An analytical evaluation of the response of steel joints under bending and axial force

L. Simões da Silva ^{a,*}, A.M. Girão Coelho ^b

^a *Departamento de Engenharia Civil, Fac. de Ciencias e Tech., Universidade de Coimbra, Polo II, Pinhal de Marrocos, 3030-290 Coimbra, Portugal*

^b *Departamento de Engenharia Civil, Instituto Superior de Engenharia de Coimbra, Quinta da Nora, Apartado 10057, 3031-601 Coimbra, Portugal*

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Abstract

This paper presents an equivalent elastic model where each bi-linear spring is replaced by two equivalent elastic springs using an energy formulation and in the context of a post-buckling stability analysis. Such a model yields analytical solutions for the evaluation of the behaviour of steel joints under compressive forces, combined axial force and bending moment, which enables the reproduction of their full non-linear behaviour. The resulting formulation is applied to a simple beam-to-column welded connection initially loaded in pure compression. Subsequent loading of the joint in combined bending and various levels of axial force clearly shows the reduction in moment capacity. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Predicting the behaviour of steel joints has been the object of intensive research over the past 15 years. Because beam-to-column and beam-to-beam joints act predominantly in bending, the major effort for these joint configurations was directed at establishing moment–rotation curves in the absence of an axial force in the beam. Hence, present estimates of moment resistance and initial stiffness in beam-to-column and beam-to-beam joints are only valid whenever the axial force in the beam does not exceed 10% of its plastic resistance, as stated, for example, in the Annex J of Eurocode 3 [1]. Yet, there are many examples of steel structures where beams carry significant axial forces, pitched-roof portal frames being a typical example. It is thus necessary to widen the scope of the current state-of-the-art in steel joints to the evaluation of its behaviour under pure compression or combined axial force and bending moment.

Adopting the philosophy of the component method [2], the overall response of a joint loaded in bending can be determined based on the properties of its parts (components). Extending the component method to take the effect of an axial force into account, as already proposed in the literature [3], the behaviour of a joint loaded in combined bending and axial force may be assessed accordingly. Therefore, the components are assembled into a mechanical model, consisting of extensional springs and rigid links, as shown in Fig. 1 for the particular case of a welded beam-to-column steel joint. Each spring (component) is characterized by a non-linear force–deformation (F – Δ) curve, adequately represented by a bi-linear approximation (Fig. 2), as explained by Simões da Silva et al. [4].

* Corresponding author. Tel.: +351-239-797-216; fax: +351-239-797-217.

E-mail address: luis_silva@gipac.pt (L. Simões da Silva).

Nomenclature

k_e	initial elastic stiffness
k_p	post-limit stiffness
$q_1; \phi$	total rotation of the joint
q_2	rotation of rigid links (compression zone)
q_3	rotation of rigid links (tension zone)
q_4	axial displacement of the joint
z	lever arm
L	length of rigid links
L_1	range of the point of application of the axial force
M	bending moment
N	axial force
F^C	resistance (limit load)
P^B	twice the limit load
V	total potential energy function
Δ	total (axial) displacement
Δ^f	collapse displacement
Δ^y	yield displacement

The analysis of the spring model of Fig. 1(b) is complex. In order to obtain solutions for such model, an incremental non-linear procedure [5] is usually required. Extending the approach proposed in Ref. [6] for pure bending, this paper presents generalized models for the evaluation of the response of steel joints under bending and axial force, based on an elastic analogy of elastic–plastic behaviour [7]. These models are open to simple analytical closed-form solutions that reproduce the full non-linear behaviour of steel joints, as described below.

2. Behaviour under pure compression

Consider the three-degree-of-freedom equivalent elastic model of Fig. 3, ($q_1 = \phi$ – total rotation of the joint; q_2 – rotation of the rigid links of length L_c ; q_4 – axial displacement of the joint; L_1 – range of the point of application of the axial force), whereby only the spring (component) in the compression zone is assumed to exhibit a bi-linear response,

Fig. 1. Welded beam-to-column steel joint: (a) connection geometry and (b) spring model.

Fig. 2. Component characterization.

