IMRT Beam Angle Optimization using Dynamically Dimensioned Search

J. Dias[1], H. Rocha[2], B. Ferreira[3,4] and M. C. Lopes[3,4]

[1] Faculty of Economics and Inesc-Coimbra, University of Coimbra, Portugal
[2] Inesc-Coimbra, Rua Antero de Quental, nº 199, 3000-033 Coimbra, Portugal
[3]IPOC-FG, EPE, Av. Bissaya Barreto 98, 3000-075 Coimbra, Portugal
[4]I3N, Campus Universitário de Santiago, 3810-193 Aveiro, Portugal

Abstract—**In Intensity Modulated Radiation Therapy (IMRT), the selection of appropriate radiation incidence directions is one of the determinants of proper tumor coverage and sparing of healthy tissues. Nevertheless, most of the times, the incidence directions used in clinical practice are equidistant or determined by a trial and error procedure that is very time consuming and does not guarantee the best possible treatment plan. This paper presents some preliminary results considering the application of DDS (Dynamically Dimensioned Search) algorithm to the problem of Beam Angle Optimization (BAO) for IMRT treatment planning. BAO is a problem known by having many local minima. DDS is a derivative-free optimization algorithm, and presents the capability of not getting trapped in these local minima as happens with, for instance, gradient descent based algorithms. In this paper we will briefly describe the problem, the algorithm, and present computational results for clinical cases of head and neck tumors.**

Keywords— **Radiotherapy, IMRT, Beam Angle Optimization, DDS.**

I. INTRODUCTION

Radiation therapy consists of the treatment of cancerous tissues using radiation, having as goal the destruction of cancerous cells and the preservation of healthy tissues. IMRT is a modern technique in radiation therapy, where the radiation beam is modulated by a multileaf collimator. Multileaf collimators enable the transformation of the beam into a grid of small beamlets of independent intensities. The treatment plan is based on the patient CT images, where the radiation oncologist delineates the target volume (PTV) and organs at risk (OAR), and declares the treatment objectives (prescription dose for the PTV and dose constraints for the OARs). To plan a given treatment it is necessary to determine how many and which incidence directions to use (BAO) and the radiation intensity from each beam angle (Fluence Map Optimization – FMO). In clinical practice, most of the times, the number of angles is considered fixed and decided *a priori*, and the remaining treatment plan parameters are determined by using an iterative and time consuming trial and error procedure until a treatment complying with the medical prescription is obtained (forward planning). An alternative approach considers using inverse planning, where given the desired medical prescription the treatment parameters are automatically determined. Inverse planning applied to radiotherapy is a fruitful ground of research with several important unresolved issues, but until now most of the efforts have been devoted at solving the FMO, and comparatively fewer research effort has been directed to the BAO problem. The BAO problem has been tackled using several different methodologies like response surface approaches [1], derivative-free approaches [2], mixed integer programming approaches [3], simulated annealing [4], particle swarm optimization [5] or genetic algorithms [6]. In this paper we apply DDS algorithm to the BAO problem. The DDS algorithm is a simple stochastic single-solution based global search algorithm [7]. In the next section we will describe the BAO problem in more detail. In section III we will briefly describe the DDS algorithm. Section IV will show the preliminary computational results. Section V will state some conclusions and possible future developments.

II. THE BEAM ANGLE OPTIMIZATION PROBLEM

In BAO we want to calculate the optimal number of beams, k, to use in a given treatment and to decide what are the best k beam angles. This is a very important step in IMRT optimization since it directly influences both the quality of the treatment delivered and the overall treatment time (the treatment time increases with the increase in the number of beams). In this paper we consider that k is determined *a priori*. So, a given solution to the BAO problem will be any set of k angles chosen from the interval [0,360]. Many authors choose to discretize this interval and interpret this problem as a combinatorial problem: the problem of choosing a combination of k angles out of a set of n possible angles, where n is determined by the degree of the discretization. In this paper, we consider each angle as a continuous variable. As pointed out in [2], it is not even necessary to consider an upper and lower bound for each variable, since an angle of -10º, for instance, is equal to 350º or an angle of 370º is equal to 10º.

