On the validity of lumped capacitance approaches for the numerical prediction of heat and mass transfer in desiccant airflow systems

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Abstract

In the present work, a detailed model of simultaneous heat and mass transfer through a desiccant micro-porous medium of a channel wall of compact heat and mass exchangers, such as desiccant wheels, was developed. The relevant phenomena considered by this model within the porous medium are surface diffusion of the adsorbed water, Knudsen diffusion of water vapour, heat conduction and the sorption process. A one-dimensional formulation of this model is used to investigate the validity of two simplifying approaches based on the lumped-capacitance method. One consists of neglecting the transversal heat and mass transfer resistances within the porous medium, the other of cancelling only the thermal resistance.

Results are presented for a wide range of values of the layer thickness of the porous medium. It is concluded that the hypothesis of null-resistances in the cross direction is valid only for layer thicknesses lower than 0.1 mm, approximately, while the approach based on the thermal lumped capacitance model is valid for layer thicknesses lower than 5 mm. Finally, a dimensional analysis of interfacial balances leads to the conclusion that the Biot number for surface diffusion is several orders of magnitude higher than the corresponding thermal Biot number.
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1. Introduction

The study of the transport phenomena in desiccant airflow systems has been addressed in numerous research works, many of them concerning combined desiccant cooling and dehumidification, air filtering, air dehumidification and energy recovery processes (e.g., [1–16]).

A significant number of studies was carried out to predict the behaviour of air dehumidifying systems using simplified mathematical models (e.g., [17–27]). In most of them, the heat and mass transport phenomena occurring inside the porous desiccant medium are not described in a very detailed way, simplified approaches being adopted instead. Pseudo-gas-side controlled models have been the most used models (e.g., [21]) based on fictitious overall heat and mass transfer coefficients when it is important to take into account the internal resistances in the desiccant. A single-blow test procedure for compact heat and mass transfer exchangers to evaluate those overall transfer coefficients is presented in [28]. An alternative to this approach consists in using the parabolic profile assumption, which validity was investigated in [29] and supports the rotary desiccant dehumidifier model used in [23].

The range of validity of such models can be investigated by using experimental techniques and by detailed numerical modelling. The adsorbate surface diffusion inside microporous media like silica gel is known to be a dominant mechanism [6]. Some studies concerning the use of correlations for solid-side mass diffusivity estimation are important references [30–33], but large discrepancies between experimental and theoretical values are observed [33].

The air stream behaviour and its interaction with the desiccant medium are also often treated in a simplified way, namely by assuming a fictitious bulk flow pattern, as well as fictitious

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Nomenclature

Bi_h Biot number for heat transfer

Bi_K Biot number for Knudsen diffusion

Bi_m Biot number for mass transfer

Bi_s Biot number for surface diffusion

C_ℓ mass concentration of adsorbed water .... kg m^{-3}

C_ℓ^+ dimensionless mass concentration of adsorbed water

C_v vapour mass concentration of the gas mixture inside the porous medium .... kg m^{-3}

C_v^+ dimensionless vapour mass concentration of the gas mixture inside the porous medium

$C_{v,f}$ vapour mass concentration in the airflow .... kg m^{-3}

$C_{v,i}$ vapour mass concentration in the gas mixture at the interface .... kg m^{-3}

$c_{p,f}$ specific heat of the air flow .... $\text{J kg}^{-1}\,{}^\circ\text{C}^{-1}$

$c_{p,p}$ specific heat of porous medium ..... $\text{J kg}^{-1}\,{}^\circ\text{C}^{-1}$

$c_{p,v}$ specific heat of water vapour .... $\text{J kg}^{-1}\,{}^\circ\text{C}^{-1}$

D_f diffusion coefficient of water vapour in the airflow .... $\text{m}^2\,\text{s}^{-1}$

$D_{K,eff}$ effective coefficient of Knudsen diffusion............................... $\text{m}^2\,\text{s}^{-1}$

D_p coefficient of water (mass) diffusion inside the porous medium........................ $\text{m}^2\,\text{s}^{-1}$

$D_{s,eff}$ effective coefficient of surface diffusion (of adsorbed water) .... $\text{m}^2\,\text{s}^{-1}$

d_{hyd} hydraulic diameter .... m

h_{ads} heat of adsorption .... J kg^{-1}

h_h convection heat transfer coefficient...................... $\text{W m}^{-2}\,{}^\circ\text{C}^{-1}$

h_{fg} latent heat of vaporization .... J kg^{-1}

h_ℓ enthalpy of liquid water .... J kg^{-1}

h_ℓ' enthalpy of adsorbed water.............. J kg^{-1}

h_m convection mass transfer coefficient .... m s^{-1}

h_v enthalpy of vapour .... J kg^{-1}

h_w' heat of wetting........................ J kg^{-1}

j_{ads} adsorption rate per unit area of the interface........................... $\text{kg s}^{-1}\,\text{m}^{-2}$

j_{app} heat or mass convective flux predicted by the simplified model

j_h convective heat flux .... W m^{-2}

j_h^+ dimensionless heat conduction flux

$j_{h,ads}^+$ dimensionless heat generation/consumption rate per unit area of the interface

j_K^+ dimensionless Knudsen flux at the interface

j_m convective mass flux .... $\text{kg s}^{-1}\,\text{m}^{-2}$

j_{ref} heat or mass convective flux predicted by the detailed model

j_s^+ dimensionless surface diffusion flux at the interface

Le Lewis number

M_v molecular mass of water .... kg kmol^{-1}

Nu Nusselt number

p pressure of the air–vapour mixture........... Pa

p_v partial pressure of water vapour .... Pa

$p_{v,sat}$ saturation pressure of water vapour .... Pa

R_{gv} particular gas constant of the air–vapour mixture .... $\text{J kg}^{-1}\,{}^\circ\text{C}^{-1}$

