

# Strange nucleon form factors: Solitonic approach to $G_M^s$ , $G_E^s$ , $\tilde{G}_A^p$ and $\tilde{G}_A^n$ and comparison with world data

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**Abstract.** We summarize the results of the chiral quark-soliton model ( $\chi$ QSM) concerning basically all form factors necessary to interpret the present data of the parity-violating electron scattering experiments SAMPLE, HAPPEX, A4 and G0. The results particularly focus on the recently measured asymmetries and the detailed data for various combinations of  $G_M^s$ ,  $G_E^s$ ,  $\tilde{G}_A^p$  and  $\tilde{G}_A^n$  at  $Q^2 = 0.1 \text{ GeV}^2$ . The calculations yield positive strange magnetic and electric form factors and a negative axial vector one, all being rather small. The results are very close to the combined experimental world data from parity-violating electron scattering and elastic  $\nu p$  and  $\bar{\nu} p$  scattering.

**PACS.** 12.40.-y Other models for strong interactions – 14.20.Dh Protons and neutrons

## 1 Introduction

The strange quark contribution to the distributions of charge and magnetization in the nucleon has been a very important issue well over decades, since it provides a vital clue in understanding the structure of the nucleon and in particular in probing the quark sea. There are some indications of about 4% contribution to the momentum sum rule of deep inelastic lepton scattering, of roughly 15% to the spin of the nucleon extracted from polarized deep inelastic scattering, or of up to 30% contribution to the mass of the nucleon, where all these numbers show rather large uncertainties. Recently, the strangeness content of the nucleon has been studied particularly intensively since parity-violating electron scattering (PVES) has demonstrated to provide an essential tool for probing the sea of  $s\bar{s}$  pairs in the vector channel [1]. In fact, various PVES experiments have been already conducted from which the strange vector form factors can be extracted [2–12]. The results from the SAMPLE, HAPPEX, PVA4, and G0 Collaborations have shown evidence for a non-vanishing strange quark contribution to the structure of the nucleon. In particular, evidence was found that the strange magnetic moment of the proton is positive [11], suggesting that the strange quarks reduce the proton's

magnetic moment. This is an unexpected and surprising finding, since a majority of theoretical studies favors a negative value. One of the models, which yield a positive strange magnetic moment of the proton, is the chiral quark soliton model ( $\chi$ QSM). It will be used in the present paper to investigate the form factors  $G_M^s$ ,  $G_E^s$ ,  $\tilde{G}_A^p$  and  $\tilde{G}_A^n$  and to compare them with world data.

Using the  $\chi$ QSM the present authors have recently investigated the set of six electromagnetic form factors ( $G_{E,M}^{u,d,s}$ ) and three axial-vector ones ( $G_A^{u,d,s}$ ) [13–16]. The results show a good agreement with the data of the SAMPLE, HAPPEX, A4 and G0 experiments. This includes parity-violating asymmetries (PVA) which have been measured by the G0 experiment over a range of momentum transfers in the forward direction [12]. We even predicted the PVAs of the future G0 experiment at backward angles [17]. In the present contribution we perform more detailed comparison including the most recent data of the HAPPEX experiment on He-4 and the results of the PVAs combined with elastic  $\nu p$  and  $\bar{\nu} p$  scattering.

## 2 Chiral quark soliton model

The  $\chi$ QSM has been used several times to calculate strange properties of the nucleon and of hyperons. It is

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an effective relativistic quark theory based on the instanton degrees of freedom of the QCD vacuum and has been derived from QCD in the large- $N_c$  limit. In the end it turns out to be the simplest possible quark theory which allows for spontaneously broken chiral symmetry. It results in an effective chiral action for valence and sea quarks both moving in a static self-consistent Goldstone background field [18–20]. For this model it is absolutely natural to have strange quark contributions to the nucleon. The  $\chi$ QSM has very successfully been applied to mass splittings of hyperons, to electromagnetic and axial-vector form factors [18] of the baryon octet and decuplet and to forward and generalized parton distributions of the nucleon. With one set of four parameters, unchanged for years, it reproduces all appropriate observables of light baryons with an accuracy of (10–30)%. This parameter set consists of an effective current mass for up- and down-quarks, a cut-off parameter in the relativistic proper-time regularization scheme, and a quark-pion coupling constant corresponding to a constituent mass. These parameters are fitted to the pion decay constant, the pion mass, and baryonic properties as proton charge radius and delta-nucleon mass splitting. In addition we assume an effective current strange quark mass of 180 MeV. A numerical iteration procedure yields then the self-consistent mean field whose lowest states get occupied until baryon number  $B = 1$  is reached. The resulting solitonic state is semiclassically rotated in space and iso-space in order to project on proper spin and hypercharge quantum numbers.