Fig. 3. Simplified model with non-linear compression zone loaded in compression.

with the remaining spring in the tension zone behaving elastically. The following total potential energy function V is obtained:

$$V(q_1, q_2, q_4) = \frac{1}{2}k_{et}\left(q_4 - \frac{z}{2}\sin q_1\right)^2 + \frac{1}{2}k_{ec}\left[q_4 + \frac{z}{2}\sin q_1 - 2L_c(1 - \cos q_2)\right]^2 + \frac{1}{2}k_{pc}\left[\frac{P_C^B}{2k_{pc}} + 2L_c(1 - \cos q_2)\right]^2 - N[q_4 + L_1(1 - \cos q_1)], \quad (1)$$

where k_e is the elastic stiffness, k_p the post-limit stiffness and P^B twice the limit load of the component, applied as a pre-compression (subscripts t and c refer to tension and compression zones, respectively). Eliminating q_4 as a passive coordinate and carrying out the differentiation with respect to the various degrees-of-freedom, after some reworking, the following equilibrium equations are yielded:

$$\frac{\partial V}{\partial q_1} = 0 \iff N = \frac{2zk_{ec}k_{et}\cos q_1[z\sin q_1 - 2L_c(1 - \cos q_2)]}{2L_1(k_{ec} + k_{et})\sin q_1 - z(k_{ec} - k_{et})\cos q_1}, \quad (2)$$

$$\frac{\partial V}{\partial q_2} = 0 \iff \sin q_2 = 0 \quad (3a)$$

$$\left[-\frac{2zL_c k_{ec} k_{et}}{k_{ec} + k_{et}} \sin q_1 + 4L_c^2 \left(k_{pc} + \frac{k_{ec} k_{et}}{k_{ec} + k_{et}}\right) (1 - \cos q_2) + P_C^B L_c - \frac{2L_c k_{ec}}{k_{ec} + k_{et}} N\right] = 0. \quad (3b)$$

The appropriate combination of Eqs. (2) and (3) provides the equilibrium paths of the system, reproduced below:

(i) Fundamental solution is

$$\begin{cases} N = \frac{2z^2 k_{ec} k_{et} \sin q_1}{2L_1(k_{ec} + k_{et}) \tan q_1 - z(k_{ec} - k_{et})}, \\ q_2 = 0, \\ q_4 = \frac{1}{k_{ec} + k_{et}} \left[-\frac{z}{2} (k_{ec} - k_{et}) \sin q_1 + \frac{z^2 k_{ec} k_{et} \sin(2q_1)}{2L_1(k_{ec} + k_{et}) \sin q_1 + z(k_{ec} - k_{et}) \cos q_1} \right]. \end{cases} \quad (4)$$

(ii) Non-linear solution in q_2 is

$$\begin{cases} N = \frac{2z k_{ec} k_{et} [z \sin q_1 - 2L_c(1 - \cos q_2)]}{2L_1(k_{ec} + k_{et}) \tan q_1 - z(k_{ec} - k_{et})}, \\ 1 - \cos q_2 = \frac{2z k_{ec} k_{et} (z + 2L_1 \tan q_1) \sin q_1 - P_C^B [2L_1(k_{ec} + k_{et}) \tan q_1 - z(k_{ec} - k_{et})]}{4L_c \{ 2L_1 [k_{ec} k_{et} + (k_{ec} + k_{et}) k_{pc}] \tan q_1 + z [k_{ec} k_{et} - (k_{ec} - k_{et}) k_{pc}] \}}, \\ q_4 = \frac{1}{k_{ec} + k_{et}} \left\{ -\frac{z}{2} (k_{ec} - k_{et}) \sin q_1 + 2L_c k_{ec} (1 - \cos q_2) + \frac{2z k_{ec} k_{et} \cos q_1 [z \sin q_1 - 2L_c(1 - \cos q_2)]}{2L_1(k_{ec} + k_{et}) \sin q_1 - z(k_{ec} - k_{et}) \cos q_1} \right\}. \end{cases} \quad (5)$$

3. Behaviour under combined bending and axial force

Now, consider the general equivalent elastic model of Fig. 4, where an extra degree-of-freedom (q_3) was brought in to deal with the non-linear tensile spring [6], corresponding to the rotation of the rigid links of length L_t . The previous expressions can be generalised to accommodate bending and compression. The potential function is written thus:

$$\begin{aligned} V(q_1, q_2, q_3, q_4) = & \frac{1}{2} k_{et} \left[q_4 - \frac{z}{2} \sin q_1 + 2L_t(1 - \cos q_3) \right]^2 + \frac{1}{2} k_{ec} \left[q_4 + \frac{z}{2} \sin q_1 - 2L_c(1 - \cos q_2) \right]^2 + \frac{1}{2} k_{pt} \left[\frac{P_T^B}{2k_{pt}} \right. \\ & \left. + 2L_t(1 - \cos q_3) \right]^2 + \frac{1}{2} k_{pc} \left[\frac{P_C^B}{2k_{pc}} + 2L_c(1 - \cos q_2) \right]^2 - Mq_1 - N[q_4 + L_1(1 - \cos q_1)]. \end{aligned} \quad (6)$$