R_v particular gas constant of water vapour......................... $\text{J kg}^{-1}\,{}^\circ\text{C}^{-1}$

r_{gv} average radius of pore volume occupied by the gaseous phase .... m

S_{ads} source-term in the energy conservation equation (3) .... W m^{-3}

Sh Sherwood number

$S_{v-\ell}$ source-term in the mass conservation equations (1) and (2) .... $\text{kg s}^{-1}\,\text{m}^{-3}$

s thickness of the desiccant layer .... m

T temperature............................... ${}^\circ\text{C}$

$\widetilde{T}$ average temperature of the porous medium.... ${}^\circ\text{C}$

T_i temperature of the interface .... ${}^\circ\text{C}$

T_f temperature of the airflow .... ${}^\circ\text{C}$

T_0 initial temperature in the porous medium ..... ${}^\circ\text{C}$

t time coordinate .... s

t_1 time to the variable $\widetilde{X}_\ell$ predicted by the detailed model attain 90% of the maximum variation .... s

w_v water-vapour content of the gas mixture inside the porous medium (d.b.) .... kg kg^{-1}

$\widetilde{w}_v$ average water-vapour content of the gas mixture in the porous medium (d.b.) .... kg kg^{-1}

X_ℓ adsorbed water content (d.b.) .... kg kg^{-1}

$\widetilde{X}_\ell$ average adsorbed water content in the porous medium (d.b.) .... kg kg^{-1}

$X_{\ell,0}$ initial adsorbed water content in the porous medium .... kg kg^{-1}

$\widetilde{X}_\ell^+$ dimensionless average adsorbed water content in the porous medium

y spatial coordinate .... m

Greek symbols

$\Delta\varepsilon$ deviation between the evolutions of the convective heat or mass fluxes, with respect to the detailed model (Eq. (19))

ε_{gv} volume fraction of the gaseous mixture within the porous medium

ϕ generic variable, i.e., temperature, adsorbed water content or water-vapour content

$\Phi_{h,ads}^+$ ratio between the heat released in the phase change of the vapour flux convected to the interface, if the phase change were completely achieved, and the convection heat flux (Eq. (26a))

φ_v vapour mass fraction of the gas mixture inside the porous medium...................... kg kg^{-1}

$\varphi_{v,eq}$ vapour mass fraction of the gas mixture within the porous medium at equilibrium .... kg kg^{-1}

$\varphi_{v,f}$ vapour mass fraction in the airflow ...... kg kg^{-1}

$\varphi_{v,i}$ vapour mass fraction of the gas mixture at the interface .... kg kg^{-1}

be analysed, namely the importance of neglecting the internal diffusive resistances.

2. Mathematical modelling

2.1. Physical model and simplifications

The physical model is schematically represented in Fig. 1, together with the boundary conditions considered.

The present study is focussed on the assessment of the internal resistances of the porous medium in the cross direction. For this purpose, it is of no interest to consider the variation of the flow properties in the streamwise direction. Therefore the physical domain is reduced to an element of the channel wall, which is considered as a homogeneous desiccant medium, having the properties of silica-gel and an infinitesimal length in the flow direction. The heat and mass transfer phenomena inside the porous medium are considered only in the y direction.

Inside the porous medium, two phases of water (vapour and adsorbed water) coexist in an equilibrium state, which is characterized by the water vapour–silica gel sorption isotherm. Due to the small dimension of the silica gel pores, only two mechanisms of mass transport are considered: surface diffusion of adsorbed water and Knudsen diffusion of water vapour [7]. The mass transport of dry air inside the pores is also neglected and the total pressure is assumed to be constant and uniform.

At the interface between the airflow and the desiccant layer, transfer of water vapour and of heat occurs. The upper face of the desiccant layer is located at the symmetry plane relatively to an adjacent channel, and consequently is treated as an adiabatic and impermeable boundary.

The air stream in contact with the infinitesimal-length wall element is treated as a well mixed flow (bulk flow), characterised by constant and uniform properties (pressure, temperature and vapour content), thus dispensing the need of solving any conservation equation in the flow domain.

2.2. Model equations, coefficients and properties

The governing equations can be mathematically deduced from appropriate balances applied to an infinitesimal control
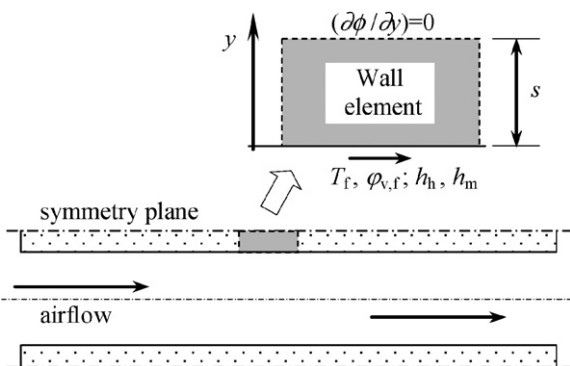

Fig. 1. Schematic representation of the plane airflow channel and of the calculation domain—a wall element—with the corresponding boundary conditions.

volume of the desiccant porous medium, leading to the following partial differential equations:

– mass conservation of water vapour:

$$\frac{\partial(\rho_{gv}^* \varphi_v)}{\partial t} = \frac{\partial}{\partial y}\left(\frac{D_{K,eff}}{\varepsilon_{gv}} \cdot \frac{\partial(\rho_{gv}^* \varphi_v)}{\partial y}\right) - S_{v\text{-}\ell} \tag{1}$$

– mass conservation of adsorbed water:

$$\frac{\partial(\rho_d^* X_\ell)}{\partial t} = \frac{\partial}{\partial y}\left(D_{s,eff}\frac{\partial(\rho_d^* X_\ell)}{\partial y}\right) + S_{v\text{-}\ell} \tag{2}$$

– energy conservation in the porous medium (gas mixture, adsorbed water and solid):

$$\frac{\partial}{\partial t}\left(\rho_d^* \cdot c_{p,p} \cdot T\right) = \frac{\partial}{\partial y}\left(\lambda_p \frac{\partial T}{\partial y}\right) + S_{ads} \tag{3}$$

Under the assumption of local thermodynamic equilibrium, only one of the above mass conservation equations must be solved. Dropping equation (1), the local value of the vapour mass fraction φ_v is specified as the equilibrium value $\varphi_{v,eq}$, obtained from the sorption isotherm as a function of the local values of T and X_ℓ [6]. Consequently, the source-term $S_{v\text{-}\ell}$ representing the adsorption rate per unit volume of the porous medium is expressed by:

$$S_{v\text{-}\ell} = \frac{\partial}{\partial y}\left(\frac{D_{K,eff}}{\varepsilon_{gv}} \cdot \frac{\partial(\rho_{gv}^* \varphi_v)}{\partial y}\right) - \frac{\partial(\rho_{gv}^* \varphi_v)}{\partial t} \tag{4}$$

