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Lattice gluon propagator in renormalizable ξ gauges

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We study the SU(3) gluon propagator in renormalizable R_ξ gauges implemented on a symmetric lattice with a total volume of $(3.25\ \mathrm{fm})^4$ for values of the gauge fixing parameter up to $\xi = 0.5$. As expected, the longitudinal gluon dressing function stays constant at its tree-level value ξ. Similar to the Landau gauge, the transverse R_ξ gauge gluon propagator saturates at a nonvanishing value in the deep infrared for all values of ξ studied.

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I. INTRODUCTION

During the last decade, *ab initio* lattice gauge computations have been instrumental for advancing our understanding of the infrared (IR) sector of QCD. In fact, once discretization and finite volume artifacts are accounted for, these calculations provide nonperturbative results that are regarded as benchmark tests for the available continuum approaches. In particular, large volume SU(2) and SU(3) lattice simulations of the gluon and ghost propagators have been carried out by different groups, using quenched and unquenched configurations gauge fixed in the Landau gauge [1–9]. These simulations have unequivocally established that in the deep IR the gluon propagator and the ghost dressing function (defined as q^2 times the propagator) saturate to a nonvanishing value.

The finding of these so-called massive (or decoupling) solutions has in turn spurred an intense activity in the continuum formulation of the theory, in order to determine a reliable picture of the fundamental QCD dynamics capable of predicting their emergence. The most successful proposals that have been developed include (i) Schwinger-Dyson studies [10–14], in which the saturation is due to the (dynamical) generation of an effective gluon mass scale [15–18]; (ii) the refined Gribov-Zwanziger picture [19–21], in which the effect is due to a mass scale generated through dimension 2 condensates induced by the presence of the Gribov horizon; (iii) functional and renormalization group studies [22,23], in which decoupling solutions appear when special boundary conditions are imposed to the relevant equations.

On the other hand, given the gauge variant nature of the objects under scrutiny, it is clearly very important to perform simulations in as many gauges as possible, in order to discern which aspects of the nonperturbative behavior of the function at hand are (or are not) affected by a gauge choice. From the continuum perspective, this is also extremely important as the results obtained can be used to further test the aforementioned proposals.

Indeed, while the gluon two-point function has been studied in covariant and noncovariant gauges [24–27] (for continuum studies in noncovariant gauges see, e.g., [28–34] and the references therein), reliable calculations in renormalizable ξ (R_ξ) gauges [35] have not been systematically pursued so far. While this class of gauges is the only one completely under control at the perturbative level, its lattice implementation has nevertheless proven to be quite complicated due to poor numerical convergence of the corresponding gauge fixing (GF) algorithm [36–41]. A GF procedure with an improved convergence rate was finally implemented in [42]; however, one still encountered significant problems, which unfortunately become more severe as the GF parameter ξ and/or the lattice volume become larger, and the number of colors N_c and/or the lattice coupling β become smaller [43–45]. As a result, there have been only preliminary studies of the R_ξ gluon propagator [41,42,45].

In this paper, we present the SU(3) gluon propagator in R_ξ gauges for a relatively large lattice volume $(3.25\ \mathrm{fm})^4$ and GF parameter up to $\xi = 0.5$. This allows us to address in some detail the IR behavior of the R_ξ gluon propagator, showing, in particular, that, similar to what has been found in the Landau gauge, the gluon propagator saturates in the IR.

The paper is organized as follows. In Sec. II we briefly review the R_ξ gauges framework, both in the continuum and in its lattice discretized version. In Sec. III we discuss the algorithm we employ to successfully implement the R_ξ GF procedure for a relatively large lattice volume $(3.25\ \mathrm{fm})^4$ and GF parameter up to $\xi = 0.5$. We present our simulation results for the SU(3) gluon propagator in R_ξ gauges in Sec. IV. Our conclusions are finally drawn in Sec. V.

II. THE R_ξ GAUGES FRAMEWORK

In the continuum, gauge fixing is achieved by adding to the $\mathrm{SU}(N_c)$ Yang-Mills gauge invariant action the term (in Minkowski space),

$$S_{\mathrm{GF}} = \int \mathrm{d}^4x \left[b^m \Lambda^m - \frac{\xi}{2}(b^m)^2 \right]. \tag{1}$$

Here ξ is a (non-negative) GF parameter, b^m are the so-called Nakanishi-Lautrup multipliers and $\Lambda^m = \Lambda^m[A]$ is the GF condition. For all fields we write $\Phi = \Phi^m t^m$, where t^m represents the $\mathrm{SU}(N_c)$ generators.