As far as strange form factors are concerned the formalism used in the present investigation can be found in the paper of Silva *et al.* [17] and references therein. We just mention for clarity some relations between form factors, which are often differently denoted in the literature. We refer to the papers of Musolf *et al.* [21], of Alberico *et al.* [22] and of Maas *et al.* [7]. Altogether we have:

$$\tilde{G}_A^p = -(1 + R_A^1)G_A^{(3)}(Q^2) + R_A^0 + G_A^s, \quad (1)$$

$$\tilde{G}_A^n = -3(1 + R_A^1)G_A^{(3)}(Q^2) + R_A^0 + G_A^s, \quad (2)$$

$$\tilde{G}_A^p = G_A^e = -\frac{1}{2}G_A^{NC} \quad (3)$$

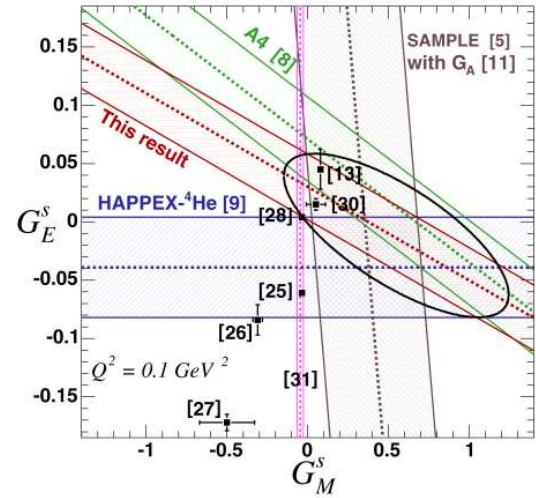
with the values for the electro-weak radiative corrections [21]:

$$R_A^1 = -0.41 \pm 0.24, \quad R_A^0 = 0.06 \pm 0.14. \quad (4)$$

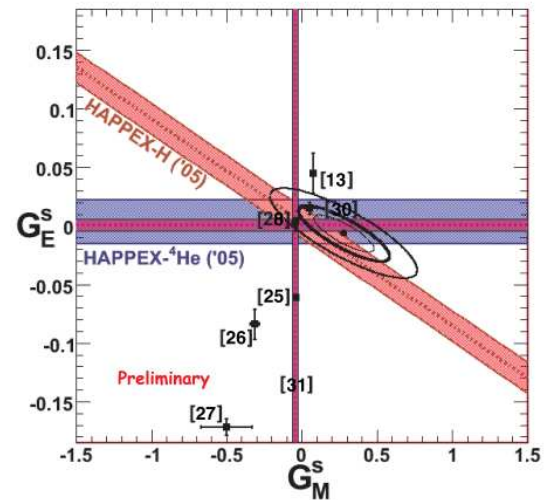
### 3 Electron and neutrino scattering data

The experimental situation is by far the best at  $Q^2 = 0.1 \text{ GeV}^2$ , where in addition to the usual linear combinations of electric and magnetic form factors the measurements of parity violation on He-4 allowed an extraction of  $G_E^s$ . The experimental results of the HAPPEX Collaboration are  $G_E^s = -0.038 \pm 0.042 \pm 0.010$  measured at  $Q^2 = 0.091 \text{ GeV}^2$  [10] and, more recently,  $G_E^s = -0.002 \pm 0.017$  at  $Q^2 = 0.1 \text{ GeV}^2$  [23].