The energy equation (3) is written in a classical form, in terms of temperature as the primary dependent variable. From the mathematical derivation of that equation, the following expression results:

$$S_{ads} = \rho_d^* \frac{\partial(X_\ell h_w')}{\partial t} + \frac{\partial}{\partial y}\left(D_{s,eff}\frac{\partial(\rho_d^* X_\ell)}{\partial y}h_\ell'\right)$$
$$+ \frac{\partial}{\partial y}\left(\frac{D_{K,eff}}{\varepsilon_{gv}} \cdot \frac{\partial(\rho_{gv}^* \varphi_v)}{\partial y}h_v\right) - \frac{\partial(\varphi_v \rho_{gv}^* h_{fg})}{\partial t} \tag{5a}$$

However, this term is usually taken as [53,55,56]:

$$S_{ads} = S_{v\text{-}\ell} \cdot h_{ads} \tag{5b}$$

Exploratory calculations showed that expressions (5a) and (5b) led both to practically the same results.

The specific enthalpy of the water vapour is given by

$$h_v = c_{p,v}T + h_{fg} \tag{6}$$

and that of the adsorbed water is given by

$$h_\ell' = h_v - h_{ads} \tag{7a}$$

or, by other means:

$$h_\ell' = h_\ell - h_w' \tag{7b}$$

where h_{ads} is the heat of adsorption and h_w' is the heat of wetting.

The instantaneous volume fraction ε_{gv} is the ratio between the instantaneous volume occupied by the air–vapour mixture in the pores and the total volume of the solid porous medium, being correlated with the local content of adsorbed water.

The effective diffusivity associated with Knudsen diffusion in a porous desiccant medium may be written as [6]:

$$D_{K,\text{eff}} = 97 r_{gv} \sqrt{\frac{T + 273.15}{R_v}} \frac{\varepsilon_{gv}}{\tau_{gv}} \qquad (8)$$

The effective diffusivity associated to the surface diffusion of adsorbed water $D_{s,\text{eff}}$ in a porous desiccant medium is directly influenced by the path tortuosity τ_{ℓ} and depends on the pair adsorbate–adsorbent. In the present work, the following expression was used [6]:

$$D_{s,\text{eff}} = \frac{D_{s0}}{\tau_{\ell}} \cdot \exp\left(\frac{-4.5 \times 10^{-4} h_{\text{ads}}}{b R_v (T + 273.15)} \right) \qquad (9)$$

where b depends on the type of adsorption bond and D_{s0} is a constant that depends on the adsorbent.

The effective thermal conductivity of the porous medium λ_p is difficult to predict accurately due to the coupled transfer phenomena. An average value is used according to the conductivity and mass fraction of each component phase. A similar procedure is used regarding the specific heat of the porous medium.

Finally, for the graphical representation of the results, the moisture content of the air–vapour mixture w_v (mass fraction of vapour on dry basis) is calculated from the vapour mass fraction φ_v through

$$w_v = \varphi_v / (1 - \varphi_v) \qquad (10)$$

2.3. Boundary conditions

The upper and the two lateral faces of the wall element are considered adiabatic, as well as impermeable to both water phases (cf. Fig. 1). According to the low-mass-transfer rate theory [57], the mass and heat fluxes evaluated on the gas side of the interface are

$$j_m = h_m \cdot \rho_f (\varphi_{v,f} - \varphi_{v,i}) \qquad (11)$$

and

$$j_h = h_h (T_f - T_i) \qquad (12)$$

respectively, where $\varphi_{v,i}$ and T_i are instantaneous values at the interface and $\varphi_{v,f}$ and T_f are instantaneous bulk flow values. The convection heat and mass transfer coefficients are taken from the values of the Nusselt and the Sherwood numbers for the fully developed laminar regime of an internal channel flow, which are related by the well-known Chilton–Colburn analogy:

$$Sh = Nu \cdot Le^{-1/3} \qquad (13)$$

the Lewis number being estimated from its definition:

$$Le = \lambda_f / (\rho_f c_{p,f} D_f) \qquad (14)$$

The convection coefficients are then obtained by the expressions:

$$h_h = Nu \cdot \lambda_f / d_{\text{hyd}} \qquad (15)$$

and

$$h_m = Sh \cdot D_f / d_{\text{hyd}} \qquad (16)$$

In a real channel the convection coefficients depend on the flow regime, the channel geometry and on the distance from the channel inlet. The interface fluxes calculated through Eqs. (11) and (12) are positive in the y direction, i.e., when entering the wall element (cf. Fig. 1).

At the interface with the airflow, the following surface balances can be written:

$$\left[-\frac{D_{K,\text{eff}}}{\varepsilon_{gv}} \cdot \frac{\partial(\rho_{gv}^* \varphi_v)}{\partial y} - D_{s,\text{eff}} \frac{\partial(\rho_d^* X_{\ell})}{\partial y} \right]_{y=0^+}$$
$$= h_m \cdot \rho_f (\varphi_{v,f} - \varphi_{v,i}) \qquad (17)$$

and

$$\left[-\frac{D_{K,\text{eff}}}{\varepsilon_{gv}} \cdot \frac{\partial(\rho_{gv}^* \varphi_v)}{\partial y} h_v - D_{s,\text{eff}} \frac{\partial(\rho_d^* X_{\ell})}{\partial y} h_{\ell}' - \lambda_p \frac{\partial T}{\partial y} \right]_{y=0^+}$$
$$= h_h (T_f - T_i) + h_m \cdot \rho_f (\varphi_{v,f} - \varphi_{v,i}) \cdot h_{v,i} \qquad (18)$$

2.4. Numerical modelling

The detailed model described above will be hereafter taken as the reference or standard model. The partial differential equations (1)–(3) are discretized using the finite volume method [58] and solved through the tri-diagonal matrix algorithm (TDMA). An iterative procedure is performed in each time step, in order to handle the strong coupling between the governing equations and the non-linearities implied by the changing properties. With appropriate control parameters, the resulting numerical model showed a rather stable and robust behaviour along the calculation procedure.