Each solution (the set of angles) will have to be assessed, so that it is possible to somehow quantify its quality. In reality, this assessment can only be done after considering how the radiation dose will be deposited into the patient cells, so the FMO problem needs to be first solved. To solve the FMO problem, we need a way to calculate accurately the radiation dose distribution deposited in the patient, measured in Gray (Gy). Each structure's volume is discretized in voxels (small volume elements) and the dose is computed for each voxel using the superposition principle, i.e., considering the contribution of each beamlet. Typically, a dose matrix D is such that each row of D corresponds to a voxel and each column to each possible beamlet. Thus, the number of rows of matrix D equals the number of voxels (V) and the number of columns equals the number of beamlets (N) from all beam directions considered. The element in row i and column j of matrix D corresponds to the dose contribution to voxel i from beamlet j with unit intensity. Therefore we can say that the total dose received by the voxel i is given by

$$\sum_{j=1}^{N} D_{ij} w_j \tag{1}$$

with w_j representing the intensity (or fluence) of beamlet j. The size of D originates large-scale problems being one of the main reasons for the difficulty of solving the FMO problem. From a mathematical point of view, we are thus in the presence of two related problems. If we define Θ as the set of all possible angles, then a basic formulation for the BAO problem can be defined as follows:

$$\min f\left(\theta_1, \theta_2, \cdots, \theta_k\right) \tag{2}$$

$$\text{subject to } \theta_1, \cdots, \theta_k \in \Theta \tag{3}$$

There are many different ways of solving the FMO problem and it is beyond the scope of this paper to discuss the appropriateness of the different approaches. We have chosen to use a convex penalty function voxel-based nonlinear model [8], where each voxel is penalized considering the square difference of the amount of dose received by the voxel and the amount of dose desired/allowed for the voxel.

$$Min_w \sum_{i=1}^{V} \left[\underline{\lambda}_i \left(T_i - \sum_{j=1}^{N} D_{ij} w_j \right)_+^2 + \overline{\lambda}_i \left(\sum_{j=1}^{N} D_{ij} w_j - T_i \right)_+^2 \right] \tag{4}$$

$$\text{s.t. } w_j \geq 0,\, j = 1, \cdots, N \tag{5}$$

where T_i is the desired dose for voxel i, $\underline{\lambda}_i$ and $\overline{\lambda}_i$ are the penalty weights of underdose and overdose of voxel i, respectively, and $(\bullet)_+ = \max\{0, \bullet\}$. This nonlinear formulation implies that a very small amount of underdose or overdose may be accepted in clinical decision making, but larger deviations from the desired/allowed doses are decreasingly tolerated.

III. DDS ALGORITHM

The DDS algorithm begins with any admissible solution of the problem, and iteratively perturbs this solution looking for a better one. Whenever a better solution is found, it becomes the current solution that, in turn, will be perturbed. The search is more global at the beginning of the algorithm, and it then becomes more focused in the local neighborhood of the best solution so far. This adjustment from global to a more local search is achieved by reducing the number of variables that are perturbed. The magnitudes of the perturbations are randomly sampled from a normal distribution with mean 0. In our implementation of the algorithm we follow [7], considering some adaptations described in [9]. The algorithm's parameters are as follows: *r* represents the initial standard deviation considered; *r_max* and *r_min* represent the maximum and minimum admissible standard deviations considered; *N* represents the maximum number of iterations (an upper limit to the number of objective function evaluations, since in each iteration at most one solution is evaluated); *l_success* and *l_failure* determine a change in the current standard deviation due to successive successful or unsuccessful iterations (a success meaning that the objective function value has improved). The algorithm has as input an admissible solution to the problem (that can be randomly generated) and returns as output an improved admissible solution.

