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Anomalous decay of pion and eta at finite density ☆

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Abstract

We study the anomalous decays π^0, $\eta \to \gamma\gamma$ in the framework of the three-flavor Nambu–Jona-Lasinio [NJL] model, in the vacuum and in quark matter in β-equilibrium. It is found that the behavior of the relevant observables essentially reflects a manifestation of the partial restoration of chiral symmetry, in nonstrange and strange sectors. The probability of such decays decreases with density, showing that anomalous mesonic interactions are significantly affected by the medium.

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1. Introduction

The structure of pseudoscalar mesons, its mass spectra and decays have attracted a lot of interest along the years, an important motivation for this interest being certainly related to the fact that the origin of these mesons is related to the spontaneous or explicit breakdown of symmetries of QCD. Furthermore, since at high densities and temperatures new phases with restored symmetries are expected to occur, the study of pseudoscalar meson observables under those conditions is specially relevant since they could provide signs for the phase transitions and the associated restoration of symmetries. As a matter of fact, major theoretical and experimental efforts have been dedicated to heavy-ion physics looking for signatures of the quark–gluon plasma, a state of matter with deconfinement of quarks and restoration of symmetries [1–3].

In the limit of vanishing quark masses, the QCD Lagrangian has 8 Goldstone bosons, associated with the dynamical breaking of chiral symmetry. The nonexistence of a ninth Goldstone boson in QCD is explained by assuming that the QCD Lagrangian has a $U_A(1)$ anomaly. The origin of the physical masses of the different pseudoscalar mesons is due to the explicit breaking of symmetries, but it presents differences that will be analyzed next. While for pions and kaons it is enough to break explicitly the chiral symmetry by giving current masses to the quarks, the breaking of the $U_A(1)$ symmetry by instantons has the effect of

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giving a mass to η' of about 1 GeV. So the mass of η' has a different origin than the masses of the other pseudoscalar mesons and it cannot be regarded as the remnant of a Goldstone boson. Concerning the other two neutral mesons, η and π^0, they are degenerated when the $U_A(1)$ anomaly and the current quark masses are turned off, but, when these effects are taken into account it turns out that a percentage of η mass is due to the anomaly, and, therefore, this meson should regarded as less "Goldstone-like" than the pion. Investigating this problem in vacuum and in medium is an important task. Besides mass spectrum and meson–quark coupling constants, the observables associated with the two photon decay of these mesons might provide useful insight into this problem, and calculations of such observables in vacuum and at finite temperature may be found in the literature [4–11].

Understanding the processes $\pi^0(\eta) \to \gamma\gamma$ is specially relevant having in mind that the great percentage of photons in the background of heavy-ion collisions is due to the decay of π^0 and η [12]. As a matter of fact the production of such mesons is indicated by the occurrence of two photon pairs with an invariant mass equal to these meson masses. Possible medium modifications of anomalous mesonic interactions is a topic that has attracted lost of interest. As pointed by Pisarski [7] and Pisarski and Tytgat [8] while for fermions the axial anomaly is not affected by the medium, the opposite is expected for anomalous mesonic interactions. In this concern, the study of $\pi^0 \to \gamma\gamma$ is particularly interesting due to its simplicity and its association with restoration of chiral symmetry. Although the lifetime of the neutral pion is much longer than hadronic time scales and it is not expected to be observed inside the fireball, the physics is the same as that of other anomalous decays ($\omega \to \pi\pi\pi, \omega \to \rho\pi$) that are relevant for experiments in the hot/dense region.

A great deal of knowledge on chiral symmetry breaking and restoration, as well as on meson properties, comes from model calculations [13]. In particular, the Nambu–Jona–Lasinio [NJL] [14,15] type models have been extensively used over the past years to describe low energy features of hadrons and also to investigate restoration of chiral symmetry with temperature or density [16–25].