It is usually convenient to adopt an on-shell formalism, eliminating the b^m fields through their (trivial) equation of motion; this yields the (gauge) condition $\xi b^m = \Lambda^m$, and the GF action

$$S_{\mathrm{GF}} = \frac{1}{2\xi}\int \mathrm{d}^4x (\Lambda^m)^2. \tag{2}$$

R_ξ gauges are obtained when choosing the linear GF condition $\Lambda^m = \partial^\mu A^m_\mu$. In this case the (nonperturbative) gluon propagator $\Delta^{mn}_{\mu\nu} = \delta^{mn}\Delta_{\mu\nu}$ can be decomposed according to

$$\Delta_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Delta_{\mathrm{T}}(q^2) + \frac{q_\mu q_\nu}{q^2}\Delta_{\mathrm{L}}(q^2). \tag{3}$$

Slavnov-Taylor identities ensure that $q^2\Delta_{\mathrm{L}} = \xi$ to all orders. Therefore, all the dynamical information is carried by the transverse form factor Δ_{T} alone.

The lattice formulation of Yang-Mills theories is obtained in terms of the Wilson gauge action, in which the dynamical variables are the gauge links U_μ, related to the gauge fields (in lattice units) through

$$U_\mu(x) = \exp[ig_0 A_\mu(x + \hat{e}_\mu/2)],$$
$$A_\mu(x + \hat{e}_\mu/2) = \left. \frac{U_\mu(x) - U^\dagger_\mu(x)}{2ig_0} \right|_{\mathrm{traceless}}, \tag{4}$$

where $\hat{e}_\mu$ is the unit vector along the direction μ, and $\beta = 2N_c/g_0^2$ is the lattice coupling which determines the lattice spacing a. Physical quantities are then obtained by the evaluation of the Euclidean path integral through Monte Carlo techniques, with a probability distribution given by the exponential of the action.

In R_ξ gauges, besides the usual integration over the link variables, one has to integrate over the Λ fields. Equation (2) implies that the integration measure is a Gaussian distribution, with variance ξ, i.e.,

$$P[\Lambda^m(x)] \propto \exp\left\{ -\frac{1}{2\xi}\sum_m [\Lambda^m(x)]^2 \right\}. \tag{5}$$

The numerical difficulty of implementing the R_ξ gauges lies in enforcing the GF condition. In fact, the standard procedure for GF requires to gauge rotate all link variables through the gauge transformation $U_\mu(x) \to g(x)U_\mu(x)g^\dagger(x + \hat{e}_\mu)$, where g represents elements of the $\mathrm{SU}(N_c)$ gauge group that minimizes a suitable functional implementing the desired GF condition. In the Landau gauge case, which is the $\xi \to 0$ limit of the gauges studied here, the simplest possible functional of the gauge links $U_\mu(x)$ leading, upon a suitable minimization procedure, to the condition $\nabla \cdot A^m = 0$ is

$$\mathcal{E}_{\mathrm{LG}}[U, g] = -\mathrm{ReTr}\sum_{x,\mu} g(x)U_\mu(x)g^\dagger(x + \hat{e}_\mu). \tag{6}$$

Contrary to this simple limit, the general case of a nonvanishing ξ was proven to have no simple GF functional suitable for minimization [36]. Nevertheless, in [42] it was shown that the functional

$$\mathcal{E}_{R_\xi}[U, g] = \mathcal{E}_{\mathrm{LG}}[U, g] + \mathrm{ReTr}\sum_x ig(x)\Lambda(x) \tag{7}$$

yields the correct condition $\nabla \cdot A^m = \Lambda^m$, provided that the following convergence algorithm is implemented. The gauge transformation g is built as a product of a sequence of infinitesimal gauge transformations $g = \prod_j \delta g_j$. For each infinitesimal transformation δg_j one minimizes the functional (7); however when moving on to the next infinitesimal transformation δg_{j+1}, the Gaussian distribution Λ^m is maintained unchanged and the link U_μ is updated through a gauge rotation.