The algorithm behavior can be described as follows:

1. Set counter *i*←1; Define the initial admissible solution *x_current* and evaluate this solution (*f_current*). *f_best*←*f_current; x_best*←*x_current; success*←0; *failure*←0.
2. Calculate the probability of any given variable be perturbed as *p(i)*=1*–ln(i)/ln(N)*. For each decision variable *x_best(j)*, *j*=1,...,*k*, add the variable to the set *J* with probability *p(i)*.
3. For every variable *x_best(j)*, *j*∈*J*, perturb randomly this variable considering a normal distribution *N*(0,*r*). This perturbed solution will constitute the new *x_current*.
4. Evaluate *x_current*. If *f_current*< *f_best*, then *f_best*←*f_current; x_best*←*x_current* ; *success*← *success* +1 and *failure*←0. Else *success*←0 and *failure*← *failure*+1.
5. If *failure* ≥ *l_failure* then *r*=min*(r/2,r_min)*. If *success* ≥ *l_success* then *r*=max*(2r,r_max)*.
6. *i*← *i* +1. If *i* ≥ *N* then stop, else go to 2.

Steps 2 and 3 of the algorithm are responsible for calculating a new current solution in a random manner (by randomly deciding which variables to perturb and the magnitude of the perturbation). Given the specificities of the BAO problem, we also guarantee that the current solution does not have two adjacent angles that are too near each other. From a clinical point of view, angles that are less than 4º apart are considered the same. The evaluation of the current solution in step 4 is done by resorting to the optimization of the FMO problem. The computational times needed to solve each of these optimization problems are considerably high.

IV. COMPUTATIONAL RESULTS

The DDS algorithm was tested considering ten clinical examples of already treated patient cases of head-and-neck tumors at the Portuguese Institute of Oncology of Coimbra (IPOC) signalized as complex cases where proper target coverage and organ sparing proved to be difficult to obtain. The spinal cord and the brainstem are some of the most critical organs at risk (OARs) in the head-and-neck tumor cases. These are organs such that if only one subunit is damaged, the whole organ functionality is compromised. Thus, it is extremely important not to exceed the tolerance dose prescribed for these types of organs. Other than the spinal cord and the brainstem, the parotid glands, the largest of the three salivary glands, are also important OARs. A common complication due to the irradiation of parotid glands is xerostomia (the medical term for dry mouth due to lack of saliva). This decreases the quality of life of patients undergoing radiation therapy of head-and-neck, causing difficulties to swallow. The parotids are parallel organs, i.e., if a small volume of the organ is damaged, the rest of the organ functionality may not be affected. Thus the organ mean dose is generally used instead of the maximum dose as an objective for planning.

We considered treatments with 5 coplanar beams, since 5 angles is the usual starting point for the trial and error procedure conducted by planners and for this beam number irradiation direction becomes increasingly important. In order to facilitate convenient access, visualization and analysis of patient treatment planning data, as well as dosimetric data input for treatment plan optimization research, we have used CERR, a computational tool developed within MATLAB [10]. Our tests were performed on a Intel Core i7 CPU 2.8 GHz computer with 4GB RAM and Windows 7. We used CERR 3.2.2 version and MATLAB 7.4.0 (R2007a). The dose was computed using CERR's pencil beam algorithm (QIB). For each of the ten head-and-neck cases, the voxel size considered was 0.3cm×0.3cm×0.3cm. To address the convex nonlinear formulation of the FMO problem we used a trust-region-reflective algorithm (fmincon) of MATLAB 7.4.0 (R2007a) Optimization Toolbox. For this set of patients, each instance of the FMO problem can take from 56 seconds to 350 seconds to be calculated. In this study, the OARs used for treatment optimization were defined as being the spinal cord, the brainstem and the parotid glands. For the head-and-neck cases in study, the PTV was separated in two parts with different prescribed doses: PTV1 and PTV2. The prescription dose for the target volumes and tolerance doses for the organs at risk considered in the optimization are presented in Table 1. The algorithm's parameters used were r=36; r_max=90; r_min=3; $l\_success$=3; $l\_failure$=k and N=200. The initial solution was always the equidistant solution.

