Gribov Copies and Gauge Fixing in Lattice Gauge Theories

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We address the problem of the gauge fixing versus Gribov copies in lattice gauge theories. For the Landau gauge, results show that a suitable combination of evolutionary algorithms with traditional steepest descent methods identifies the global maximum of the optimisation function. We discuss the performance of the combined algorithm on small cubic lattices for SU(2) and SU(3).

1. INTRODUCTION AND MOTIVATION

The formulation of gauge theories on the lattice does not require gauge fixing. However, to study Green's functions of the fundamental fields, it is unavoidable to pick a gauge. Moreover, the correlation functions of the fundamental fields can be used to compute renormalization constants or non-perturbative coefficients used in numerical simulations. For a general overview on gauge fixing in lattice gauge theories and related issues see [1].

On the lattice, gauge fixing can be viewed as an optimisation problem. Typically, we have a maximising function with many local maxima, the Gribov copies, and we want to identify its global maximum. The global maximum is not always unique defined. From the numerical point of view, global optimisation is not a trivial task, and among the various techniques Genetic Algorithms (GA) [2] seems to be a good global searching method.

Genetic Algorithms require an evolutive population and assume a number of rules for reproduction. The tuning of basic parameters, the probabilities of the genetic operators and the size of the evolutive population, is crucial to have good performance. The size of the population is an important parameter and can easily drive the method to use as much memory as available.

Our investigation started using pure GA to SU(3) and SU(2) Landau gauge fixing on 4^4 and 8^4 lattices. We conclude that the method was unable to identify conveniently the global maximum. This negative result is due to the large number of variables involved and to the structure of the optimisation function. Fortunately, a combination of GA with steepest descent method (SD) [3] seems to be able to converge to the global maximum.
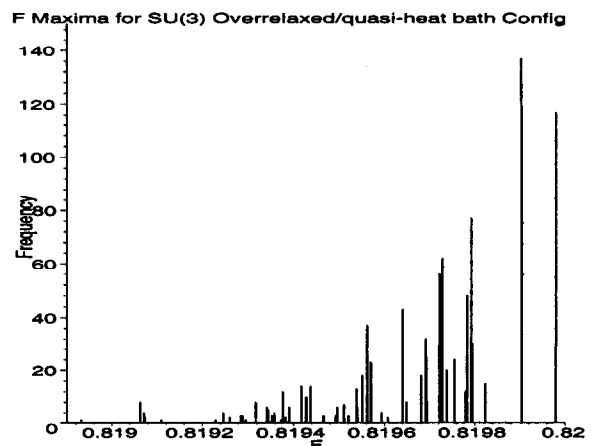

Figure 1. Local maxima from an 8^4 SU(3) configuration obtained after 1000 SD starting at randomly chosen different points.
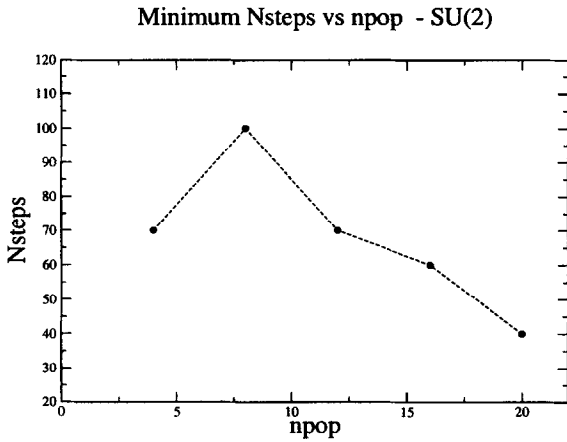
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Minimum Nsteps vs npop - SU(2)

Minimum Nsteps vs npop - SU(3)

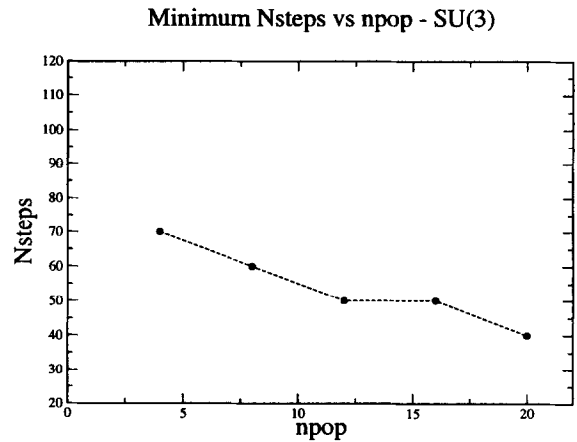Figure 2. N_{steps} as function of *npop* for a random SU(2) configuration.

Figure 3. N_{steps} as function of *npop* for $\beta = 5.7$ SU(3) configurations.

2. EVOLUTION PROGRAM AND LANDAU GAUGE FIXING

Landau gauge fixing is implemented by global maximising the function of the links

$$F[U] = \sum_{x,\mu} \text{Re}\left\{ \text{Tr}\left[g(x)\, U_\mu(x)\, g^\dagger(x+\mu) \right] \right\} \quad (1)$$

over the gauge orbits.

