European Journal of Operational Research 139 (2002) 317–326

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/dsw

Decision Aiding

Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure

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Received 28 February 1999; accepted 2 May 2001

Abstract

In a decision aiding context, knowing the preferences of the Decision Maker (DM) and determining weights of criteria are very hard questions. Several methods can be used to give an appropriate value to the weights of criteria. J. Simos proposed a very simple procedure, using a set of cards, allowing to determine indirectly numerical values for weights. The purpose of this paper is first to explain why the above method needs to be revised, and second, the revised version we propose. This new version takes into account a new kind of information from the DM and changes certain computing rules of the former method. A software has been implemented based on the revised Simos' procedure whose main features are presented in this paper. The new method has been applied to different real-life cases (public transportation problems, water resources problems, environment problems, etc); it proved to be successful.

Keywords: Multicriteria decision aiding (MCDA); Weights of criteria; ELECTRE type methods

1. Introduction

Simos (1990a,b) proposed a technique allowing any DM (not necessarily familiarized with multicriteria decision aiding) to think about and express the way in which he wishes to hierarchise the different criteria of a family F in a given context. This procedure also aims to communicate to the analyst the information he needs in order to attribute a numerical value to the weights of each criterion of F, when they are used in an ELECTRE type method (Roy and Mousseau, 1996; Roy and Bouyssou, 1993). The procedure has been applied to different real-life contexts; it proved to be very well accepted by DMs and we believe that the information obtained by this procedure is very significant from the DM's preference point of view. However, the way Simos recommends to process the information needs a revision for two main reasons:

1. It is based on an unrealistic assumption. This occurs by the lack of an essential information (as it was already underlined by Schärlig (1996)).

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0377-2217/02/$ - see front matter
PII: S0377-2217(01)00370-8

2. It leads to process criteria having the same importance (i.e., the same weight) in a not robust way.

The revision proposed about those aspects has been applied successfully in various institutions such as Agence de l'Environnement et de la Maîtrise de l'Energie (Ah Yan Chung Food Yan, 1996), Institut National Environnement Industriel et Risques (Merad, 2000), Institut National de la Recherche Agronomique (Arondel, 1996), Agence de l'Eau Loire-Bretagne, Délégation du Mans (Arondel, 2000) and Régie Autonome des Transports Parisiens (Denieul, 1996). Several consulting companies (Bureau d'Aide à la Décision, Lausanne, Institut Technique des Céréales et des Fourrages, Boigneville, Cabinet d'Etudes Adage-Environnement, Paris) use this model, but their work did not give rise to publication.

This procedure is very well adapted to the contexts where, for different reasons (multiple decision makers, robustness analyses, etc.) we need to bring to the fore not only one but several sets of weights. In such cases, it is important to exploit the information quickly under different assumptions.

In Section 2, we will show how to collect and to exploit the information in order to attribute a numerical value to the weights of each criterion by using Simos' procedure. We will complete this section pointing out why Simos' procedure needs to be revised. In Section 3, we will present a revised version of Simos' procedure. The new procedure is different from the former method by three main aspects: (1) collecting a new kind of information; (2) processing the information in order to obtain the normalized weights; (3) using a new technique that normalizes the weights minimizing the rounding off errors.

Finally, Section 4 will be devoted to the SRF software which will be illustrated by a small example. We point out the possibility of SRF to work in multiple users contexts and to lead to a robustness analysis (Roy, 1998).

Let us underline that, like the former method, the revised version aims to attribute an intrinsic weight to each criterion (as it is the case in any aggregation procedure of ELECTRE type methods), i.e., which does not depend either on the range of the scale or on the encoding (in particular the unit selected) to express the evaluation (score) on this scale. Real-life applications show that when one asks the DM what importance he wishes to assign to each criterion, he/she expresses his/her preferences spontaneously without knowing neither the range of the scale nor the procedure used to encode this scale. Nevertheless, several aggregation procedures, in particular MAUT (Keeney and Raiffa, 1993) and the weighted sum, use weights not having this intrinsic characteristic. In such case, the fact that the weight of a given criterion is greater than the weight of another one is not significant at all because some changes in the range or in the unit of the scale can reverse the positions of these two criteria.