Writing $\delta g_j = 1 + i\sum_m w^m_j t^m$, the variation of the functional (7) with respect to the coefficients w^m_j then reads

$$\mathcal{E}_{R_\xi}[U, \delta g_j] - \mathcal{E}_{R_\xi}[U, 1] = \mathrm{Tr}\sum_{x,m} w^m_j(x) t^m \Delta(x),$$
$$\Delta(x) = \sum_\mu g_0[A_\mu(x + \hat{e}_\mu/2) - A_\mu(x - \hat{e}_\mu/2)] - \Lambda(x). \tag{8}$$

Choosing $w^m_j = \alpha_j \Delta^m$, with α_j being a relaxation parameter to be optimized, will reduce Δ. Our goal is to converge to a vanishing Δ in all lattice points x, which in turn implies [36]

$$\theta = \frac{1}{N_c L^4}\sum_x \mathrm{Tr}[\Delta(x)\Delta^\dagger(x)] \to 0. \tag{9}$$

Whenever this condition is fulfilled (which, based on the experience of Landau GF [46], means to have $\theta < 10^{-15}$), then the configuration can be considered to be R_ξ gauge fixed.

III. LATTICE SETUP AND ALGORITHM

In order to study the gluon propagator we use 50 configurations generated through importance sampling of the SU(3) Wilson action [47]. We opt for a symmetric lattice of size $L = 32$ and $\beta = 6.0$, with associated lattice spacing $a = 0.1016(25)$ fm measured from the string tension [48]. The simulated volume is therefore $(3.25\ \mathrm{fm})^4$, large enough to resolve the onset of nonperturbative effects in the propagator's transverse form factor. The values of ξ chosen are $\xi = 0.1, 0.2, 0.3, 0.4$, and 0.5. For comparison, we also report the Landau gauge, obtained as the $\xi \to 0$ limit of the gauges studied.

As explained above, to gauge fix the configurations, we need to minimize the functional $\mathcal{E}_{R_\xi}[U, g]$ through a suitable succession of gauge rotations of the link variables. In practice, for every lattice site x, eight real valued numbers $\Lambda^m(x)$ are generated with the Gaussian probability distribution (5) and combined in the SU(3) algebra element Λ (for the generation of the Gaussian distribution we rely on the standard Box-Muller algorithm). Then, for each link and Λ field, the GF functional (7) is minimized over the link gauge orbit. For the Λ integration, we consider 50 different Λ's for each configuration. Furthermore, to reduce the correlations in the evaluation of the path integral, the Λ's are generated independently for each gauge configuration.

For each combination of link/Λ's one expects the functional $\mathcal{E}_{R_\xi}[U, g]$ to have several minima and that the problem of the Gribov copies is, for the linear covariant gauges, at least as complicated as for the Landau gauge. Given the exploratory nature of this work (and that the R_ξ GF problem is computationally much more demanding than the corresponding Landau GF problem, see below), possible Gribov copies effects will be simply neglected, and a single minimum (or copy) will be studied.

The GF procedure represents clearly the hardest and most time consuming part of the whole calculation. In fact, while in Landau gauge the minimization over the gauge orbit is similar to finding the energy minimum of a spin glass system, in linear covariant gauges the minimization resembles finding the energy minimum of a spin glass in a Gaussian random distributed external field (our Λ). From the computational point of view the latter problem is much harder than the first one. In addition, minimization in the Landau case happens in a compact space, whereas for linear covariant gauges the contribution of the "external field" Λ is unbounded; therefore for sufficiently large ξ (recall that the width of the distribution of the Gaussian distributed Λ increases with ξ), it can happen that the minima of the energy is at the boundary of the SU(3) group and not necessarily at a point where the derivative of the energy vanishes.

For minimization purposes we first tried to apply three different standard optimized techniques used in the Landau case [46]: the fast Fourier transform–accelerated steepest descent (FFT), over relaxation (OVR), and stochastic relaxation (STR). Each one of the techniques we have employed shows some convergence problems when minimizing the functional θ. A typical case is shown in Fig. 1 for the FFT algorithm, where one can see a convergence rate of around ~75% for $\xi = 0.3$ and up to 50 000 iterations. A similar behavior is seen for OVR and STR, with the convergence rate dropping to ~40% for $\xi = 0.5$ in all three cases. Thus we opted to cycle through all convergence techniques when the procedure stalls. Indeed, by cycling through FFT, OVR, and STR, for our hardest case of $\xi = 0.5$, we increase the convergence success rate up to ~90%; for the remaining 10% of cases,