This work comes in the sequel of previous studies on the behavior of neutral mesons in hot and dense matter [23–25]. The study of phase transitions in quark matter simulating neutron matter in β equilibrium, at zero and finite temperature, as well as the discussion of behavior of pseudoscalar mesons (in particular neutral mesons) in such media, in connection with the restoration of symmetries, has been done in [23–25], by analyzing only the mass spectrum. In particular, it was shown that the behavior of the masses of π^0 and η in this concern reflects mainly the restoration of chiral $SU(2)$ symmetry and manifests strongly in the pion. This not so evident for the η, due to its strangeness content and the dependence of the anomaly. The aim of this Letter is to investigate the anomalous decays of π^0 and η in the vacuum and in quark matter in β-equilibrium and discuss our results in connection with the breaking and restoration of symmetries. We also compare our results with those obtained by other authors who studied the effect of temperature on these anomalous mesonic interactions [5–7,11].

2. Formalism

We consider the three-flavor NJL type model containing scalar–pseudoscalar interactions and a determinantal term, the 't Hooft interaction, with the following Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\gamma^{\mu}\partial_{\mu} - \hat{m}\right)q + \frac{1}{2}g_S\sum_{a=0}^{8}\left[\left(\bar{q}\lambda^a q\right)^2 + \left(\bar{q}i\gamma_5\lambda^a q\right)^2\right] + g_D\left\{\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right\}. \quad (1)$$

Here $q = (u, d, s)$ is the quark field with three flavors, $N_f = 3$, and three colors, $N_c = 3$. $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark masses matrix and λ^a are the Gell-Mann matrices, $a = 0, 1, \ldots, 8$, $\lambda^0 = \sqrt{2/3}\,\mathbf{I}$.

The last term in (1) is the lowest six-quark dimensional operator and it has the $SU_L(3) \otimes SU_R(3)$ invariance but breaks the $U_A(1)$ symmetry. This term is a reflection of the axial anomaly in QCD and can be put in a form suitable to use the bosonization procedure ([20,24] and references therein):

$$\mathcal{L}_D = \frac{1}{6}g_D D_{abc}\left(\bar{q}\lambda^c q\right) \times \left[\left(\bar{q}\lambda^a q\right)\left(\bar{q}\lambda^b q\right) - 3\left(\bar{q}i\gamma_5\lambda^a q\right)\left(\bar{q}i\gamma_5\lambda^b q\right)\right] \quad (2)$$

with constants $D_{abc} = d_{abc}$ if $a, b, c \in \{1, 2, \ldots, 8\}$, where d_{abc} are the $SU(3)$ structure constants, and $D_{000} = \sqrt{2/3}$, $D_{0ab} = -\sqrt{1/6}\,\delta_{ab}$.

The usual procedure to obtain a four quark effective interaction from the six quark interaction is to contract one bilinear $(\bar{q}\lambda_a q)$ making a shift $(\bar{q}\lambda^a q) \to (\bar{q}\lambda^a q) + \langle\bar{q}\lambda^a q\rangle$ with the vacuum expectation value $\langle\bar{q}\lambda^a q\rangle$. Then, an effective Lagrangian can be written as:

$$\mathcal{L}_{\text{eff}} = \bar{q}(i\gamma^\mu\partial_\mu - \hat{m})q + S_{ab}\big[(\bar{q}\lambda^a q)(\bar{q}\lambda^b q)\big] + P_{ab}\big[(\bar{q}i\gamma_5\lambda^a q)(\bar{q}i\gamma_5\lambda^b q)\big], \tag{3}$$

where

$$S_{ab} = g_S\delta_{ab} + g_D D_{abc}\langle\bar{q}\lambda^c q\rangle,$$
$$P_{ab} = g_S\delta_{ab} - g_D D_{abc}\langle\bar{q}\lambda^c q\rangle. \tag{4}$$

Integration over quark fields in the functional integral with (3) gives the meson effective action

$$W_{\text{eff}} = -i\,\mathrm{Tr}\ln\big(i\partial_\mu\gamma_\mu - \hat{m} + \sigma_a\lambda^a + i\gamma_5\phi_a\lambda^a\big) - \frac{1}{2}\big(\sigma_a {S_{ab}}^{-1}\sigma_b + \phi_a\ {P_{ab}}^{-1}\phi_b\big). \tag{5}$$

The fields σ^a and ϕ^a are the scalar and pseudoscalar meson nonets.

