Reply to "Comment on 'Lattice gluon and ghost propagators and the strong coupling in pure $SU(3)$ Yang-Mills theory: Finite lattice spacing and volume effects' "

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The quenched gluon and ghost propagator data published in [Duarte et al. Phys. Rev. D **94**, 014502 (2016).] is reanalyzed following the suggestion of [Boucaud et al. Phys. Rev. D **96**, 098501 (2017).] to resolve the differences between the infrared data of the simulations. Our results confirm that the procedure works well either for the gluon or for the ghost propagator but not for both propagators simultaneously as the observed deviations in the data follow opposite patterns. Definitive conclusions require improving the determination of the (ratios) of lattice spacings. A simple procedure for the relative calibration of the lattice spacing in lattice simulations is suggested.

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The lattice studies of the gluon [1–10] and ghost [3,4,7,11–15] propagators in pure Yang-Mills gauge theories have been thoroughly pursued in the past years. The emerging picture is a finite and non-vanishing gluon propagator in the infrared region, a manifestation of a nonperturbative mechanism responsible for the generation of a gluon mass scale, and a ghost propagator which follows closely its tree level value.

The production of high precision data for the propagators, modulo possible Gribov copies effects [16], requires understanding the finite volume V and finite lattice spacing a artefacts. In [1,9,17] an attempt was made to estimate the combined effects of using a finite volume and a finite spacing on the simulations. For hypercubic lattices with a size $La \gtrsim 6.5$ fm, in the perturbative scaling window, the lattice propagators associated to simulations with a $\beta \gtrsim 5.7$ collapse into a single curve for momenta $p \gtrsim 1$ GeV. In the infrared region the lattice data differ by far more than one standard deviation, revealing a systematic effect which remains to be understood. An analysis of the discretization effects on the strong coupling limit can be found in [18,19].

In [20] the authors suggest that the observed differences can be attributed to the uncertainties in setting the scale in lattice simulations. Moreover, an example is given that by "*recalibration*" of the lattice spacing, compatible with the magnitude of the statistical error on a, two different gluon data that were initially incompatible in the infrared region collapse into a unique curve.

Despite the statistical error associated to any definition of the lattice spacing, the simulations for the propagators performed so far never considered this effect on the final result. Note that this "uncertainty" is not related to lattice artefacts or to Gribov copies effects. From Table I in [1] the lattice spacing reads $a = 0.1838(11)$ fm for $\beta = 5.7$, $a = 0.1016(25)$ fm for $\beta = 6.0$ and $a = 0.0627(24)$ fm for $\beta = 6.3$ which translates into a relative statistical error of 0.6%, 2.5% and 3.8%.

The aim of this reply is to redo the analysis of the data published in [1] for the gluon and ghost propagators assuming the point of view of [20]. In order to avoid and reduce possible systematics due to the use of a finite lattice spacing, our first step is to renormalize the data of [1] at a different kinematical point and we set $D_R(\mu^2) = 1/\mu^2$ with $\mu = 1.5$ GeV for both propagators.

The renormalized lattice gluon propagator and ghost dressing function can be seen on Fig. 1 as a function of tree level improved momentum $p_\mu = (1/a)\hat{p}_\mu$ and $\hat{p}_\mu = 2\sin(\pi n_\mu/L)$, with $n_\mu = -L/2, -L/2+1, \ldots, 0, \ldots, L/2-1$. For the conversion into physical units we used the central value of a reported in Table I [1]. Clear differences between the various simulations are seen in the infrared gluon data. In the ghost data, the renormalization at a lower momenta, compared to the choice used in [1] where $\mu = 4$ GeV, translates into milder differences in the infrared but strong differences in the ultraviolet between the various simulations. In what concerns the dependence with the lattice spacing, the pattern observed in [1] is clearly seen. In particular, the dependence on a for the gluon and ghost data is opposite, with the coarser lattice being below (above) the remaining data for the gluon (ghost) propagator.