Our implementation of the combined GA and SD algorithm uses the genetic operators (after re-unitarisation): *i)* random crossover, *ii)* random blending, and the mutation operators

$$g(x) \longrightarrow g(x) + \epsilon A, \quad (2)$$

$$g(x) \longrightarrow A, \quad (3)$$

$$g(x) \longrightarrow g(x)(1 + \epsilon A), \quad (4)$$

where ϵ ($|\epsilon| \leq 0.025$) is a random number and A is a random SU(N) matrix.

As fitness function we take

$$\widetilde{F}[g] = N_{\text{steps}} \text{ SD iterations on } F[U^g], \quad (5)$$

i.e. the fitness function is obtained from (1) after N_{steps} steepest descent iterations.

The code starts by generating an uniformly distributed initial population 2.5 times larger than

the evolutive population. The first generation is chosen from this initial population. The mating selection for reproduction, and the choice of the first evolutive generation, uses roulette wheel sampling favouring the best members of the population.

On the final generation we apply SD requiring $|\partial A| < 10^{-10}$ to all members of the population.

3. RESULTS AND DISCUSSION

The combined algorithm was tested with SU(2) 4^4 random configurations, and 8^4 ($\beta = 5.7$) SU(3) configurations generated with version 6 of the MILC code [4].

All runs report results after 400 GA generations. Concerning CPU time we are not yet in the position on giving numbers. This is because we performed a large number of convergence tests at intermediate generations. Final results, including more details about the method and CPU time information will be reported elsewhere soon.

For 3 of the SU(3) configurations and for the SU(2) configurations, the global maximum was computed by performing a large number (1000) of SD starting from randomly chosen different points. In our case, this should not be a prob-
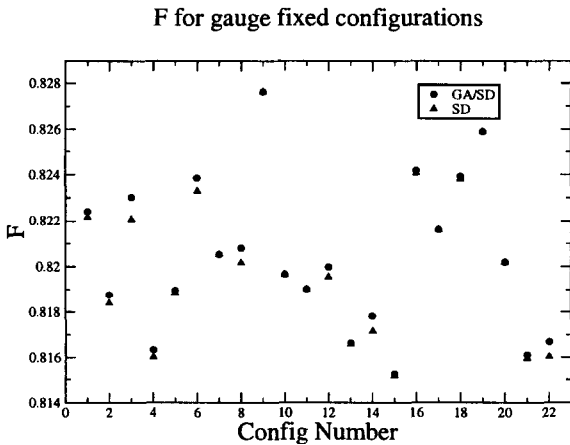
F for gauge fixed configurations

Figure 4. F for gauge fixed SU(3) configurations.

Gluon Propagator - SU(3) configurations

Figure 5. SU(3) gluon propagator.

lem since we are working with relatively small lattices. If we compare the results of the combined algorithm with random search, we have never observed maxima larger than that obtained by random search.

Configurations show a large number of local maxima - see figure 1. For the combined algorithm, the convergence of the best member of the population to the global maxima of F depends on the size of the evolutive population ($npop$) and on N_{steps}. Although one can arrive at the proper answer with quite small evolutive populations, the number of single steepest descent iterations defining the fitness function (CPU time) increases as the size of the population decreases. In figures 2 and 3 one can see the relation between $npop$ and N_{steps} for SU(2) and SU(3) configurations.

Results are encouraging but further studies of how the algorithm scales with volume and β should be done. The tests were performed using a serial version of the code. A parallel implementation of the algorithm is about to start testing.

To address the problem of Gribov copies we look at the simplest correlation function which can be computed, namely the gluon propagator. In order to compute the propagator, 22 over-relaxed/quasi heat bath SU(3) configurations where generated and the correlation func-
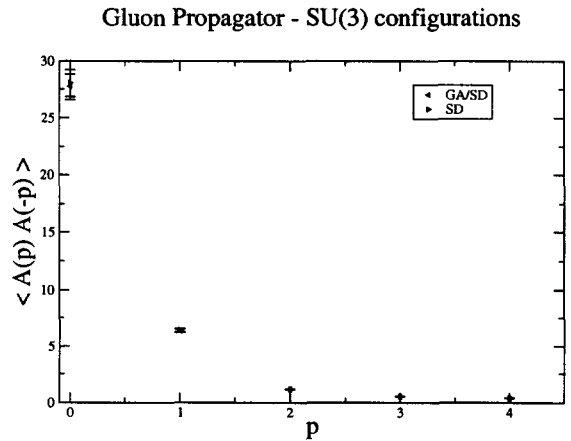
tion from SD compared to the correlation function from the combined GA/SD algorithm. Results are given in figure 5. The two methods show essentially the same behaviour. However, due to the large number of maxima in some configurations, a careful analysis should be done before drawing conclusions. Furthermore, if an analysis of the gluon propagator certainly must be done, investigations of the quark and ghost propagators should not be forgotten.

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