A Graph-based Technique for Higher Order Topological Data Structure Visualisation

J.-P. de Almeida\textsuperscript{a,b,8}, J.G. Morley\textsuperscript{a}, I.J. Dowman\textsuperscript{a}

\textsuperscript{a} Dept of Geomatic Engineering, University College London, Gower Street, LONDON WC1E 6BT, UK - (almeida, jmorley, idowman)@ge.ucl.ac.uk
Tel. +44 (0)20 76792740 Fax +44 (0)20 73800453
http://www.ge.ucl.ac.uk

\textsuperscript{b} Section of Geomatic Engineering, Dept of Mathematics, Faculty of Science and Technology, University of Coimbra, Largo D. Dinis, Apartado 3008, 3001-454 COIMBRA, Portugal
Tel. +351 239 791150 Fax +351 239 832568
http://www.mat.uc.pt

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1. Introduction

Interpretation and analysis of spatial phenomena is a highly time consuming and laborious task in several fields of the Geomatics world (Anders et al., 1999). That is why the automation of those tasks is especially needed in areas such as Geographical Information Science (GIScience). Carrying out these tasks in the context of an urban scene is particularly challenging given its complexity: relatively small component elements and their generally complex spatial pattern (Eyton, 1993, and Barr & Barnsley, 1996, both cited in Barnsley and Barr, 1997).

Topology is a particularly important research area in the field of GIScience, for it is a central defining feature of a geographical information system (GIS). But, as far as topological relationships between spatial objects are concerned, “generally speaking contemporary desktop GIS packages do not support further information beyond the first level of adjacency” (Theobald, 2001). Therefore, this research project focused on scene analysis by building up a technique for the better understanding of topological relationships between vector-based GIS objects, beyond the first level of adjacency.

Another initial interest was to investigate the possible use of graph theory for this purpose. To date, this mathematical framework has been used in different applications in a wide range of fields to represent connections and relationships between spatial entities. Several authors (including Laurini and Thompson, 1992) have maintained that “this particular tool is extremely valuable and efficient in storing and describing the spatial structure of geographical entities and their spatial arrangement”. Theobald (2001) added that “concepts of graph theory allow us to extend the standard notion of adjacency”.

The aim of retrieving structured information translated into more meaningful homogeneous regions, for instance from an initial unstructured data set, may be achieved by identifying meaningful structures within the initial random collection of objects and by understanding

\textsuperscript{8} Corresponding author.
the spatial arrangement between them. We believe that applying graph theory and carrying out graph analysis may accomplish this.

2. The graph-based analysis tool

2.1 Preparation of the polygon data base

As an example scenario, LiDAR data are being used to test the graph-based technique. It is an unstructured data set with no patterns pre-defined and meaningless in terms of urban scene. Eventually we are also interested in examining how this technique performs with structured data, for instance OS MasterMap®.

To start with, several polygonal regions were constructed through the initial random collection of objects (see Figure 1). This task was based on a generated triangulated irregular network (TIN) and on a binary classification of respective facets according to its slopes (de Almeida et al., 2004a, 2004b).

![Figure 1. Equal interval binary classification of TIN facets.](image)

2.2 Graph construction

A routine was implemented in Arc Macro Language (ArcGIS 8 environment) to retrieve polygon adjacencies. This implied a combination of information spread over two lists: polygon component arcs list (information referring to area definition) and the arc adjacent polygons list (information referring to connectivity of arcs and contiguity of polygons), (de Almeida et al., 2004b).

The network of connectivity throughout the map of polygonal regions (Figure 1) was established by using graph theory: each graph node represents each one of the polygons, and graph edges link up nodes corresponding to adjacent polygons. The adjacency-list representation is the data structure used to represent the graphs of adjacencies in the computer (Sedgewick, 1998). This was implemented in C programming language foreseeing the advantages and potentialities of pointer structures in C for graph analysis.

2.3 Graph analysis: the depth-first search

It is possible to retrieve further geographical information by analysing different paths within the generated graph of adjacencies. A simple visual observation of the sequences of levels of adjacency and containment between nodes along some graph paths, may tell us that, for instance, a node in the tail of a path, representing the highest level of adjacency
and containment, is a candidate to be either a hole in the ground or something on top of an urban feature, say a building (de Almeida et al., 2004a, 2004b).

Accomplishing this sort of analysis might be possible namely by traversing the whole graph and systematically visit all its nodes. For this purpose, the depth-first search algorithm (Sedgewick, 1998) was implemented in such a way that, for seek of flexibility, it is possible to traverse the graph starting from any of its nodes. Nevertheless, we might be ultimately interested in considering this analysis starting from the useful external border (ground polygon), from where sequences of adjacency also make sense in terms of containment and, thus, meaningful in terms of scene (vd. Figure 2).

