Flexural response of continuous concrete beams prestressed with external tendons

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Abstract: This article presents the results of a numerical investigation of the flexural behavior of continuous externally prestressed concrete beams. Aspects of behavior studied include the increase in stress in external tendons, moment redistribution in the post-elastic range and secondary moments due to prestressing. A finite element model for the full-range analysis of continuous externally prestressed concrete beams is introduced. The model predictions agree well with the experimental results. The analysis shows that the ultimate stress increase in external tendons of continuous beams is dependent on both the number and rotation of plastic hinges that can be developed at failure load. The degree of moment redistribution is significantly influenced by the nonprestressed tension steel and the pattern of loading. An approach based on linear transformation concept is designed to examine the secondary moments over entire loading up to the ultimate. The results indicate that the secondary moments increase linearly with the prestressing force and can be conveniently calculated by an elastic analysis.

CE Database subject headings: Continuous structures; Prestressing; Tendons; Structural behavior; Numerical analysis

Introduction

External prestressing is increasingly used for the rehabilitation and also in the construction of concrete bridges, which are usually continuous over multiple spans in engineering practice. For the rational design of this type of structures, a good understanding of the behavior of continuous concrete beams with external prestressing is needed.

During the last 20 years, many experimental and theoretical works have been carried out to study the behavior of simple beams with external prestressing, particularly, the second-order effects caused by the change in tendon eccentricity with varying member deflection (Tan and Ng 1997; Harajli et al. 1999; Pisani 2005; Ng and Tan 2006; Au et al. 2008; Lou and Xiang 2010) and the ultimate stress in external tendon (Ghallab and Beeby 2005; He and Liu 2010). Due to continuity, the behavior of continuous externally prestressed concrete beams may be different from that of simple ones. Moreover, the continuous prestressed concrete beams have some additional characteristics, such as redistribution of moments in the post-elastic range and secondary moments due to prestressing. In ACI 318-11 (ACI Committee 318 2011), the secondary moment was included in the design moment in consideration of incomplete redistribution of moments, while in an earlier version (ACI Committee 318 1971) this moment was neglected, with the explanation that a continuous beam has converted into a statically determine structure after the formation of plastic hinges.

Some investigators have devoted their works to continuous beams with external prestressing. Harajli et al. (2002) conducted an experimental program and proposed an analytical model to evaluate the flexural response of externally prestressed continuous beams. Roberts-Wollmann et al. (2005) presented the results of a research originally performed by MacGregor, which included the test of a three-span continuous segmental girder post-tensioned with external tendons, and the development of an equation for calculating the ultimate stress in external tendons. Aravinthan et al. (2005) tested a series of prestressed concrete beams with large eccentricity external tendons, including six two-span continuous beams and three simple beams, to investigate the flexural behavior of the beams. Tan and Tjandra (2003,2007) conducted a set of experimental works in which the shear and flexural behavior of continuous reinforced concrete beams strengthened by external prestressing were studied.

The above literature review shows that only a few works have been undertaken to reveal the comprehensive behavior of continuous concrete beams prestressed with external tendons. In particular, the moment redistribution and secondary moment behavior, which are important for the analysis and design of this type of structures, have not been adequately addressed yet. Also, most of the existing studies were based on the laboratory tests with limited number and size of specimens. Over past years, a number of analysis methods have been proposed to predict the behavior of concrete beams prestressed with external tendons (Ramos and Aparicio 1996; Ariyawardena and Ghali 2002; Dall’Asta et al. 2007; Pisani 2009). However, few of the methods have been used to evaluate the overall response of continuous externally prestressed concrete beams.

This article describes a numerical study that is carried out to evaluate the flexural response of continuous externally prestressed concrete beams, including the load-deflection characteristics, increase in stress in external tendons, moment redistribution and secondary moments. The main parameters considered include the amount of nonprestressed steel, the pattern of loading, the type of beams and the layout of external tendons. A procedure based on linear transformation concept is designed to examine the actual secondary moments over entire loading up to the ultimate. Some conclusions are drawn based on the results obtained from the analysis.

Numerical model and its validation

Material models

The stress-strain ($f_c - \varepsilon_c$) relationship for concrete in compression suggested by Hognestad (1951) is adopted in this numerical evaluation. It is expressed by

for $0 \le \varepsilon_c \le 0.002$,

$$f_c = f_c^{'}\left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002}\right)^2\right] \tag{1a}$$

for $0.002 < \varepsilon_c \le \varepsilon_u$,

$$f_c = f_c^{'}\left[1 - 0.15\left(\frac{\varepsilon_c - 0.002}{\varepsilon_u - 0.002}\right)\right] \tag{1b}$$

as shown in Fig. 1(a). Also shown in this figure is the stress-strain diagram for concrete in tension (Kwak and Kim 2002), which is composed of elastic and strain-softening portions. In Fig. 1(a), $f_c^{'}$=concrete cylinder compressive strength;

ε_u=ultimate concrete compressive strain; f_t=concrete tensile strength; ε_{t0}=10 f_t / E_c, where E_c=elastic modulus of concrete.

The stress-strain ($f_p - \varepsilon_p$) curve for prestressing steel proposed by Menegotto and Pinto (1973) is used in this study. It is shown in Fig. 1(b) and expressed as

$$f_p = E_p \varepsilon_p \left[Q + \frac{1-Q}{\{1+[\varepsilon_p E_p / (K f_{py})]^R\}^{1/R}} \right] \le f_{pu} \tag{2}$$

where E_p, f_{py} and f_{pu}=modulus of elasticity, yield stress and ultimate strength of prestressing steel, respectively; and *K*, *Q* and *R*=empirical parameters, which are taken as 1.0618, 0.01174 and 7.344, respectively, in this numerical evaluation.

The stress-strain relationship for nonprestressed steel is assumed to be elasto-perfectly plastic both in tension and in compression.

Description of the nonlinear analysis

The finite element model developed by Lou and Xiang (2006) is used in this study. The method of proposed analysis is based on the assumptions of linear strain distribution across the concrete section, negligible shear deformations and negligible friction forces between external tendons and deviators. The concrete beam is divided into a number of beam elements interconnected by nodes. Each node has three degrees of freedom, namely, axial and transverse displacements and rotation. The cross section of a beam element is subdivided into discrete concrete and steel layers to include varied material properties. In this study, a total of 36 beam elements with equal element length are used for a two-span continuous beam, and 10 concrete layers and two steel layers (one for bottom steel bars and one for top steel bars) are adopted for a rectangular section. The finite element formulation is established based on the

Euler-Bernoulli beam theory. The contribution of external prestressing to the concrete beam is made by transforming the current prestressing force into equivalent nodal loads. At every step during the solution process, the eccentricities of external tendons are updated in terms of the current positions of these tendons, which are determined according to the current nodal displacements at anchorage and deviator points, and of the concrete beam, thus allowing the second-order effects to be considered. During the analysis, when the concrete strain at the critical section(s) reaches the ultimate compressive strain, which is taken as 0.0033 in this study, the beam fails due to the loss of the resisting force in the concrete at the critical section(s).

The finite element model can conduct the geometric and material nonlinear analysis of externally prestressed concrete beams, both simple and continuous, from zero loads up to the ultimate. The geometric nonlinearity includes the variation of the tendon eccentricity as well as the large displacement effect of the structure. In previous studies (Lou and Xiang 2006; Lou et al. 2011), the computer model was calibrated by experimental results of a total of 15 simply supported externally prestressed concrete specimens from different sources. In the present study, some continuous externally prestressed concrete specimens are collected to further verify the proposed analysis.

Comparisons between computational and experimental results

In a study by Harajli et al. (2002), nine two-span continuous externally prestressed concrete beams were tested. The test variables included the amount of external tendons and nonprestressed steel, the tendon profile and the deviator configuration.

The beams were of a rectangular section with 150mm in width and 200mm in depth. Among these specimens, B6D1 and B6D2 had two 6mm longitudinal bonded reinforcement with yield strength of 347MPa over critical regions, and had draped external tendons consisting of 5mm wires and 8mm seven-wire strands, respectively; B12D1 and B12D2 had two 12mm longitudinal bonded reinforcement with yield strength of 582MPa over critical regions, and had draped external tendons consisting of 5mm wires and 8mm seven-wire strands, respectively; B10S1A and B10S1B had two 10mm longitudinal bonded reinforcement with yield strength of 568MPa over critical regions, and had straight external tendons consisting of 5mm wires. Specimen B10S1A had deviators at midspans while Specimen B10S1B had no deviators at midspans. The effective prestress of the specimens mentioned above was of about 0.55 times the ultimate strength of the prestressing steel, which was of 1607MPa for the 5mm wires and 1986MPa for the 8mm seven-wire strands. The concrete strengths were of about 40MPa.

The test beams were provided with stirrups so the concrete was confined. To simulate the confinement of concrete, the stress-strain relationship for concrete in compression proposed by Scott et al. (1982) is used in the analysis of the test beams. For unconfined concrete, the model proposed by Scott et al. (1982) is similar to the Hognestad model (1951) as indicated by Eq. (1).

Figs. 2-3 show the comparisons between numerical predictions and experimental results regarding the load-deflection curves and the increase in stress in external tendons with applied load. It can be seen from the figures that the proposed analysis reproduces the experimental results of continuous externally prestressed concrete beam specimens, over entire loading up to the ultimate, with satisfactory agreement.

Evaluation of the behavior of continuous beams

Two-span continuous externally prestressed concrete beams are adopted here for the numerical analysis. The structure, section dimensions and steel layout of the beams are shown in Fig. 4. Two patterns of loading are used: point loading either at two spans (symmetrical loading) or at one span (unsymmetrical loading). The pattern of loading and the area of nonprestressed steel for each beam are given in Table 1. Two simple beams are also used for comparison. The simple beams have a span length of 10m, and have a single draped tendon profile with effective tendon depths of 200mm at midspan and 0mm at the anchorage points, which is similar to that of continuous beams with a proper linear transformation. A moderate initial prestressing force is selected. It is not the aim of this article to discuss design options associated with the definition of the initial prestressing force, but an acceptable value for this force could result from imposing a zero deflection situation for a target level of the service load. This level of loading would depend on the code for actions to be used. As for instance, if Eurocode 0 (CEN 2002) is used, a target of 80% of the quasi-permanent combination of loads could be an acceptable value. The material parameters are as follows: for prestressing steel, the area A_p=400mm^2, effective prestress f_{pe} =1120MPa, f_{pu} =1860MPa, f_{py} =0.9 f_{pu} , E_p =195GPa; for nonprestressed steel, yield strength f_y=450MPa, elastic modulus E_s=200GPa; for concrete, $f_c^{'}$=40MPa, ε_u=0.0033, f_t=3MPa.

Overall behavior

In this analysis, all the continuous beams fail by crushing of concrete at midspan. Prior to the final failure, the beams loaded at two spans except CB2S experience sequentially four typical phases, namely, first cracking at the center support, second cracking at midspan, first formation of plastic hinge (yielding of nonprestressed steel) at the center support, and second formation of plastic hinge at midspan. For CB2S, the first plastic hinge appears at midspan, attributed to relatively lower amount of nonprestressed tension steel in the midspan section as compared to the center support section. On the other hand, for the continuous beams loaded at one span, the first cracking appears at the loaded midspan. All beams but Beam CB3U have only one plastic hinge at the loaded midspan at the ultimate limit state. Beam CB3U, which has lower amount of nonprestressed steel in the center support section than the midspan section, also forms a plastic hinge at the center support before the beam concrete is crushed.

