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## A comparative study of continuous beams prestressed with bonded FRP and steel tendons

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### ABSTRACT

This paper presents a numerical investigation of the performance of continuous prestressed concrete beams with bonded fiber reinforced polymer (FRP) and steel tendons. A finite element model has been developed and is validated against the available test data. A numerical test is carried out on two-span continuous bonded prestressed concrete beams. Three types of tendons are considered for a comparative study: aramid FRP (AFRP), carbon FRP (CFRP) and prestressing steel. Various levels of  $\omega_p$  (reinforcement index of prestressing tendons) are used. The results indicate that, with increasing  $\omega_p$ , the failure mode of FRP prestressed concrete beams would transit from tendon rupture to concrete crushing while the crushing failure always takes place in steel prestressed concrete beams. Moreover, FRP tendons mobilize significantly different behavior regarding the crack pattern, deformation characteristics and neutral axis depth compared to steel tendons. The moment redistribution at ultimate in AFRP prestressed concrete beams is comparable to that in steel prestressed concrete beams while, at a low  $\omega_p$  level, CFRP tendons register obviously lower redistribution than steel tendons.

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### 1. Introduction

Fiber reinforced polymer (FRP) composites have been widely employed for reinforcing concrete structures instead of traditional steel reinforcement [1–3]. FRP materials offer attractive benefits, including high tensile strength, noncorrosive and nonmagnetic properties, favorable fatigue resistance and low weight [4]. In the field of prestressing systems, FRP tendons are a promising alternative to traditional steel tendons which are susceptible to corrosive damage. One of the primary concerns for practical applications of FRP tendons is related to anchorages [5]. Over past years, many studies have been undertaken regarding the anchorage systems for FRP tendons [6–10]. There are three basic composite materials that may be used for prestressing tendons: glass FRP (GFRP), aramid FRP (AFRP) and carbon FRP (CFRP). GFRP composites are not recommended for bonded tendons because of the poor resistance to alkaline environment and also to creep under sustained loads [11]. AFRP and CFRP composites are both desired for composite tendons and have been widely used for prestressing applications.

FRP tendons possess different material properties compared to steel tendons. FRP composites are brittle in nature, with linear elastic behavior up to rupture. AFRP tendons have usually a much

lower modulus of elasticity than steel tendons, while the elastic modulus of CFRP tendons covers a wide range from 80 to 500 MPa [12]. Therefore, the common knowledge established from conventional steel prestressed concrete beams may not be applicable to FRP prestressed concrete beams. Many efforts have been made to understand the behavior of concrete beams prestressed with FRP tendons. Pisani [13] described a numerical study on the flexural behavior of simply supported bonded and unbonded prestressed concrete beams with steel and FRP tendons. Park and Naaman [14] reported the test results regarding the shear behavior of CFRP prestressed concrete simply supported beams. Toutanji and Saafi [15] tested a series of four simply supported concrete beams prestressed AFRP tendons to study the flexural performance of these beams. The tests indicated that the use of combined bonded and unbonded AFRP tendons or of additional nontensioned rebars can improve notably the ductile behavior. Stoll et al. [16] presented the results of a test program performed on two full-scale simply supported high-strength concrete bridge girders prestressed with CFRP tendons. Dolan and Swanson [17] conducted a theoretical and experimental study on the flexural capacity of simply supported beams prestressed with vertically distributed FRP tendons. Kim [18] examined the effect of prestress level on the behavior of simply supported AFRP prestressed concrete beams and also checked the applicability of several design codes and existing predictive equations.

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84 Although extensive works have been carried out regarding the  
85 behavior of prestressed concrete beams with bonded FRP tendons,  
86 most of the past works dealt only with the simply support condition.  
87 Few attempts have so far been made to investigate the behavior of  
88 continuous FRP prestressed beams. The authors [19–21] have  
89 recently conducted a set of theoretical studies to evaluate the  
90 response of continuous external FRP tendon systems, with particular  
91 emphasis placed on the redistribution of moments. Because of the  
92 bond effects and nonexistence of second-order effects, internal  
93 bonded tendon systems would behave differently from external  
94 tendon systems.  
95 This paper describes a numerical study that is conducted to  
96 reveal the flexural behavior of two-span continuous concrete

beams prestressed with internal bonded FRP and steel tendons. A  
numerical model for geometric and material nonlinear analysis of  
bonded prestressed concrete continuous beams has been developed.  
The proposed model is verified with the experimental results  
available in the literature. A comparative study on the performance  
of AFRP, CFRP and steel prestressed concrete continuous beams is  
carried out by using the proposed model. Various amounts of  
prestressing tendons are used for the numerical evaluation. Some  
findings are concluded.

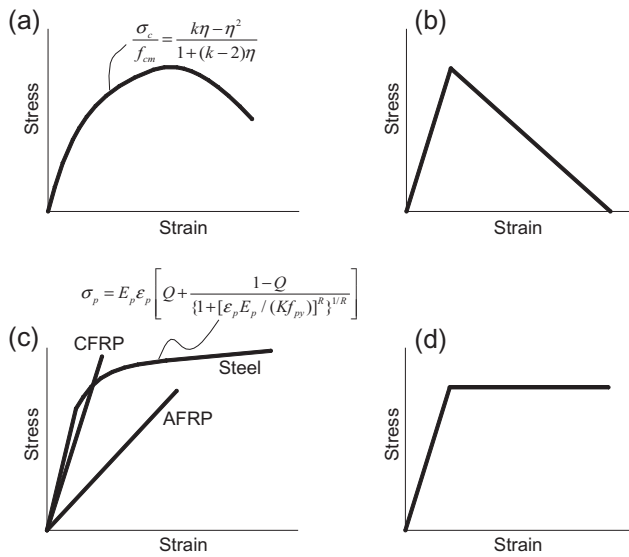


Fig. 1. Stress-strain curves of materials. (a) Concrete in compression; (b) concrete in tension; (c) prestressing tendons; (d) nonprestressed steel.

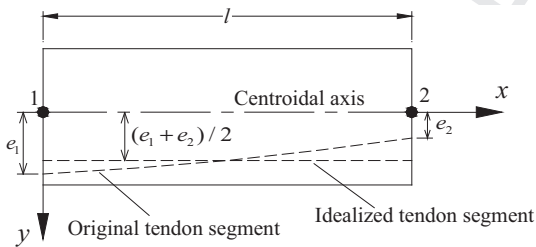


Fig. 2. Beam element with idealized tendon segment.

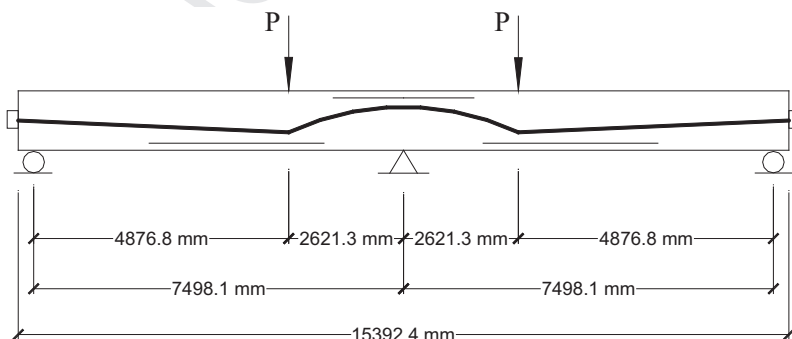


Fig. 3. Test beams by Lin [26].