The calculation domain is divided into control-volumes, forming a Cartesian and non-uniform grid, which is gradually expanded from the solid-flow interface towards the adiabatic/impermeable upper surface. A very short distance from the first inner grid point to the interface is imposed (0.5–1 μm), thus ensuring an accurate numerical representation of the boundary conditions expressed by Eqs. (11) and (12). These boundary conditions are introduced via the corresponding source-terms in the discretization equations [58]. Since the present study is focused on the assessment of the cross-direction resistances of the porous medium, the model is assumed one-dimensional and only one column of nodes is solved.

The time discretization consists of a step-by-step evolution, each time interval being automatically generated along the numerical simulation of the transient process. Basically, the sharper is the time evolution of the dependent variables, the shorter are the time steps imposed, and conversely. This generation algorithm guarantees a suitable representation of the transient evolutions, while minimizing the computing time needed to simulate a complete adsorption/desorption process until the steady-state.

Two simplifying approaches based on the lumped-capacitance method are numerically investigated. One corresponds to the theoretical analysis of the system neglecting the internal resistances to the heat and mass diffusion (approach A—"Null Res."). This global lumped-capacitance approach is achieved by specifying great enough values to all diffusion coefficients

($D_{K,eff}$, $D_{s,eff}$, and λ_p). The other approach (B—"Null Thermal Res.") corresponds to the thermal lumped-capacitance method and it is implemented by specifying a great enough value to the thermal conductivity λ_p of the medium.

3. Results and discussion

The use of dimensional analysis in the present situation is difficult, due to the strong variations of some parameters along the transient process and, on the other hand, to the complexity inherent to the coupled phenomena involved.

Starting from a given initial state, the response of the desiccant wall to a step change of the airflow conditions is simulated until global equilibrium is achieved. Table 1 summarizes the input data for the calculations, namely the state of the airflow and the initial conditions of the desiccant wall.

The value 2.45 was assigned to the Nusselt number corresponding to heat convection between a uniform temperature wall and a fully-developed laminar flow inside a corrugate sinusoidal-type channel of a compact exchanger [25]. The channel cross-section area was 4.5 mm^2, with an internal perimeter of 10.6 mm and a hydraulic diameter of 1.69 mm, approximately.

Runs were conducted covering a wide range of values of the layer thickness (0.01 mm to 5 mm) in order to study the validity of neglecting internal thermal and mass resistances (approach A —"Null Res.") and only the thermal resistance (approach B— "Null Thermal Res.").

The fluid properties of the binary air–vapour mixture depend on the primary variables T and φ_v. Appropriate φ_v-weighted functions were developed that take into account the individual dry-air and water–vapour properties available from thermodynamic tables [59]. The properties of the desiccant medium are referred to silica gel RD [6].

3.1. Tests for grid and time-step dependence

Exploratory calculations were made for the adsorption case referred in Table 1 and a wall-thickness of 5 mm, in order to analyse the dependence of the results on the space and time discretization. It was concluded that no more that 10 nodes in the y direction were needed to guarantee grid-independent solutions. All subsequent calculations were then performed with a single column of 10 control-volumes. On the other hand, with the procedure described in Section 2.4, the transient solutions showed very little dependence on the size and variation of the

time-interval, provided that a sufficiently short period (0.5 s, typically) is taken for the first 2 or 3 time-steps.

3.2. Significance of the internal resistances

A particular adsorption case was selected for the detailed analysis of the transient evolutions of the variables inside a desiccant layer with 1 mm of thickness. Time-evolving profiles are shown in Figs. 2(a)–(c) as calculated with the "normal" internal resistances (i.e., using the "standard" model, as described in Section 2). It is seen that the temperature field presents small gradients, meaning that the internal resistance to heat diffusion

Fig. 2. Time-varying profiles of the dependent variables along the adsorption process in a desiccant layer of $s = 1$ mm, considering the "normal" internal resistances: (a) temperature T; (b) vapour content w_v; (c) content of adsorbed water X_ℓ.
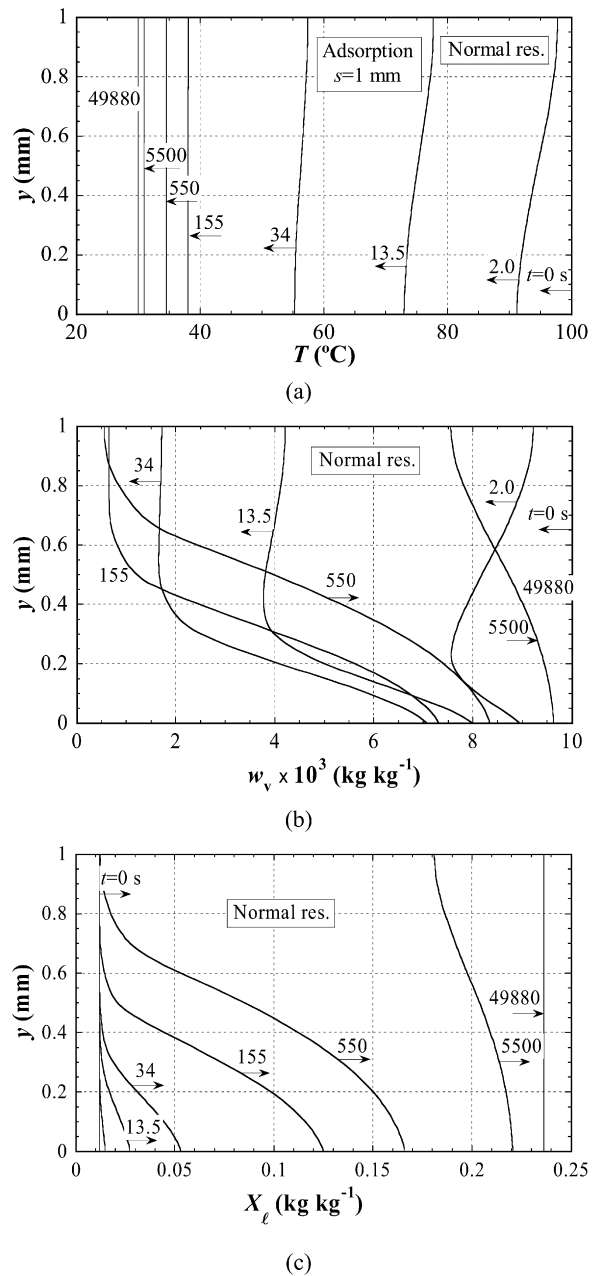
Table 1
Airflow and initial conditions of the desiccant wall for both the adsorption and desorption processes simulated

	Adsorption	Desorption
T_f (°C)	30	100
$w_{v,f}$ (kg kg^{-1})	0.01	0.01
p (Pa)	101 325	101 325
T_0 (°C)	100	30
$X_{\ell,0}$ (kg kg^{-1})	0.0126	0.2363

is almost insignificant, contrarily to those restricting the mass diffusion.