Let us follow [20] and allow for a small deviation in the lattice scale $a \to a' = (1+\delta)a$. This rescaling of a translates into a rescaling of the momenta (in physical units) $p \to p' = p/(1+\delta) = (1+\Delta)p$; for small corrections $\Delta \sim -\delta$. The propagators have to rescale accordingly but, instead, we require the renormalization condition $D_R(\mu) = 1/\mu^2$ to be always fulfilled. The renormalized propagators, in physical units, computed after the change of

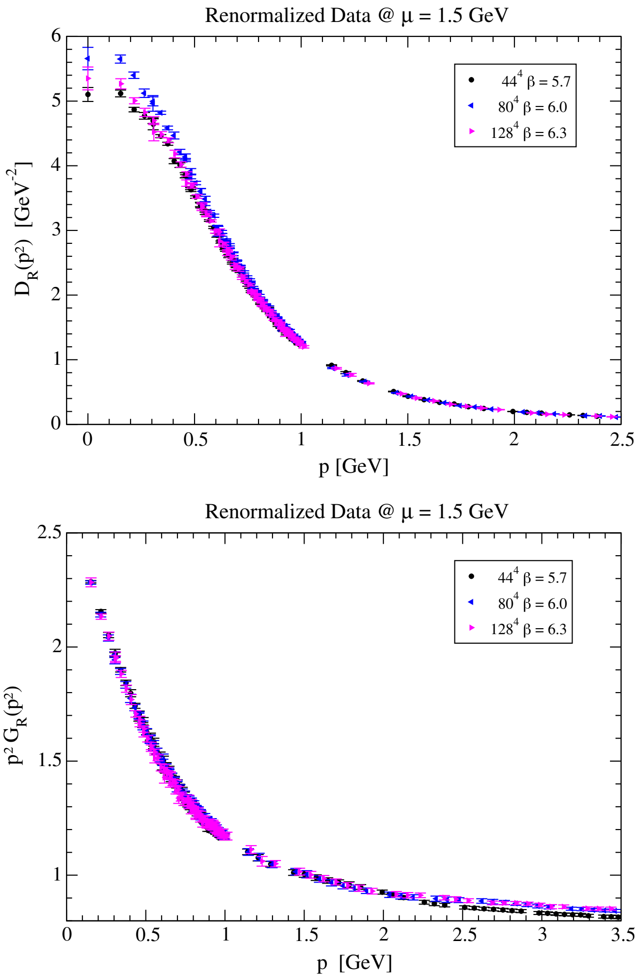

FIG. 1. Renormalized gluon propagator (top) and ghost dressing function (bottom).