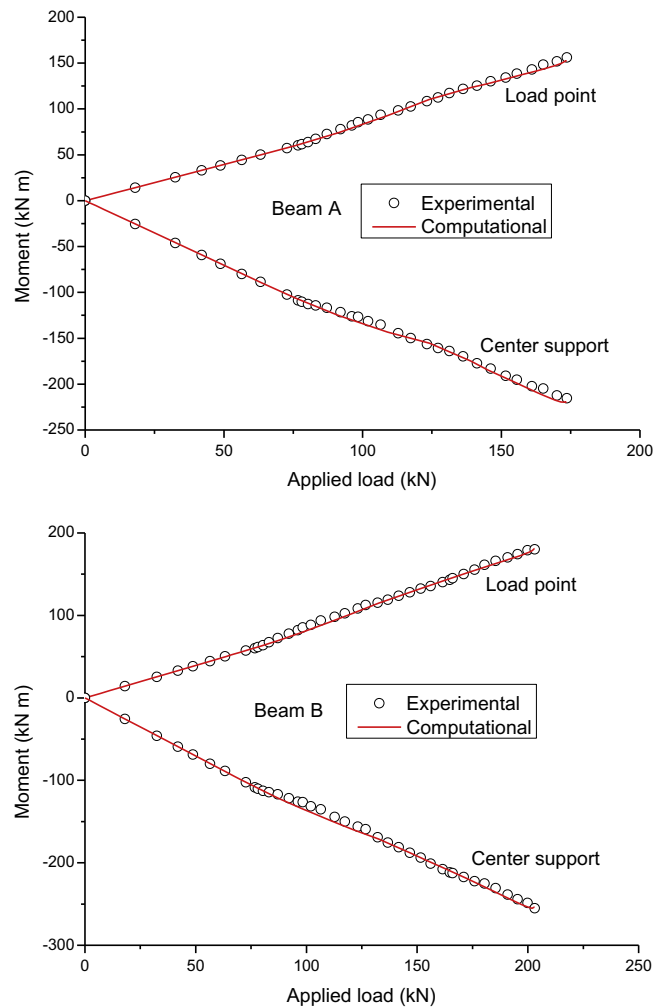


Fig. 4. Comparison between experimental and computational results regarding the load-moment response for the test beams.

2. Nonlinear model

2.1. Stress–strain curves of materials

A stress–strain equation for concrete in uniaxial compression is recommended in Eurocode 2 [22] for structural analysis of concrete members. The stress–strain curve is shown in Fig. 1(a), where  $\eta = \varepsilon_c / \varepsilon_{c0}$ ;  $\sigma_c$  and  $\varepsilon_c$  are the concrete stress and strain, respectively;  $k$  is a coefficient depending on the concrete modulus of elasticity  $E_c$ , strain at peak stress  $\varepsilon_{c0}$  and mean compressive strength  $f_{cm}$ . The concrete is assumed to be crushed when its strain reaches the specified ultimate compressive strain  $\varepsilon_u$ .

The concrete in tension is assumed to be linear elastic prior to cracking and linear strain-softening after cracking, as shown in Fig. 1(b). The concrete tensile strain at the end of strain-softening is taken as  $10 \varepsilon_{cr}$ , where  $\varepsilon_{cr}$  is the cracking strain.

The stress–strain equation for prestressing steel proposed by Menegotto and Pinto [23] is used in this study. The stress–strain curve is shown in Fig. 1(c), where  $\sigma_p$  and  $\varepsilon_p$  are the tendon stress and strain, respectively;  $E_p$  is the tendon modulus of elasticity;  $f_{py}$  is the yield stress of prestressing steel; and  $K, Q$  and  $R$  are empirical parameters which can be determined by experimental data.

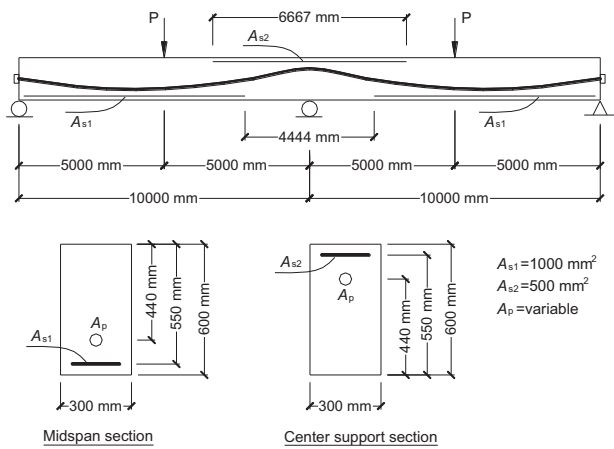


Fig. 5. Details of continuous prestressed concrete beam for numerical evaluation.

Table 1  
Material properties of FRP and steel tendons.

Tendons	Tensile strength (MPa)	Elastic modulus (GPa)	Ultimate strain (%)
AFRP	1500	68	2.2
CFRP	1840	147	1.25
Steel	1860	195	>3,5

Table 2  
Normalized ultimate tendon stress and concrete strains as well as failure mode of the beams.

Tendons	$\omega_p$	$\sigma_p / f_{pu}$	$\sigma_p / f_f$	$\varepsilon_{c1} / \varepsilon_u$	$\varepsilon_{c2} / \varepsilon_u$	Failure mode
AFRP	0.024	–	1.0	0.56	0.62	Rupture
	0.048	–	1.0	0.68	0.72	Rupture
	0.084	–	1.0	0.85	0.87	Rupture
	0.108	–	0.99	1.0	0.97	Crushing
CFRP	0.144	–	0.91	1.0	0.99	Crushing
	0.204	–	0.83	1.0	0.99	Crushing
	0.024	–	1.0	0.45	0.51	Rupture
	0.048	–	1.0	0.6	0.63	Rupture
	0.084	–	1.0	0.84	0.82	Rupture
	0.108	–	0.98	1.0	0.98	Crushing
	–	–	–	–	–	Approx. rupture
Steel	0.144	–	0.89	1.0	0.98	Crushing
	0.204	–	0.77	1.0	1.0	Crushing
	0.024	0.97	–	1.0	0.89	Crushing
	0.048	0.96	–	1.0	0.89	Crushing
	0.084	0.95	–	1.0	0.92	Crushing
	0.108	0.93	–	1.0	0.97	Crushing
	0.144	0.89	–	1.0	0.98	Crushing
	0.204	0.78	–	1.0	1.0	Crushing

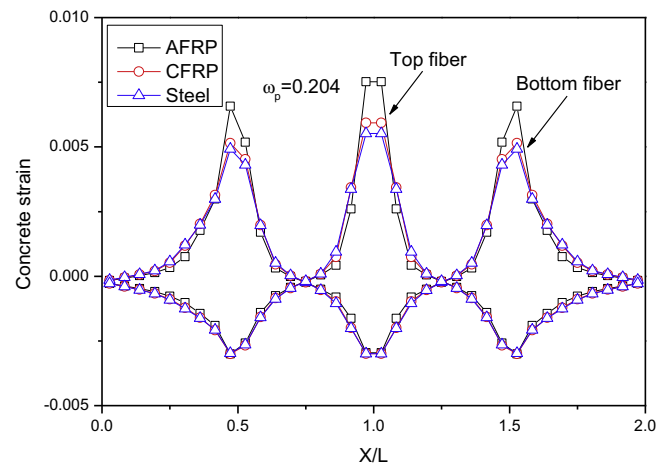
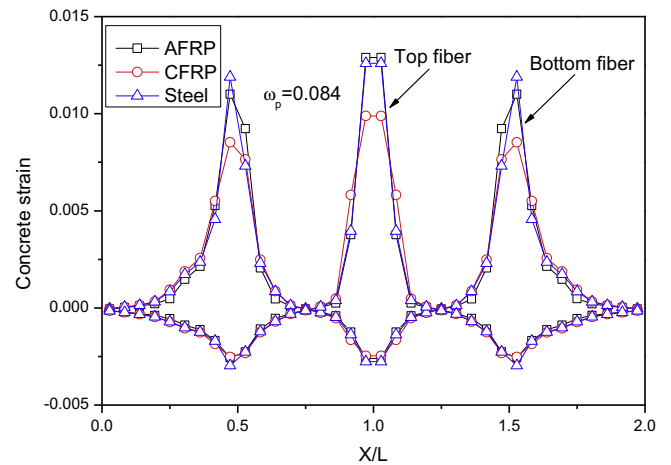
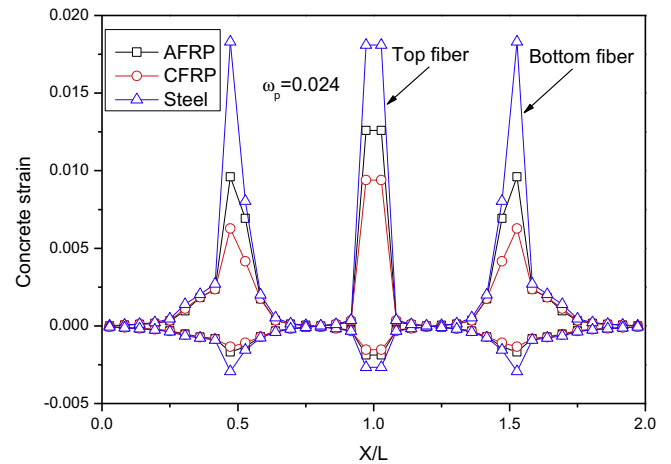