The same case was simulated using two different simplified versions of the model, taking thickness values in the range of 0.01 to 5 mm: one cancelling both thermal and mass resistances inside the desiccant wall (approach A), the other cancelling only the thermal resistance (approach B). The diagrams in Figs. 3 and 4 show the time evolutions of a set of relevant variables for the adsorption and the desorption processes referred in Table 1, respectively. It can be observed that the results of approach A (dashed lines) show a good agreement with those of the detailed model (full lines) only for very thin desiccant layers ($s \leqslant 0.01$ mm), and that they diverge as the desiccant layer thickness increases. This indicates that significant inaccuracies may result when the internal resistances to both heat and mass diffusion are neglected. In particular, according to Figs. 3(c) and (d) and Figs. 4(c) and (d), this hypothesis may lead to unrealistic estimation of the convection fluxes at the interface. It can be observed that the results of approach B (dashed-lines with marker) agree well with the detailed model for all studied wall thicknesses, except for $s = 5$ mm. In all cases, no significant deviation is observed in the evolutions of the adsorbed water content, as can be seen in Figs. 3(b) and 4(b).

To better assess the deviations of the results with each of the approaches A and B relatively to the detailed modelling, a global indicator is defined, on the basis of the interfacial heat and mass fluxes:

$$\Delta \varepsilon = \frac{\int_0^{t_1} (j_{A,B} - j_{norm}) \, dt}{\int_0^{t_1} j_{norm} \, dt} \quad (19)$$

where t_1 is chosen as the time needed in each case for the variable $\widetilde{X}_\ell$ to reach 90% of its maximum variation, when predicted by the detailed model (index 'norm'). Fig. 5 allows to conclude that the global lumped-capacitance model is reasonably valid for $s \leqslant 0.1$ mm ($\Delta \varepsilon \leqslant 4\%$), while the thermal lumped-capacitance approach is valid for all studied values of the wall thickness.

In a general way, the transient sorption process depends on the convective heat and mass transfer coefficients and, mainly, on the total mass of the wall and on the coefficients for the internal diffusion of heat and mass, which exhibit opposed variations along the adsorption and the desorption processes. This is the reason for the faster evolution of the desorption process, as it can be inferred from the comparison of Figs. 4(a) and (b).

3.3. Final remarks

The lumped capacitance method can be often used to solve problems of transient heat conduction in solids. The assessment of its validity is based on the magnitude of the Biot number:

$$Bi_h = \frac{h_h s}{\lambda_p} \quad (20)$$

Likewise, in transient mass diffusion problems inside a porous wall with no phase change, a Biot number for mass transfer can be used:

