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Weak convergence in $L^p(0,1)$ of the uniform empirical process under dependence

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Abstract

The weak convergence of the empirical process of strong mixing or associated random variables is studied in $L^p(0,1)$. We find minimal rates of convergence to zero of the mixing coefficients or the covariances, in either case, supposing stationarity of the underlying variables. The rates obtained improve, for p not too large, the corresponding results in the classical $D(0,1)$ framework. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Let $(U_n,\ n\geqslant 1)$ be a stationary sequence of uniform random variables on $[0,1]$ and define, as usual, the uniform empirical process ξ_n by

$$\xi_n(t)=n^{-1/2}\sum_{i=1}^{n}(\mathbb{1}_{[0,t]}(U_i)-t),\quad t\in[0,1]. \tag{1}$$

When the U_i are independent, $(\xi_n,\ n\geqslant 1)$ is well known to converge weakly in the Skorokhod space $D(0,1)$ to the Brownian bridge. This functional central limit theorem has been widely extended to dependent U_i's, giving a limiting process ξ, which is Gaussian centered with covariance

$$\Gamma(s,t)=s\wedge t-st+\sum_{k=2}^{\infty}(\mathrm{P}(U_1\leqslant s,U_k\leqslant t)-st)+\sum_{k=2}^{\infty}(\mathrm{P}(U_k\leqslant s,U_1\leqslant t)-st).$$

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In each particular dependence context, results are stated using some "dependence rate", which can be a mixing coefficient or the covariance $\mathrm{Cov}(U_1, U_k)$ in case of associated U_i's. A natural bound for this dependence rate is clearly the best-known rate to obtain the finite-dimensional CLT for the random variables $\mathbb{1}_{(s,t]}(U_i)$. Those bounds were obtained under ρ-mixing (see Shao and Yu, 1996) and under β-mixing up to a logarithmic factor (see Doukhan et al. (1995)). As for the strong mixing case or the associated one, this achievement remains an open question. Yoshihara (1975) proved the weak $D(0,1)$ convergence of $(\xi_n,\ n \geqslant 1)$ under the strong mixing rate $\alpha(n) = \mathrm{O}(n^{-a})$ with $a > 3$, which was later improved by Shao (1986) requiring only $a > 2$. For associated U_i's, Yu (1993) obtained the weak $D(0,1)$ convergence of $(\xi_n,\ n \geqslant 1)$ under the assumption $\mathrm{Cov}(U_1, U_n) = \mathrm{O}(n^{-a})$ with $a > 7.5$. Shao and Yu (1996) improved this result proving that it is enough to have $a > (3 + \sqrt{33})/2 \simeq 4.373$.

The classical framework to study the weak convergence of $(\xi_n, n \geqslant 1)$ to a Gaussian process is the Skorokhod topology. This approach provides useful results about the asymptotic behaviour of a large class of functionals. From a statistical point of view, one can be interested in weakening the dependence rate assumptions for a class of functionals of paths continuous in a weaker topology than Skorokhod's one. For instance, the Cramér-von Mises ω^2 test statistic, the Watson (1996) statistic, the Shepp (1982) statistic, the Anderson Darling (1952) statistics, some von Mises functionals, some functionals of the type $\int G(t, \xi_n(t))\, \mu(\mathrm{d}t)$ require only the L^2 or L^p topology on the space of paths (Cremers and Kadelka, 1986). In Oliveira and Suquet (1995), the authors obtained the $L^2(0,1)$ weak convergence of (ξ_n) to the same process ξ as above, under the hypotheses

$$\sum_{n \geqslant 1} \alpha(n) < \infty \quad \text{in the } \alpha\text{-mixing stationary case,} \tag{2}$$

$$\sum_{n \geqslant 1} \mathrm{Cov}^{1/3}(U_1, U_n) < \infty \quad \text{in the associated stationary case.} \tag{3}$$

Let us observe that (2) is simply the condition giving the Ibragimov's unidimensional CLT for bounded strong mixing random variables (Ibragimov and Linnik, 1971). In this case the sequence $(\mathbb{1}_{(s,t]}(U_n),\ n \geqslant 1)$ is strong mixing with mixing coefficients dominated by those of $(U_n,\ n \geqslant 1)$. In the associated case, the best-known dependence rate to obtain the unidimensional CLT for the U_i's is the summability of the sequence $\mathrm{Cov}(U_1, U_n)$. Unfortunately, the sequence $(\mathbb{1}_{(s,t]}(U_n),\ n \geqslant 1)$ does not inherit the association property from $(U_n,\ n \geqslant 1)$ and the control of covariances of indicators explains the extra exponent $\frac{1}{3}$.