Fig. 6. Concrete strain distribution over the length for different tendon types and various  $\omega_p$  levels.

The prestressing steel used in the present numerical investigation is assumed to be Grade 270, 7-wire low relaxation strands and the values of  $K$ ,  $Q$  and  $R$  are 1.0618, 0.01174 and 7.344, respectively. The prestressing FRP tendons are linear elastic up to rupture, as shown in Fig. 1(c).

The nonprestressed steel is assumed to be linear elastic prior to yielding and perfectly plastic after yielding, as shown in Fig. 1(d).

2.2. Finite element model

A finite element model based on the Euler–Bernoulli beam theory has been developed. The model is an extension of a previously

developed model for unbonded prestressed concrete continuous beams [24], in which the contribution of unbonded tendons was not included in the stiffness matrix but was made by transforming the prestressing force into equivalent nodal loads. For analysis of bonded prestressed concrete beams, however, the contribution of bonded tendons to the stiffness matrix must be considered because of the compatibility of strains between prestressing tendons and the surrounding concrete.

Consider a two-node plane beam element to be described in the local coordinate system  $(x, y)$ , as shown in Fig. 2. The node points are assumed to be at the centroid of the cross-sections. The prestressing tendon segment in the beam element is idealized to be

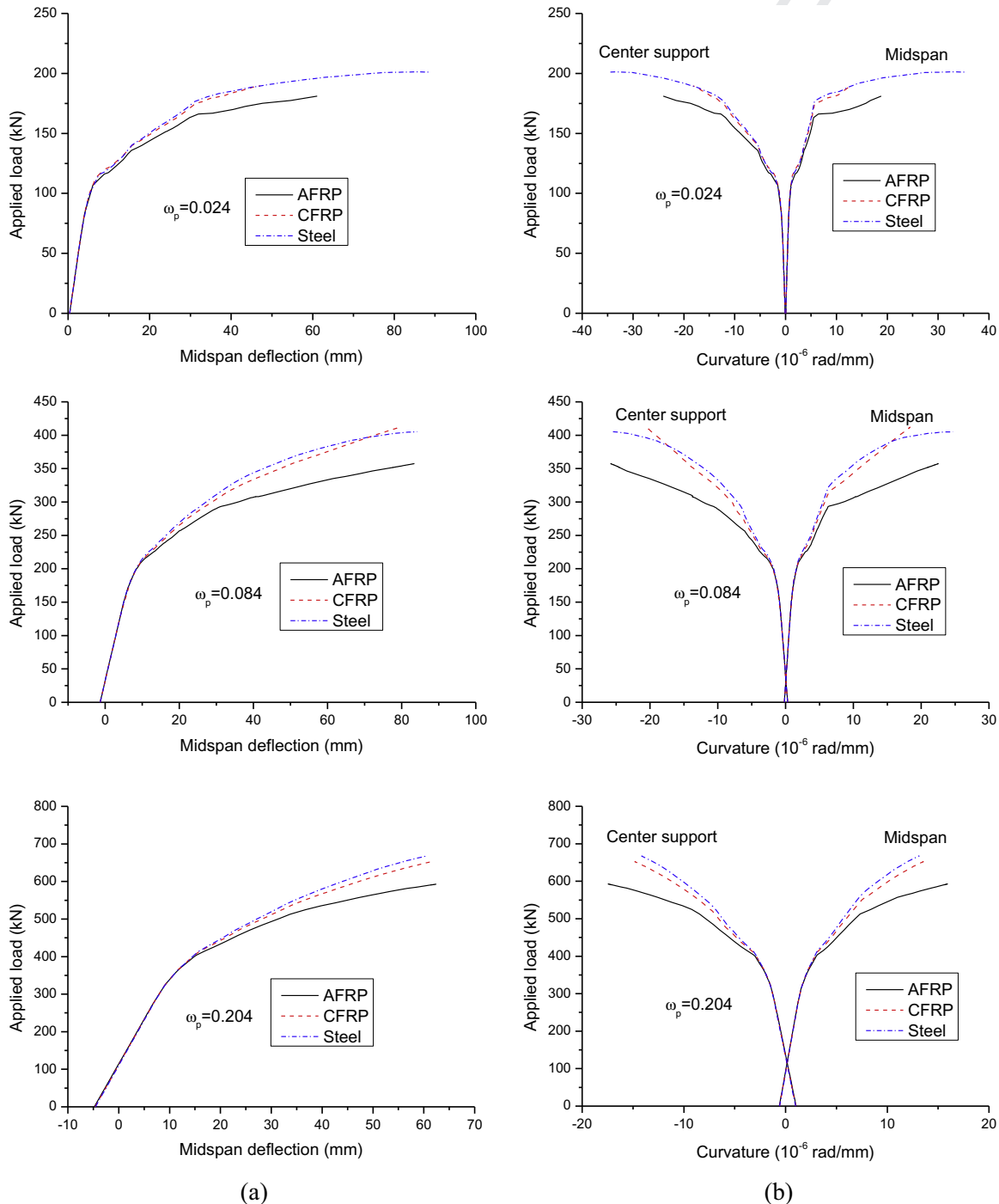


Fig. 7. Load-deformation response for different tendon types and various  $\omega_p$  levels. (a) Load–deflection response; (b) load–curvature response.

parallel with the centroidal axis, with eccentricity equal to the mean value of the eccentricities at the end nodes. Each node has three degrees of freedom: axial displacement  $u$ , transverse displacement  $v$ , and rotation  $\theta$ . Assuming that  $u$  and  $v$  are a linear function and a cubic polynomial of  $x$ , respectively, they can be expressed in terms of element nodal displacements as follows:

$$u = (1 - \xi)u_1 + \xi u_2 \quad (1a)$$

$$v = (1 - 3\xi^2 + 2\xi^3)v_1 + l\xi(1 - \xi)^2\theta_1 + (3\xi^2 - 2\xi^3)v_2 + l(-\xi^2 + \xi^3)\theta_2 \quad (1b)$$

where  $l$  = length of the beam element; and  $\xi = x/l$ .