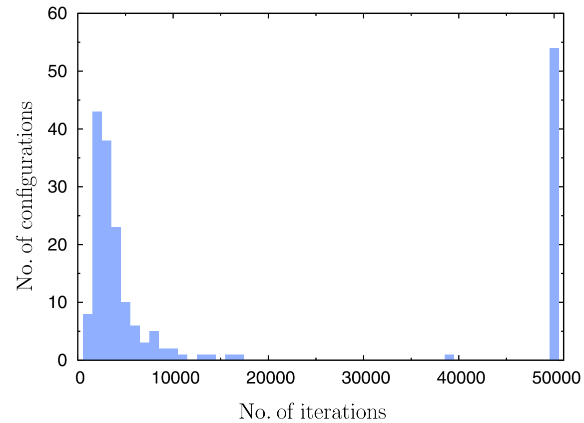

FIG. 1 (color online). Number of iterations required to meet our convergence goal $\theta < 10^{-15}$ with a FFT steepest descent minimization procedure (similar results hold for the OVR and STR minimization procedures). We illustrate the case of 200 uncorrelated configurations for the value $\xi = 0.3$. Notice that at 50000 iterations the minimization is considered to have failed.

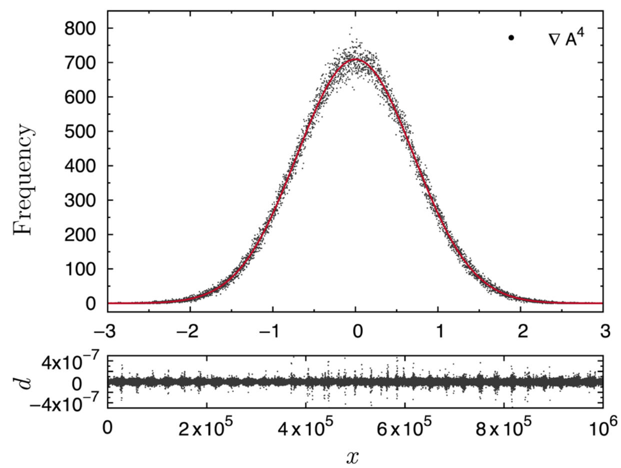

FIG. 2 (color online). (Top panel) The 32^4 values of $\nabla \cdot A^4$ evaluated for a configuration gauge fixed at $\xi = 0.5$, grouped in 5000 bins, compared with a Gaussian with standard deviation $\sqrt{\xi} \simeq 0.316$. (Bottom panel) Plot of $d = \nabla \cdot A^4 - \Lambda^4$; the two distributions coincide within $\sqrt{\theta}$ precision.

restarting the combined algorithm, after performing finite random gauge transformations, leads to convergence for all cases (technical details will be presented elsewhere [49]).

In Fig. 2 we compare the distribution obtained from the values of $\nabla \cdot A^m$ with the one expected for Λ^m for a given configuration and a given color index ($m = 4$) after GF has succeeded; as one can appreciate the GF is within the precision defined above.

IV. SIMULATION RESULTS

The lattice gluon two-point correlation function reads

$$\langle A^m_\mu(\hat{q}) A^n_\nu(\hat{q}')\rangle = \delta^{mn}\Delta_{\mu\nu}(q) L^4 \delta(\hat{q}+\hat{q}'), \quad (10)$$

where $\Delta_{\mu\nu}$ is given in Eq. (3) The lattice momenta $\hat{q}$ (used for Fourier transforms) and q are defined according to [9,50]

$$q_\mu = \frac{2}{a}\sin\frac{\hat{q}_\mu}{2}; \quad \hat{q}_\mu = \frac{2\pi n_\mu}{L}, \quad n_\mu = 1,2,\ldots,L. \quad (11)$$

From Eq (3) and (10) it follows that the transverse and longitudinal SU(3) propagator form factors can be estimated using

$$\Delta_{\mathrm{T}}(q^2) = \frac{1}{24L^4}\sum_{\mu,\nu,m}(\delta_{\mu\nu} - q_\mu q_\nu/q^2)\langle A^m_\mu(\hat{q})A^m_\nu(-\hat{q})\rangle,$$

$$\Delta_{\mathrm{T}}(0) = \frac{1}{32L^4}\sum_{\mu,m}\langle A^m_\mu(0)A^m_\mu(0)\rangle,$$

$$\Delta_{\mathrm{L}}(q^2) = \frac{1}{8L^4}\sum_{\mu,\nu,m} q_\mu q_\nu/q^2\langle A^m_\mu(\hat{q})A^m_\nu(-\hat{q})\rangle. \quad (12)$$