As one can see, there is an hiatus for intermediate results allowing applications to a larger set of functionals than $L^2(0,1)$ continuous ones, at a lower cost than $D(0,1)$'s conditions. For α-mixing stationary underlying variables U_i, Suquet (1996) proves the $L^p(0,1)$-weak convergence of ξ_n under the conditions $\alpha(n) = \mathrm{O}(n^{-a})$ with $a > p/2$. The main tool used in Oliveira and Suquet (1995) and Suquet (1996) is some special wavelets multiresolution analysis of L^p spaces. In this paper we propose a more classical approach of $L^p(0,1)$ weak convergence allowing a unified treatment of strong mixing and associated cases obtaining a general condition which gives us back the results of Suquet (1996).

Our results are stated in Section 2 together with some tightness conditions which can have their own interest. Proofs are given in Section 3 and some applications to functionals of paths are presented in Section 4.

2. Results

Theorem 1. *Let $(\xi_n,\ n \geqslant 1)$ be a sequence of random elements in $L^p(0,1)$, $p \geqslant 1$, verifying:*
(i) *For some $\gamma > 1$, $\sup_{n \geqslant 1} \mathbb{E}\|\xi_n\|_1^\gamma < \infty$,*
(ii) $\lim_{h \to 0} \sup_{n \geqslant 1} \mathbb{E}\|\xi_n(\cdot + h) - \xi_n(\cdot)\|_p^p = 0$.
Then $(\xi_n,\ n \geqslant 1)$ is tight in $L^p(0,1)$.

Corollary 2. *A sufficient condition for the sequence of random functions* $(\xi_n,\ n\geqslant 1)$ *to be tight in* $L^p(0,1)$, $p\geqslant 1$, *is the existence of positive exponents* $q\leqslant p<r$ *such that*
(a) *For some constant* c,

$$\mathbb{E}|\xi_n(t)|^r\leqslant c,\quad t\in[0,1],\ n\geqslant 1,$$

(b) *For some function* $\varepsilon(h)=\mathrm{o}(1)$ *as* $h\to 0$,

$$\mathbb{E}|\xi_n(t+h)-\xi_n(t)|^q\leqslant\varepsilon(h),\quad 0\leqslant h<1,\ 0\leqslant t\leqslant 1-h,\ n\geqslant 1.$$

In the next theorem, we suppose that $p\geqslant 2$, $\{U_n,\ n\geqslant 1\}$ is a stationary sequence of $[0,1]$-uniformly distributed random variables and ξ_n denotes the uniform empirical process defined by (1).

Theorem 3. *Assume the sequence* $\{U_n,\ n\geqslant 1\}$ *is either strong mixing with rate*

$$\sum_{n\geqslant 1}\alpha(n)<\infty\quad \textit{if } p=2, \tag{4}$$

$$\alpha(n)=\mathrm{O}(n^{-a})\quad \textit{for some } a>p/2,\ \textit{if } p>2, \tag{5}$$

either associated with

$$\sum_{n\geqslant 1}\mathrm{Cov}^{1/3}(U_1,U_n)<\infty\quad \textit{if } p=2, \tag{6}$$

$$\mathrm{Cov}(U_1,U_n)=\mathrm{O}(n^{-a})\quad \textit{for some } a>3p/2,\ \textit{if } p>2. \tag{7}$$

Then the uniform empirical process ξ_n *weakly converges in* $L^p(0,1)$ *to a centered Gaussian process* ξ *with a.s. continuous paths and covariance* Γ:

$$\Gamma(s,t)=s\wedge t-st+\sum_{k=2}^{\infty}(\mathrm{P}(U_1\leqslant s,U_k\leqslant t)-st)+\sum_{k=2}^{\infty}(\mathrm{P}(U_k\leqslant s,U_1\leqslant t)-st).$$