Fig. 5 illustrates the concrete strain distribution over the beam length, at the ultimate limit state, for the beams loaded at two spans. The failure mode and cracking pattern of the beams can be observed from the figure. The midspan concrete is crushed when its compressive strain reaches 0.0033, while at failure load the concrete over the center support doesn’t yet reach its capacity. Due to low nonprestressed steel ratio, crack concentration can be observed at the critical sections of the beams. The beams with higher amount of nonprestressed steel over the critical regions display better crack distribution in the regions, namely, more cracks but smaller crack width, as compared to the beams with lower amount of nonprestressed steel.

Load-deflection response and stress increase in external tendons

The load-deflection response and stress increase in external tendons are shown in Figs. 6 and 7, respectively, for the continuous beams loaded at two spans, and in Figs. 8 and 9, respectively, for the continuous beams loaded at one span. The values of the ultimate load P_u, deflection Δ_u and stress increase in external tendons Δf_{ps} are given in Table 1.

It is seen from Figs. 6-9 and Table 1 that, because the failure of the beams occurs at midspan, increasing the amount of nonprestressed steel over midspan region is more effective than over center support region to enhance the ultimate load-carrying capacity. This phenomenon is particularly obvious for the beams loaded at one span, where the center support section is non-critical and, therefore, the amount of nonprestressed steel over center support region has insignificant effect on the ultimate load of the beams. Also, it is seen that a higher amount of center support nonprestressed steel registers a lower ultimate deflection and stress increase in external tendons as expected. However, a higher amount of midspan nonprestressed steel may result in a higher ultimate deflection and stress increase in external tendons. In addition, it is seen that, for the beams loaded at one span, there appear upward displacements at the non-loaded span, the values of which are related to the amount of nonprestressed steel: the higher the amount of nonprestressed steel, the larger the upward displacements at the non-loaded span.

Fig. 10 shows the comparison between simple and continuous beams for the stress increase in external tendons. It is seen that at a certain load level, the continuous beams mobilize much less increase in stress in external tendons as compared to the simple beam. At the ultimate limit state, because of less number of plastic hinges, the continuous beams loaded at one span register much lower increase in stress in external tendons, as compared to continuous beams loaded at two spans. It is also observed that, although the continuous beams loaded at two spans (CB1S and CB4S) have three plastic hinges (two positive at midspans and one negative at the center support) at failure, the ultimate stress increase in external tendons of them is obviously lower than that of simple beams. This is attributed to the fact that, when the continuous beams collapse by crushing of concrete at midspan, the plastic hinge at the center support is still far from its full rotation capacity, as can be observed from Fig. 5. If the midspan sections have sufficient rotation capacity so that the negative plastic hinge can attain its full rotation capacity, namely, the concrete strain at bottom fiber of the center support section reaches the ultimate strain of 0.0033, the tendon stress increases of continuous beams, indicated by the dotted lines of Fig. 10, would be higher than those of the corresponding simple beams. Therefore, it may be concluded that a two-span continuous beam in which two spans are loaded would produce higher ultimate stress increase in external tendons than a simple beam, provided that three plastic hinges are fully developed (This is achievable by proper confinement of concrete over the critical regions). On the other hand, if one of the plastic hinges fails to reach its full rotation capacity, the continuous beam may produce a lower ultimate stress increase in external tendons as compared to the simple beam.

Moment redistribution

For simple beams, there is always a linear relationship between the applied load and the moment of a section throughout the whole loading history. For continuous beams, however, due to redistribution of moments this relationship losses its linearity when the critical section begins to assume nonlinear behavior. The relationships between the applied load and moments at the midspan and center support are shown in Figs. 11 for the beams loaded at two spans, and in Fig. 12 for the beams loaded at one span. Both the elastic moments, which are calculated based on the linear-elastic theory, and the actual moments obtained from the current nonlinear finite element analysis are plotted in the figures.

It is seen that, at the early stage of loading, the actual moment increases linearly, just as the elastic moment, with the applied load up to the cracking of concrete, indicating that there is no redistribution of moments at this stage. In this analysis, for the beams loaded at two spans, the first crack appears at the center support. Therefore, once the crack appears, the moment is redistributed from the center support to the midspan, resulting in a diminution of the increase rate of moments at the center support and a consequent growth of the increase rate of moments at the midspan, as shown in Figs. 11. On the other hand, for the beams loaded at one span, the first crack occurs at the loaded midspan, leading to a diminution of the increase rate of moments at the loaded midspan and a consequent growth of the increase rate of moments at the center support, as shown in Figs. 12. After that, the beams experience some other phases which may affect the progress of moments. As expected, the formation of plastic hinges, marked in Fig. 11(c) as an example, has very important effect on the redistribution of moments. However, in this study, the second cracking does not appear to exhibit noticeable effect on the moment evolution in continuous beams. This observation is consistent with an earlier experimental study by Lopes et al. (1997).

The degree of moment redistribution can be expressed by: $\beta = 1 - M / M_e$, where M is the actual moment in the post-elastic range, and M_e is the elastic moment based on the linear-elastic theory. A list of actual and elastic moments and the degree of moment redistribution, at the ultimate limit state, is given in Table 2. It is seen in Table 2 and Figs. 11-12 that, over the center support, there are negative redistributions of moments for the beams loaded at one span, and positive ones for the beams loaded at two spans except CB2S. Beam CB2S has a negative redistribution of moments over the center support, because the moment is prone to redistributed from the lower reinforced midspan section to the higher reinforced center support section. Also, the center support section has higher degree of moment redistribution as compared to the midspan section, particularly for the beams loaded at one span. It is also seen that the degree of moment redistribution is significantly influenced by the pattern of loading and by A_{s1}/A_{s2}, a parameter defined by the ratio of the amount of nonprestressed tension steel over the midspan region to that over the center support region. Over the center support, the beams loaded at one span have higher degree of moment redistribution than do the beams loaded at two spans, particularly for lower values of A_{s1}/A_{s2}. Also, for equal amount of $A_{s1}+A_{s2}$, a higher value of A_{s1}/A_{s2} results in higher degree of moment redistribution for the beams loaded at two spans, but lower degree of moment redistribution for the beams loaded at one span. For equal value of A_{s1}/A_{s2}, a larger amount of nonprestressed tension steel leads to lower degree of moment redistribution, as expected.

Linear transformation and secondary moments

Linear transformation

When a tendon line is moved over the interior support(s) without changing its intrinsic shape, as illustrated in Fig. 13, this tendon line is termed to be linearly transformed (Lin and Burns 1981). It was stated that linear transformation of the tendon line does not change the ultimate load-carrying capacity of continuous beams (Lin and Burns 1981). To examine this statement, the tendon line of the continuous beam illustrated in Fig. 13 is linearly transformed into various profiles. The cross-section and reinforcement details of the beam are the same as Beam CB4S as illustrated in Fig. 4 and listed in Table 1. Apart from one-point loading, third-point and uniform loading are also used. Linear transformation is made by moving the tendon line over the center support by Δ (correspondingly by $\Delta/2$ over the midspan). The analysis shows that the continuous beams with various linearly transformed tendon profiles have the same ultimate load-carrying capacity as well as the same flexural characteristics, including deflection, curvature, strain and stress in external tendons, nonprestressed steel and concrete, over the entire loading up to failure. Fig. 14 shows the load-deflection response and stress increase in external tendons for two typical linearly transformed tendon beams under different types of loading. Identical responses for the beams throughout the entire loading history can be observed from the figure. On the other hand, linear transformation would cause changes of secondary moments, indicating that a non-concordant tendon profile can be linearly transformed into a concordant profile, which does not produce secondary moments, without changing the basic flexural characteristics of continuous beams. This provides an approach to examine the actual secondary moments over the whole loading process as discussed in the following section.

Secondary moments

In a continuous beam with non-concordant tendon profile, it is known that the prestressing would produce secondary moments. Regarding the behavior of secondary moments at the plastic stage, however, there have been different viewpoints among researchers and no agreement has been reached yet. In this study, the behavior of secondary moments is examined by nonlinear finite element analysis combining with the above-mentioned linear transformation concept.

For a beam with non-concordant tendon profile, the reaction at a support consists of two components: the reaction R_{load} caused by external loads (live and dead loads) and the secondary reaction R_{sec} caused by prestressing. For a beam with a concordant tendon profile, on the other hand, there is no secondary reaction R_{sec} and only the reaction R_{load} by external load exists. Since linear transformation does not change the flexural characteristics throughout the entire loading history, it can be recognized that, at a given load, the reaction R_{load} of the beam with non-concordant tendon profile is equal to that of the beam with linearly transformed concordant tendon profile. Therefore, the secondary reaction at a support can be obtained from the value difference between the reactions of the beams with non-concordant and linearly transformed concordant tendon profiles. Based on the above discussion, the procedure of computing the secondary reactions or moments for a continuous beam with non-concordant tendon profile is summarized as follows:

(1) Determine the linearly transformed concordant tendon profile.

(2) Compute the support reactions of the continuous beams with non-concordant and linearly transformed concordant tendon profiles over entire loading process by nonlinear element finite analysis.

(3) Calculate the secondary reactions (and thereby secondary moments) by subtracting the support reactions of the beam with concordant tendon profile from those of the beam with non-concordant tendon profile.

The continuous beam (with cross-section and reinforcement details as CB4S) shown in Fig. 13 is used here to illustrate the results obtained from the analysis. When Δ is equal to 149.2mm, that is, e_2 is equal to 149.2mm and e_1 is equal to 125.4mm, the tendon profile is concordant. Four linearly transformed non-concordant tendon profiles (Δ=0, 100, 200, and 300mm) are selected. Figs. 15 and 16 show the applied load versus secondary reaction response for the beams with non-concordant tendon profiles during the whole loading process. This response exhibits a trend very similar to the increase in stress in external tendons with applied load. It consists of three approximately straight lines. Cracking or yielding leads to much quicker increase in the secondary reactions. Compared to the secondary reactions at the end support (Fig. 15), the secondary reactions at the center support (Fig. 16) are twice in magnitude and opposite in direction. This confirms the validity of the proposed method for computing the secondary moments.

Figs. 17 and 18 show respectively the change in secondary reactions at the end and center supports with increasing tendon stress from effective prestress to the ultimate for different linearly transformed tendon profiles. There remains almost a linear relationship between the secondary reactions and the stress in external tendons up to the ultimate. It may be inferred from these results that the secondary reactions or moments at the plastic or ultimate stage could be conveniently computed by an elastic analysis using the current external prestressing force and neglecting the self-weight of the beams. To prove this conjecture, the end and center support secondary reactions computed by the elastic analysis are compared with those by the proposed method, which is based on linear transformation concept and nonlinear finite element analysis, in Figs. 19 and 20, respectively. It is seen that, from effective prestress up to the ultimate, the results determined by the elastic analysis are completely identical to those by the proposed method.