Also the combined data  $G_E^s = -0.006 \pm 0.016$  [23] are consistent with zero. Experimental evidence from

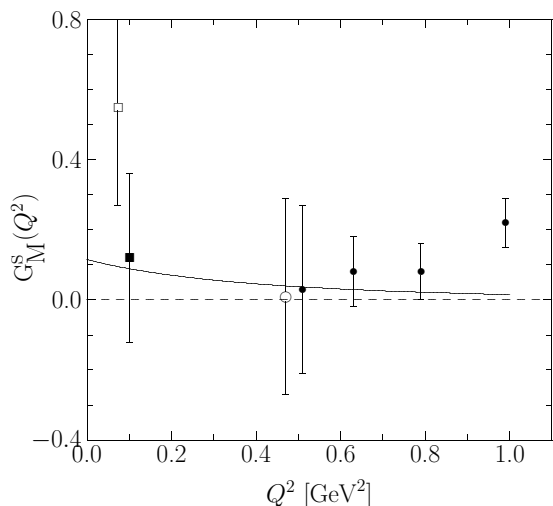


**Fig. 1.** The world data on the strange form factors  $G_M^s$  and  $G_E^s$  at  $Q^2 = 0.1 \text{ GeV}^2$  including the HAPPEX data on He-4 of 2004. The figure is taken from ref. [11]. The numbers indicate the references of theoretical calculations. The  $\chi$ QSM is given by [13].

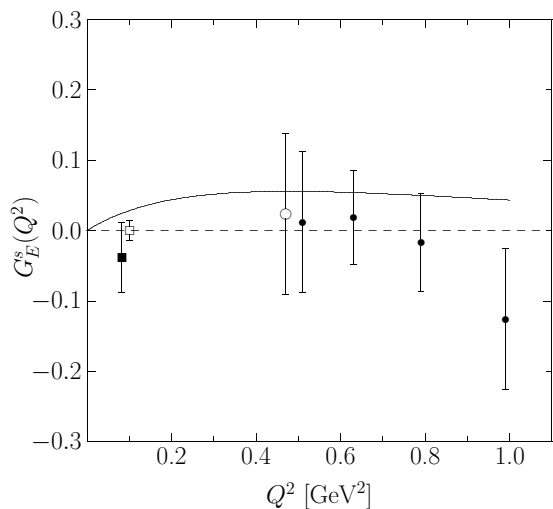


**Fig. 2.** The world data on the strange form factors  $G_M^s$  and  $G_E^s$  at  $Q^2 = 0.1 \text{ GeV}^2$  including the HAPPEX data on He-4 of 2004 and of 2005 (preliminary). The numbers indicate the references of theoretical calculations. The figure is taken from ref. [23]. The  $\chi$ QSM is given by [13].

the SAMPLE and HAPPEX Collaborations gives a positive value of the strange magnetic form factor  $G_M^s$  at  $Q^2 = 0.1 \text{ GeV}^2$  of  $G_M^s = 0.37 \pm 0.20 \pm 0.26 \pm 0.07$  [24],  $G_M^s = 0.55 \pm 0.28$  [11] and  $G_M^s = 0.12 \pm 0.24$  [23], respectively. The overall comparison of the  $\chi$ QSM calculation [13] with the world data and with other model calculations [25–29], and lattice gauge calculations [30,31] is given in fig. 1. If one adds the preliminary 2005 data of the HAPPEX-He-4 experiment the regions of confidence get smaller and one obtains fig. 2. Only those theoretical calculations are selected which are somehow in the vicinity of



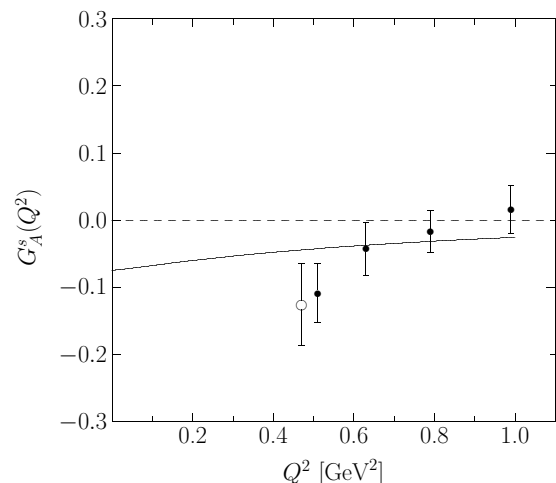
**Fig. 3.** The strange magnetic form factor  $G_M^s(Q^2)$  of the nucleon: The  $\chi$ QSM is compared with the analysis of Pate *et al.* [36] involving simultaneously parity-violating  $ep$  data and data from  $\nu$  and  $\bar{\nu}$  scattering. The open circle is from a combination of HAPPEX and E734 data, while the closed circles are from a combination of G0 and E734 data. The open square is from ref. [11] and involve parity-violating  $ep$  data only, similarly as the closed square from refs. [23] taken from [29].