3. Proofs

Proof of Theorem 1. We shall use a convolution approximation. To avoid notational complications, we define $\xi_n(t)=0$ for any t outside $[0,1]$. Let κ be a probability density with support $[-1,1]$. We suppose moreover that κ is Lipschitz, that is

$$\|\kappa\|_{\mathrm{Lip}}:=\sup_{s\neq t}\frac{|\kappa(t)-\kappa(s)|}{|t-s|}<\infty.$$

For any positive integer j, define $\kappa_j(t)=j\kappa(jt)$. The sequence $(\kappa_j,\ j\geqslant 1)$ is an approximate identity.

Using the classical trick

$$\kappa_j*\xi_n(x)-\xi_n(x)=\int_{|t|\leqslant 1/j}\{\xi_n(x-t)-\xi_n(x)\}\kappa_j(t)\,\mathrm{d}t$$

and the Jensen inequality with respect to the probability measure $\kappa_j(t)\,\mathrm{d}t$, we easily obtain

$$\mathbb{E}\int_{\mathbb{R}} |\kappa_j * \xi_n(x) - \xi_n(x)|^p \,\mathrm{d}x \leqslant \mathbb{E}\int_{|t|\leqslant 1/j} \kappa_j(t)\,\mathrm{d}t \int_{\mathbb{R}} |\xi_n(x-t) - \xi_n(x)|^p \,\mathrm{d}x$$

$$\leqslant \sup_{|t|\leqslant 1/j} \mathbb{E}\|\xi_n(\cdot) - \xi_n(\cdot + t)\|_p^p.$$

Hence by (ii),

$$\lim_{j\to\infty} \sup_{n\geqslant 1} \mathbb{E}\|\kappa_j * \xi_n - \xi_n\|_p^p = 0. \tag{8}$$

Now, it is easily checked that

$$\mathbb{E}\|\kappa_j * \xi_n\|_\infty \leqslant \|\kappa_j\|_\infty \sup_{i\geqslant 1} \mathbb{E}\|\xi_i\|_1.$$

and for any $0\leqslant s<t\leqslant 1$:

$$\mathbb{E}|\kappa_j * \xi_n(t) - \kappa_j * \xi_n(s)|^\gamma \leqslant \|\kappa_j\|_{\mathrm{Lip}}^\gamma \sup_{i\geqslant 1} \mathbb{E}\|\xi_i\|_1^\gamma\, |t-s|^\gamma.$$

It follows (see for instance Theorem 12.3, p. 95 in Billingsley (1968)) that for each fixed j the sequence $(\kappa_j * \xi_n,\ n\geqslant 1)$ is tight in $C(0,1)$ and hence also in $L^p(0,1)$.

For fixed $\eta>0$, define $\eta_i = 2^{-i}\eta$ $(i\geqslant 1)$ and choose a sequence of positive ε_i decreasing to 0. By (8) and the Markov inequality, we can extract a subsequence of indexes j_i such that

$$P(\|\xi_n - \kappa_{j_i} * \xi_n\|_p > \varepsilon_i) < \eta_i, \quad n\geqslant 1,\ i\geqslant 1. \tag{9}$$

By the $L^p(0,1)$-tightness of the sequence $(\kappa_{j_i} * \xi_n,\ n\geqslant 1)$, there is some compact C_i in $L^p(0,1)$ such that

$$P(\kappa_{j_i} * \xi_n \notin C_i) < \eta_i, \quad n\geqslant 1,\ i\geqslant 1. \tag{10}$$

From (9) and (10) we deduce

$$P(\xi_n \in A) > 1 - 2\eta, \quad n\geqslant 1,$$

where

$$A = \{f\in L^p(0,1),\ \kappa_{j_i} * f \in C_i \text{ and } \|\kappa_{j_i} * f - f\|_p \leqslant \varepsilon_i,\ i\geqslant 1\}.$$