The first variation of the action (5) leads to the set of gap equations for constituent quark masses M_i:

$$M_i = m_i - 2g_S\langle\bar{q}_i q_i\rangle - 2g_D\langle\bar{q}_j q_j\rangle < \bar{q}_k q_k\rangle \tag{6}$$

with $i, j, k = u, d, s$ cyclic and $\langle\bar{q}_i q_i\rangle = -i\,\mathrm{Tr}\,S_i(p)$ are the quark condensates. Here the symbol Tr means trace in color and Dirac spaces and integration over momentum p with a cut-off parameter Λ to regularize the divergent integrals. The pseudoscalar meson masses M_H $(H = \pi^0, \eta)$ are obtained from the condition $(1 - P_{ij}\Pi^{ij}(P_0 = M_H, \mathbf{P} = 0)) = 0$, where $\Pi^{ij}(P_0 = M_H, \mathbf{P} = 0)$ is the polarization operator at the rest frame

$$\Pi^{ij}(P_0) = 4\big[\big(I_1^i + I_1^j\big) - \big(P_0^2 - (M_i - M_j)^2\big)I_2^{ij}(P_0)\big], \tag{7}$$

and integrals are given by

$$I_1^i = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{\mathsf{p}^2}{E_i}\,d\mathsf{p}, \tag{8}$$

$$I_2^{ij}(P_0) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{\mathsf{p}^2\,d\mathsf{p}}{E_i E_j}\frac{E_i + E_j}{P_0^2 - (E_i + E_j)^2} + i\frac{1}{2\pi}\frac{p^*}{(E_i^* + E_j^*)}, \tag{9}$$

where $E_{i,j} = \sqrt{\mathsf{p}^2 + M_{i,j}^2}$ and $E_{i,j}^* = \sqrt{(\mathsf{p}^*)^2 + M_{i,j}^2}$ are the quark energies. The momentum p^* is defined by $p^* = \sqrt{(P_0^2 - (M_i - M_j)^2)(P_0^2 - (M_i + M_j)^2)}/2P_0$.

The quark–meson coupling constant is evaluated as

$$g_{H\bar{q}q}^{-2} = -\frac{1}{2M_H}\frac{\partial}{\partial P_0}\big[\Pi_{ij}(P_0)\big]_{|P_0 = M_H}, \tag{10}$$

where the bound state contains quark flavors i, j.

Having the on-shell quark–meson coupling constant we can calculate the meson decay constant f_H according to the definition

$$f_H = N_c g_{H\bar{q}q}\frac{P_\mu}{P^2}\int\frac{d^4p}{(2\pi)^4} \times \mathrm{tr}\big[(i\gamma_5)S_i(p)(\gamma_\mu\gamma_5)S_j(p + P)\big]. \tag{11}$$

Our model parameters, the bare quark masses $m_d = m_u, m_s$, the coupling constants and the cutoff in three-momentum space, Λ, are fitted to the experimental values of masses for pseudoscalar mesons ($M_{\pi^0} = 135.0$ MeV, $M_K = 497.7$) and $f_\pi = 92.4$ MeV.

Here we use the following parameterization [21,24,25]: $\Lambda = 602.3$ MeV, $g_S\Lambda^2 = 3.67$, $g_D\Lambda^5 = -12.39$, $m_u = m_d = 5.5$ MeV and $m_s = 140.7$ MeV.

We also have $M_\eta = 514.8$ MeV, $\theta(M_\eta^2) = -5.8°$, $g_{\eta\bar{u}u} = 2.29$, $g_{\eta\bar{s}s} = -3.71$.