It is seen from Figs. 15-20 that the magnitude and direction of the secondary moments are dependent on the layout of external tendons. When the tendon line is below the concordant line, the prestressing produces a positive reaction at the end support, and correspondingly a negative center support reaction with twice the magnitude of the end support reaction, resulting in positive moments in beam sections.

Therefore, in this case, the secondary moment is favorable at the center support but unfavorable at the midspan. On the other hand, when the tendon line is above the concordant line, the prestressing induces negative end support reactions and consequently negative secondary moments. Thereby, the secondary moment turns to be unfavorable at the center support while favorable at the midspan. Also, it is seen that a larger deviation of tendon line from the concordant line leads to a higher amount of secondary moments, as expected.

Conclusions

A numerical study is undertaken to examine the behavior of continuous prestressed concrete beams with external tendons, including the load-deflection response, stress increase in external tendons, redistribution of moments in the post-elastic range and secondary moments due to external prestressing. The following conclusions can be drawn from the current study:

1. Because two-span continuous beams fail at midspan, increasing the amount of nonprestressed steel over midspan region is more effective than over center support region to enhance the ultimate load-carrying capacity, particularly for the beams loaded at one span.

2. The ultimate stress increase in external tendons in continuous beams is dependent on both the number and rotation of plastic hinges that can be developed at failure load. A two-span continuous beam in which three plastic hinges are fully developed would produce higher ultimate tendon stress increase than does a simple

beam. On the other hand, if one of the plastic hinges fails to reach its full rotation capacity, the continuous beam may produce a lower ultimate tendon stress increase as compared to the simple beam.

3. Over the center support, the beams loaded at one span register higher degree of moment redistribution than do the beams loaded at two spans, particularly obvious for lower values of A_{s1}/A_{s2}. For equal amount of $A_{s1}+A_{s2}$, a higher value of A_{s1}/A_{s2} results in higher degree of moment redistribution for the beams loaded at two spans, but lower degree of moment redistribution for the beams loaded at one span.

4. The analysis indicates that linear transformation does not change the ultimate load-carrying capacity of continuous externally prestressed concrete beams as well as the basic flexural characteristics over entire loading up to the ultimate. Based on linear transformation concept, a method for identifying the secondary moments is proposed, and is validated by the results obtained from the analysis.

5. The secondary reaction or moment exhibits nearly tri-linear behavior with varying applied load up to failure. Cracking of concrete and yielding of nonprestresed steel lead to much quicker increase of the secondary reaction or moment.

6. The secondary moments should be considered in the ultimate design of continuous beams prestressed with external tendons, where the full redistribution of moments is rare. The secondary moments increase linearly with the stress in external tendons and can be computed conveniently based on an elastic analysis using the current prestressing force and neglecting the self-weight of the beams.

7. When the prestressing tendon line is below the concordant line, the secondary

moment is favorable at the center support but unfavorable at the midspan. On the other hand, when the tendon line is above the concordant line, the secondary moment becomes unfavorable at the center support while favorable at the midspan.

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Tan, K. H., and Tjandra, R. A. (2007). “Strengthening of RC continuous beams by external prestressing.” ASCE Journal of Structural Engineering, 133(2): 195-204.

Table 1 Parameters and typical computational results for beams

Beams	Type of beams	Loading	A_{s1} (mm^2)	A_{s2} (mm^2)	A_{s3} (mm^2)	P_u (kN)	Δ_u (mm)	Δf_{ps} (MPa)
CB1S	Continuous beams	Sym.	540	540	360	194.4	67.0	265.4
CB2S			540	1080	360	218.8	65.3	254.0
CB3S			1080	540	360	247.5	72.0	278.1
CB4S			1080	1080	360	271.5	69.5	263.1
CB1U		Unsym.	540	540	360	175.1	70.6	116.2
CB2U			540	1080	360	181.5	70.0	109.5
CB3U			1080	540	360	227.5	75.4	122.0
CB4U			1080	1080	360	235.6	74.4	113.8
SB1	Simple beams	Center point	540	-	360	141.2	120.2	377.8
SB4			1080	-	360	190.8	119.7	363.7

Table 2 Results of actual, elastic moments and degree of moment redistribution at ultimate limit state

Beams	M (kN-m)		M_e (kN-m)		β (%)	
	Midspan	Center support	Midspan	Center support	Midspan	Center support
CB1S	362.6	-359.6	343.7	-397.2	-5.50	9.47
CB2S	360.8	-485.0	380.2	-446.0	5.10	-8.74
CB3S	490.9	-368.4	428.8	-492.5	-14.48	25.20
CB4S	488.6	-492.8	464.4	-541.2	-5.21	8.94
CB1U	339.8	-308.5	395.4	-197.4	14.06	-56.28
CB2U	338.9	-342.4	408.2	-203.7	16.98	-68.09
CB3U	467.0	-316.0	502.3	-245.6	7.03	-28.66
CB4U	465.8	-358.8	518.2	-254.0	10.11	-41.26

Figure

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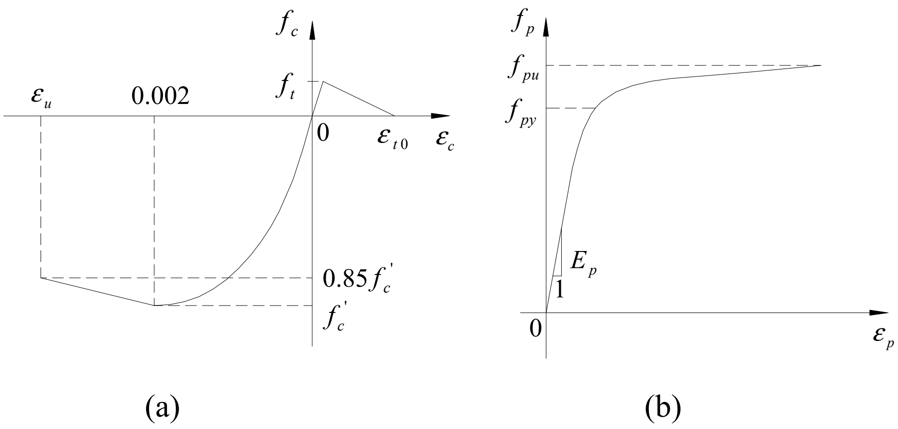

Fig. 1 Stress-strain curves for materials: (a) concrete; (b) prestressing steel

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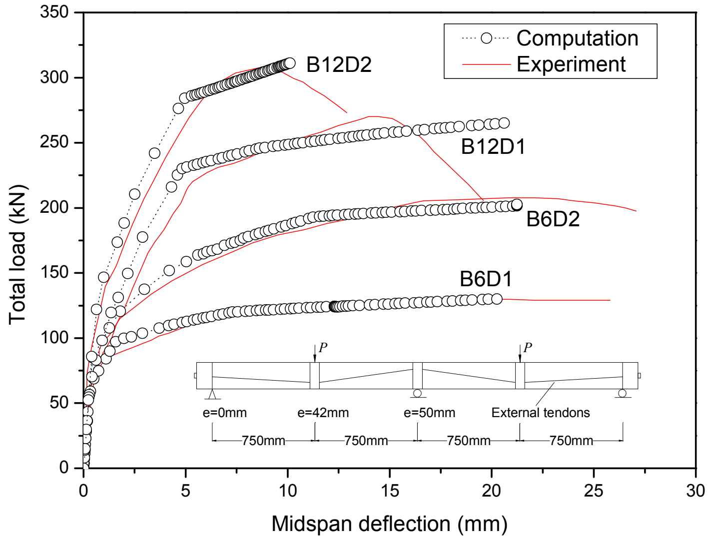
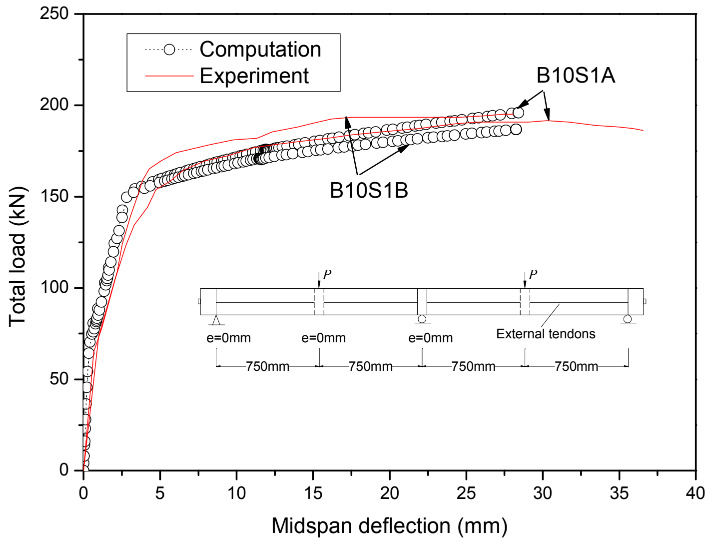


(a)



(b)

Fig. 2 Comparison of predicted load-deflection curves with experimental results: (a) specimens with draped external tendons; (b) specimens with straight external tendons

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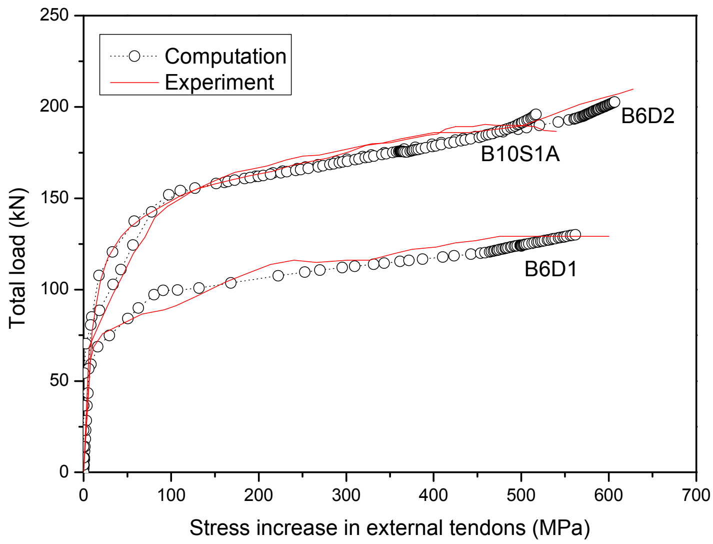

Fig. 3 Comparison of predicted load versus stress increase in external tendons with experimental results

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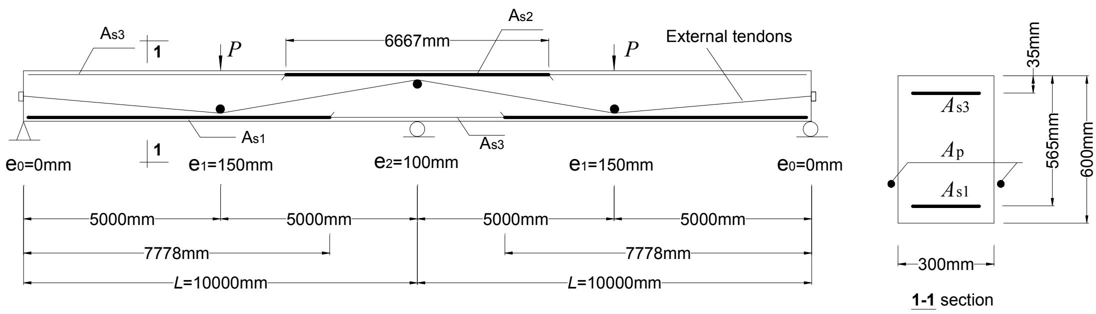