In order to obtain a relevant information (output) in the context of such procedures, it is crucial that the output takes into account the nature and the encoding (as it was mentioned by Keeney and Raiffa). Nevertheless this is not always the case. For instance, the Analytic Hierarchy Process (AHP) by Saaty (1980, 1984) asks the user to think about and to express his/her preferences without reference to the range or the encoding of the criteria scales (like in the Simos' method). However, the weights thus obtained are used in a weighted sum aggregation technique. In these cases, we are afraid that the way how we exploit the output is not coherent with it means. For more details about the question on how to attribute numerical values to the parameters which must reflect the relative importance of criteria, see Mousseau (1993, 1995), Roy and Mousseau (1996), and Bana e Costa and Vansnick (1994, 1997). Let us also to mention the software DIVAPIME (by Vincent Mousseau, LAMSADE) and MACBETH (by Carlos Bana e Costa, SAEG-IST and Jean-Marie De Corte and Jean-Claude Vansnick, University of Mons-Hainaut).

2. The procedure proposed by Simos

The main innovation in this approach consists of associating a "playing card" with each criterion. The fact that the person being tested has to handle the cards in order to ranking them, inserting the white ones, allows a rather intuitive understanding of the aim of this procedure. After explaining how to collect the information (Section 2.1), we recall the way Simos suggests to make use of it (Section 2.2). Finally we show (Section 2.3) why in our opinion his technique is not a satisfactory one.

2.1. Collecting the information

The technique used to collect information consists of the following three steps:

1. We give to the person being tested (the user) a set of cards: the name of each criterion is written on each card together with some other (complementary) information, if necessary. Therefore, we have n cards, n being the number of criteria of a family F. These cards should exhibit no number what-so-ever in order not to induce the answers. We also give a set of white cards with the same size. The number of the latter will depend on the user's needs.

2. We ask the user to rank these cards (or criteria) from the least important to the most important. So, the user will rank in ascending order according to the importance he wants to ascribe to the criteria: the first criterion in the ranking is the least important and the last criterion in the ranking is the most important. According to the user's point of view, if some criteria have the same importance (i.e., the same weight), he should build a subset of cards holding them together with a clip or a rubber band. Other ways of ranking the cards may be used (e.g., purely displaying them flat on a table) being it a simple matter of preference of the user. Consequently, we obtain a *complete pre-order* on the whole of the n criteria. Let $\bar{n}$ be the number of ranks of this pre-order (most of these ranks being reduced to one card only, i.e., to one criterion). The first rank is named *Rank 1*, the second one *Rank 2*, and so on.

3. We ask the user to think about the fact that the importance of two successive criteria (or two successive subsets of *ex aequo* criteria) in the ranking can be more or less close. The determination of the weights must take into account this smaller or bigger difference in the importance of successive criteria. So, we ask him/her to introduce white cards between two successive cards (or subsets of *ex aequo* cards). The greater the difference between the mentioned weights of the criteria (or the subsets of *ex aequo* criteria), the greater the number of white cards:

- *No white card* means that the criteria have not the same weight and that the difference between the weights can be chosen as the unit for measuring the intervals between weights. Let u denotes this unit.
- *One white card* means a difference of *two times* u.
- *Two white cards* mean a difference of *three times* u, etc.

2.2. Determining the weights of criteria with a Simos' procedure

The way Simos proposes to process the information collected in order to attribute numerical values to the weights of criteria is presented in Maystre et al. (1994) using an example reproduced hereafter.

Let us consider a family F with 12 criteria:

$$F = \{a, b, c, d, e, f, g, h, i, j, k, l\}.$$

Let us suppose that the user groups together the cards associated to the criteria having the same importance (i.e., the same weight) into six different subsets of *ex aequo*. Table 1 shows the ranking of these cards taking into account the number of white cards inserted between two successive subsets of *ex aequo*.

In order to convert the ranks into weights, Simos proposes the following algorithm:

1. Ranking the subsets of *ex aequo* from the least good to the best according to the white cards.

Table 1
Presentation of the information given by the set of cards

Rank	Subset of *ex aequo*	Number of cards according to the rank
1	$\{c, g, l\}$	3
2	$\{d\}$	1
3	White card	1
4	$\{b, f, i, j\}$	4
5	$\{e\}$	1
6	$\{a, h\}$	2
7	$\{k\}$	1

Source: Maystre et al. (1994, Table 11.7, p. 175).