The axial strain  $\varepsilon_0$  on any point of the beam element is given by:

$$\varepsilon_0 = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial v}{\partial x} \right)^2 \quad (2)$$

The second term of the right side of the above equation represents the large displacement effects. Assuming that the shear deformation of the element is negligible, which is a reasonable approximation for slender (Euler–Bernoulli) beams commonly used for prestressing applications, the section curvature  $\phi$  is expressed by

$$\phi = -\frac{\partial^2 v}{\partial x^2} \quad (3)$$

Assuming that a plane section remains plane during bending and that perfect bond exists between the reinforcement and the surrounding concrete, the strain  $\varepsilon$  at any fiber of the section is given by:

$$\varepsilon = \varepsilon_0 + y\phi \quad (4)$$

The element nodal displacement vector  $\mathbf{u}^e$  and element nodal force vector  $\mathbf{P}^e$  may be written as

$$\mathbf{u}^e = \{u_1 \quad v_1 \quad \theta_1 \quad u_2 \quad v_2 \quad \theta_2\}^T \quad (5)$$

$$\mathbf{P}^e = \{N_1 \quad V_1 \quad M_1 \quad N_2 \quad V_2 \quad M_2\}^T \quad (6)$$

According to the total Lagrangian description, the tangent stiffness equations for an element can be determined by applying the principle of virtual work as follows:

$$d\mathbf{P}^e = \mathbf{K}_t^e d\mathbf{u}^e = (\mathbf{K}_1^e + \mathbf{K}_2^e + \mathbf{K}_3^e) d\mathbf{u}^e \quad (7)$$

$$\mathbf{K}_1^e = \int_l \mathbf{B}_l^T \mathbf{D}_t \mathbf{B}_l dx \quad (8a)$$

$$\mathbf{K}_2^e = \int_l \mathbf{B}_l^T \mathbf{D}_t \mathbf{B}_n dx + \int_l \mathbf{B}_n^T \mathbf{D}_t \mathbf{B}_l dx + \int_l \mathbf{B}_n^T \mathbf{D}_t \mathbf{B}_n dx \quad (8b)$$

$$\mathbf{K}_3^e = \int_l \mathbf{N} \mathbf{J}^T \mathbf{J} dx \quad (8c)$$

$$\mathbf{B}_l = \begin{bmatrix} -\frac{1}{l} & 0 & 0 & \frac{1}{l} & 0 & 0 \\ 0 & \frac{6}{l^2} - \frac{12\xi}{l} & \frac{4}{l} - \frac{6\xi}{l} & 0 & -\frac{6}{l^2} + \frac{12\xi}{l} & \frac{2}{l} - \frac{6\xi}{l} \end{bmatrix} \quad (9a)$$

$$\mathbf{B}_n = [1 \quad 0]^T \mathbf{u}^e \mathbf{J}^T \mathbf{J} \quad (9b)$$

$$\mathbf{J} = \begin{bmatrix} 0 & -\frac{6\xi}{l} + \frac{6\xi^2}{l} & 1 - 4\xi + 3\xi^2 & 0 & \frac{6\xi}{l} - \frac{6\xi^2}{l} & -2\xi + 3\xi^2 \end{bmatrix} \quad (10)$$

$$\mathbf{D}_t = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \quad (11)$$

$$d_{11} = \sum_i E_{tci} A_{ci} + \sum_j E_{tsj} A_{sj} + \sum_k E_{tpk} A_{pk} \quad (12a)$$

$$d_{12} = d_{21} = \sum_i E_{tci} A_{ci} y_{ci} + \sum_j E_{tsj} A_{sj} y_{sj} + \sum_k E_{tpk} A_{pk} y_{pk} \quad (12b)$$

$$d_{22} = \sum_i E_{tci} A_{ci} y_{ci}^2 + \sum_j E_{tsj} A_{sj} y_{sj}^2 + \sum_k E_{tpk} A_{pk} y_{pk}^2 \quad (12c)$$

$$N = \sum_i \sigma_{ci} A_{ci} + \sum_j \sigma_{sj} A_{sj} + \sum_k \sigma_{pk} A_{pk} \quad (13)$$

where the summation symbol signifies that the cross section is divided into layers to employ a layered approach; the subscripts  $ci$ ,  $sj$  and  $pk$  represent the  $i$ th concrete layer,  $j$ th nonprestressed steel layer and  $k$ th tendon layer, respectively;  $E_t$  is the tangent modulus for materials;  $A$  corresponds to the area and  $\sigma$  corresponds to the stress.

After assembling the equilibrium equations for the structure in the global coordinate system and imposing an appropriate boundary condition, a load or displacement control incremental method in combination with the Newton–Raphson iterative algorithm is used for the solution of the nonlinear equilibrium equations. During the solution process, when any of the constituent materials reaches its ultimate strain or strength capacity, failure is assumed to take place and the analysis is therefore terminated. The proposed model is capable of conducting the geometric and material nonlinear analysis of continuous concrete beams prestressed with bonded steel and FRP tendons over the entire loading range up to the ultimate. Time-dependent effects such as tendon relaxation and concrete creep are not covered in the present study, but the modeling of these effects may be seen elsewhere [25].

### 3. Verification of the proposed model

In order to validate the proposed nonlinear model, two of the continuous prestressed concrete beams tested by Lin [26] have been analyzed. These beams were designated as Beams A and B, and were tested under static loads up to failure. The structure and section details of the specimens are shown in Fig. 3. The beams were designed to be identical except for the content of nonprestressed steel: Beam B contained two 14 mm tensile steel bars over the midspan and center support regions while Beam A did not. The beams had a rectangular section (203.2 × 406.4 mm), and were 15392.4 mm long with two equal spans to which two concentrated loads were applied at 2621.3 mm from the center support. The tendon profile, which was designed to be concordant, was curved over the center support region and straight over other regions. The tendon consisted of 32 parallel 5 mm high-strength steel wires. The

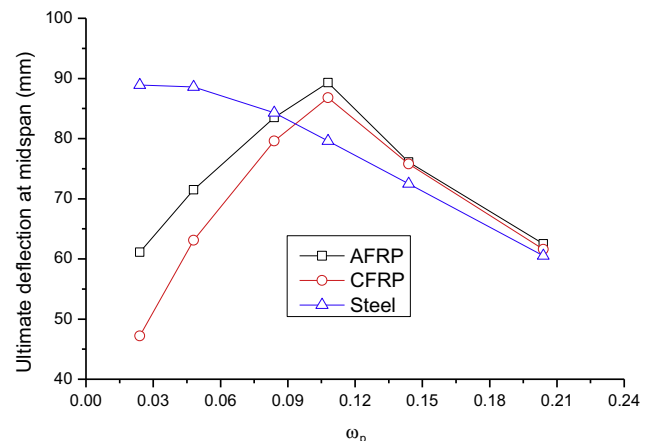


Fig. 8. Variation of ultimate deflection with  $\omega_p$  level for different tendon types.

262 ultimate tensile strength, yield stress and elastic modulus of the  
263 prestressing tendon were 1765, 1572 MPa and 200 GPa, respec-  
264 tively. The yield strength and elastic modulus of nonprestressed  
265 steel were 314 MPa and 196 GPa, respectively. The cylinder com-  
266 pressive strengths of concrete at age of 28 days were 36.2 MPa  
267 for Beam A and 41.3 MPa for Beam B. The tendon was tensioned  
268 to have an initial prestress of 979 MPa when the beams were  
269 14 days old. About two weeks later the beams were loaded up to  
270 collapse.

271 Fig. 4 shows a comparison between experimental and computa-  
272 tional results regarding the load versus moment responses at the  
273 center support and load point for the two test beams. The experi-

274 mentally obtained moments were calculated according to the  
275 experimental values of the reaction at the end support reported  
276 in the literature. It can be seen in the graphs that proposed analysis  
277 reproduces with remarkable accuracy the moment evolution in  
278 both of the continuous prestressed concrete beams throughout  
279 the loading history up to failure.