Clearly for any fixed $\delta>0$, A can be recovered by a finite number of balls with radius 2δ. As $L^p(0,1)$ is complete, the compacity of A follows and $(\xi_n,\ n\geqslant 1)$ is tight in $L^p(0,1)$. □

Proof of Corollary 2. If $q=p$ it is evident that (b) implies (ii). Suppose now that $q<p$ and denote by $1/u$ and $1/v$ the barycentric coordinates of p in the segment (q,r), that is

$$p = \frac{1}{u}q + \frac{1}{v}r, \quad \frac{1}{u} + \frac{1}{v} = 1, \quad u,v>0.$$

The Hölder inequality applied to any nonnegative random variable Y writes:

$$\mathbb{E}Y^p \leqslant \mathbb{E}^{1/u}Y^q\,\mathbb{E}^{1/v}Y^r.$$

Choosing $Y = |\xi_n(t+h) - \xi_n(t)|$ and integrating with respect to t, we get

$$\begin{aligned}\mathbb{E}\|\xi_n(\cdot+h) - \xi_n(\cdot)\|_p^p &\leqslant \int \mathbb{E}^{1/u}|\xi_n(t+h) - \xi_n(t)|^q \mathbb{E}^{1/v}|\xi_n(t+h) - \xi_n(t)|^r \,\mathrm{d}t \\ &\leqslant (2^{r-1}c)^{1/v}\varepsilon(h)^{1/u},\end{aligned}$$

using (a) and (b). Hence $\mathbb{E}\|\xi_n(\cdot+h) - \xi_n(\cdot)\|_p^p$ converges to zero, uniformly in n, as h goes to zero and the hypotheses of Theorem 1 are satisfied. □

Proof of Theorem 3. We first check the tightness of $(\xi_n,\ n\geqslant 1)$.

Case $p=2$: By stationarity of $(U_i,\ i\geqslant 1)$, we have

$$\mathbb{E}|\xi_n(t) - \xi_n(s)|^2 = g_1(s,t) + 2\sum_{k=2}^{n}\left(1-\frac{k}{n}\right)g_k(s,t),$$

where

$$g_k(s,t) = \mathrm{Cov}(\mathbb{1}_{(s,t]}(U_1), \mathbb{1}_{(s,t]}(U_k)), \quad s,t\in[0,1],\ k\geqslant 1.$$

In the strong mixing case, the bound $|g_k(s,t)|\leqslant\alpha(k-1)$ follows obviously from the definition of the mixing coefficients. In the associated case, Lemma 4.5 of Yu (1993) provides the uniform bound

$$|g_k(s,t)|\leqslant 16\,\mathrm{Cov}^{1/3}(U_1,U_k), \quad s,t\in[0,1]. \tag{11}$$

So under (4) or (6),

$$\mathbb{E}|\xi_n(t) - \xi_n(s)|^2 \leqslant g_1(s,t) + 2\sum_{k=2}^{\infty}|g_k(s,t)| \tag{12}$$

where the right-hand side is a normally convergent series of continuous functions. Its sum, say $g(s,t)$, is uniformly continuous on the square $[0,1]^2$ and vanishes on the diagonal. This gives the uniform estimate

$$\mathbb{E}|\xi_n(t+h) - \xi_n(t)|^2 \leqslant \sup_{0\leqslant s\leqslant 1} g(s,s+h) = \varepsilon(h), \quad t\in[0,1],\ n\geqslant 1. \tag{13}$$

So we have condition (b) of Corollary 2 and hence (ii) of Theorem 1 is verified. Condition (i) with $\gamma=2$ is easily obtained choosing $s=0$ in (12).

Case $p>2$: We use Corollary 2. From (13), condition (b) is satisfied with $q=2$.

By Theorem 2 of Yokoyama (1980) condition (a) holds in the strong mixing case if the sequence $(n^{r/2-1}\alpha(n),\ n\geqslant 1)$ is summable. Choosing $p<r<2a$, this follows from our assumption (5).

In the associated case, the random variables $\mathbb{1}_{[0,t]}(U_i)$ are associated as nonincreasing functions of associated variables. So combining the moment inequality of Birkel (1988) with Eq. (11), we obtain condition (a) of Corollary 2 by choosing $p<r<2a/3$.