$$Bi_m = \frac{h_m s}{D_p} \quad (21)$$

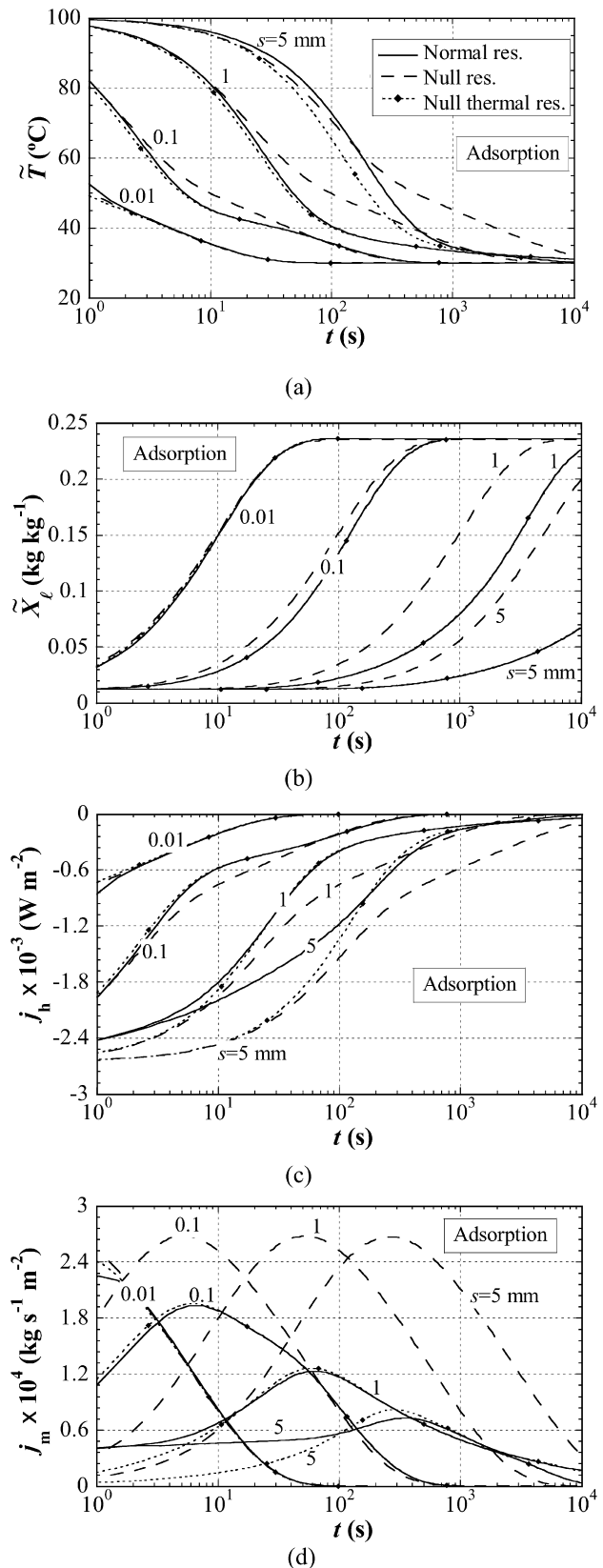Fig. 3. Effect of imposing internally null mass and thermal resistances and null thermal resistance on the predicted evolution of the adsorption process (v. Table 1), by comparison of the time variations of: the space-averaged (a) temperature and (b) content of adsorbed water; the (c) heat and (d) mass fluxes at the interface. ((——) normal, (– – –) null thermal and mass and (—♦—) null thermal resistances.)
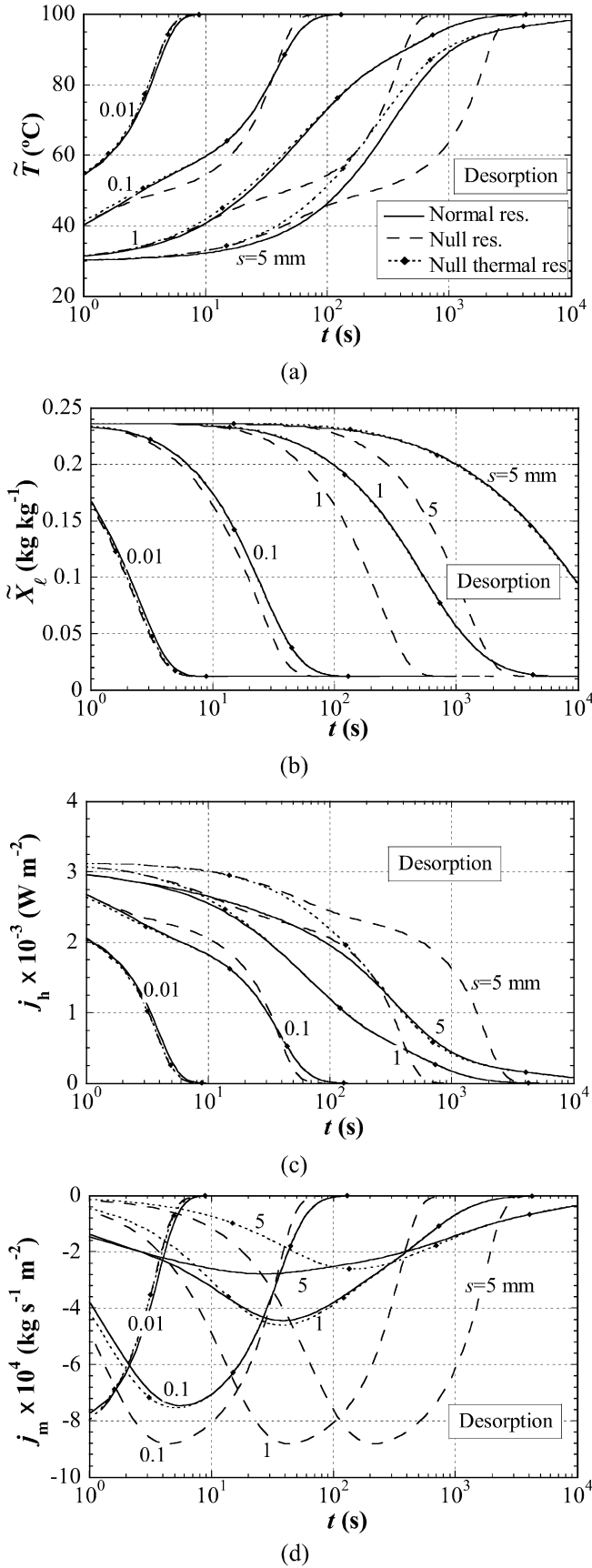
(a)

(b)

(c)

(d)

Fig. 4. Effect of the null thermal and mass resistances and null thermal resistance assumptions on the predicted time evolution of the desorption process (v. Table 1): (a)–(d) as in Fig. 5.
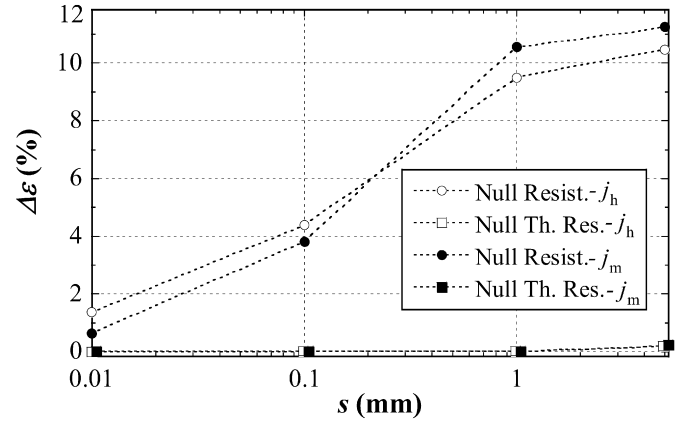

Fig. 5. Deviation of the evolutions of the convective fluxes of the null resistances and of null thermal resistance approaches relatively to the predicted evolutions of the adsorption process with normal resistances.

where D_p is a coefficient of mass diffusion within the porous medium and s is the half-thickness of the wall. In the present case, the heat and mass fluxes convected to the surface split into two terms within the porous medium, due to the phase change imposed by the sorption phenomenon, as shown by the balance equations (17) and (18). These may be rewritten as:

$$h_m(C_{v,f} - C_{v,i}) = \left[-D_{K,eff} \cdot \frac{\partial C_v}{\partial y} - D_{s,eff} \frac{\partial C_\ell}{\partial y} \right]_{y=0^+} \quad (22)$$

and

$$h_h(T_f - T_i) = \left[-j_{ads} h_{ads} - \lambda_p \frac{\partial T}{\partial y} \right]_{y=0^+} \quad (23a)$$

where

$$j_{ads} = -D_{s,eff} \frac{\partial C_\ell}{\partial y} \quad (23b)$$

represents the adsorption rate per unit area of the interface, after the mass balance expressed by Eq. (22). The resulting normalized forms of the mass and energy balances at the interface are, respectively:

$$\left[-\frac{1}{Bi_K} \cdot \frac{\partial C_v^+}{\partial y^+} - \frac{1}{Bi_s} \cdot \frac{\partial C_\ell^+}{\partial y^+} \right]_{y=0^+} = 1 \quad (24)$$

and

$$\left[-\frac{\Phi_{h,ads}^+}{Bi_s} \frac{\partial C_\ell^+}{\partial y^+} - \frac{1}{Bi_h} \frac{\partial T^+}{\partial y^+} \right]_{y=0^+} = 1 \quad (25)$$

where the dimensionless mass concentrations are $C_v^+ = C_v/(C_{v,f} - C_{v,i})$ and $C_\ell^+ = C_\ell/(C_{v,f} - C_{v,i})$ and three Biot numbers appear: one for heat transfer with the usual definition, $Bi_h = h_h s/\lambda_p$, and two Biot numbers for mass transfer, respectively: $Bi_K = h_m s/D_{K,eff}$ for the Knudsen diffusion, and $Bi_s = h_m s/D_{s,eff}$ for the surface diffusion. The parameter

$$\Phi_{h,ads}^+ = \frac{h_m(C_{v,f} - C_{v,i})h_{ads}}{h_h(T_i - T_f)} \quad (26a)$$

represents the ratio between the heat released in the phase change of the vapour flux convected to the interface, if the phase
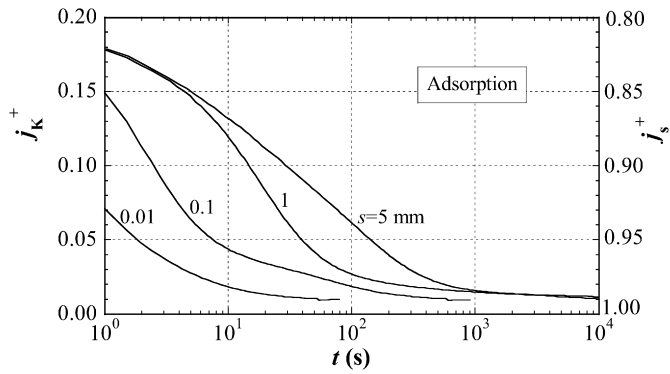
Fig. 6. Dimensionless mass fluxes by Knudsen and by surface diffusion at the interface defined as $j_K^+ = -(1/Bi_K) \cdot (\partial C_v^+/\partial y^+)$ and $j_s^+ = -(1/Bi_s) \cdot (\partial C_\ell^+/\partial y^+)$, respectively.
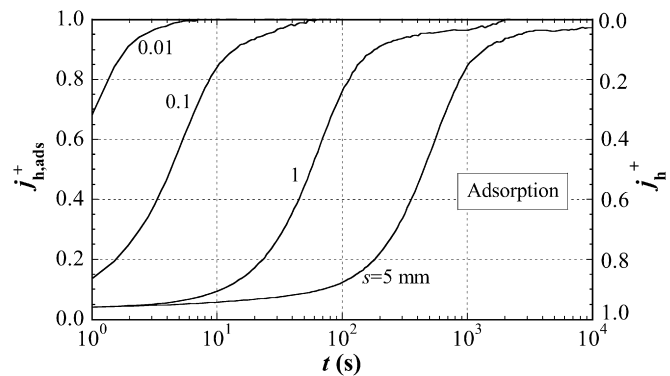

Fig. 7. Dimensionless heat generation/consumption rate per unit area of the interface and heat conduction flux, defined by $j_{h,ads}^+ = -(\Phi_{h,ads}^+/Bi_s) \cdot (\partial C_\ell^+/\partial y^+)$ and $j_h^+ = -(1/Bi_h) \cdot (\partial T^+/\partial y^+)$, respectively.

change were completely achieved, and the convection heat flux. It may also be expressed as:

$$\Phi_{h,ads}^+ = \frac{h_{ads} D_f (C_{v,f} - C_{v,i})}{\lambda_f (T_i - T_f)} Le^{-1/3} \qquad (26b)$$

after the Chilton–Colburn analogy (cf. Eqs. (1), (15), (16)), which provides a quick assessment of its order of magnitude.

Eqs. (24) and (25) indicate that the internal gradients of temperature and of the mass concentration of both water phases are coupled. This means that the relevance of the internal gradients can not be directly assessed by a unique dimensionless parameter.

The time evolutions of the terms on the left side of Eqs. (24) and (25) are plotted in Figs. 6 and 7 for the adsorption case and different wall thicknesses. It can be seen from Fig. 6 that, after a short initial period, the contribution of Knudsen diffusion is negligible, meaning that the surface diffusion is the dominant mass transport phenomenon inside the porous medium, which is in agreement with [6]. On the other hand, Fig. 7 shows the relative importance of the two terms in Eq. (25) along the process: j_h^+ is generally more important in the beginning, while the phase change term $j_{h,ads}^+$ becomes gradually dominant.