2. Attributing a *position* (called *weight* by Simos) to each criterion and to each white card: the least qualified card receives *Position 1*, the next one *Position 2*, and so on.
3. Determining the *non-normalized weight* (called *average weight* by Simos) of each rank by dividing the sum of the positions of this rank by the total number of criteria belonging to this rank.
4. Determining the *normalized weight* (called *relative weight* by Simos) of each criterion by dividing the non-normalized weight of the rank by the total sum of the positions of the criteria (without taking into account the white cards). Note that the normalized weights are written with no decimals. The technique consists of rounding off to the lower or higher nearest integer value.

2.3. The main objections to the way Simos' procedure determines numerical values for weights

2.3.1. Excluding certain subsets of weights

Let us consider, for example, four criteria a, b, c and d. If the user ranks these criteria in the following order: a, b, c, d, and if he/she considers that the difference of importance between a and b, b and c, c and d is the same, then it makes no sense to insert some white cards between two successive criteria. Consequently, the weight of the criterion d must be four times greater than the weight of the criterion a. This ratio is determined automatically by the procedure without taking into account the user's preference point of view. However, other sets of weights like 3, 4, 5, 6 or 6, 7, 8, 9 could fit better his/her opinion about the relative importance of criteria. Such sets of weights cannot be obtained by the Simos' procedure.

Let us suppose that the user wishes to distinguish the differences between weights in such a way that each difference is always greater than the previous one. Thus, he could insert two white cards between a and b, three white cards between b and c and four white cards between c and d. We obtain the following weights: 1, 4, 8, 13. Consequently, the criterion d is 13 times more important than the criterion a. A set of weights like for example 10, 12, 15, 19 may be more suitable to the user's wishes, but this set of numerical values cannot be obtained when we process the information by using Simos' procedure.

The procedure recommended to convert the ranks into weights limits the set of the feasible weights because it determines automatically the ratio between the weight of the most important criterion and the weight of the least important one in the ranking. Thus, if there is no *ex aequo* cards in the first and last ranks, the ratio is equal to the total number of cards, T. But, if we have a subset with q most important criteria and a subset with p least important criteria in the ranking, we obtain the following ratio:

$$z = \frac{\left(\sum_{i=0}^{q-1} (T-i)\right)p}{\left(\sum_{i=0}^{p-1} (1+i)\right)q}.$$

In the example presented in Table 2, $T = 13$, $q = 1$, $p = 3$, which leads to the ratio $z = \frac{13}{2} = 6.5$. The person being tested is not obviously conscious of this computing rule which can have certain unattended effects. In the ELECTRE type methods the weights must be interpreted as the number of voices (votes) assigned to each criterion. Those votes can be added together to determine the weight of a coalition. The ratio between the maximum number of votes and the minimum number of votes ascribed to the different criteria is a kind of information we obtained automatically from the set of cards when built in this way. This drawback excludes many realistic sets of weights.

2.3.2. Processing the subsets of ex aequo

Let us go back to the example of Table 2. Now, let us suppose that c is the single least important criterion, and g and l are the *ex aequo* criteria in rank 2. The weight of c is reduced from 2 to 1, but the weights of g and l are increased from 2 to 2.5. The remaining weights do not change, and we can ponder about the foundations of this increase in the weights of g and l. If we take c and g as the least important criteria, and l and d as a subset of *ex aequo*, then we obtain 1.5 for c and g, and 3.5 for l and d. Like in the former case, in the new ranking the remaining weights do not change and we are surprised because the weight of the criterion d is reduced from 4 to 3.5. This fact occurs because

Table 2
Converting the ranks into weights by using Simos' procedure

Subsets of *ex aequo*	Number of cards	Positions	Non-normalized weights	Normalized weights	Total
$\{c, g, l\}$	3	1, 2, 3	$\frac{1+2+3}{3} = 2$	$\frac{2}{86} \times 100 \approx 2.326 \to 2$	$3 \times 2 = 6$
$\{d\}$	1	4	4	$\frac{4}{86} \times 100 \approx 4.651 \to 5$	$1 \times 5 = 5$
White	1	(5)	...	...	...
$\{b, f, i, j\}$	4	6, 7, 8, 9	$\frac{6+7+8+9}{4} = 7.5$	$\frac{7.5}{86} \times 100 \approx 8.721 \to 9$	$4 \times 9 = 36$
$\{e\}$	1	10	10	$\frac{10}{86} \times 100 \approx 11.628 \to 12$	$1 \times 12 = 12$
$\{a, h\}$	2	11, 12	$\frac{11+12}{2} = 11.5$	$\frac{11.5}{86} \times 100 \approx 13.372 \to 13$	$2 \times 13 = 26$
$\{k\}$	1	13	13	$\frac{13}{86} \times 100 \approx 15.116 \to 15$	$1 \times 15 = 15$
Sum	13	86∗	...	...	100