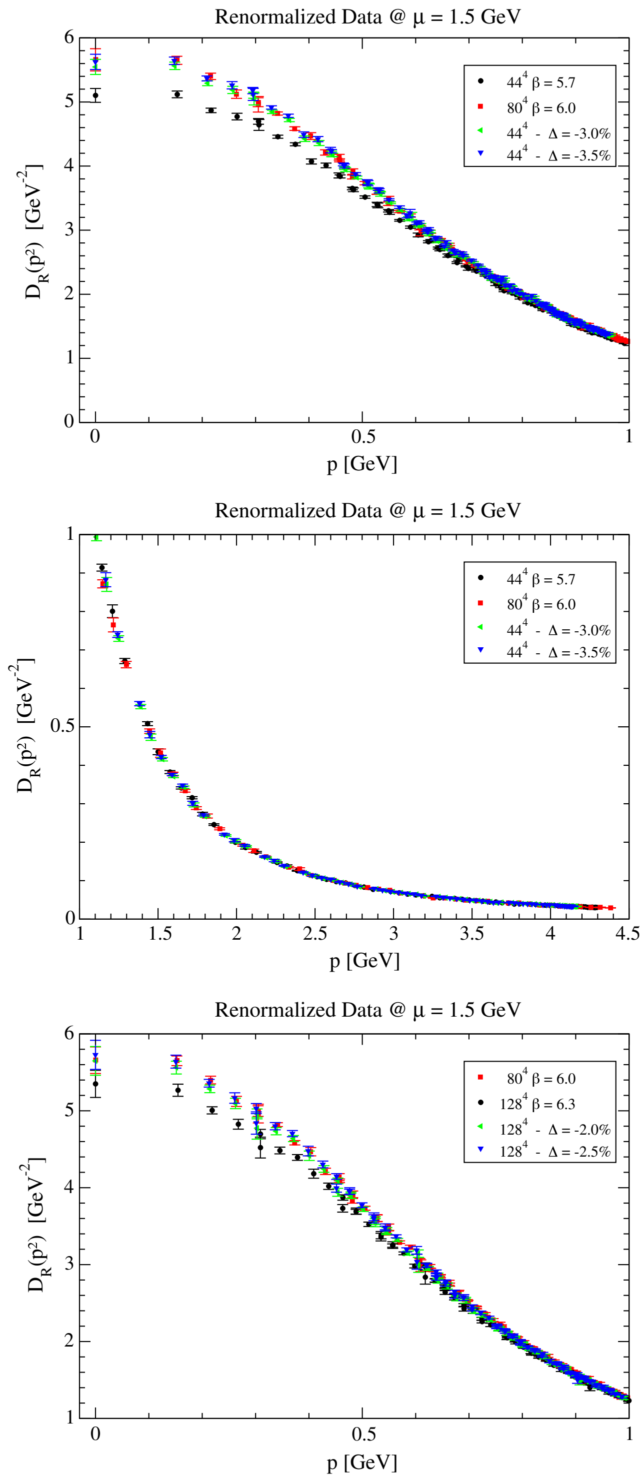

FIG. 2. "*Recalibrated*" gluon propagator: (top) 44^4 and $\beta = 5.7$ data for momenta below 1 GeV; (middle) above 1 GeV; (bottom) 128^4 and $\beta = 6.3$ data for momenta below 1 GeV.

scale are named below as *recalibrated* propagators. As reference data we take the propagators of the simulation performed using $\beta = 6.0$ and the 80^4 lattice.

The *recalibrated* gluon data for the $\beta = 5.7$ and $\beta = 6.3$ can be seen on Fig. 2. For the first (coarser) lattice data we show the infrared data separately from those with $p \geq 1$ GeV. Similar curves could be drawn for the (finer lattice) $\beta = 6.3$ data. A systematic deviation in the scale setting of the same order of magnitude as the statistical errors associated to the lattice spacing settles the differences observed on Fig. 1 both in the infrared and ultraviolet regions.

The resolution of the differences between the gluon propagator data over the full range of momenta provides a way of setting the relative values of the lattice spacing either by identifying a particular momenta or by matching the lattice data for different simulations. A candidate kinematical point being the maximum of the gluon dressing function, see Fig. 3. A naive fit of the data to the Padé approximation $z(p^2+m_1^2)/(p^4+m_2p^2+m_3^4)$ for $p \in [0.5, 1.5]$ GeV, gives $p \sim 0.84$ GeV ($\beta = 5.7$), 0.85 GeV ($\beta = 6.0$) and 0.86 GeV ($\beta = 6.3$) for the maximum of the dressing function. An "exact" determination of $p_{\max}$ demands a detailed and careful analysis.

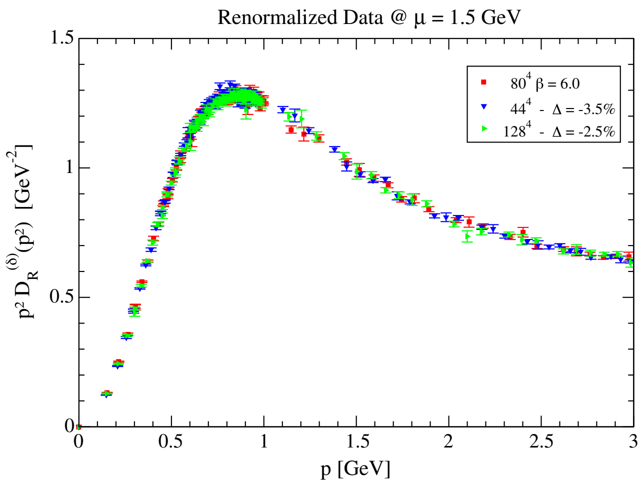

FIG. 3. *Recalibrated* gluon dressing function.