If the assertions supported by Fig. 6 are included in Eq. (24), it can be concluded that the relevance of the internal gradients
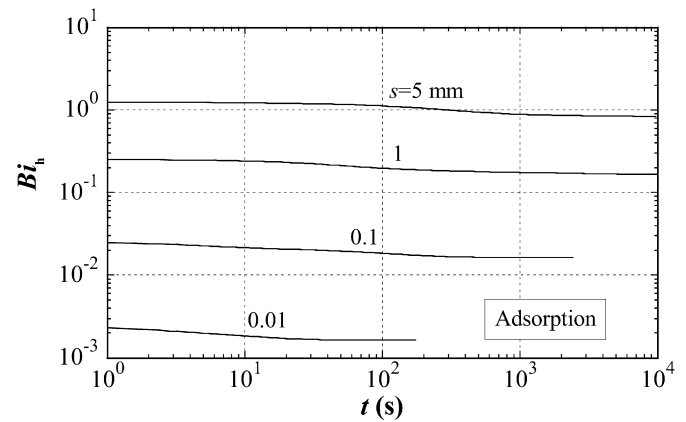
(a)
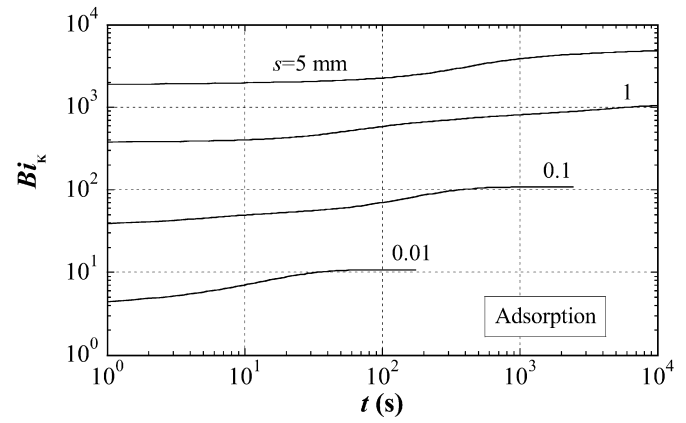
(b)
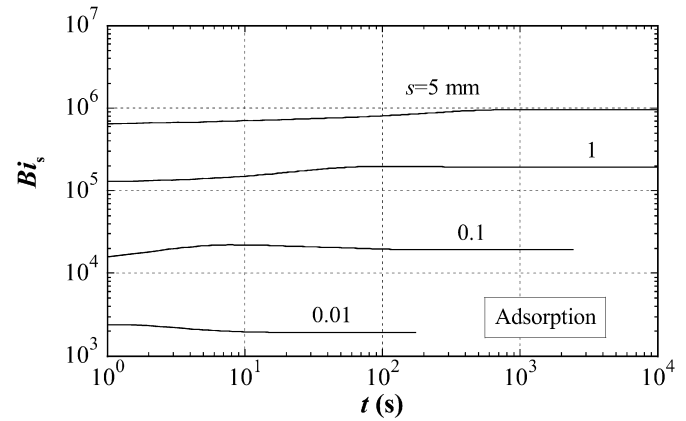
(c)

Fig. 8. Biot numbers for: (a) heat conduction, (b) Knudsen and (c) surface diffusion.

of mass concentration of adsorbed water may be directly represented by the magnitude of Bi_s. On the contrary, the observation of Fig. 7 together with Eq. (24) shows that both terms have importance, even at different stages of the process, the thermal gradients being affected simultaneously by Bi_h and by the parameter $Bi_s/\Phi_{h,ads}^+$. The calculated time evolutions of the three Biot numbers defined previously are plotted for the same cases in Figs. 8(a)–(c), where an obvious influence of the desiccant layer thickness can be observed. Besides, these parameters experience significant variations along the process: Bi_h decreases about 30%, while Bi_K nearly doubles its value and Bi_S increases

Fig. 9. Ratio between the heat released in the phase change and the convection heat flux, $\Phi_{h,ads}^{+}$.

almost 50% (except for $s \leqslant 0.01$ mm, with changes of about $\pm 10\%$). In Fig. 9, the calculated time evolutions of the parameter $\Phi_{h,ads}^{+}$ are presented. It can be observed that they are very close to the evolutions obtained for the parameter $j_{h,ads}^{+}$, which is in agreement with the fact that the surface diffusion is the dominant mass transport phenomenon.

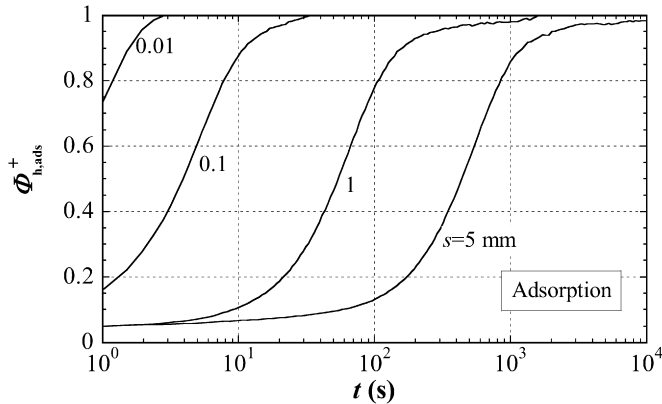The analysis of the results of Section 3.2, in particular those represented in Fig. 5, leads to the conclusion that the thermal lumped capacitance model is valid when $Bi_h \leqslant 1$ ($s = 5$ mm), approximately, and that the validity of the global lumped capacitance model is reasonable when $Bi_s \leqslant 2 \times 10^4$, this value being several orders of magnitude higher than the critical thermal Biot number.

4. Conclusions

A detailed mathematical model was formulated in order to describe the behaviour of a wall element of a compact heat and mass exchanger, like a desiccant wheel. A one-dimensional version of it was used under specific conditions, aiming to determine the validity field of the hypotheses of negligible internal resistances of the porous medium in the cross direction, which are frequently used in simplified models.

The capabilities of the model were demonstrated, namely the detailed analysis of the transient distributions of the primary variables (temperature, content of adsorbed water and water vapour content) across the desiccant layer. The time evolutions of the averaged variables, as well as the transient heat and mass fluxes at the interface, predicted by the detailed standard model were compared with those based on the hypothesis of "null" internal resistances for heat and mass diffusion (approach A) and of null thermal resistances (approach B).

From a dimensional analysis, it was possible to conclude that in a sorption process the relevance of the internal temperature and mass concentration gradients can only be assessed through the order of magnitude of a set of four dimensionless numbers, which includes at least three Biot numbers.

It was also concluded that, for the adsorption/desorption cases simulated, the heat and mass lumped capacitance assumption supporting the pseudo gas-side model is valid only for desiccant layers thinner than about 0.1 mm, corresponding

approximately to the range of $Bi_s < 2 \times 10^4$. The use of the thermal lumped capacitance model in this kind of coupled heat and mass transfer problems appears to be reasonable when $Bi_h < 1$.

In a 2-D modelling, the characteristic dimension associated to the internal diffusion phenomena occurring in the axial direction is the channel length, which is typically two or three orders of magnitude greater than the thickness of the desiccant layer. Therefore, the assumption of null resistances to diffusion in the axial direction is not applicable. In this case, an interesting hypothesis to be numerically investigated consists of assuming infinite resistance values (or, by other words, neglecting diffusion) in the streamwise direction, meaning that the internal axial gradients are only due to the convective interaction with the air flow. This will be addressed in a future work, when using the 2-D formulation of the model.

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