The numerical values presented in this table were taken from Maystre et al. (1994, Table 11.7, p. 175). We have slightly changed the titles to avoid ambiguities of meaning. In this table the symbol "right arrow" (→) means the conversion of the normalized weight of each criterion to the rounded value, and the symbol "star" (∗) means that the sum does not include the positions in brackets.

l, which in a first ranking was *ex aequo* together with c and g, is now *ex aequo* with d.

In Table 2, we can easily understand the previous drawback. If we consider two successive criteria (or subsets of *ex aequo* criteria) in the set of cards and if there is no white card between them, then the difference of weights assigned to these criteria is not constant. We can observe a difference of 2 between c, g, l and d, 2.5 between b, f, i, j and e, and only 1.5 between e and a, h. This fact occurs because the difference of weights between two successive subsets of criteria is automatically influenced by the existence of the *ex aequo* cards in these successive subsets. The user have not a real or absolute perception of the way in which the numerical values are determined by the procedure. Moreover, the technique that Simos proposes to process the subsets of *ex aequo* allows to determine the ratio z by the formula (cf. Section 2.3.1):

$$z = \frac{\left(\sum_{i=0}^{q-1} (T - i)\right)p}{\left(\sum_{i=0}^{p-1} (1 + i)\right)q}$$

thus giving p and q an importance that seems to be insufficiently founded and completely uncontrolled by the user.

2.3.3. The technique of rounding off to the next integer value

In real-life decision aiding contexts, when the normalized weights are determined, it is very difficult for the DM to accept Simos' procedure because, in many cases, the sum of all normalized weights does not correspond to 100, as we can see in Table 3. It should be noted that it is very easy to construct other data sets where a positive or negative difference in relation to 100 can reach several units.

Table 3
Converting the ranks into weights for a subfamily F' of F

Subsets of *ex aequo*	Number of cards	Positions	Non-normalized weights	Normalized weights	Total
$\{c, g, l\}$	3	1, 2, 3	$\frac{1+2+3}{3} = 2$	$\frac{2}{40} \times 100 \approx 5.000 \to 5$	$3 \times 5 = 15$
$\{d\}$	1	4	4	$\frac{4}{40} \times 100 \approx 10.000 \to 10$	$1 \times 10 = 10$
Blank	1	(5)	...	...	...
$\{b, f, i, j\}$	4	6, 7, 8, 9	$\frac{6+7+8+9}{4} = 7.5$	$\frac{7.5}{40} \times 100 \approx 18.750 \to 19$	$4 \times 19 = 76$
Sum	9	40∗	...	...	101

The numerical values presented in this table correspond to the four first lines of Table 2. In this table the symbol "right arrow" (→) means the conversion of the normalized weight of each criterion to the rounded value, and the symbol "star" (∗) means that the sum does not include the positions in brackets.

3. The revised procedure

The revised version of Simos' procedure takes into account a new kind of additional information from the decision maker and changes certain computing rules of the former method. The new kind of additional information concerns the ratio between the weights of the most important criterion and the least important one in the ranking.

3.1. Outline of the new procedure

The revised Simos' procedure uses the data collection method described in Section 2.1. In several cases, it seems very well adapted. Generally, it is very easy for the user to express his/her preferences as an ordering of criteria. It can happen that he assigns directly a numerical value to each criterion. Unfortunately, those values are not easily interpretable in terms of weights. This information collection procedure is simple and fast. Thus, it is well fitted for decision aiding contexts with multiple DMs.

In order to overcome the drawback presented in Section 2.3.1, we introduce a new kind of information asking the user to state *how many times the last criterion is more important than the first one in the ranking*. Let z be the value of this ratio. This ratio is not usually very well defined from the user's point of view. So, it is very important to analyze, in an easy way, the effect on the output of the changes in z.

The algorithm presented in the following subsections:

- takes into account an additional kind of information concerning the value of z (this value was an implicit assumption of the former method, but it will be now eliminated);
- eliminates the misprocessing of the subsets of *ex aequo* of the former method;
- processes the rounding off of the numerical values in an optimal way as it will be presented in Section 3.3.