280 **4. Numerical investigation**

281 A bonded prestressed concrete rectangular beam continuous  
282 over two equal spans to which two centre-point loads are symmet-  
283 rically applied, as shown in Fig. 5, is used as a control beam for the

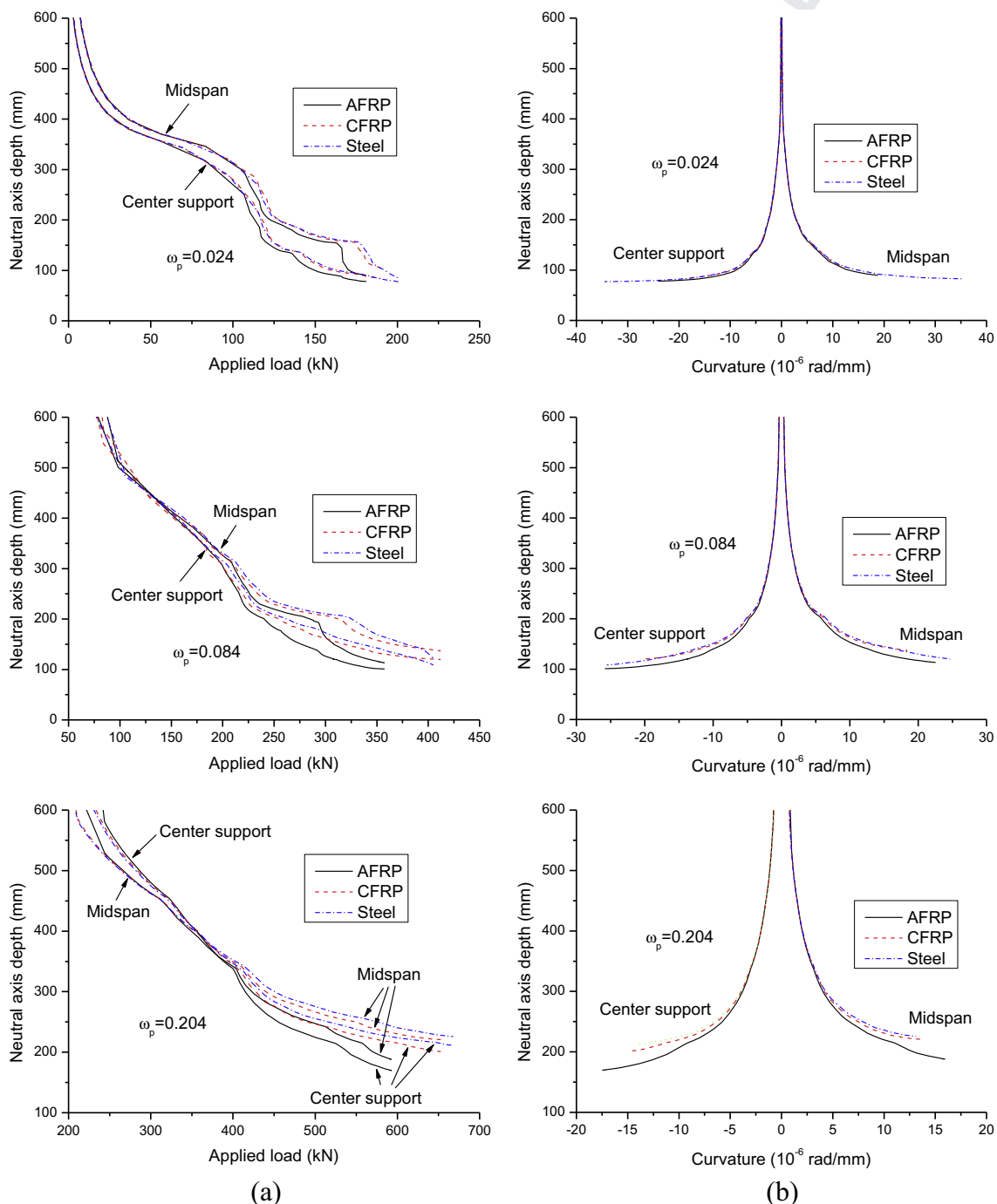


Fig. 9. Development of neutral axis depth for different tendon types and various  $\omega_p$  levels. (a) Load versus neutral axis depth; (b) curvature versus neutral axis depth.

numerical evaluation. The prestressing tendons are assumed to have an idealized parabolic profile with eccentricities at the end support, midspan and center support of 0, 140 and 140 mm, respectively. The beam contains nonprestressed tensile steel over the positive moment region ( $A_{s1} = 1000 \text{ mm}^2$ ) and negative moment region ( $A_{s2} = 500 \text{ mm}^2$ ). The yield strength and elastic modulus of nonprestressed steel are 450 MPa and 200 GPa, respectively. The concrete characteristic cylinder compressive strength  $f_{ck}$  (note:  $f_{ck} = f_{cm} - 8$ ) is taken as 60 MPa. Three types of prestressing tendons are used: AFRP, CFRP and steel tendons. The material properties (tensile strength, elastic modulus and ultimate strain) for each type of tendons are summarized in Table 1. The yield stress of prestressing steel is taken as 90% of the tensile strength. The initial prestress  $f_{p0}$  is taken as 950 MPa. Various tendon areas  $A_p$  are used so as to generate various levels of reinforcement index of prestressing tendons  $\omega_p$ , which is defined as

$$\omega_p = \frac{A_p f_{p0}}{b d_p f_{ck}} \quad (14)$$

where  $b$  is the section width and  $d_p$  is the effective depth of prestressing tendons at the center support or midspan. In this study, the  $\omega_p$  level ranges from a minimum of 0.024 to a maximum of 0.204. In the finite element idealization, the concrete beam is divided into 36 two-node beam elements (18 elements for each span) with length of 555.56 mm each. Each element is subdivided into 10 concrete layers, 1 tendon layer and 1 nonprestressed steel layer.

#### 4.1. Failure mode and crack pattern

A summary of the normalized tendon stress ( $\sigma_p/f_{pu}$  or  $\sigma_p/f_f$ ) and concrete strains ( $\varepsilon_{c1}/\varepsilon_u$  and  $\varepsilon_{c2}/\varepsilon_u$ ) at failure as well as the failure mode of the beams is given in Table 2, where  $\sigma_p$  is the ultimate stress in tendons;  $f_{pu}$  and  $f_f$  are the tensile strengths of steel and FRP tendons, respectively;  $\varepsilon_{c1}$  and  $\varepsilon_{c2}$  are the concrete strains at ultimate in extreme compressive fiber of the midspan and center support sections, respectively. A normalized ultimate tendon stress of 1.0 indicates a rupture failure while a normalized ultimate concrete strain of 1.0 signifies a crushing failure. According to the present numerical tests, all the steel prestressed concrete beams have failed due to crushing of concrete at the critical section. At the lowest  $\omega_p$  level of 0.024, failure of the steel prestressed beam takes place at midspan while at failure the center support section is still far below its ultimate strain capacity. As  $\omega_p$  increases, the exploitation of the center support is enhanced. At the highest  $\omega_p$  level of 0.204, the concrete is crushed both at midspan and at the center support. For FRP prestressed concrete beams, on the other hand, failure may take place either due to concrete crushing or tendon rupture, depending on the  $\omega_p$  level. According to the present analysis, the tendon rupture and concrete crushing take place simultaneously when  $\omega_p$  reaches around 0.108. A rupture failure appears for  $\omega_p$  lower than 0.108 while a crushing failure occurs for  $\omega_p$  higher than 0.108. At a low  $\omega_p$  level (e.g., 0.024), the critical sections of a FRP prestressed concrete beam are far below their ultimate capacities when the tendons are ruptured. At a high  $\omega_p$  level (e.g., 0.204), on the other hand, the FRP tendons are far below their ultimate tensile strength when the concrete is crushed.