Table 1 Prescribed doses for all the structures considered

Structure	Mean dose	Maximum Dose	Prescribed Dose
Spinal cord	–	45 Gy	–
Brainstem	–	54 Gy	–
Left parotid	26 Gy	–	–
Right parotid	26 Gy	–	–
PTV1	–	–	70.0 Gy
PTV2	–	–	59.4 Gy
Body	–	80 Gy	–

Table 2 presents the computational results, considering the improvement in the objective function value in relation to the equidistant solution. For each patient, five runs of the DDS algorithm were performed.

Table 2 Computational Results

Patient	*equi* solution	Average DDS Solution	Standard Deviation	Average % improvement
1	387.3	374.5	0.8	3.3%
2	72.9	68.0	0.5	6.7%
3	187.6	172.2	2.0	8.2%
4	156.4	149.2	1.0	4.6%
5	277.6	258.1	1.7	7.0%
6	165.6	154.5	0.6	6.7%
7	40.4	34.8	1.0	13.8%
8	165.0	154.1	2.6	6.6%
9	124.3	117.1	1.0	5.8%
10	186.4	181.0	2.8	2.9%

The quality of the results can be perceived considering a variety of metrics. A metric usually used for plan evaluation is the volume of PTV that receives 95% of the prescribed dose. Typically, 95% of the PTV volume is required as a minimum. These metrics are displayed for the ten cases in Fig. 1, considering the best solution out of the 5 solutions generated. The horizontal lines represent 95% of the prescribed dose. Satisfactory treatment plans should obtain results above these lines. By simple inspection we can verify the advantage of DDS treatment plans that have an improved tumor irradiation metric for most cases compared to *equi* treatment plans.

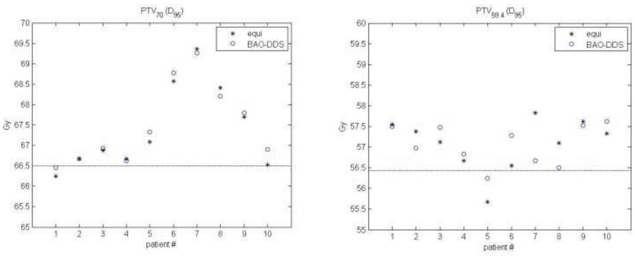

Fig. 1 Comparison of target irradiation metrics obtained by DDS and *equi* treatment plans.

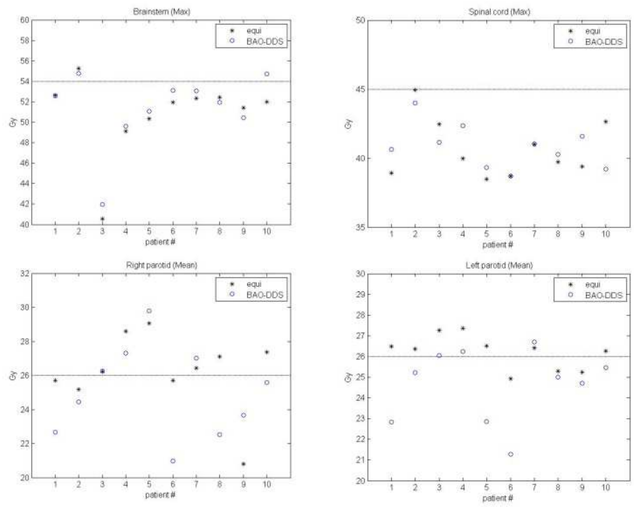

Fig. 2 Comparison of organ sparing metrics obtained by BAO-DDS and *equi* treatment plans.