Note, that $\theta(M_\eta^2)$ is the mixing angle which represents the mixing of λ^8 and λ^0 components in the η-meson state (for details see [24,25]).

For the quark condensates we have: $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(241.9\text{ MeV})^3$, $\langle\bar{s}s\rangle = -(257.7\text{ MeV})^3$, and $M_u = M_d = 367.7$ MeV, $M_s = 549.5$ MeV, for the constituent quark masses. In this section we have described the NJL model with the 't Hooft determinant. The model describes well the vacuum properties related with chiral symmetry and its spontaneous breaking including their flavor dependence.

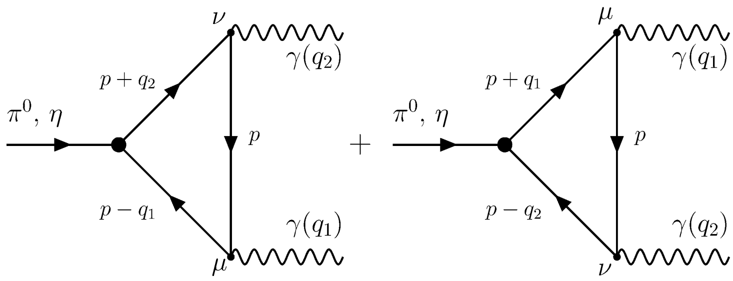

Fig. 1. The quark triangle diagram for the $H \to \gamma\gamma$ (direct and exchange process).

3. The decay $H \to \gamma\gamma$

For the description of the decays $H \to \gamma\gamma$ we consider the triangle diagrams for the electromagnetic meson decays. They are shown in Fig. 1. The corresponding invariant amplitudes are given by

$$\widetilde{\mathcal{T}}_H(P, q_1, q_2) = i \int \frac{d^4 p}{(2\pi)^4} \mathrm{Tr}\{\Gamma_H S(p - q_1)\hat{\epsilon}_1 S(p)\hat{\epsilon}_2 S(p + q_2)\} + \text{exchange}. \quad (12)$$

Here the trace $\mathrm{Tr} = \mathrm{tr}_c\, \mathrm{tr}_f\, \mathrm{tr}_\gamma$, must be performed over color, flavor and spinor indices. The meson vertex function Γ_H has the $i\gamma_5$ form in the Dirac space, contains the corresponding coupling constant $g_{H\bar{q}q}$ (see (10)) and presents itself the 3×3 matrix form in the flavor space. $S(p)$ is the quark propagator $S(p) = \mathrm{diag}(S_u, S_d, S_s)$, $\hat{\epsilon}_{1,2}$ is the photon polarization vector with momentum $q_{1,2}$. The trace over flavors leads to different factors for different mesons H: $Q_{H\bar{q}q}$. This factor depends on the electric charges and flavor of quarks into the meson H: $Q_\pi = 1/3$, $Q_{\eta_u} = 5/9$ and $Q_{\eta_s} = -\sqrt{2}/9$.

For this evaluation, we move to the meson rest frame and use the kinematics $P = q_1 + q_2$ and $P = (M_H, \mathbf{0})$. Taking the trace in (12) we can obtain

$$\widetilde{\mathcal{T}}_H(P, q_1, q_2) = \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu \epsilon_2^\nu q_1^\alpha q_2^\beta \mathcal{T}_H\big(P^2, q_1^2, q_2^2\big), \quad (13)$$

where

$$\mathcal{T}_{\pi^0}\big(P^2 = M_{\pi^0}^2, q_1^2, q_2^2\big) = 32\alpha\pi\, g_{\pi^0\bar{u}u} I_{\pi^0}^u \quad (14)$$

and

$$\mathcal{T}_\eta\big(P^2 = M_\eta^2, q_1^2, q_2^2\big) = \frac{32\alpha\pi}{3\sqrt{3}}\big[\cos\theta\big(5g_{\eta\bar{u}u} I_\eta^u - 2g_{\eta\bar{s}s} I_\eta^s\big) - \sin\theta\sqrt{2}\big(5g_{\eta\bar{u}u} I_\eta^u + g_{\eta\bar{s}s} I_\eta^s\big)\big], \quad (15)$$

where α is the fine structure constant. The integrals $I_H^i \equiv I_H^i(P)$ are given by

$$I_H^i(P) = i M_i \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - M_i^2)} \times \frac{1}{[(p - q_1)^2 - M_i^2][(p + q_2)^2 - M_i^2]}. \quad (16)$$