**Fig. 4.** The strange electric form factor  $G_E^s(Q^2)$  of the nucleon: The  $\chi$ QSM is compared with the analysis of Pate *et al.* [36]. The open circle is from a combination of HAPPEX and E734 data, while the closed circles are from a combination of G0 and E734 data. The open square is from ref. [10] and involve parity-violating  $ep$  data only, similarly as the closed square from refs. [23] taken from [29].

the experimental data. Actually they do not give a consistent picture. This is also true for lattice-QCD calculations (LQCD). For example those of Lewis *et al.* [30] advocate a positive magnetic strange moment, whereas the recent results of Leinweber *et al.* [31–33], indicate a negative one.

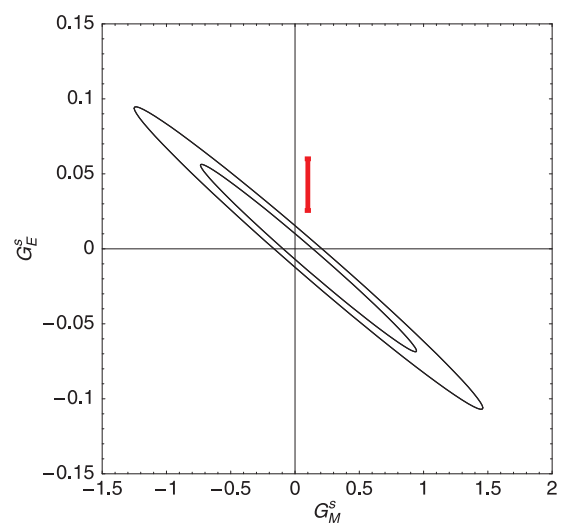
It is interesting to combine the data from PV electron scattering with the data from elastic  $\nu p$  and  $\bar{\nu} p$  scattering off protons [34], which provides independent information



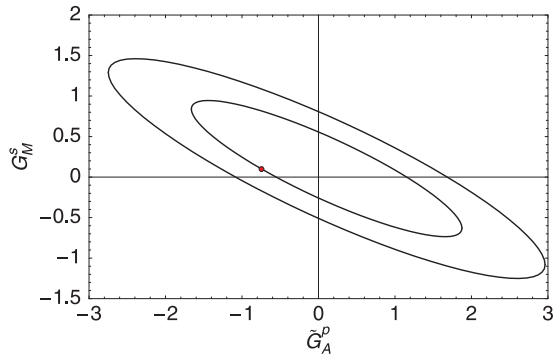
**Fig. 5.** The strange axial vector form factor  $G_A^s(Q^2)$  of the nucleon: The  $\chi$ QSM is compared with the analysis of Pate *et al.* [36]. The open circle is from a combination of HAPPEX and E734 data, while the closed circles are from a combination of G0 and E734 data.

on the strange form factors [35]. A comparison with the results of such an extraction [36] can be seen in figs. 3, 4 and 5. Apparently the  $\chi$ QSM is compatible with more or less all data available up to  $Q^2 = 1 \text{ GeV}^2$ , which is the range where the  $\chi$ QSM can provide form factors. It is not excluded that the experiments favor a negative  $G_E^s(Q^2)$  whereas the  $\chi$ QSM yields a positive one.