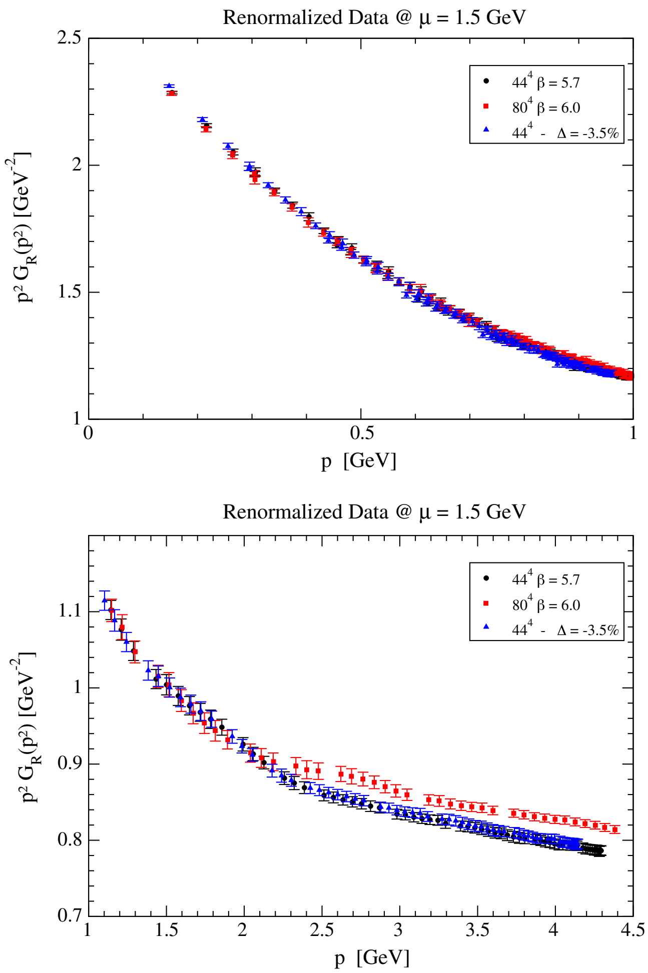

FIG. 4. *Recalibrated* ghost propagator data: (top) 44^4 and $\beta = 5.7$ for momenta below 1 GeV; (bottom) above 1 GeV.

In what concerns the ghost propagator, a *recalibration* of the lattice spacing does not change significantly the conclusions reported in [1]. As reported in [1], the 44^4 data provides the largest $G_R(p^2)$, contrary to the gluon propagator data where it provides the lowest $D_R(p^2)$. The effects of the lattice spacing on the ghost and gluon propagators are in opposite directions. The procedure of [20] does not seem to be able to improve the agreement between simulations simultaneously for both propagators. On Fig. 4 we report the *recalibrated* ghost data for the coarser lattice ($\beta = 5.7$). Similar curves could be reported for $\beta = 6.3$ and the larger lattice 128^4.

In conclusion, our reanalysis of the lattice propagator data published in [1] confirms that the procedure of [20] softens the differences between the lattice gluon data for simulations with various lattice spacings. However, for the ghost propagator, the recipe does not improve the agreement between the lattice data, as the deviations are in the opposite direction of the gluon data. Definitive conclusions concerning the topic discussed here, require a method that provides a good (relative) calibration of the lattice spacing or, equivalently, provide a precise lattice measurement of the beta function. In this sense, a possible method is discussed here, and further work is under development.

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[1] A. G. Duarte, O. Oliveira, and P. J. Silva, Phys. Rev. D **94**, 014502 (2016).
[2] A. Cucchieri and T. Mendes, *Proc. Sci.*, LAT2007 (2007) 297 [arXiv:0710.0412].
[3] A. Cucchieri, T. Mendes, O. Oliveira, and P. J. Silva, Phys. Rev. D **76**, 114507 (2007).
[4] I. L. Bogolubsky, E. M. Ilgenfritz, M. Müller-Preussker, and A. Sternbeck, Phys. Lett. B **676**, 69 (2009).
[5] D. Dudal, O. Oliveira, and N. Vandersickel, Phys. Rev. D **81**, 074505 (2010).
[6] A. Cucchieri, D. Dudal, T. Mendes, and N. Vandersickel, Phys. Rev. D **85**, 094513 (2012).
[7] E.-M. Ilgenfritz, C. Menz, M. Müller-Preussker, A. Schiller, and A. Sternbeck, Phys. Rev. D **83**, 054506 (2011).
[8] O. Oliveira and P. Bicudo, J. Phys. G **38**, 045003 (2011).
[9] O. Oliveira and P. J. Silva, Phys. Rev. D **86**, 114513 (2012).

[10] A. Sternbeck and M. Müller-Preussker, Phys. Lett. B **726**, 396 (2013).
[11] A. Sternbeck, E. M. Ilgenfritz, and M. Müller-Preussker, Phys. Rev. D **73**, 014502 (2006).
[12] O. Oliveira and P. J. Silva, Braz. J. Phys. **37**, 201 (2007).
[13] A. Cucchieri and T. Mendes, Phys. Rev. D **78**, 094503 (2008).
[14] A. Cucchieri and T. Mendes, Phys. Rev. D **88**, 114501 (2013).
[15] A. Cucchieri, D. Dudal, T. Mendes, and N. Vandersickel, Phys. Rev. D **93**, 094513 (2016).
[16] P. J. Silva and O. Oliveira, Nucl. Phys. **B690**, 177 (2004).
[17] O. Oliveira and P. J. Silva, *Proc. Sci.*, LAT2009 (2009) 226 [arXiv:0910.2897].
[18] A. Sternbeck and L. von Smekal, Eur. Phys. J. C **68**, 487 (2010).
[19] A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).
[20] P. Boucaud, F. De Soto, J. Rodríguez-Quintero, and S. Zafeiropoulos, preceding Comment, Phys. Rev. D **96**, 098501 (2017).