Fig. 4 Details of two-span continuous beams used for numerical evaluation

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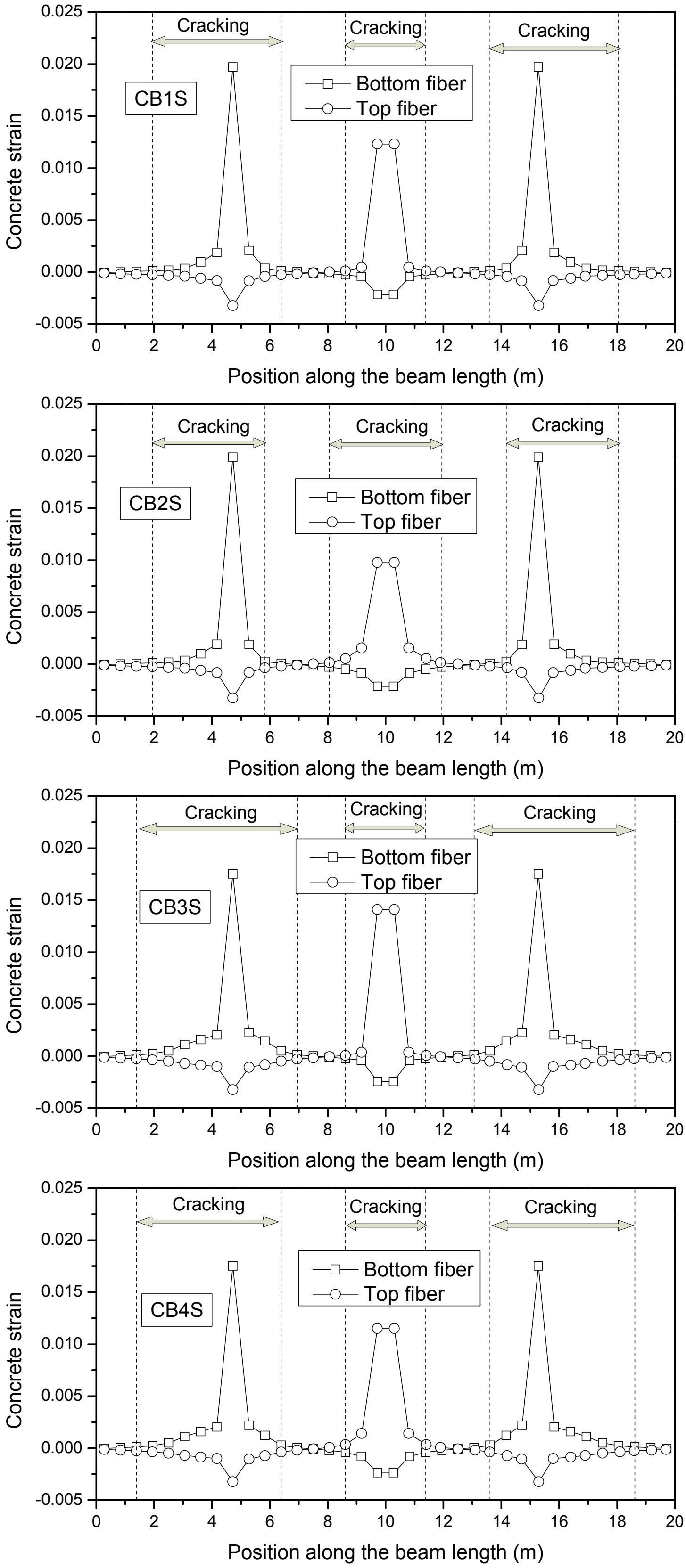

Fig. 5 Concrete strain distribution over the length for continuous beams loaded at two spans at failure load

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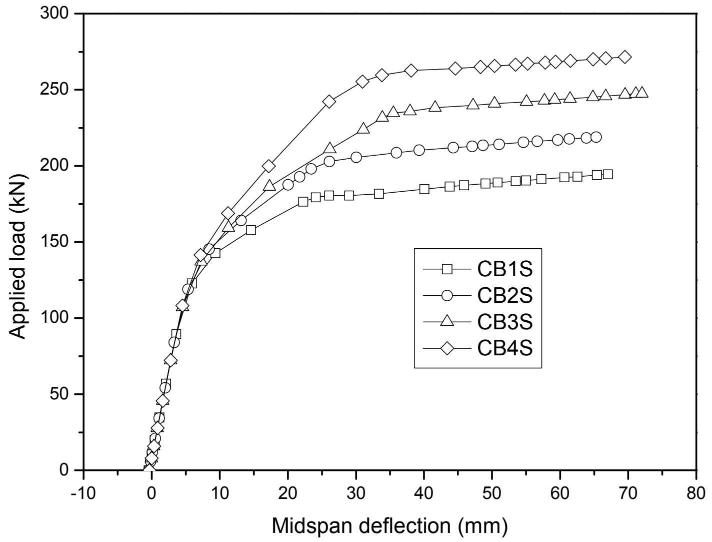

Fig. 6 Load-deflection response for continuous beams loaded at two spans

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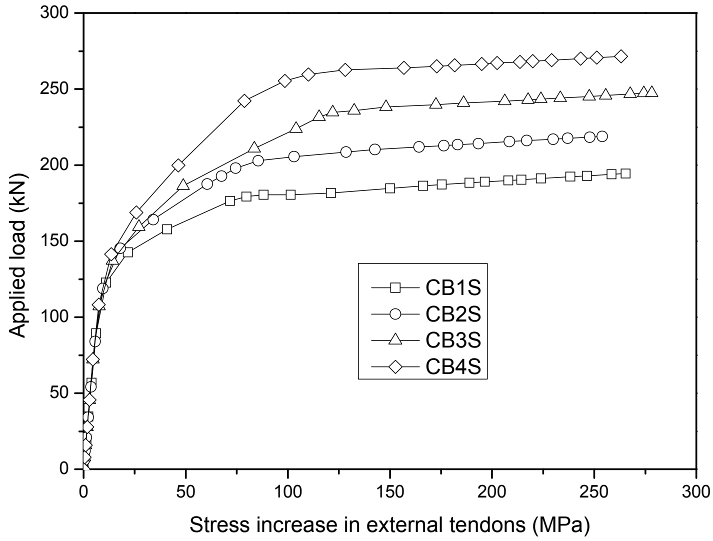

Fig. 7 Stress increase in external tendons for continuous beams loaded at two spans

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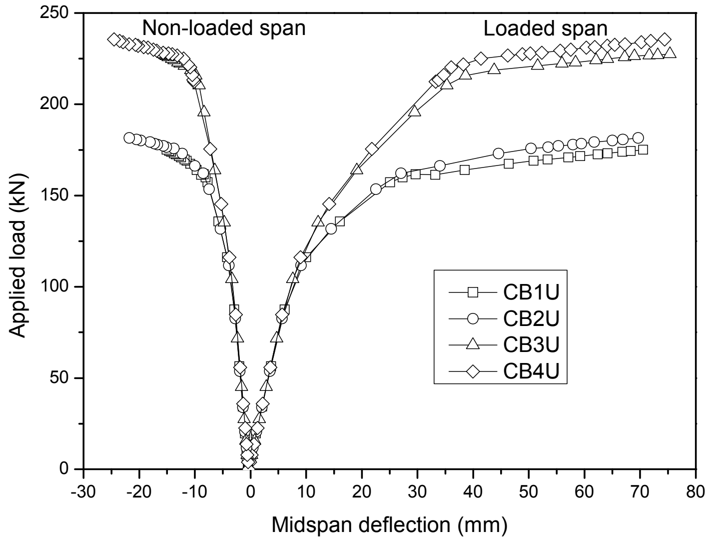

Fig. 8 Load-deflection response for continuous beams loaded at one span

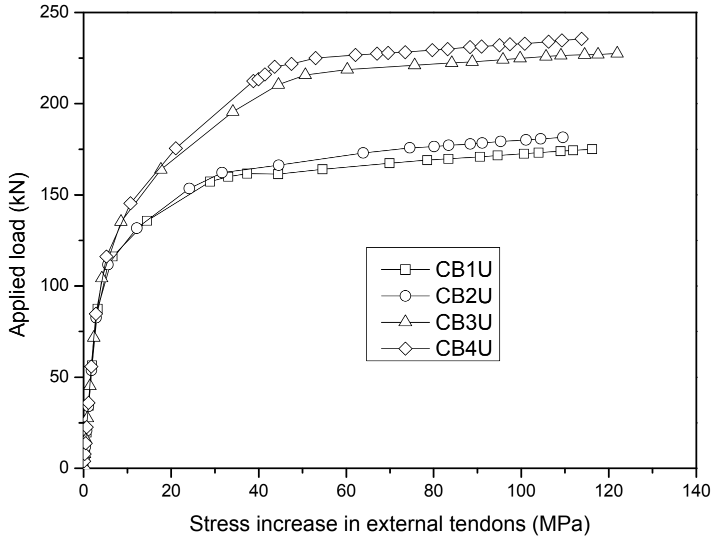

Fig. 9 Stress increase in external tendons for continuous beams loaded at one span

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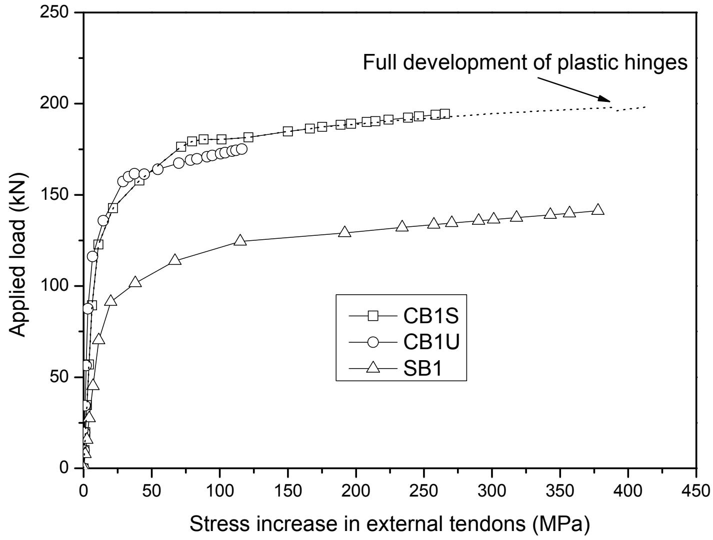