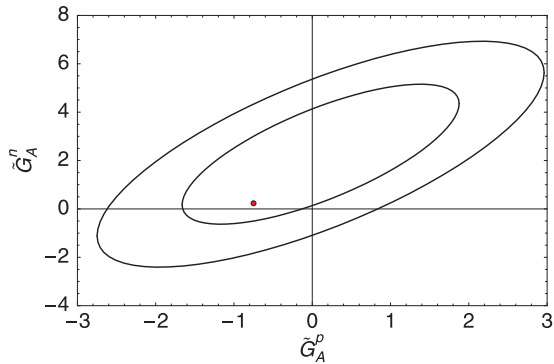
It is also interesting to compare the present calculations with the analysis of Young *et al.* in ref. [37]. These authors use systematic expansions of all the unknown form factors to simultaneously analyze the current data sets and extract the values at  $Q^2 = 1 \text{ GeV}^2$ , indepen-



**Fig. 6.** The contours display the 68% and 95% confidence intervals for the joint determination of  $G_M^s$  and  $G_E^s$  at  $Q^2 = 0.1 \text{ GeV}^2$ . The result of the  $\chi$ QSM is indicated. The ellipses originate from a theory-independent combined fit to all parity-violating data by Young *et al.* [37].



**Fig. 7.** The contours display the 68% and 95% confidence intervals for the joint determination of  $G_M^s$  and  $\tilde{G}_A^p$  at  $Q^2 = 0.1 \text{ GeV}^2$ . The result of the  $\chi$ QSM is indicated. The ellipses originate from a theory-independent combined fit to all parity-violating data by Young *et al.* [37].



**Fig. 8.** The contours display the 68% and 95% confidence intervals for the joint determination of  $\tilde{G}_A^n$  and  $\tilde{G}_A^p$  at  $Q^2 = 0.1 \text{ GeV}^2$ . The result of the  $\chi$ QSM is indicated. The ellipses originate from a theory-independent combined fit to all parity-violating data by Young *et al.* [37].

dent of theoretical input, except assuming the constraint of charge symmetry. Figure 6 shows this analysis for  $G_M^s$  and  $G_E^s$ , Figure 7 for  $G_M^s$  and  $\tilde{G}_A^p$  and Figure 8 for  $\tilde{G}_A^n$  and  $\tilde{G}_A^p$ . The error bar of the  $\chi$ QSM-result in fig. 6 is caused by a systematic error of the model in case of a purely strange observable. It originates from the inability of the  $\chi$ QSM to describe simultaneously mesonic tails with different Yukawa masses [13]. For quantities, which are not purely strange, this systematic error is usually negligible.

## 4 Summary

In the present theoretical work, we have investigated various form factors which are relevant for the analysis of parity-violating electron scattering experiments SAMPLE, HAPPEX, A4 and G0 and the scattering of  $\nu$  and  $\bar{\nu}$  scattering off nucleons. These form factors are  $G_M^s$ ,  $G_E^s$ ,  $\tilde{G}_A^n$  and  $\tilde{G}_A^p$ . We used for the study the electromagnetic and strange vector and axial vector form factors calculated in the chiral quark soliton model, yielding both small but positive magnetic and electric strange

form factors, see refs. [13–16]. All these  $\chi$ QSM form factors were obtained with one fixed set of four model parameters, which has been adjusted several years ago to basic mesonic and baryonic observables. As seen already in a previous paper of the present authors [17] the parity-violating asymmetries obtained in the present work are in a good agreement with the experimental data, which implies that the present model produces reasonable form factors of many different quantum numbers. We also predicted in that paper the parity-violating asymmetries for the future G0 experiment at backward angles. In the present paper we demonstrated that our theoretical numbers reproduce also form factors from a combined analysis of parity-violating electron scattering and  $\nu$  and  $\bar{\nu}$  scattering. Altogether, comparing the results of the  $\chi$ QSM with the overall world data one observes a remarkable agreement. It seems that the chiral quark soliton model, which has been applied over several years with an accuracy of (10–30)% to many observables of baryons in the octet and decuplet, is formulated in terms of proper effective degrees of freedom. The model is in fact the simplest quark model which describes spontaneous breaking of chiral symmetry. It is based on the  $N \rightarrow \infty$  expansion of the QCD and appears to describe the properties of the light baryons reasonably well. Perhaps one can learn from the present comparison of the  $\chi$ QSM with experiment that the degree of freedom of spontaneous breaking of chiral symmetry governs not only the up and down sector of the nucleon but also its strange quark content.

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