In order to introduce finite density effects, we apply the Matsubara technique [6,26], and the integrals relevant for our calculation ((8) and (9)) are then modified in a standard way [23–25]. In particular $I_H^i(P)$ (16) takes the form:

$$I_H^i(P_0, \mathbf{P} = 0) = -\frac{M_i}{4\pi^2} \int_{\lambda_i}^{\infty} dp\, \frac{p}{E_i^2} \frac{1}{4E_i^2 - P_0^2} \ln\left(\frac{E_i + p}{M_i}\right), \quad (17)$$

where λ_i is the Fermi momentum.

Finally, the decay width is obtained from

$$\Gamma_{H\to\gamma\gamma} = \frac{M_H^3}{64\pi} |\mathcal{T}_{H\to\gamma\gamma}|^2 \quad (18)$$

and the decay coupling constant is

$$g_{H\to\gamma\gamma} = \frac{\mathcal{T}_{H\to\gamma\gamma}}{e^2}. \quad (19)$$

4. Discussion and conclusions

We present in Table 1 our results for $\mathcal{T}_{H\to\gamma\gamma}$, $\Gamma_{H\to\gamma\gamma}$ and $g_{H\gamma\gamma}$ ($H=(\pi^0,\eta)$) in the vacuum, in comparison with experimental results [4,9,27], and we can see that there is a good agreement.

Some comments are in order concerning our calculation of the integral $I_H^i(P)$ (17). The fermionic action of NJL model (5) has ultraviolet divergences and requires a regularizing cutoff. However, the integral $I_H^i(P)$ (17) is not a divergent quantity. We chose to regularize the action from the beginning, so that there is no need to cut the integral I_H^i—only the ultraviolet integrals I_1^i (8) and I_2^{ij} (9) are regularized [22]. The advantages of using $\Lambda\to\infty$ in nondivergent integrals was already shown in [5,6,22], where a better agreement with experiment of several observables is obtained with this procedure.

Now let us discuss our results at finite density. We consider here the case of asymmetric quark matter imposing the condition of β equilibrium and charge neutrality through the following constraints, respectively on the chemical potentials and densities of quarks and electrons: $\mu_d=\mu_s=\mu_u+\mu_e$ and $\frac{2}{3}\rho_u-\frac{1}{3}(\rho_d+\rho_s)-\rho_e=0$, with $\rho_i=\frac{1}{\pi^2}(\mu_i^2-M_i^2)^{3/2}\theta(\mu_i^2-M_i^2)$ and $\rho_e=\frac{\mu_e^3}{3\pi^2}$ [23–25].

As discussed by several authors, this version of the NJL model exhibits a first order phase transition [19, 23,28]. As shown in [28], by using a convenient parameterization [21] the model may be interpreted as having a mixing phase—droplets of light u,d quarks at a critical density $\rho_c=2.25\,\rho_0$ (where $\rho_0=0.17\ \text{fm}^{-3}$ is the nuclear matter density) surrounded by a nontrivial vacuum—and, above this density, a quark phase with partially restored chiral symmetry [23,28]. An interesting feature of quark matter in weak equilibrium is that at densities above $\rho_s\sim 3.8\,\rho_0$ the mass of the strange quark becomes lower than the chemical potential what implies the occurrence of strange quarks in this regime and this fact leads to meaningful effects on the behavior of meson observables [23–25].