In the case of the transverse form factor, renormalization is performed by fitting the bare lattice propagator to the one-loop inspired result

$$\Delta_{\mathrm{T}}(q^2) = \frac{K}{q^2}\left(\ln\frac{q^2}{\Lambda^2}\right)^{-\gamma}, \quad (13)$$

where $\gamma = 13/22$ is the gluon anomalous dimension. The fits, performed using the largest momentum range starting around 2.5 and going up to 5 GeV, provide the constants K and Λ, which are then used to compute the renormalization constant Z_A via

$$\Delta_{\mathrm{T}}(q^2) = Z_A \Delta^{\mathrm{lat}}_{\mathrm{T}}(q^2), \quad (14)$$

after requiring that the renormalized propagator is such that $\Delta_T(\mu^2) = 1/\mu^2$. Our renormalization scale has been set to $\mu = 4.317$ GeV.

To begin with, we show in Fig. 3 the gluon dressing functions. Within statistical fluctuations, the longitudinal dressing function $q^2\Delta_{\mathrm{L}}(q^2)$ should be a constant function, coinciding with the variance ξ of the Λ probability distribution. This is evidently true for all cases analyzed. In the right panel of Fig. 3 we plot the renormalized transverse dressing function $q^2\Delta_{\mathrm{T}}(q^2)$ for the different ξ values studied. Clearly, no significant deviation from the Landau gauge case is observed; in particular, we find no evidence of the effects reported in the recent continuum study [51] where a dressing function in which the height of the peak rapidly increases and its location moves towards higher q^2 values with increasing ξ was observed.

Next, we turn our attention to the transverse form factor Δ_{T}. The left panel of Fig. 4 shows the renormalized $\Delta_{\mathrm{T}}(q^2)$ for the various ξ values. As was already the case in the Landau gauge, one can see that the R_ξ transverse propagators show an inflection point, implying that the associated spectral density is not positive definite. Indeed, the associated (Euclidean) Schwinger function violates the

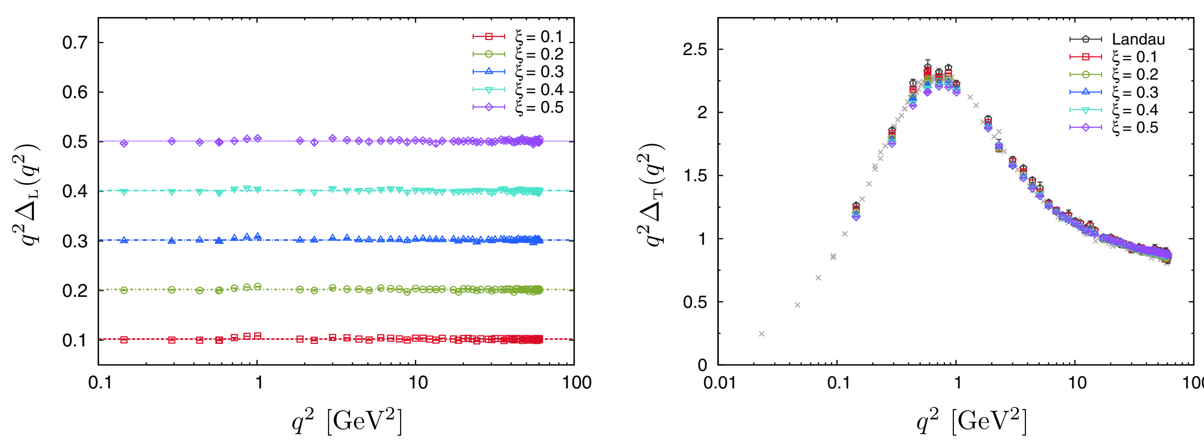

FIG. 3 (color online). (Left panel) The R_ξ longitudinal dressing function $q^2\Delta_{\mathrm{L}} \equiv \xi$; a fit of the data to a constant yield $\xi = 0.103(2)$, 0.203(2), 0.302(3), 0.402(3), and 0.502(3) respectively. (Right panel) The R_ξ gluon transverse dressing function $q^2\Delta_{\mathrm{T}}$. Landau gauge results obtained for a symmetric lattice of $L = 80$ and $\beta = 6.0$ (gray crosses) are also plotted [9].

