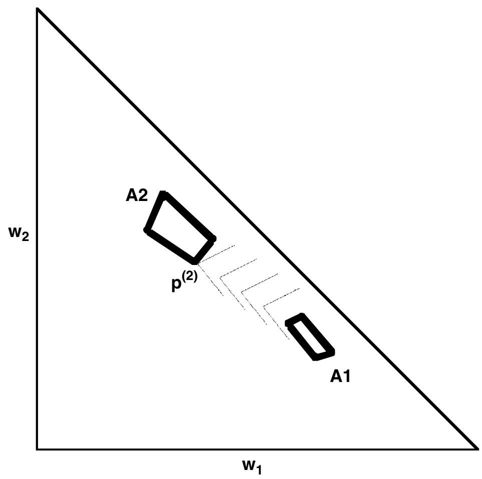

Figure 3. Computation of $p^{(2)}$.

$w_1/w_2 \geq 2$ and $w_1/w_3 \geq 4$ (i.e. the constraints imposing a high value for w_1). The critical constraints for actor 2 are $w_2/w_1 \geq 1.2$ and, for Process 1, also $w_2/w_3 \geq 2$.

Comparing the values of $p^{(1)}$ and $p^{(2)}$ obtained by Process 2, it is clear that the conflict lays mostly on the trade-off between the first and second criteria. To proceed towards an agreement, actor 1 should decrease w_1 and increase w_2, and actor 2 should do the contrary. Furthermore, given the low importance they seem to attach to w_3, they could try to devise agreements that perform well under the first two criteria, even if this means sacrificing the performance under the third criterion.

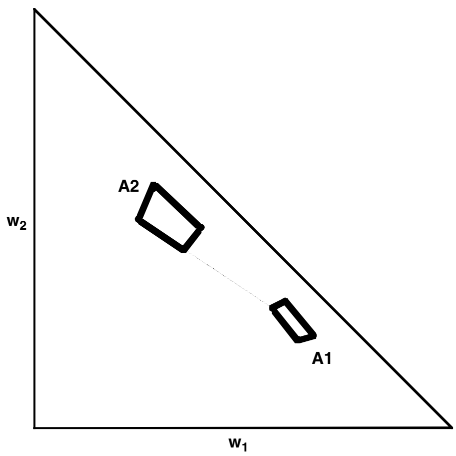

Figure 4. Path according to process 1.

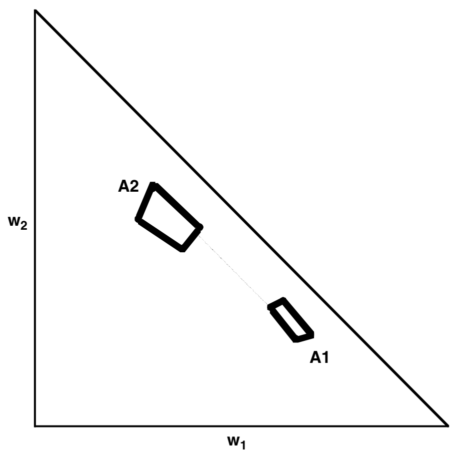

Figure 5. Path according to process 2.

6. Conclusion

This paper addressed the way in which VIP-G may evolve to support situations where, instead of cooperative group decision making, conflicting interests are present and voting is not an option. Therefore, negotiation is to be understood as meaning that all group members have to agree on a decision by using persuasion and making concessions, while trying to find common interests. The approaches proposed are not intended to support negotiations such as court settlements or buyer/seller, in which the parties not even agree on which criteria are to maximize or to minimize (e.g., the buyer wishes to minimize the price, while the seller wishes to maximize it).

Conflict lays in the evaluation of the relative importance of the criteria. With this in mind, we have proposed new procedures that consider explicitly the weights space, which can be added to the ones already proposed for VIP-G (the latter focusing on the results space). The main advantages of the procedures proposed is that they maintain the idea of tolerance introduced in VIP Analysis and in such a way that the tolerances of the different actors need not be compared. The idea of using paths of convergence also seemed adequate, since it does not choose any point in this path. In extending this approach to a multilateral setting, this may provide flexibility in the cases where the number of actors supporting a region in the weights space is much greater than the number of actors supporting a different region. For instance, if four DMs are in agreement to accept a region W_1 in the weights space, whereas a fifth DM supports a different (non-intersecting) region W_2, then taking the midpoint of a path between W_1 and W_2 might be hard to sustain. One important development for future research is indeed the extension of the tools proposed here to negotiation settings with many parties.

Future work is also planned to develop an Internet-based decision support system implementing the proposed extensions of VIP Analysis for group decisions and negotiation support. The ultimate goal is to build an easy to use system available for students, researchers and decision-makers around the world, similarly to Interneg/Aspire/Inspire (Kersten and Noronha 1999; Kersten and Lo 2003) or Decisionarium (Hämäläinen 2003).

In the envisaged VIP-G system, each DM uses a client application similar to the standalone VIP Analysis to perform private analyses, using the server to disclose and to gather information regarding the inputs he/she is considering and the results he/she is getting. The server's role encompasses the management of a database of the DM's inputs, which may be broadcasted to all the DMs (i.e. to the client applications). A communications module is responsible for all the communications between the user's working environment and the server. Communications may be structured as models-passing messages or results-passing messages. Other modules will perform aggregation of inputs (aggregation of models defined as constraints on weights) and aggregation of outputs (namely, finding robust conclusions taking into account the tolerance-acceptability tradeoff). Depending on the characteristics of the group and the decision problem, a user-driven approach may suffice, or a facilitator may be needed to establish deadlines and to encourage participation. The communications module will also allow unstructured communications (as in e-mail) to enable informal discussions both about the decision problem (e.g., to defend an opinion, or to propose the exclusion of an alternative) and the decision-making process (e.g., to establish a deadline, or to prompt others to discuss an open issue).

Acknowledgements

This research was supported by FCT/FEDER projects POSI/SRI/37346/2001 and POCI/EGE/58371/2004.

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