Table 1
Comparison of the experimental values with numerical results obtained in the NJL model

		NJL	Exp.
π^0	$\lvert\mathcal{T}_{\pi^0\to\gamma\gamma}\rvert$ [eV]$^{-1}$	2.5×10^{-11}	$(2.5\pm0.1)\times10^{-11}$
	$\Gamma_{\pi^0\to\gamma\gamma}$ [eV]	7.65	7.78(56)
	$g_{\pi^0\gamma\gamma}$ [GeV]$^{-1}$	0.273	0.274 ± 0.010
	$\tau_{\pi^0\to\gamma\gamma}$ [s]	8.71×10^{-17}	8.57×10^{-17}
η	$\lvert\mathcal{T}_{\eta\to\gamma\gamma}\rvert$ [eV]$^{-1}$	2.54×10^{-11}	$(2.5\pm0.06)\times10^{-11}$
	$\Gamma_{\eta\to\gamma\gamma}$ [keV]	0.440	0.465
	$g_{\eta\gamma\gamma}$ [GeV]$^{-1}$	0.278	0.260
	$\tau_{\eta\to\gamma\gamma}$ [s]	1.52×10^{-18}	1.43×10^{-18}

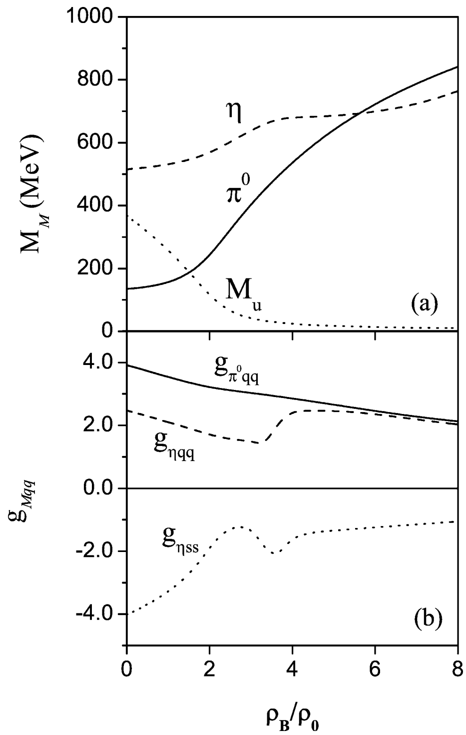

Fig. 2. Mesonic and quark masses (a) and meson–quark–quark coupling constant, (b) as function of the baryonic density.

In order to evaluate the transition amplitude of the decay $H\to\gamma\gamma$ in function of the density, we require the behavior of M_H and $g_{H\bar{q}q}$ with density, that are plotted in Fig. 2(a) and (b), respectively. As we will discuss in the sequel, this behavior is essentially a manifestation of the partial restoration of chiral symmetry. The difference between the behavior at $T=0$, $\rho\neq0$ and $T\neq0$, $\rho=0$ is that, in the last case the mesons are no more bound states above the critical point since they dissociate in $\bar{q}q$ pairs at the Mott temperature. At finite density they continue to be bound states but with a weaker coupling to the