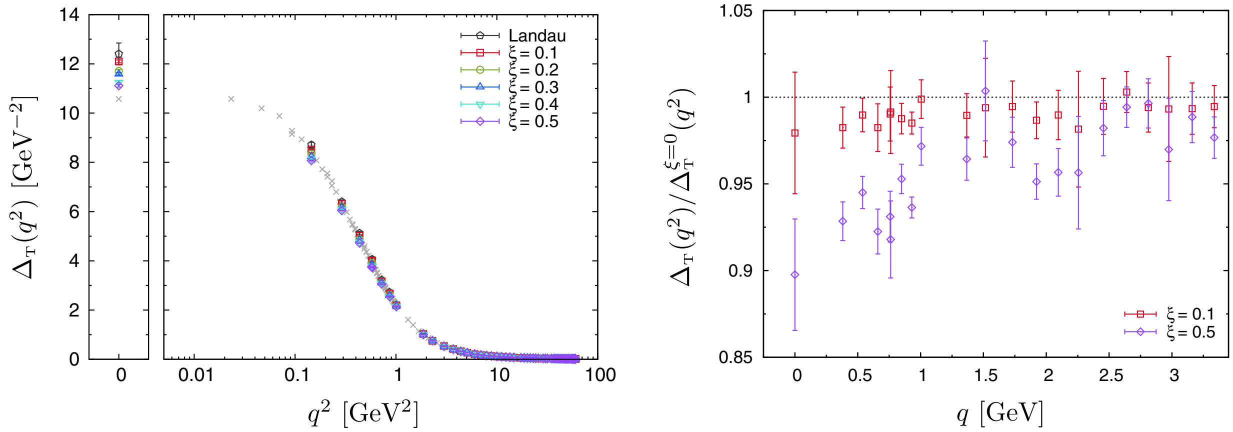

FIG. 4 (color online). (Left panel) The R_ξ transverse propagator Δ_T renormalized at $\mu = 4.317$ [GeV]. The gray crosses (the same as Fig. 3 right [9]) provide an estimate for the volume effects expected at $q^2 = 0$. (Right panel) The ratio $\Delta_T(q^2)/\Delta_T^{\xi=0}(q^2)$.

property of reflection positivity [52,53], which could be interpreted as a manifestation of confinement [54–58]. In addition, they have a marked tendency to flatten towards the small momentum region, thus providing strong evidence that also in the $\xi \neq 0$ case the behavior of the zero-momentum modes of the lattice gluon field are tamed by some nontrivial IR dynamics. The data confirms an IR hierarchy such that Δ_T (slightly) decreases for increasing values of the gauge fixing parameter [40,42]. This is better seen in Fig. 4 (right panel) where we plot the ratio of the transverse propagator to the Landau gauge propagator $\Delta_T^{\xi=0}$ as a function of the momentum for the two values $\xi = 0.1$ and $\xi = 0.5$, observing a maximum difference of about 10%.

In order to estimate the finite volume effects on Δ_T, we included in the corresponding plot the Landau gauge ($\xi = 0$) transverse form factor computed from an 80^4 lattice with $\beta = 6.0$ and a physical volume of about 8.1 fm [9] (gray crosses). These data show that simulations on larger physical volumes have the tendency to suppress the gluon propagator in the infrared region. This effect is clearly seen for momenta of about 300 MeV and smaller. For the R_ξ gauges studied here, a similar scaling with lattice volume is expected. This suggests a systematic overall decrease of ∼10%–15% for Δ_T in the small momentum region.

V. CONCLUSIONS

In this paper the lattice SU(3) gluon propagator in R_ξ gauges was computed for values of the GF parameter ξ up to 0.5 and a lattice volume large enough to access its IR region. From the numerical point of view, the most intensive task turned out to be the gauge fixing, due to the large number of GFs required and convergence issues. Nevertheless, at least within the set of parameters simulated here, a proper combination of various methods (FFT, OVR, and STR) solved the minimization problem associated with the GF in R_ξ gauges.

Our R_ξ propagators show very similar characteristics to the one found in the Landau gauge, being characterized by an inflection point in the few hundreds of MeV region followed by a rapid saturation to a finite nonvanishing value in the IR. In particular, we find qualitative agreement with the recent continuum analysis of [59], where predictions for the behavior of the R_ξ gauge gluon propagator were derived within the context of a dynamically generated gluon mass scenario.

Our study suggests that the IR saturation of the gluon propagator represents a remarkable generic feature of all Yang-Mills theories quantized in R_ξ gauges, which every model of the underlying IR dynamics ought to be able to explain.

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