At the failure loads, the concrete strain distribution over the length for different tendon types and various  $\omega_p$  levels is displayed in Fig. 6, where  $X/L$  is the ratio of the distance from the end support to the span. From the graphs in this figure the crack pattern may be deduced. At  $\omega_p$  of 0.024, the steel prestressed concrete beam develops very large tensile strains in the critical positive and negative moment regions, while the tensile strains over other regions are small. This observation indicates that there are a few main cracks with large widths in the critical regions while the cracks in other

regions are insignificant. In other words, there appears crack concentration in the beam containing a low amount of steel reinforcement. As  $\omega_p$  increases, the crack pattern of the steel prestressed concrete beam appears to be improved, that is, the maximum crack widths are reduced and the crack zones are extended. The crack pattern for FRP prestressed concrete beams is related to the failure mode. At a low  $\omega_p$  level (e.g., 0.024), failure is caused by rupture of FRP tendons and, therefore, crack concentration is not as important as the case of the steel prestressed concrete beam. At a high  $\omega_p$  level (e.g., 0.204), crushing failure happens. In this case, the crack pattern of the CFRP prestressed concrete beam is very similar to that of the steel prestressed concrete beam, while the AFRP prestressed concrete beam develops obviously bigger crack widths at the critical regions than the CFRP or steel prestressed concrete beam.

#### 4.2. Deformation characteristics

Prior to the application of external loads, there are initial deformations as a result of combined prestressing and self-weight. At the lowest  $\omega_p$  level of 0.024, the self-weight effect prevails over the prestressing effect, thereby resulting in a downward deflection of the beam as well as a sagging curvature at midspan and a hogging curvature at the center support. For  $\omega_p$  greater than 0.048, the prestressing effect overrides the self-weight effect, causing a camber of the beam as well as a hogging curvature at midspan and a sagging curvature at the center support. The entire load-deflection and load-curvature responses for different tendon types and various  $\omega_p$  levels are shown in Fig. 7(a) and (b), respectively. Before cracking, the behavior is primarily controlled by concrete and, therefore, the responses for different tendon types are almost identical. After cracking, the contribution of prestressing reinforcement is increasingly important. Due to lower modulus of elasticity, AFRP tendons mobilize smaller bending stiffness of a beam than CFRP or steel tendons. At a high  $\omega_p$  level (in the case of crushing failure), the entire load-deformation response characteristics of the CFRP prestressed beam are similar to those of the steel prestressed beam. However, at a low  $\omega_p$  level, the load-deformation behavior of FRP prestressed beams may be undesirable because of premature failure caused by tendon rupture.

Fig. 8 shows the variation of ultimate deflection with the  $\omega_p$  level for different tendon types. At the lowest  $\omega_p$  level of 0.024, the ultimate deflections of AFRP and CFRP prestressed beams are respectively 31.3% and 46.9% lower than that of the steel prestressed beam. As  $\omega_p$  increases, the ultimate deflection for steel tendons gradually decreases while the ultimate deflections for

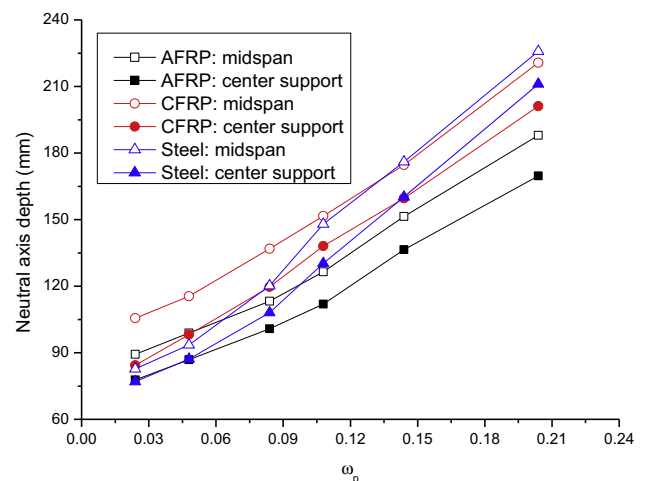
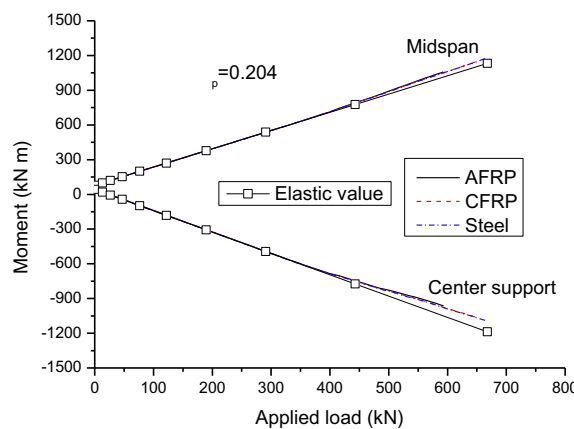
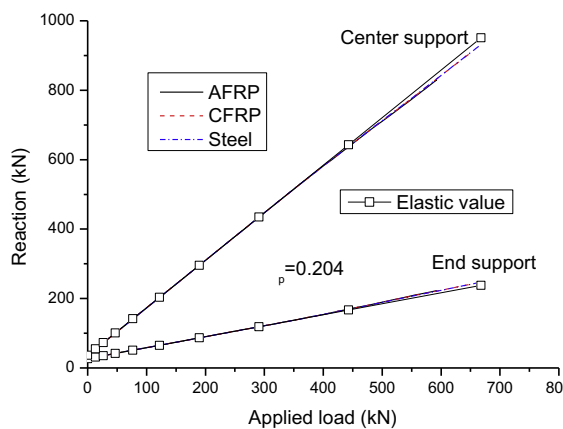
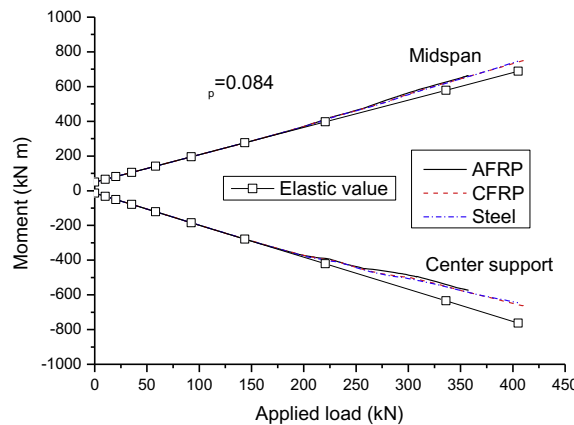
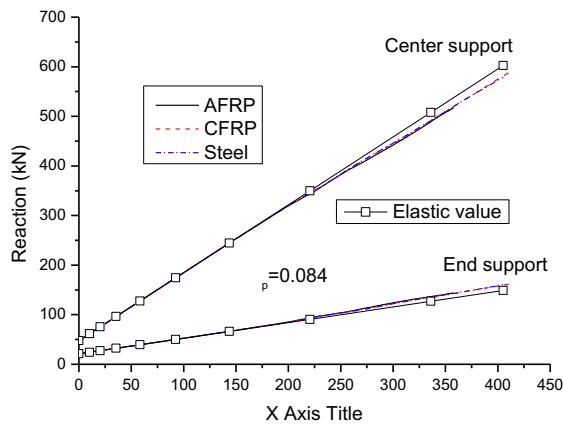
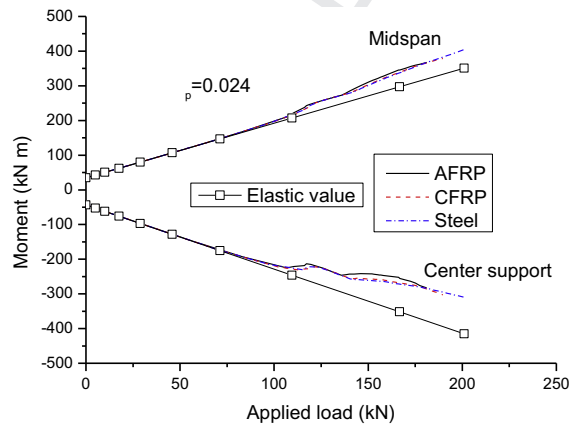
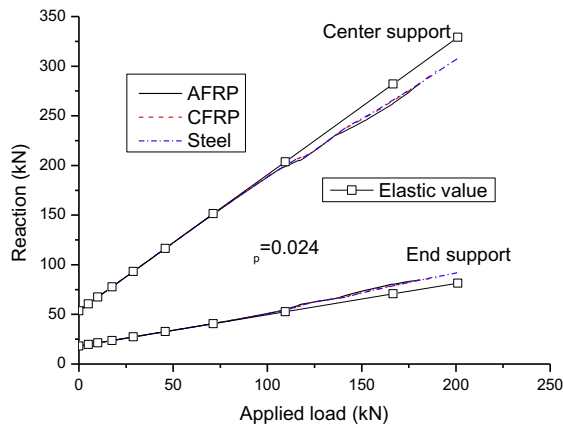


Fig. 10. Variation of neutral axis depth with  $\omega_p$  level for different tendon types.