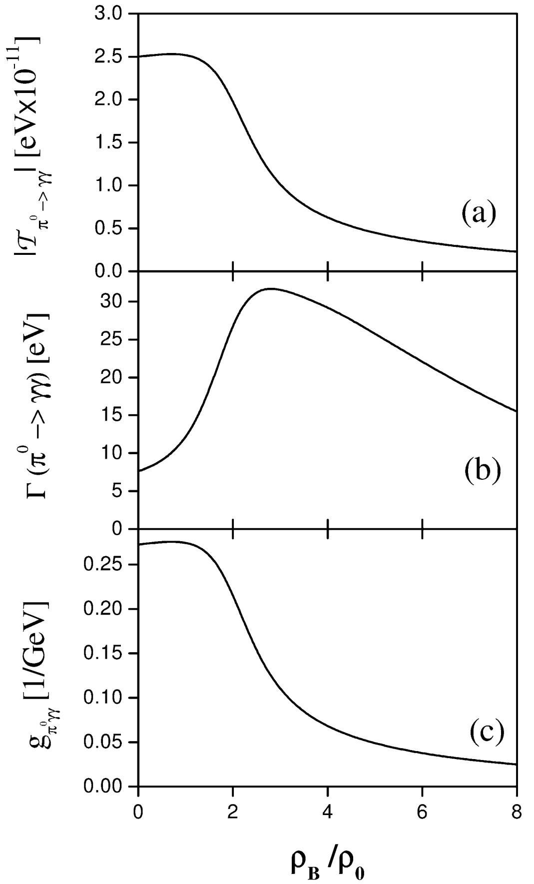

Fig. 3. The decay $\pi^0 \to \gamma\gamma$: (a) transition amplitude; (b) decay width; (c) coupling constant.

quarks as it can be seen in Fig. 2(b). So, it is natural to expect that the effect of density on $H \to \gamma\gamma$ decay observables be qualitatively similar to the effect of finite temperature.

We discuss now our results for the medium effects on the two photon decay of π^0: $\mathcal{T}_{\pi^0\to\gamma\gamma}$, $\Gamma_{\pi^0\to\gamma\gamma}$ and $g_{\pi^0\gamma\gamma}$ that are plotted in Fig. 3(a)–(c). In order to understand this results let us remember the central role of this meson in connection with the breaking and restoration of chiral symmetry in the $SU(2)$ sector. A sign for the restoration of this symmetry is that the mass of the pion increases with density and this meson becomes degenerated with σ meson and the pion decay constant f_π goes asymptotically to zero. The behavior of $\pi^0 \to \gamma\gamma$ observables with density is closely related with the restoration of chiral symmetry in the $SU(2)$ sector. The fact that $\mathcal{T}_{\pi^0\to\gamma\gamma}$, as well as $g_{\pi^0\to\gamma\gamma}$ decrease with density reflects the fact that M_u and $g_{\pi^0\bar{q}q}$ decrease with density. As it can be seen from Fig. 2, the quark mass decreases sharply while $g_{\pi^0\bar{u}u}$ decreases more slowly. In the region $\rho < \rho_c$ the behavior of the transition amplitude is dictated by a compromise between the behavior of the mass and the meson quark coupling. Above ρ_c the mass decrease seems to be the dominant effect. Concerning $\Gamma_{\pi^0\to\gamma\gamma}$ it has a maximum at about the critical density, since there are two competitive effects, on one side the decrease of the transition amplitude and, on the other side, the increase of the pion mass. Above the critical density the two photon decay of the pion becomes less favorable, what reflects that it turns a weaker bound $\bar{q}q$ pair, as already mentioned before. This is compatible with recent experimental results indicating that pionic degrees of freedom are less relevant at high densities [29].

In order to clarify the connection between the behavior of the anomalous couplings and the restoration of chiral symmetry one can do the simple exercise of calculating the transition amplitude in the chiral limit, by setting the external momenta equal to zero. Then, similarly to [7,8], where such analysis was done with temperature, we get $|\mathcal{T}_{\pi^0\to\gamma\gamma}| \propto \frac{M_u{}^2}{f_\pi}\frac{1}{\lambda_u{}^2}$, the mass decreases being the dominant effect, which leads to a vanishing of the transition amplitude at the critical density.