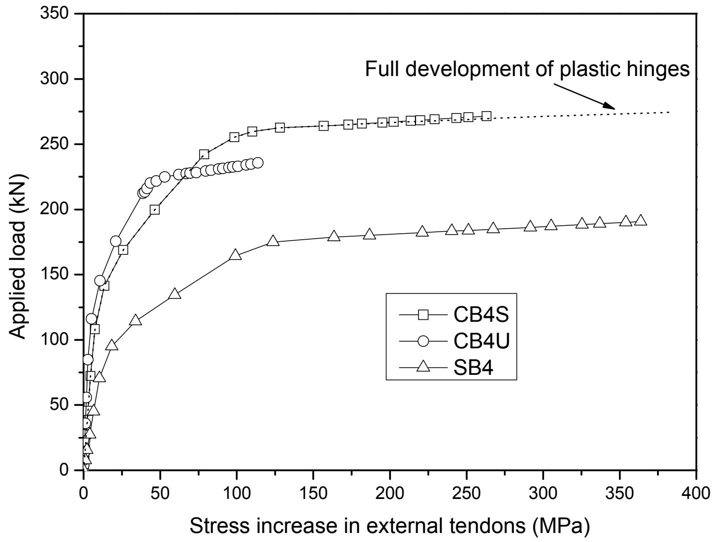

Fig. 10 Comparison between continuous and simple beams for stress increase in external tendons

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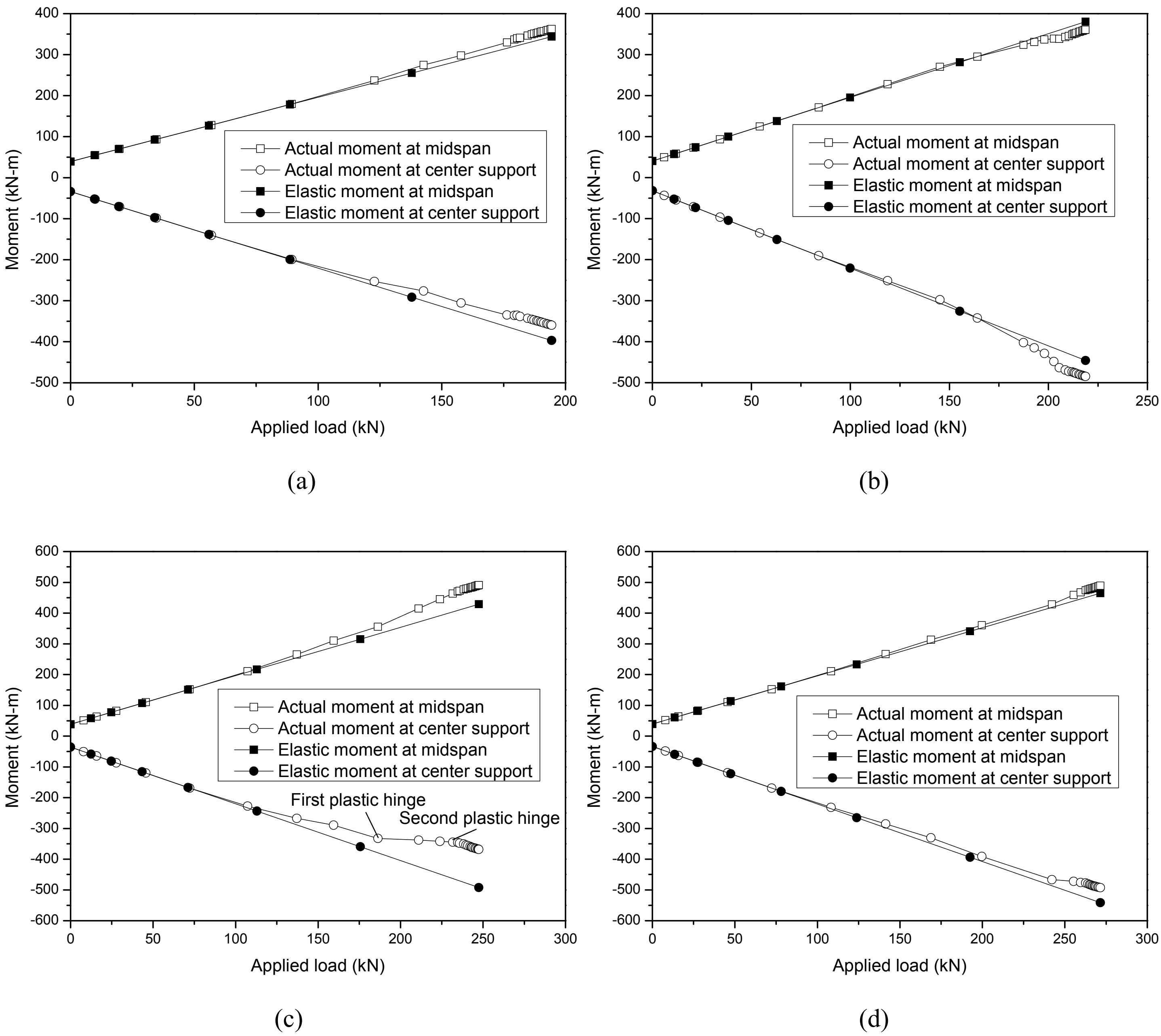

Fig. 11 Actual and elastic moments versus applied load for continuous beams loaded at two spans: (a) CB1S; (b) CB2S; (c) CB3S; (d) CB4S

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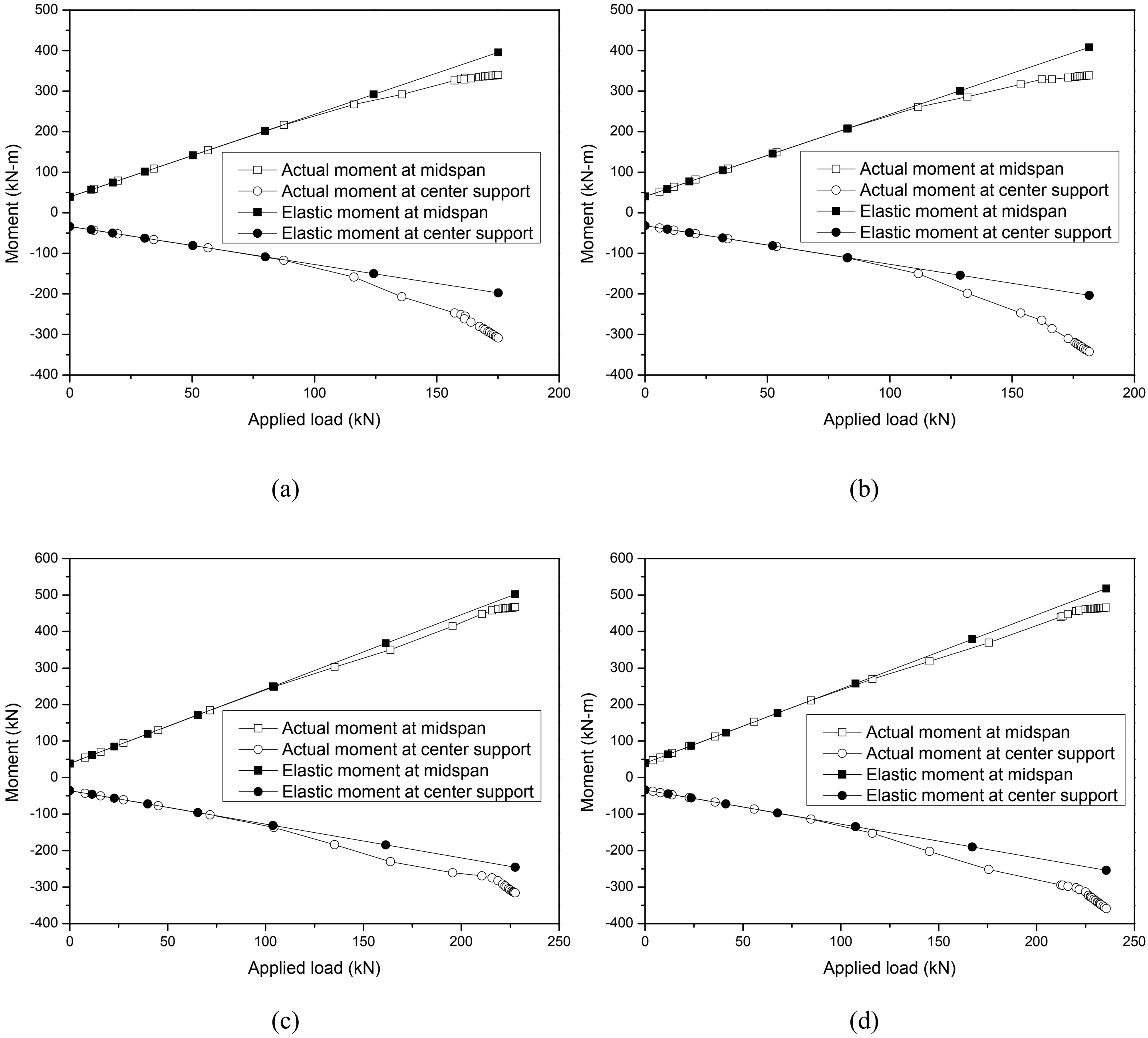

Fig. 12 Actual and elastic moments versus applied load for continuous beams loaded at one span: (a) CB1U; (b) CB2U; (c) CB3U; (d) CB4U

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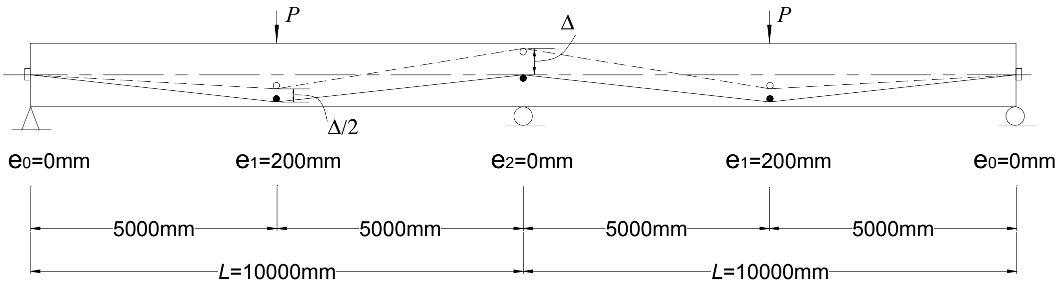