3.2. The algorithm

The algorithm must attribute a numerical value to the weights of each criterion g_i for $i = 1, \dots, n$. It must determine successively:

(a) The *non-normalized weights* $k(1), \dots, k(r), \dots, k(\bar{n})$ associated to each subset of *ex aequo* according to its rank, setting (by convention) $k(1) = 1$ (i.e., assigning 1 to the least important criterion or subset of *ex aequo* criteria). This convention is not a restrictive one because the output of the ELECTRE type methods does not change when we multiply the weights of criteria by a constant.

(b) The *normalized weights* which require other convention (usually, more intelligible and favorable to comparisons): $\sum_{i=1}^{n} k_i = 100$; k_i denotes the normalized weight of each criterion g_i for $i = 1, \dots, n$.

3.2.1. Determining the non-normalized weights $k(r)$

Let e'_r be the number of white cards between the ranks r and $r + 1$. Set

$$\begin{cases} e_r = e'_r + 1 \quad \forall r = 1, \dots, \bar{n} - 1, \\ e = \sum_{r=1}^{\bar{n}-1} e_r, \\ u = \frac{z-1}{e} \end{cases}$$

(for u, retain six decimal places). We obtain

$$k(r) = 1 + u(e_0 + \cdots + e_{r-1}) \quad \text{with } e_0 = 0$$

(for these weights, retain only two decimal places by using the rounding off technique to the nearest lower or upper value). If there exist several criteria *ex aequo* in the rank r, then all of those criteria must have the same weight $k(r)$. Table 4 shows the output concerning the data given by Table 1, for $z = 6.5$, which gives $u = 0.916666$ since $e = 6$.

3.2.2. Determining the normalized weights k_i

Let g_i be a criterion of rank r and k'_i be the weight of this criterion in its non-normalized expression $k'_i = k(r)$. Set

$$\begin{cases} K' = \sum_{i=1}^{n} k'_i, \\ k^*_i = \frac{100}{K'} k'_i. \end{cases}$$

Let us derive k''_i from k^*_i by deleting some of its decimal figures. We will consider three options characterized by w as follows:

Table 4
Non-normalized weights for $z = 6.5$

Rank r	Criteria in the rank r	Number of white cards according to rank r, e'_r	e_r	Non-normalized weights $k(r)$	Total
1	$\{c, g, l\}$	0	1	1.00	$1.00 \times 3 = 3.00$
2	$\{d\}$	1	2	1.92	$1.92 \times 1 = 1.92$
3	$\{b, f, i, j\}$	0	1	3.75	$3.75 \times 4 = 15.00$
4	$\{e\}$	0	1	4.67	$4.67 \times 1 = 4.67$
5	$\{a, h\}$	0	1	5.58	$5.58 \times 2 = 11.16$
6	$\{k\}$	...	...	6.50	$6.50 \times 1 = 6.50$
Sum	12	1	6	...	42.25

$$\begin{cases} w = 0: & \text{takes into account no figures after the decimal point;} \\ w = 1: & \text{take into account only one figure after the decimal point;} \\ w = 2: & \text{take into account only two figures after the decimal point.} \end{cases}$$

By using this rounding off technique, we obtain the following result:

$$\begin{cases} K'' = \sum_{i=1}^{n} k''_i \leqslant 100, \\ \epsilon = 100 - K'' \leqslant 10^{-w} \times n. \end{cases}$$

In fact, the value $v = 10^w \times \epsilon$ is an integer at most equal to n. Now, if we set $k_i = k''_i + 10^{-w}$ for v criteria suitably selected and k''_i for the other $n - v$ criteria, we have $\sum_{i=1}^{n} k_i = 100$ with the normalized weights k_i showing the required number of decimal places (which was our objective).

In order to set a minimum distortion of the weights, the choice of the v criteria which we must add 10^{-w} is carried out by the following algorithm:

1. Determining, for each criterion g_i, the ratios:

$$d_i = \frac{10^{-w} - (k^*_i - k''_i)}{k^*_i} \quad \text{and} \quad \bar{d}_i = \frac{(k^*_i - k''_i)}{k^*_i},$$

where $k^*_i = 100 \times k'_i / K'$ and k''_i is determined from k^*_i retaining only the first w decimal places ($w = 0, 1, 2$). Let us remark that the ratio d_i represents the dysfunction concerning the relative error rounded upwards to the nearest whole number, and the ratio $\bar{d}_i$ represents the dysfunction concerning the relative error rounded downwards to the nearest whole number.