Concerning the $\eta \to \gamma\gamma$ decay, although qualitatively similar to $\pi^0 \to \gamma\gamma$, there are differences that we will examine in detail and that are related to the evolution of the strange quark content of this meson and the behavior of the strange quark in this regime. The quark content of η is given by

$$|\eta\rangle = \cos\theta \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle - \sin\theta\sqrt{\frac{2}{3}}\,|\bar{u}u + \bar{d}d + \bar{s}s\rangle. \quad (20)$$

It was show in [24] that the mixing angle ($\theta = -5.8^\circ$ in the vacuum) decreases with density, has a minimum ($\simeq -25^\circ$) at $\rho \simeq 2.8\,\rho_0$, equals to zero at $\rho \simeq 3.5\,\rho_0$, then it increases rapidly up to the value $\sim 30^\circ$, when strange valence quarks appear in the medium ($\rho \simeq 3.8\,\rho_0$). $g_{\eta\bar{s}s}$ (Fig. 1(b)) reflects this evolution of the strange quark content. So, one sees that at high densities the η is governed by the behavior

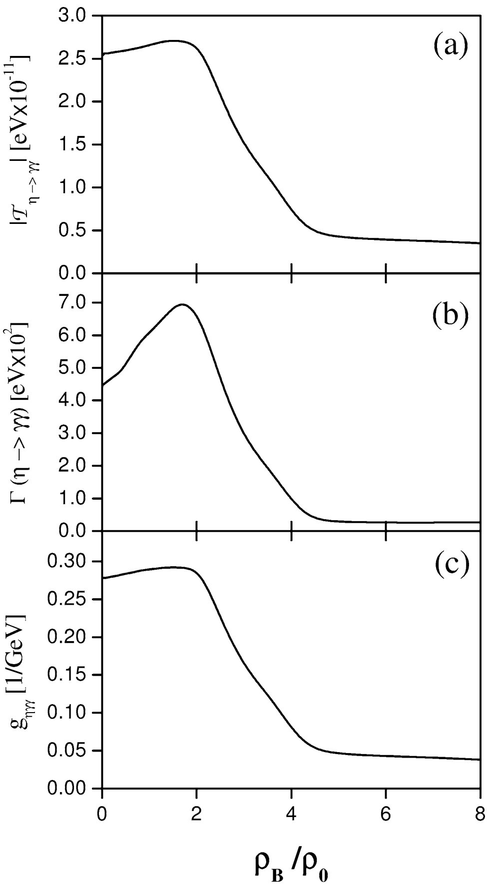

Fig. 4. The decay $\eta \to \gamma\gamma$: (a) transition amplitude; (b) decay width; (c) coupling constant.

of the strange quark mass. Since after $\rho \simeq 3.8\,\rho_0$ there is a tendency to the restoration of chiral symmetry in the strange sector, although less pronounced then for nonstrange quarks, this effect should manifest itself in the behavior of η observables. Therefore, although the η is less "Goldstone like" than the pion, one expects results qualitatively similar for the two photon decay, which are shown in Fig. 4(a)–(c). The main difference is that the width $\Gamma_\eta \to \gamma\gamma$ almost vanishes above $\rho = 3.8\,\rho_0$. This behavior of the η seems to indicate that the role of the $U_A(1)$anomaly for the mass of this meson at high densities is less important.

In conclusion, we studied the behavior of two photon decay observables for the neutral mesons π^0, and η in quark matter in weak equilibrium and we discussed the results in connection with restoration of symmetries. We have shown that, in spite of the different quark structure of these mesons, at high densities they share a behavior which is mainly a manifestation of restoration of chiral symmetry. We show that these anomalous decays are significantly affected by the medium, however the relevance of this results from the experimental point of view should be discussed. Recent experimental results from PHENIX [29] show that π^0 production is suppressed in the central region of Au + Au collisions as compared to the peripherical region. This means that $\pi^0 \to \gamma\gamma$ decay could only be interesting for experimental heavy-ion collisions at intermediate densities. However, although the peak of the π^0 width is at a moderate density (see Fig. 3) its life time is here of the order of 2.08×10^{-17} s, much longer than the expected lifetime of the fireball in the hadronic phase, 10^{-22} s, so the decay should occur outside of the fireball. The same considerations apply to the η, although its maximum lifetime is 9.36×10^{-19} s. However, since the physics underlying these processes is the same of other anomalous processes interesting from the experimental point of view, the modification of anomalous mesonic interactions by the medium might, in principle, be observed.

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