FRP tendons increase quickly. When  $\omega_p$  reaches 0.108 (transition from a rupture failure to a crushing failure), the deflections for FRP tendons turn to be obviously higher than that for steel tendons. With the continuing increase of  $\omega_p$ , the deflections for both steel and FRP tendons decrease, but the decrease in deflection for steel tendons tends to be slower than those for FRP tendons. At the highest  $\omega_p$  level of 0.204, the discrepancy between the deflections for steel and FRP tendons appears to be insignificant. At a low level of  $\omega_p$ , the deflection for CFRP tendons is smaller than that for AFRP tendons, but the difference gradually diminishes with the increase of  $\omega_p$ , and appears to be not noticeable as  $\omega_p$  increases to 0.144 or above.

4.3. Neutral axis depth

The initial neutral axis depth for a prestressed concrete beam under prestressing and self-weight may be positive or negative, depending on the  $\omega_p$  level. At  $\omega_p$  of 0.024, the initial neutral axis depths are about 900 mm at midspan and 680 mm at the center support, that is, the neutral axis is initially located below the bottom fiber of the midspan section and above the top fiber of the center support section. At  $\omega_p$  of 0.204, the initial neutral axis depths are about -50 mm at midspan and 90 mm at the center support, that is, the neutral axis is initially located outside the midspan section (above the top fiber) while inside the center support section.



(a)

(b)

Fig. 11. Development of support reaction and bending moment for different tendon types and various  $\omega_p$  levels. (a) Load-reaction response; (b) load-moment response.



The neutral axis moves rapidly as the external loads are applied and gradually increased.

The evolution of neutral axis depth with the applied load and curvature, after the neutral axis reaches the extreme tension fiber of the section, for different tendon types and various  $\omega_p$  levels is shown in Fig. 9(a) and (b), respectively. It is seen in Fig. 9(a) that at a low  $\omega_p$  level, the development of neutral axis depth exhibits a significantly nonlinear manner and the neutral axes at the center support and midspan move at almost the same rate. As  $\omega_p$  increases, the nonlinear manner is obviously reduced and the movement of neutral axis at the center support may become faster than that at midspan. After the evolution of cracks stabilizes, the shift of neutral axis in the AFRP prestressed concrete beam appears to be faster than that in the CFRP or steel prestressed concrete beam. From Fig. 9(b) it is seen that the neutral axis moves rapidly at first but the movement tends to slow down as the hogging or sagging curvature increases. After stabilizing of the crack development, at a given curvature, AFRP tendons mobilizes lower neutral axis depth than CFRP or steel tendons, particularly obvious at a high  $\omega_p$  level.

Fig. 10 shows the variation of neutral axis depth at ultimate with the  $\omega_p$  level for different tendon types. It is seen that the neutral axis depth increases as  $\omega_p$  increases. The rate of increase in neutral axis depth for steel tendons tends to be faster than that for FPP tendons. At a given  $\omega_p$  level, the midspan section develops higher neutral axis depth than the center support section. AFRP tendons mobilize lower neutral axis depth at the midspan or center support section than CFRP tendons. At a low  $\omega_p$  level, the neutral axis depth for steel tendons is close to that for AFRP tendons, while at a high  $\omega_p$  level the neutral axis depths for steel and CFRP tendons are comparable.

#### 4.4. Development of support reaction and bending moment

The development of reactions at the end and center supports for different tendon types and various  $\omega_p$  levels is illustrated in Fig. 11(a), while the evolution of bending moments at the midspan and center support is shown in Fig. 11(b). The elastic values, calculated based on the linear-elastic theory, are also displayed in the graphs. The secondary reactions and moments due to prestressing are included in the values shown in these graphs. The magnitude and direction of secondary reactions and moments are dependent on the layout of tendons. For the beams analyzed in this study, the tendon line is below its linearly transformed concordant line. As a result, the secondary reaction at the end support is positive but negative at the center support, causing positive secondary moments in the beams.

It can be seen in Fig. 11 that at initial loading, the actual reaction and moment develop linearly with the applied load, indicating there is no redistribution of moments in this elastic stage. After cracking, the actual values appear to deviate from the elastic ones due to redistribution of moments. Because cracking appears firstly at the center support, upon cracking the moments tend to be redistributed from the center support zone towards the midspan zone. As a consequence, there is a diminution of the rate of increase in reaction and moment at the center support and, correspondingly, a growth of the rate of increase in end support reaction and midspan moment. In the inelastic ranges, the deviation of the actual reaction and moment from the elastic values is notable at a low  $\omega_p$  level but tends to diminish as  $\omega_p$  increases, as can be observed in Fig. 11.

#### 4.5. Degree of moment redistribution

Fig. 12 shows the development of the degree of moment redistribution for different tendon types and various  $\omega_p$  levels. The

degree of moment redistribution  $\beta$  is defined by:  $\beta = 1 - (M/M_e)$ , where  $M$  and  $M_e$  are the actual and elastic moments corresponding to a load level, respectively. The degree of moment redistribution is equal to zero until cracking. After cracking, the redistribution of moments takes place. The redistribution is positive at the center support while negative at midspan. At low and medium  $\omega_p$  levels, the evolution of redistribution after cracking is obviously affected by several typical phases, namely, the stabilization of crack devel-