2. Creating two lists L and $\bar{L}$ defined as follows:
- List L is built by the pairs (i, d_i) ranked according to the increasing values of the ratio d_i.
- List $\bar{L}$ is built by the pairs $(i, \bar{d}_i)$ ranked according to the decreasing values of the ratio $\bar{d}_i$.

Set $M = \{i / d_i > \bar{d}_i\}$, $|M| = m$.

3. Partitioning the n criteria of F into two subsets F^+ and F^- where $|F^+| = v$ and $|F^-| = n - v$. The criteria of F^+ will be rounded upwards to the nearest whole number and the criteria of F^- will be rounded downwards to the nearest whole number. The partition of F is carried out as follows:

(a) If $m + v \leqslant n$, then construct F^- with the m criteria of M plus the $n - v - m$ last criteria of $\bar{L}$ not belonging to M. So, the list F^+ will be built by the first v criteria of $\bar{L}$ not belonging to M.

(b) If $m + v > n$, then construct F^+ with the $n - m$ criteria of L not belonging to M plus the $v + m - n$ first criteria of L not belonging to M. So, the list F^- will be built by the $n - v$ last criteria of L not belonging to M.

However, we have just two small difficulties:
- Sometimes, the increase of 10^{-w} should be done only for certain criteria of a given subset of *ex aequo* but not all. In such case and since the choice of the criteria to which we must add 10^{-w} is arbitrary, we select the criteria g_i with the higher subscripts i.
- The procedure can round downwards to zero the weights of the criteria belonging to the first ranks. The software presented in Section 4 overcomes this drawback by attributing 10^{-w} to the normalized weights $k''_i < 10^{-w}$, consequently up-

Table 5
Determining the normalized weights of each criterion for $w = 1$ and $z = 6.5$

Rank	Criteria	N.	Normalized weights k_i^*	Normalized weights k_i''	Ratio d_i	Ratio $\bar{d}_i$	Normalized weights k_i
1	*c*	3	2.366863905	2.3	0.014000000	0.028250000	2.4(3)
1	*g*	7	2.366863905	2.3	0.014000000	0.028250000	2.4(2)
1	*l*	12	2.366863905	2.3	0.014000000	0.028250000	2.4(1)
2	*d*	4	4.544378698	4.5	0.012239583	0.009765625	4.5
3	*b*	2	8.875739645	8.8	0.002733333	0.008533333	8.9(7)
3	*f*	6	8.875739645	8.8	0.002733333	0.008533333	8.9(6)
3	*i*	9	8.875739645	8.8	0.002733333	0.008533333	8.9(5)
3	*j*	10	8.875739645	8.8	0.002733333	0.008533333	8.9(4)
4	*e*	5	11.053254438	11.0	0.004229122	0.004817987	11.0
5	*a*	1	13.207100592	13.2	0.007034050	0.000537634	13.2
5	*h*	8	13.207100592	13.2	0.007034050	0.000537634	13.2
6	*k*	11	15.384615385	15.3	0.001000000	0.005500000	15.3
Sum	12	...	100	99.3	...	...	100

The third column of this table named "N." allows to identify the *Number of each criterion*. In the last column entitled "Normalized weights k_i", the numbers in brackets are used to identify all the criteria belonging to the list F^+ defined hereafter.

dating v. Nevertheless, this supposes that v does not become negative, which must normally be the case.

Table 5 shows the normalized weights of each criterion for $w = 1$. In Roy and Figueira (1998), we can find the weights concerning the options $w = 0$ and $w = 2$ as well as the output related to other data sets.

Table 6 presents the lists L and $\bar{L}$.