Fig. 13 Linearly transformed tendon beams for examination of secondary moments

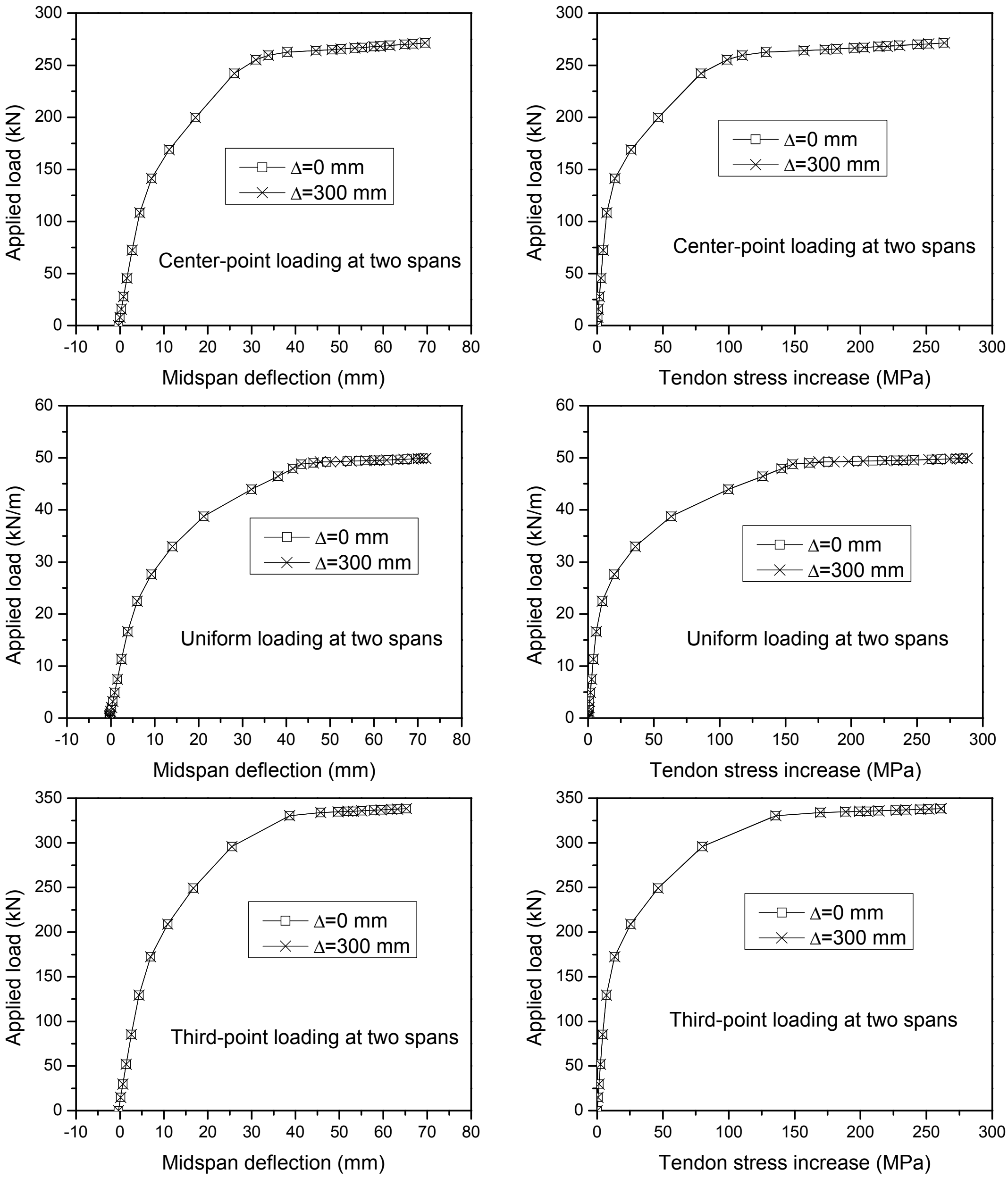

Fig. 14 Responses of two typical linearly transformed tendon beams

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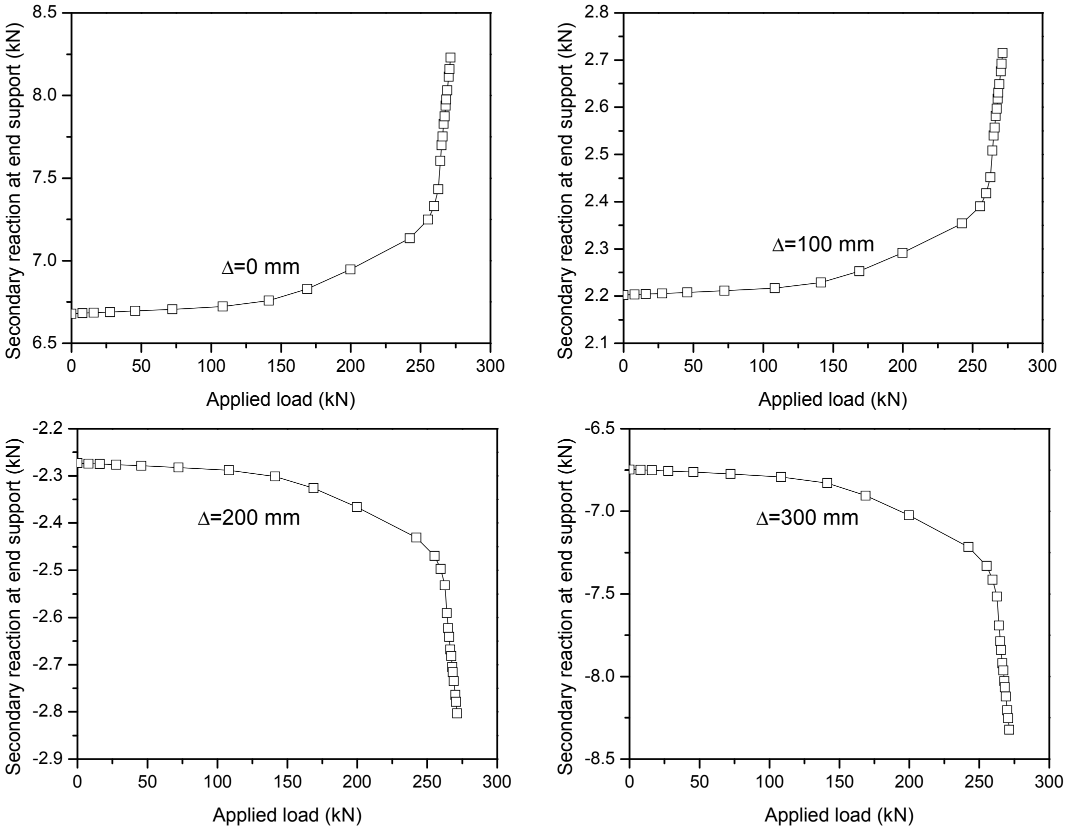

Fig. 15 Relationship between secondary reaction at end support and applied load

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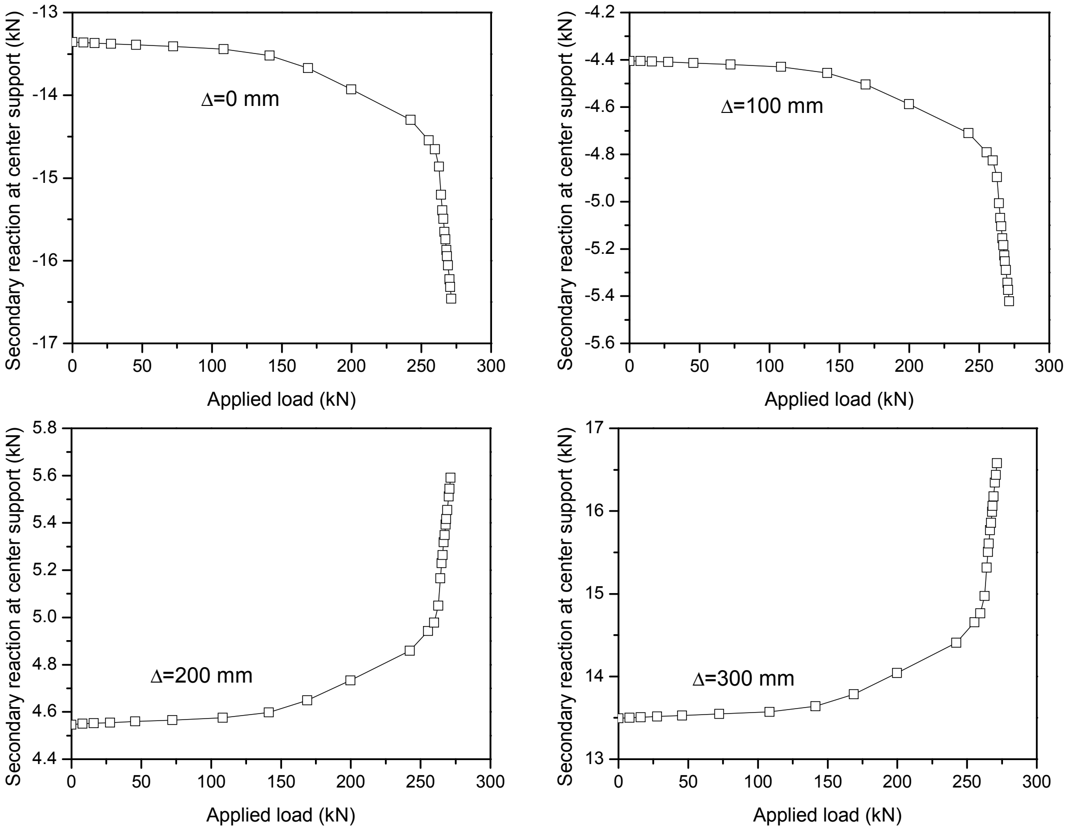

Fig. 16 Relationship between secondary reaction at center support and applied load

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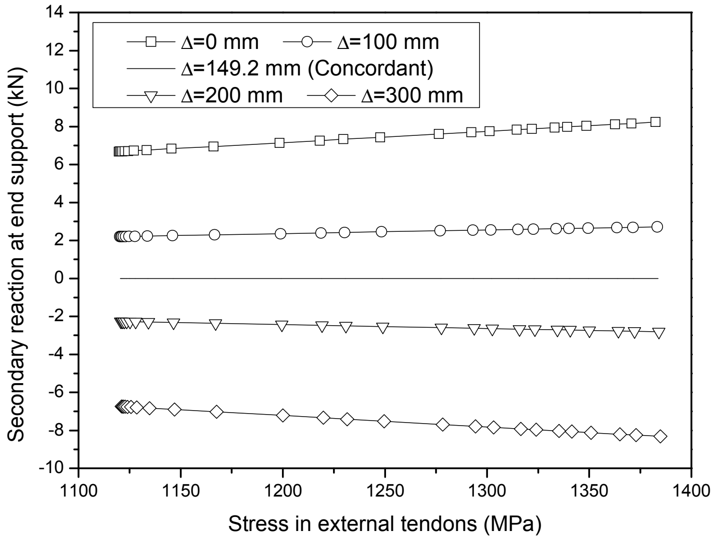

Fig. 17 Relationship between secondary reaction at end support and tendon stress

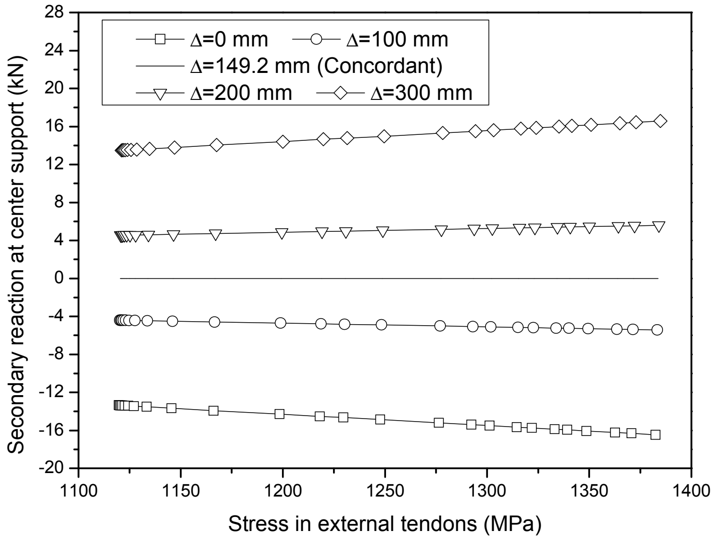

Fig. 18 Relationship between secondary reaction at center support and tendon stress