In order to verify organ sparing, mean and/or maximum doses of OARs are usually displayed. These metrics are displayed for the ten cases in Fig. 2. The horizontal lines represent the tolerance mean or maximum dose for the corresponding structures. Satisfactory treatment plans should obtain results under these lines. For spinal cord and brainstem, treatment plans fulfill the maximum dose tolerance in almost all tested cases. However, as expected, the mean dose limit for parotids was achieved less times, mostly by DDS treatment plans. Moreover, observing Fig. 2, it is perceivable that DDS treatment plans outperform *equi* treatment plans in terms of mean dose obtained. In fact, in average, DDS treatment plans reduced the parotid's mean dose irradiation in 1Gy compared to the *equi* treatment plans.

V. CONCLUSIONS

The preliminary results of applying DDS to BAO show that it is possible to improve organ sparing without jeopardizing tumor coverage. Adaptations to the algorithm can be considered, like considering accepting a worse solution with a given probability (similar to simulated annealing). Another possibility is the use of surrogate models like radial basis functions [2] or neural networks [6], allowing the evaluation of more than one solution in each iteration of the algorithm, without compromising computational times.

ACKNOWLEDGMENT

This work was supported by FEDER, COMPETE, iCIS (CENTRO-07-ST24-FEDER-002003), Pest-C/EEI/UI0308/2011, PEst-C/EEI/UI0308/2011, SFRH/BSAB/1301/2012. The work of H. Rocha was supported by the European Social Fund and Portuguese Funds.

REFERENCES

1. Aleman, D., H. Romeijn, and J. Dempsey. A Response Surface-Based Approach to Beam Orientation Optimization in IMRT Treatment Planning. in IIEAnnual Conf. Exposition. 2006. Orlando, FL.
2. Rocha, H., J. Dias, B.C. Ferreira, and M.C. Lopes, Selection of intensity modulated radiation therapy treatment beam directions using radial basis functions within a pattern search methods framework. Journal of Global Optimization, 2012. accepted for publication.
3. Lee, E.K., T. Fox, and I. Crocker, Simultaneous beam geometry and intensity map optimization in intensity-modulated radiation therapy. International Journal of Radiation Oncology, Biology, Physics, 2006. 64(1): p. 301-320.
4. Pugachev, A. and L. Xing, Pseudo beam's eye-view as applied to beam orientation selection in intensity-modulated radiation therapy. International Journal of Radiation Oncology* Biology* Physics, 2001. 51(5): p. 1361-1370.
5. Li, Y., D. Yao, J. Yao, and W. Chen, A particle swarm optimization algorithm for beam angle selection in intensity-modulated radiotherapy planning. Physics in medicine and biology, 2005. 50: p. 3491.
6. Dias, J., H. Rocha, B. Ferreira, and M.d.C. Lopes, A genetic algorithm with neural network fitness function evaluation for IMRT beam angle optimization. Central European Journal of Operations Research, 2013. Accepted for publication.
7. Tolson, B. and C. Shoemaker, Dynamically dimensioned search algorithm for computationally efficient watershed model calibration. Water Resources Research, 2007. 43: p. W01413.
8. Aleman, D.M., A. Kumar, R.K. Ahuja, H.E. Romeijn, and J.F. Dempsey, Neighborhood search approaches to beam orientation optimization in intensity modulated radiation therapy treatment planning. Journal of Global Optimization, 2008. 42(4): p. 587-607.
9. Regis, R.G. and C.A. Shoemaker, Combining radial basis function surrogates and dynamic coordinate search in high-dimensional expensive black-box optimization. Engineering Optimization, 2013. 45(5): p. 529-555.
10. Deasy, J.O., A.I. Blanco, and V.H. Clark, CERR: A computational environment for radiotherapy research. Medical Physics, 2003. 30: p. 979-985.