The criteria of M are marked by (†). The list F^+ is thus the following:

$$F^+ \leftarrow \langle 12, 7, 3, 10, 9, 6, 2 \rangle.$$

3.3. Justification of the new computational rules

The information that the set of cards has got to collect and transmit is, on the one hand, based on the ranking (order, equality) of the weights and, on the other hand, on the ratio of the differences among the consecutive weights. These differences must be proportional to the number of intervals which separate these two subsets of consecutive cards (the subsets are usually reduced to one card). The numbers $k(r)$ obviously reflect exactly the pre-order materialized by the set of cards. Through the formulas of Section 3.2.1 one can immediately deduce the following equality:

$$\frac{k(r+1) - k(r)}{k(s+1) - k(s)} = \frac{e_r}{e_s},$$

Table 6
Lists L and $\bar{L}$ for $w = 1$

N. crit.	d_i
11	0.001000000
10	0.002733333
9	0.002733333
6	0.002733333
2	0.002733333
5	0.004229122
8(†)	0.007034050
1(†)	0.007034050
4(†)	0.012239583
12	0.014000000
7	0.014000000
3	0.014000000

N. crit.	$\bar{d}_i$
12	0.028250000
7	0.028250000
3	0.028250000
4(†)	0.009765625
10	0.008533333
9	0.008533333
6	0.008533333
2	0.008533333
11	0.005500000
5	0.004817987
8(†)	0.000537634
1(†)	0.000537634

where e_r (e_s, respectively) is the number of intervals which separate the subsets of cards belonging to ranks $r+1$ and r ($s+1$ and s, respectively).

The formulas assure the conformity of the result and the collecting information concerning the ratio z given that

$$k(\bar{n}) = 1 + u(e_0 + e_1 + \cdots + e_{\bar{n}-1}) = 1 + ue = z = zk(1).$$

It is obvious that the rounding off procedure presented in Section 3.2.2 leads to the normalized weights whose sum is exactly 100. The criterion that this procedure intends to optimize brings into action the sequence of deformations d_i and $\bar{d}_i$, which are produced by the rounding offs by surplus of by deficit. The preconized solution minimizes not only the largest of these deformations, but also the second largest one and so on, in a lexicographical way. This result is a particular case of the more general problem described in Roy (2000).

4. The SRF software

The software SRF (coded with Borland Delphi 3) is used to determine the weights of criteria in the ELECTRE type methods. This software is an implementation of the revised Simos' procedure described in this paper. It must allow to any DM (not necessarily familiarized with multicriteria decision aiding):

- to think about and to express a ranking of criteria;
- to introduce some complementary information in the software in order to obtain the weights of the criteria.

For more details about SRF, see Roy and Figueira (1998).

This software is very well adapted in the contexts where, for different reasons (multiple users, robustness analyses, etc.), we need to determine several sets of weights. In such cases, SRF allows to collect different data sets and to process the information quickly. The main characteristics of the SRF software are the following:

1. It allows to take into account *multiple users* working with the same family of criteria. This feature can be very pertinent in group decision making contexts.
2. Each user can build and work with *several "ranking tables"*. Therefore, the user can now overcome his/her own hesitation between including a criterion in a certain subset of *ex aequo* criteria, or in another subset of *ex aequo* criteria (generally, the previous or the following one).
3. For each "ranking table", the user can also build *several "interval tables"* and, obviously, insert a variable number of white cards between the same two successive subsets of *ex aequo* criteria placed in different "interval tables". Note that, in practice, it is very difficult for the user to express the difference of the relative importance between the weights of two successive subsets of *ex aequo* criteria by introducing a single set of white cards between them.
4. This software allows the user to introduce *different values concerning the ratio z* (between the weight of the most important criterion and the weight of the least important one in the ranking) since it is very difficult to express this ratio using a single constant value. Remark that the user can analyze, in an easy way, the effect of the changes in z on the output.
5. Display the output in a table format or in a graphical format by ranks or by criteria.
6. Printing out, in the same table, the weights obtained after defining z as above.

5. Conclusions

The revised Simos' procedure proposed in this paper has been applied to different real-life contexts (public transportation problems, environment problems, water resources problems, etc.). It proved to be successful and we strongly believe that the information obtained by using this new procedure is very significant from the user's preference point of view. The software developed allows not only an easy collection of different data sets but also a quick processing of the information thus obtained.

It should be noted that the revised Simos' procedure and the software can be used not only to determine the weights of criteria in the ELECTRE type methods but also in other contexts like, for example, to build an interval scale or a ratio scale on any ordered set (cf. Roy, 1999). Therefore, in this latter case, it is necessary that the interval between any pair of consecutive levels in the ordered set can be compared, by the user, to any other similar interval. In multicriteria decision aiding contexts, the new procedure and the software can also be used to adapt or convert a scale of a given criterion into an interval scale or a ratio scale.

Acknowledgements

This paper was partly supported by the Luso-French grant n328J4 (ICCTI/Ambassade de France au Portugal).

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