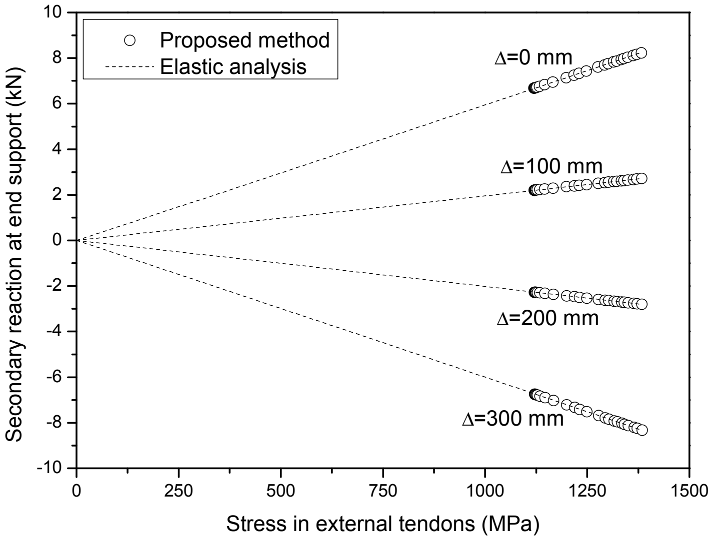

Fig. 19 Comparison of end support secondary reactions from proposed method with those from elastic analysis

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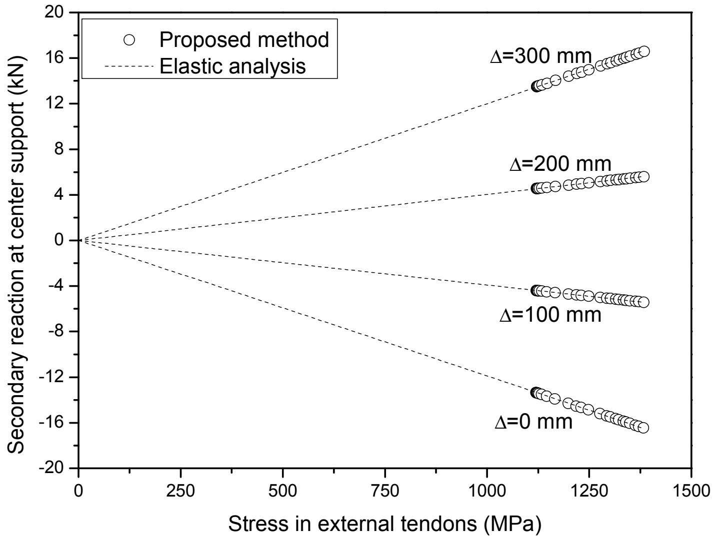

Fig. 20 Comparison of center support secondary reactions from proposed method with those from elastic analysis

List of Figures captions:

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Fig. 20 Comparison of center support secondary reactions from proposed method with those from elastic analysis.

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Manuscript Authors: TIEJIONG LOU, SERGIO M.R. LOPES, ADELINO V. LOPES

Corresponding Author Name and Address: SERGIO LOPES, DEPARTMENT OF CIVIL ENGINEERING, FCTUC - POLO 2, UNIVERSITY OF COIMBRA, COIMBRA 3030-788, PORTUGAL

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up to 34 lines = 315 word equiv.	up to 34 lines = 630 word equiv.
up to 51 lines = 473 word equiv.	up to 51 lines = 945 word equiv.
up to 68 text lines = 630 word equiv.	up to 68 text lines = 1260 word equiv.

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Reviewer 1:

	Reviewer Comments	Reaction by the authors
1	**Overall comments:** The revised manuscript is almost the same as the original one with minor amendments. The literature survey has been expanded and there are minor changes in the other parts. The FE in-house code by Lou and Xiang from (2006) has been compared with experiments of external prestressed simply-supported concrete beams and here with continuous beams, see Figs. 2 and 3. The comparison presents load-versus deflections or stress in tendons at specific sections. The authors evaluate numerically only the behavior of the continuous beam in terms of overall response, load-deflection/stress in tendons, moment's redistribution, linear transformation and secondary moments in the linear and non-linear regimes. In the case of the linear transformation and secondary moments the authors assume that the flexure rigidity remains unchanged up to failure - "Linear Transformation does not change the basic flexural behavior of beams over entire loading up to failure" (see answer to comment 9 in comment part - page 3). Finally some conclusions are drawn.	**The authors want to thank this reviewer for reading and commenting again the manuscript. However, some of his comments are very difficult to answer. (see the reactions below)**
2	As I have already said in my previous referee report the paper is merely a numerical study that uses an in-house FE code rather than commercial codes such as Ansys, Adina or Abaqoues. The topic has been dealt extensively in the 90's including experiments and sophisticated computational models, see [2-3] below, superior to the one used by the authors, see FE paper in Lou and Xiang (2006). In addition, a paper with a very similar title that deals with continuous externally prestressed beams has been published recently, 2009, in the European Journal of Scientific Research, see [1], but using ANSYS instead of the in-house FE code. The contribution of this paper, see [1], is much more important than the proposed paper since it discusses the issues of modeling external prestressing in commercial FE codes rather than in an in-house code.	There are a number of computer models for concrete beams prestressed with external tendons. However, few of the models have been used to evaluate the comprehensive behavior of continuous externally prestressed concrete beams which this study focused on. The authors did not find any study that covered the same topic presented in this article. The topics of the reference papers listed by the reviewer are completely different from that of this article.

3	The concept of Linear Transformation has been defined first by Guyon in the early 50's for the elastic range only and in accordance with the various code EC2 and ACI the ultimate load-carrying capacity is affected by the shifting (linear transformation) of the cable.	The authors understand that cable shifting does affect the ultimate load-carrying capacity, but "linear transformation" is not coincident to "cable shifting". When a cable line is moved over the interior support(s) without changing its intrinsic shape, as illustrated in Fig. 13 of this manuscript, this cable line is termed as "linear transformation". That is to say, linear transformation is just a particular case of cable shifting. If it is not the case of linear transformation, cable shifting would certainly affect the ultimate load-carrying capacity. It was stated in a classic book (Lin and Burns 1981, p.388): "Linear transformation of the c.g.s. line does not change the ultimate load-carrying capacity of a continuous beam". This statement was later proved by an experimental work published in the ACI Structural Journal (Aravinthan et al. 2005) and here by the numerical work presented in this manuscript.
4	In the non-linear regime there is a change in the flexure rigidity along the beam due to cracking of the concrete. Hence, the linear transformation occurs in a structure that is already cracked, which means that the linear transformation adds additional internal stress resultants such as bending moments and shear forces that may increase or reduce that cracking zones and the depth of the cracks. Hence, the linear transformation may change the flexure rigidity and therefore it cannot be used in the non-linear regime.	The results obtained from the current analysis indicate that linear transformation does not change the ultimate load-carrying capacity of continuous externally prestressed concrete beams as well as the basic flexural characteristics (including flexural rigidity) over entire loading up to the ultimate. (see Fig. 14 of the manuscript). This observation is consistent with the viewpoint of Lin and Burns (1981) and also with an earlier experimental study by Aravinthan et al. (2005). In fact, it is an easy work to verify the above statement by performing the analysis of beams with various linearly transformed tendon profiles using any available computer models. If the reviewer could conduct this analysis, the authors believe that he would change his viewpoint on linear transformation and would agree with the above statement.
5	Another problem that the authors have not addressed is the ability of the cable to move relative to the concrete. In general when the cable is prestressed it yields large friction loads at the deviators. These friction loads do not necessary allow the cable to slip relative to the deviator for loads smaller than the friction load and in opposite directions. Hence, up to a certain level of external loads the cable behaves as a bonded tendon and above it as an unbounded one. This topic has not been addressed at all.	One of the assumptions adopted in this numerical work is that the friction forces between external tendons and deviators are negligible. This assumption is commonly accepted and adopted for the modeling of external prestressing in existing literature.

6	The conclusions drawn by the authors are general although their study is limited to a prescribed beam with some specific date and a specific cable configuration. They may be correct for the specific configuration but not necessary correct in general. I am not sure that they are correct or valid for other type of concrete strength, other type continuous beams, other cable layout, etc. Hence, the conclusions are too general in my opinion.	The conclusions are valid for general beams that meet the basic assumptions adopted in this study.
7	Finally, a reliable FE code should be verified through comparisons with experiments and commercially available codes. The comparison must include: deflections, strains and stresses in concrete and various reinforcements along the structure and not only at certain sections. In this particular case since the FE code evaluation process is not complete and therefore its results are unreliable than any numerical study that is based on is unreliable too.	The FE code has been verified with typical experimental results (load-deflection response and stress increase in external tendons) of both simply supported and continuous beams. The authors think that the set of results (from simply supported and continuous beams) is sufficient to ensure a high level of confidence on the correctness of the computer program.
8	The presentation of a numerical study that is based on a mathematical formulation, which yielded an in-house code, and that cannot be criticized or evaluated is an unreliable procedure. But, if the FE formulation and the numerical study have been presented in the same paper then the principles of the mathematical could be evaluated and criticized properly. In my opinion, numerical studies only that do not yield any new scientific information and in error are useless. In addition, please notice that are many segmental bridges that have been already built using external prestressing. The authors could refer to such bridges and to compare their analysis with the design. The paper is merely an unsophisticated FE numerical study where similar ones exist in the literature and superior to this one. In conclusion, the scientific quality and value of the paper are minor and it is of low quality with no new information of scientific quality. Thus, my recommendation has not been changed and the paper should be declined from the journal.	This study was designed to examine the comprehensive behavior of continuous externally prestressed concrete beams. The subject of the paper, in the authors' opinion, is important and practically significant. The study was conducted using a FE model that has been verified with experimental results of both simply supported and continuous beams. Some original findings and new knowledge on the subject were presented in the manuscript. The authors believe that the work is a valid contribution to continuous beams with external prestressing and is sufficiently important to be published in ASCE Journal.