![Figure 2. Detail of the graph of adjacencies for the map in Figure 1. Example of a meaningful graph path in the context of the urban scene: a building (de Almeida et al., 2004a, 2004b).](image)

By counting the levels of adjacency while traversing the whole graph, the application also analyzes the depth of the depth-first search tree, and hence it is possible to know how many levels of adjacency/containment a graph node is away from the root node. This particular information should be useful, for example, in identifying man-made structures like buildings.
3. The visualisation tool

3.1 Visualisation of analyses

In this research we give particular emphasis to the visualisation aspects of analyses. The human brain is utterly sensitive to any visual representation of real scenes. Any possible visual analysis carried out by the user following previous analytical analysis might be revealed to be extremely useful.

![Diagram of polygonal regions and respective graph of adjacencies.]

Figure 3. Example of a simple map of polygonal regions and respective graph of adjacencies.

We are seeking to incorporate in the application capabilities for visual representation not only of the initial graph (Figure 3), but, in particular, the recursive trees depicting the sequence of the traverse function calls. We strongly believe that, given the dimension and complexity of the original graphs of adjacencies, the observation of these recursive trees appears to be useful in detecting and identifying urban structures.

As it was explained in section 2.3, it is possible to draw different traverse trees depending on the root node chosen. Ultimately, as said above, the user might be interested in observing and analysing paths in the graph starting in particular from the useful external border. As an example, Figure 4 shows two different traverse trees for the scene pictured in Figure 3, starting from two different nodes, 1 (the universe polygon) and 2.

For the completion of these representations, “loop-back” edges (i.e. not visited by the recursive function) will be also included in a further implementation step. We must point out that this aim raises several issues as, for an elegant layout, crossings edges should be avoided. But the initial representation of the traverse tree constitutes *a priori* a considerable constraint for that, as it can be seen in Figure 4. We are still searching for tools based on appropriate algorithms for this purpose that could be incorporated in this application.
In addition, different colours and symbols should be used in these traverse trees to represent, for instance, different levels of adjacency, containment or any other particular attributes especially of the polygons.

3.2 An interactive visualisation tool

We are also investigating the possibility of linking up this application with the GIS environment, for we believe that the utility of the visual representations described in previous section 3.1 should be enhanced, in terms of scene analysis, if the visualisation tool is coupled with the original map of polygonal regions. In fact, the user could carry out any particular visual inspection, say in the original map of polygons, and simultaneously be capable of obtaining the respective traverse tree starting from the useful external border identified in the GIS environment, or *vice versa*.

Furthermore, this capability would be extremely useful in dealing with complex scenes, like the existence of a discontinuous ground polygon. In this situation, the corresponding graph of adjacencies will consist of different sub-graphs connected to each other by one linking edge. In such a tool it appears to be very interesting the possibility of analysing what sub-graph corresponds to what area in the map and, simultaneously, obtaining the respective traverse tree.

4. Conclusion and further work

The implementations carried out so far towards the development of a graph-based technique for scene analysis have been presented in this paper. Currently, we are working on the tool for the visualisation of topological data structures generated from adjacency graph analyses.

We shall point out that this application seeks at the same time to be an investigative tool in developing iteratively rules for scene analysis and graphic representation of results, either by incorporating existing algorithms or by developing and implementing new rules.

Further work will entail the implementation of other possible rules enabling the analysis process explained in section 2.3, eventually leading to the aggregation of graph nodes into identified meaningful structures. These, in turn, should be clustered into homogenous regions. After the delineation of cluster shapes, an analysis process will have to be accomplished, either by pattern recognition or interpretation procedures. The aim of the

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**Figure 4.** Example of two different traverse trees for the same graph, starting from different roots.
ultimate cluster shapes analysis is the retrieval of higher level information, e.g. sets of buildings, vegetation areas, and say land-use parcels.  

The results we are expecting to obtain should be useful to support land-use mapping, image understanding or, in more general terms, to support clustering analysis and cartographic generalisation processes.

References


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Biography

José Paulo de Almeida is a full-time, third year PhD student at the Department of Geomatic Engineering of the University College London. He graduated in Geomatic Engineering and holds an MSc degree in Civil Engineering (specialisation in Urban Engineering) both from the University of Coimbra, Portugal. Since Dec 1994 he has been working as Teaching Assistant for the Department of Mathematics and its Section of Geomatic Engineering, at the University of Coimbra.