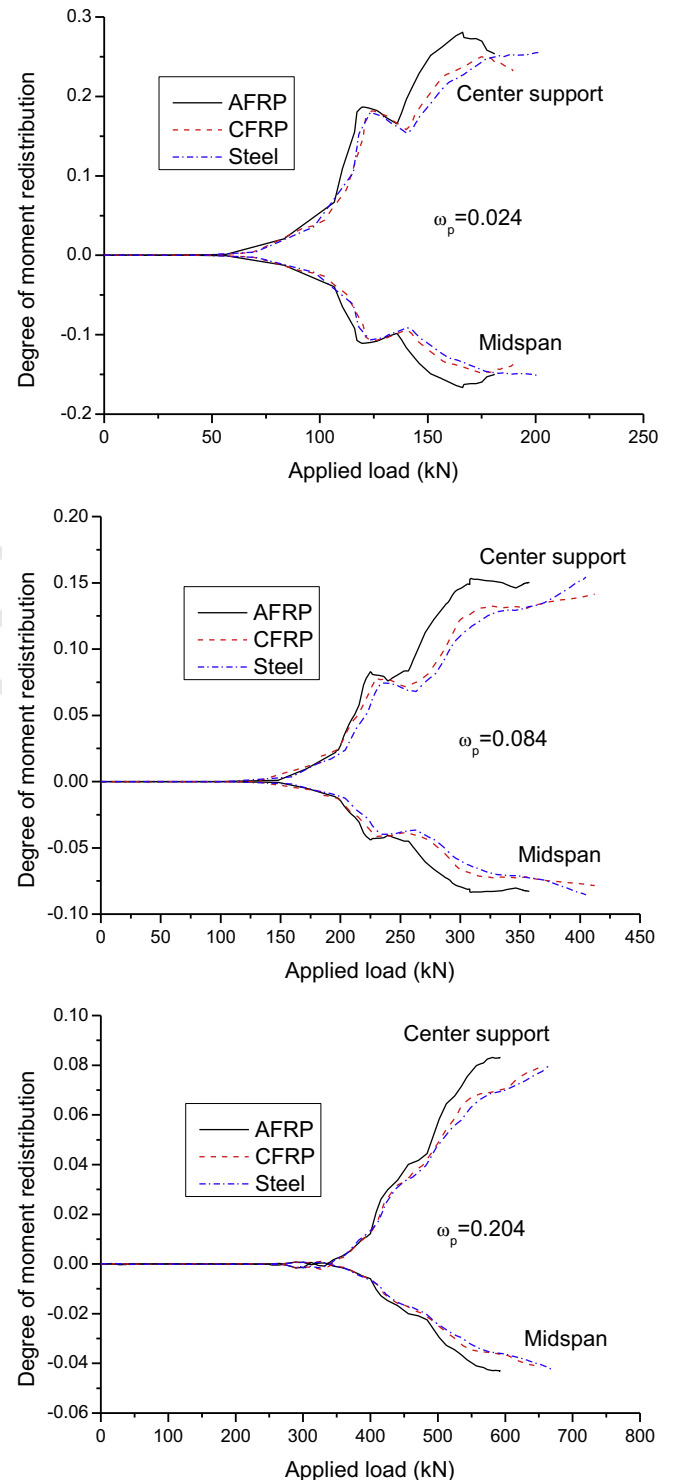


Fig. 12. Development of the degree of moment redistribution for different tendon types and various  $\omega_p$  levels.

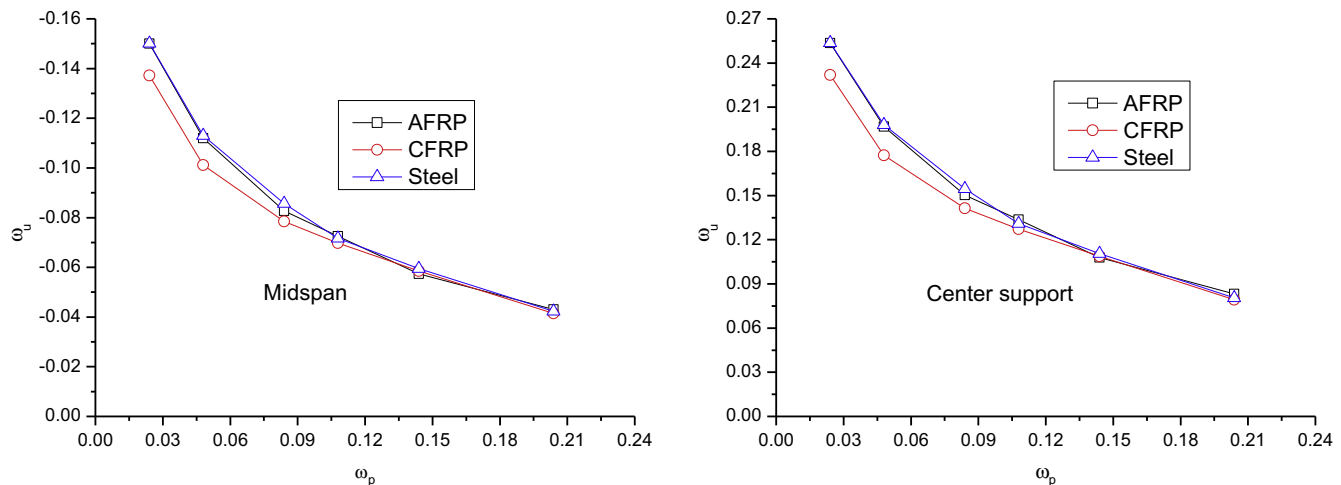


Fig. 13. Variation of the  $\beta_u$  value with  $\omega_p$  level for different tendon types.

oment, yielding of nonprestressed steel at the center support and yielding of nonprestressed steel at midspan. At a high  $\omega_p$  level, on the other hand, the influence of these phases on the redistribution development appears to be not so noticeable. For steel tendons, the maximum redistribution occurs at the ultimate limit state irrespective of the  $\omega_p$  level. For FRP tendons, on the other hand, the maximum redistribution may not take place at failure at a low  $\omega_p$  level.

Fig. 13 shows the variation of  $\beta_u$  (degree of moment redistribution at ultimate) with the  $\omega_p$  level for different tendon types. It is seen that the  $\beta_u$  value decreases as  $\omega_p$  increases. At a given  $\omega_p$  level, the  $\beta_u$  values for AFRP and steel tendons are almost identical. The redistribution for CFRP tendons is obviously lower than that for steel tendons at a low  $\omega_p$  level, but the difference between the  $\beta_u$  values for CFRP and steel tendons becomes negligible when  $\omega_p$  is greater than 0.108.

## 5. Summary and conclusions

A numerical investigation has been carried out to reveal the flexural behavior of continuous concrete beams prestressed with bonded FRP and steel tendons. A finite element model based on the layered Euler–Bernoulli beam theory for full-range nonlinear analysis of continuous FRP and steel prestressed concrete beams has been developed. The model is validated against the experimental results. A comparative study is conducted on two-span continuous prestressed concrete beams with bonded AFRP, CFRP and steel tendons using the proposed model. A wide range of  $\omega_p$  is used. Typical aspects of behavior of the beams are examined, including the failure mode and crack pattern, deformation characteristics, neutral axis depth and moment redistribution. Based on the results obtained from the analysis, the follow conclusions may be drawn:

- Failure of steel prestressed concrete beams is always attributed to crushing of concrete. On the other hand, FRP prestressed concrete beams may fail due to crushing of concrete or rupture of FRP tendons, depending on the  $\omega_p$  level: when  $\omega_p$  reaches around 0.108, concrete crushing and tendon rupture take place simultaneously; a rupture failure happens for  $\omega_p$  lower than 0.108 while a crushing failure occurs for  $\omega_p$  greater than 0.108.
- At a low reinforcement index, crack concentration appears in the steel prestressed concrete beam while this phenomenon is not so important in FRP prestressed concrete beams. At a high reinforcement index, AFRP tendons mobilize larger crack width than CFRP and steel tendons.

- At a low  $\omega_p$  level, the ultimate deflection for FRP tendons is significantly lower than that for steel tendons. As  $\omega_p$  increases, the ultimate deflection for steel tendons consistently decreases while the deflection for FRP tendons quickly increases up to  $\omega_p$  of 0.108 and then turns to decrease. The deflection difference between FRP and steel tendons appears to be insignificant at a high  $\omega_p$  level.
- Due to a lower modulus of elasticity, AFRP tendons register lower neutral axis depth at ultimate than CFRP tendons. At a low  $\omega_p$  level, the neutral axis depth mobilized by steel tendons is close to that mobilized by AFRP tendons while, at a high  $\omega_p$  level, it is close to that mobilized by CFRP tendons.
- The maximum redistribution of moments in steel prestressed concrete beams takes place at ultimate, but this may not true for FRP prestressed concrete beams. At a given  $\omega_p$  level, the redistribution at ultimate for AFRP tendons is almost identical to that for steel tendons. At a low  $\omega_p$  level, CFRP tendons mobilize obviously lower moment redistribution than steel tendons but the redistribution difference between CFRP and steel tendons is negligible at a high  $\omega_p$  level.

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