Reviewer 4:

	Reviewer Comments	**Reaction by the authors**
	Overview: Manuscript BEENG-853R1 presents a selection of response results for continuous reinforced concrete beams prestressing with external tendons obtained from nonlinear finite element analysis up to collapse. The finite element model, previously presented in another journal paper (Luo and Xiang, Engineering Structures, 2006) and already validated by comparisons with experimental results for simply supported beams, is here validated by comparisons with experimental results for continuous beams. The response results are clearly presented and critically discussed, giving new interesting insights into the ultimate state behavior of this structural typology. Thus, it is opinion of this Reviewer that manuscript BEENG-853R1 should be accepted for publication. However, some minor revisions are necessary to clarify some issues and improve some points, as commented below.	The authors deeply appreciate the reviewer's encouraging and valuable comments, which are very helpful in improving the quality of the article. In the revision, all of the comments raised by the reviewer have been taken into account.
	Technically	
1	1) Introduction The state of the art review in the introduction is rather incomplete. Although the Authors are not supposed to quote and comment every paper dealing with the nonlinear analysis of concrete beams prestressed with external tendons, they should at least acknowledge other papers that dealt in the past with similar issues, i.e., ultimate behavior of externally prestressed concrete bridges (simply supported and continuous) and the influence of tendon path and ordinary reinforcements. Just an example of two papers published on ASCE journals discussing similar structural aspects: Ramos, G., and Aparicio, A.C. (1996). Ultimate analysis of monolithic and segmental externally prestressed concrete bridges, Journal of Bridge Engineering ASCE, 1(1):10-17. Dall'Asta, A., Ragni, L., and Zona, A. (2007). Simplified method for failure analysis of concrete beams prestressed	In the revision, relevant papers including the two papers listed by the reviewers were acknowledged. (p.3, second paragraph)

	with external tendons, Journal of Structural Engineering ASCE, 133(1):121-131.	
2	2) Model Although the adopted finite element model was already presented in detail in a previous journal paper (Luo and Xiang, Engineering Structures, 2006), some important details should be more clearly discussed in the short review illustrated in the sub-paragraph "Description of the nonlinear analysis". Structural assumptions that are currently not clear: a) Please clarify if tendons can slip at deviators with or without friction (in the latter case give friction coefficient) or if tendon-deviator slips are prevented (if this is the case please give the stress difference between the tendon tracts with the highest and lowest stress); b) Please clarify the assumed strain field in the reinforced concrete beams, e.g., axial shortening included or only flexural deformations included? Is the coupling between axial and flexural deformations included within the geometric nonlinearity or the only geometric nonlinear effect is the variation of the tendon geometry? The clarification of the above points would greatly facilitate the understanding of the presented model to the interested reader, without forcing him/her to necessary go through the quoted paper (Luo and Xiang, Engineering Structures, 2006), at least in a first stage of his/her study.	a) In this study, the friction between external tendons and deviators is assumed negligible. This point was clarified in the revision. (p.5, last paragraph) b) In this study, the deformation field in the reinforced concrete beams includes both axial and flexural deformations. This point was clarified in the revision. (p.5, last paragraph) The geometric nonlinearity includes both the variation of the tendon eccentricity (second-order effects) and the coupling between axial and flexural deformations (large displacement effect). This point was clarified in the revision. (p.6, second paragraph)
3	3) Response results As already commented, this part presents some interesting results that give more insight into the nonlinear behavior up to collapse of continuous reinforced concrete beams with external prestressing tendon. The critical discussion on secondary reactions and relevant secondary bending moments is the main original contribution of manuscript BEENG-853R1. However, in the design of prestressing there are two fundamental variables: tendon layout and prestressing force. Attention in the submitted manuscript is focused on tendon layout, but no mention is made of the initial assigned prestressing force. While the Authors are not supposed to study every design parameter, they should at very least clarify the design criterion of the initial	In the revision, the design criterion of the initial prestressing force was clarified in accordance with the reviewer's suggestion. (p.8) Following the reviewer's suggestion, the authors have conducted the analysis of continuous beams with different levels of initial prestressing forces. Some typical results (load-deflection response, stress increase in external tendons and moment redistribution) were given at the end of this response letter, but were not presented in the manuscript due to length limits. This manuscript has almost been approaching the maximum length (10,000 word equivalent). The factors evaluated in this study include the amount of nonprestressed steel (over positive and negative moment regions), the pattern of loading (symmetrical and unsymmetrical), the type of beams (continuous and simple)

	prestressing force (maybe balance with the service state loads or balance of the self-weight?). In addition, it would be interesting to read the Authors' opinion regarding the influence of the initial prestressing force on the observed results. This last point could not be trivial as the structural system is nonlinear, due to both material behavior and geometric nonlinear effects.	and the layout of external tendons (concordant tendon profile and various non-concordant tendon profiles). These factors are exclusive for continuous beams, and in the authors' opinion, are the most representative to present the results of the investigation on the topic.

Numerical example: Influence of initial prestressing force

Beam CB3S, a two-span continuous beam with details shown in Fig. 4 and listed in Table 1 of the manuscript, is selected as the control beam for the numerical evaluation. Various tendon areas are used so as to produce different levels of initial prestressing force N_{p0}.
It can be seen in Figs. A1 and A2 that a higher level of the initial prestressing force leads to higher loads at cracking, yielding and ultimate, but develops lower ductility and stress increase in external tendons. From Fig. A3 and Table A1, it can be observed that a higher level of the initial prestressing force results in lower degree of moment redistribution.

Table A1 Influence of initial prestressing force on the degree of moment redistribution

N_{p0} (kN)	M (kN-m)		M_e (kN-m)		β (%)	
	Midspan	Center support	Midspan	Center support	Midspan	Center support
224	385.9	-272.8	327.5	-389.7	-17.83	30.00
448	490.9	-368.4	428.8	-492.5	-14.48	25.20
672	591.3	-459.8	526.0	-590.4	-12.41	22.12

Note: M=actual moment obtained from the nonlinear FEM analysis; M_e=elastic moment obtained from the elastic analysis; β=degree of moment redistribution=1-M/M_e

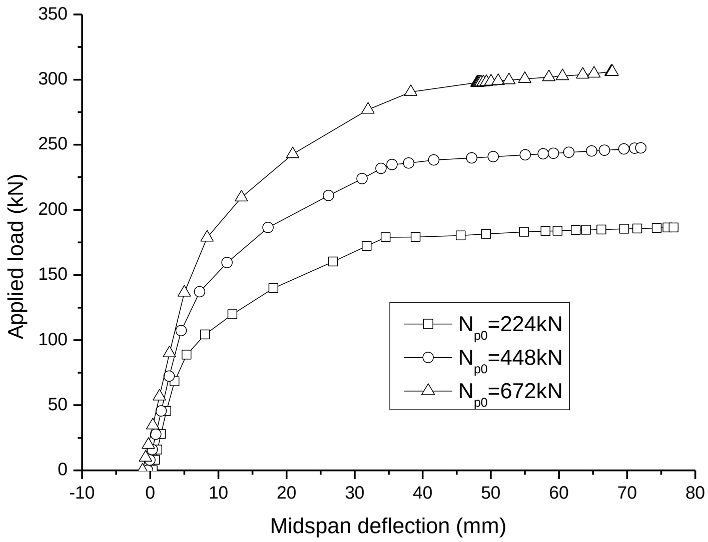

Fig. A1 Influence of initial prestressing force on the load-deflection response

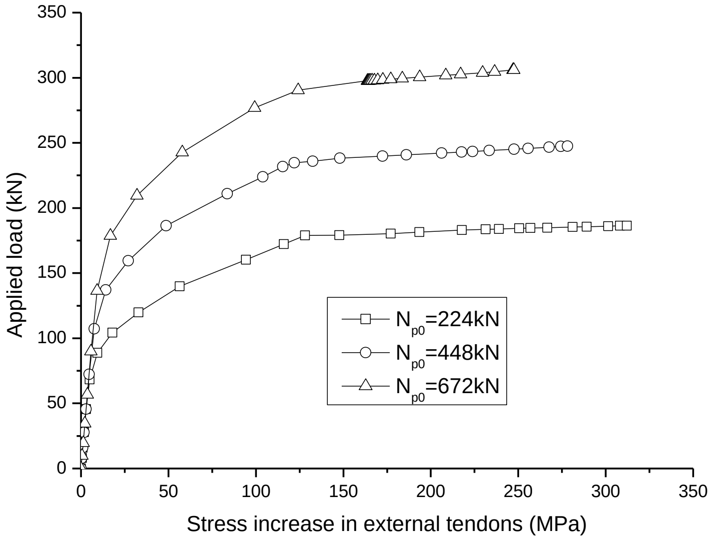

Fig. A2 Influence of initial prestressing force on the stress increase in external tendons

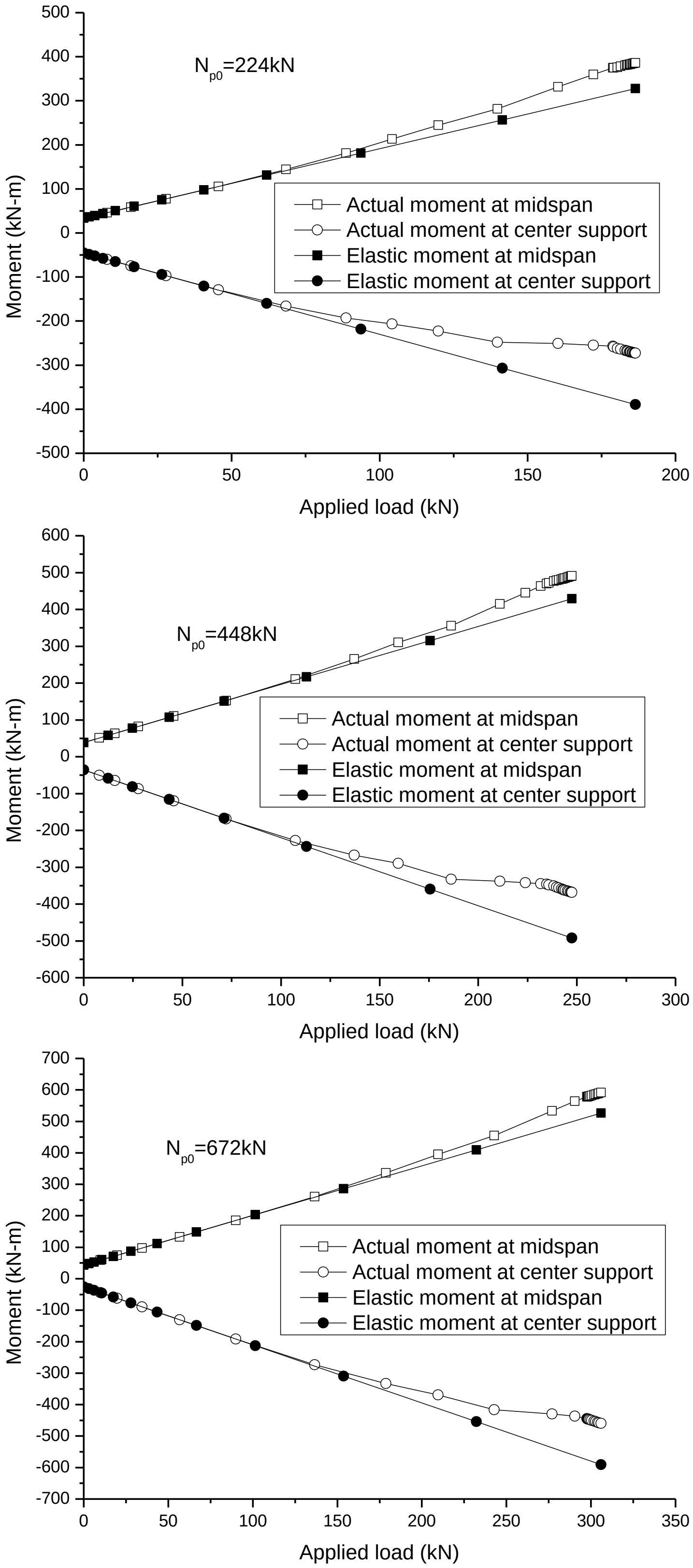

Fig. A3 Influence of initial prestressing force on the moment redistribution