Once the tightness of (ξ_n) established, to prove the weak $L^p(0,1)$ convergence of ξ_n to ξ, it suffices to verify the convergence in distribution of $\int_0^1 f(t)\xi_n(t)\,\mathrm{d}t$ to $\int_0^1 f(t)\xi(t)\,\mathrm{d}t$ for each f in $L^{p'}(0,1)$ the dual space of $L^p(0,1)$. Observe now that

$$\int_0^1 f(t)\xi_n(t)\,\mathrm{d}t = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(\int_{U_i}^1 f(t)\,\mathrm{d}t - \mathbb{E}\int_{U_i}^1 f(t)\,\mathrm{d}t\right).$$

As $L^{p'}(0,1)\subset L^1(0,1)$, the function $F(x)=\int_x^1 f(t)\,\mathrm{d}t$ is absolutely continuous. In the strong mixing case, Ibragimov's (1971) central limit theorem for strong mixing bounded random variables applies to the $F(U_i)$'s and gives the desired convergence and the covariance function of ξ. In the associated case, the same conclusion is obtained from the following theorem. □

Theorem 4. (Newman, 1984). *Let $(Y_n)_{n\geqslant 1}$ be strictly stationary associated random variables and $X_n = F(Y_n)$, where F is an absolutely continuous function. Put $\bar{X}_n = \bar{F}(Y_n)$ where $\bar{F}(t) = \int_{[0,t]} |F'(u)|\,\mathrm{d}u$. If*

$$\int_{[0,1]^2} F'(x)F'(y)\Gamma(x,y)\,\mathrm{d}x\,\mathrm{d}y < \infty, \tag{14}$$

Γ being defined by

$$\Gamma(x,y) = \Gamma_1(x,y) + \sum_{k\geqslant 2} (\Gamma_k(x,y) + \Gamma_k(y,x)),$$

where

$$\Gamma_k(x,y) = \mathrm{P}(Y_1 > x, Y_k > y) - \mathrm{P}(Y_1 > x)\mathrm{P}(Y_k > y),$$

then $\bar{X}_1$ is square integrable and $n^{-1/2}\sum_{j=1}^{n}(X_j - \mathbb{E}X_j)$ converges weakly to a centered Gaussian random variable with variance

$$\sigma^2 = \int_{[0,1]^2} F'(x)F'(y)\Gamma(x,y)\,\mathrm{d}x\,\mathrm{d}y.$$

4. Some functionals of paths

As an application, we consider now some examples of functionals of paths $T(\xi_n)$ whose convergence in distribution follows from the $L^p(0,1)$ weak convergence of the empirical process ξ_n.

Example 1. Watson statistic.

Watson (1961) proposed the following statistic for testing the "goodness of fit" on a circle in the i.i.d. case:

$$W_n^2 = \frac{1}{2}\int_0^1\int_0^1 [\xi_n(t) - \xi_n(s)]^2\,\mathrm{d}s\,\mathrm{d}t = \int_0^1 \xi_n(t)^2\,\mathrm{d}t - \left(\int_0^1 \xi_n(t)\,\mathrm{d}t\right)^2.$$

Clearly, this functional of paths is continuous in the $L^2(0,1)$ topology.

Example 2. Weighted Mallows statistics.

The convergence of the Mallows statistic $M_n = \int_0^1 |\xi_n(t)|\,\mathrm{d}t$ needs only the $L^1(0,1)$ weak convergence of ξ_n. More generally, for any nonnegative weight function ψ in $L^{p'}(0,1)$, the dual space of $L^p(0,1)$, we have the convergence in distribution of

$$M_{n,\psi} = \int_0^1 \psi(t)|\xi_n(t)|\,\mathrm{d}t,$$

under $L^p(0,1)$ weak convergence of ξ_n.

Example 3. Some Anderson Darling statistics.

