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RPA Equation Embedded into Infinite-Dimensional Fock Space F_∞*

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Abstract—To clear up both algebraic and geometric structures for integrable systems derived from self-consistent field theory, in particular, geometric aspect of the random-phase-approximation (RPA) equation is exhibited on the basis of the viewpoint of symmetry of the evolution equation. The RPA equation for an infinite-dimensional Grassmannian is constructed. © *2002 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

A conventional standard description of fermion many-body systems starts with the most basic approximation that is founded on the independent-particle picture, i.e., the self-consistent field (SCF) for the motion of fermions. Hartree–Fock (HF) theory is a typical one of such an approximation for ground states of fermion systems. Excited states are treated within the well-known random-phase approximation (RPA) if only a small fluctuation in the time-dependent HF (TDHF) mean field is taken into account around a stationary HF ground-state solution [1]. The TDHF equation is a nonlinear equation owing to its SCF character and may have no unique solution. A set of particle–hole-type pair operators of fermions with n single-particle states is closed under the Lie multiplication and forms a basis of a Lie algebra u_n [2]. The Lie algebra u_n of the pair operators generates a set of canonical transformations to a Slater determinant (S-det), i.e., the Thouless transformation [3], which induces a representation of the corresponding $U(n)$ group. It provides an exact generator coordinate representation of fermion state vectors. The RPA is a standard method for describing collective excitations in a fermion system with small quantum fluctuations.

In [4, 5], we studied the relation between TDHF theory [6] and the τ-functional method in soliton theory [7]. To go beyond a perturbative method with respect to periodic collective variables [8], we aimed at constructing TDHF theory on the associative affine Kac–Moody algebra along the soliton theory on the infinite-dimensional fermions. They are introduced through the Laurent expansion of finite-dimensional fermion operators with respect to degrees of freedom of fermions related to the mean-field potential. We attempted to embed the HF Lie algebra u_n into an infinite-dimensional Lie algebra gl_∞ with the aid of the Laurent expansion of fermion operators with respect to parameter z. Thus, the TDHF equation on the finite-dimensional Grassmannian Gr_m is embedded into the infinite-dimensional Grassmannian. We gave an expression for TDHF theory on the τ-functional space. We also showed that the TDHF equation on F_∞ under level one is nothing else but the Laurent expansion of the TDHF equation on Gr_m. The construction of the TDHF equation on F_∞ presents us explicit algebraic structures as a gauge theory inherent in SCF theory. From these facts, the SCF theory can be regarded as a method for determining self-consistently both quasiparticle energies and boson energies of collective motions which are unified into a gauge phase. Thus, we could obtain a common language, the infinite-dimensional Grassmannian and the Lie algebra, together with the associative affine Kac–Moody algebra.

2. GEOMETRIC ASPECT OF THE RPA EQUATION

First, we recapitulate the fundamental idea in a series of papers [9]. In viewing symmetries of time-evolution equations, let us consider an abstract evolution equation $\partial_t u(t) = K(u(t))$ for a function u depending only on time t. Suppose that there exists a transformation that converts a solution for u to another solution. Introducing a parameter s, we assume another kind of evolution equation with respect to s, i.e., $\partial_s u(t,s) = \bar{K}(u(t,s))$. Then, an integrability condition for the existence of the transformation is given by $\partial_s K(u(t,s)) = \partial_t \bar{K}(u(t,s))$.

In a differential-geometry approach to nonlinear problems, the above integrability condition is transcribed into zero curvature of the connection on the corresponding Lie groups of systems. Nonlinear evolution equations, e.g., famous soliton equations, such as KdV, KP, and sine/sinh-Gordon equations, originate from the well-known Lax equation [10],

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