Anderson and Darling (1952) studied the asymptotic behaviour of the statistics

$$A_{n,\psi}^2 = \int_0^1 \xi_n(t)^2\psi(t)\,dt$$

for nonnegative weight functions ψ. The choice of the suitable $L^p(0,1)$ space depends on ψ. Indeed, to check the continuity of the functional $T : f \mapsto \int_0^1 f^2\psi\,dt$, Hölder inequality provides the bound

$$|Tf - Tg| \leqslant \|\psi\|_r \cdot \|f-g\|_{2s} \cdot \|f+g\|_{2s}, \qquad \frac{1}{r} + \frac{1}{s} = 1.$$

So if ψ belongs to $L^r(0,1)$ we can take $p=2s$. For instance, choosing the weight $\psi(t)=t^{-a}(1-t)^{-a}$, with $a<1$, we obtain the convergence in distribution of $A_{n,\psi}^2$ under the $L^p(0,1)$ weak convergence of ξ_n for $p>2/(1-a)$. The case $a=1$ seems out of reach of the $L^p(0,1)$ method. Its study would require more knowledge about the regularity of the limiting process ξ.

Example 4. Hölder functions and local time.

Define the local time of ξ_n (Shorack and Wellner, 1986, p. 398) by

$$L_n(x) = n^{-1/2}\{\text{number of times } \xi_n(t)=x,\ 0\leqslant t\leqslant 1\}$$

For any suitable Borel function f we have

$$\int_0^1 f(\xi_n(t))\,dt = \int_{-\infty}^{+\infty} f(x)L_n(x)\,dx.$$

Suppose now $f\in \dot{C}^p(\mathbb{R})$, $p\notin\mathbb{N}$, the homogeneous Hölder space of order p (Meyer, 1990, p. 177), so we have the representation

$$f(x) = \sum_{k=0}^{[p]} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + R(x_0,x), \quad x_0, x\in\mathbb{R},$$

where $|R(x_0,x)|\leqslant C|x-x_0|^p$, together with the estimate

$$|f^{([p])}(x_0)| \leqslant C'(f^{([p])}(0) + |x_0|^{p-[p]}).$$

It follows that $|f^{(k)}(x_0)|\leqslant C''\left(1+|x_0|^{p-k}\right)$ for $0\leqslant k\leqslant [p]$, so the existence and continuity of the functional $T : h\mapsto \int_0^1 f(h(t))\,dt$ on $L^p(0,1)$ is easily checked.

Consider now the occupation measure μ of ξ defined by

$$\mu(B) = \lambda\{t\in[0,1]:\ \xi(t)\in B\}$$

for any Borel subset B of $\mathbb{R}$ (λ denotes the Lebesgue measure). Following Berman ((1969), Ex. 3.2), a sufficient condition for μ to be absolutely continuous with respect to λ is

$$\int_{[0,1]^2} Q(s,t)^{-1/2}\,ds\,dt<\infty, \quad \text{where } Q(s,t)=\Gamma(s,s)+\Gamma(t,t)-2\Gamma(s,t).$$

Define analogously $Q_1(s,t)$ with $\Gamma_1(s,t)=s\wedge t - st$ and $\tilde{Q}(s,t)$ with $\tilde{\Gamma}(s,t)=\Gamma(s,t)-\Gamma_1(s,t)$. Then $\tilde{Q}$ is nonnegative on $[0,1]^2$, so the integrability of $Q^{-1/2}$ follows from integrability of $Q_1^{-1/2}$ that is from

$$\int_{[0,1]^2} |t-s|^{-1/2}(1-|t-s|)^{-1/2}\,ds\,dt<\infty,$$

which is easily checked. So the occupation measure μ has a density L (the local time of ξ) and for each $f \in \dot{C}^p(\mathbb{R})$

$$\int_0^1 f(\xi(t))\,\mathrm{d}t = \int_0^1 f(t)\,\mu(\mathrm{d}t) = \int_{\mathbb{R}} f(x)L(x)\,\mathrm{d}x.$$

Finally by $L^p(0,1)$ continuity of the functional T for any $f \in \dot{C}^p(\mathbb{R})$, the moments $\int_{\mathbb{R}} f(x)L_n(x)\,\mathrm{d}x$ of the local time L_n converge in distribution to the corresponding moments of the local time L of ξ.

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