Applying the same procedure as for the system under pure compression, the following equilibrium paths are obtained:

(i) Fundamental solution is

$$\begin{cases} M = \frac{z^2}{2} \frac{k_{ec} k_{et}}{k_{ec} + k_{et}} \sin(2q_1) - N \left[L_1 \sin q_1 - \frac{z(k_{ec} - k_{et})}{2(k_{ec} + k_{et})} \cos q_1 \right], \\ q_2 = 0 = q_3, \\ q_4 = \frac{1}{k_{ec} + k_{et}} \left[-\frac{z}{2} (k_{ec} - k_{et}) \sin q_1 + N \right]. \end{cases} \quad (7)$$

(ii) Non-linear solution in q_2 is

Fig. 4. General equivalent elastic model loaded in bending and axial compression.

$$\begin{cases} M = \frac{zk_{ec}k_{et}}{k_{ec}+k_{et}} \left\{ z \sin q_1 - \frac{2zk_{ec}k_{et} \sin q_1 - (k_{ec}+k_{et})P_C^B}{2[(k_{ec}+k_{et})k_{pc}+k_{ec}k_{et}]} \right\} \cos q_1 - N \left\{ L_1 \sin q_1 - z \frac{(k_{ec}-k_{et})k_{pt}-k_{ec}k_{et}}{2[(k_{ec}+k_{et})k_{pt}+k_{ec}k_{et}]} \cos q_1 \right\}, \\ 1 - \cos q_2 = \frac{2zk_{ec}k_{et} \sin q_1 - (k_{ec}+k_{et})P_C^B + 2k_{ec}N}{4L_c[(k_{ec}+k_{et})k_{pc}+k_{ec}k_{et}]}, \\ q_3 = 0, \\ q_4 = \frac{1}{k_{ec}+k_{et}} \left[-\frac{z}{2}(k_{ec}-k_{et}) \sin q_1 + 2L_c k_{ec}(1 - \cos q_2) + N \right]. \end{cases} \quad (8)$$

(iii) Non-linear solution in q_3 is

$$\begin{cases} M = \frac{zk_{ec}k_{et}}{k_{ec}+k_{et}} \left\{ z \sin q_1 - \frac{2zk_{ec}k_{et} \sin q_1 - (k_{ec}+k_{et})P_T^B}{2[(k_{ec}+k_{et})k_{pt}+k_{ec}k_{et}]} \right\} \cos q_1 - N \left\{ L_1 \sin q_1 - z \frac{(k_{ec}-k_{et})k_{pt}-k_{ec}k_{et}}{2[(k_{ec}+k_{et})k_{pt}+k_{ec}k_{et}]} \cos q_1 \right\}, \\ q_2 = 0, \\ 1 - \cos q_3 = \frac{2zk_{ec}k_{et} \sin q_1 - (k_{ec}+k_{et})P_T^B - 2k_{et}N}{4L_t[(k_{ec}+k_{et})k_{pt}+k_{ec}k_{et}]}, \\ q_4 = \frac{1}{k_{ec}+k_{et}} \left[-\frac{z}{2}(k_{ec}-k_{et}) \sin q_1 - 2L_t k_{et}(1 - \cos q_3) + N \right]. \end{cases} \quad (9)$$

(iv) Non-linear solution in q_2 and q_3 is

$$\begin{cases} M = \frac{zk_{ec}k_{et}}{k_{ec}+k_{et}} [z \sin q_1 - 2L_c(1 - \cos q_2) - 2L_t(1 - \cos q_3)] \cos q_1 - N \left(L_1 \sin q_1 - \frac{z}{2} \frac{k_{ec}k_{et}}{k_{ec}+k_{et}} \cos q_1 \right), \\ 1 - \cos q_2 = \frac{2zk_{ec}k_{et} \sin q_1 - P_C^B(k_{ec}+k_{et}) - 4L_t k_{ec}k_{et}(1 - \cos q_3) + 2k_{ec}N}{4L_c[(k_{ec}+k_{et})k_{pc}+k_{ec}k_{et}]}, \\ 1 - \cos q_3 = \frac{2zk_{ec}k_{et}k_{pc} \sin q_1 - k_{ec}k_{et}(P_T^B - P_C^B) - (k_{ec}+k_{et})k_{pc}P_T^B - 2k_{et}(k_{ec}+k_{pc})N}{4L_t[(k_{ec}+k_{et})k_{pc}k_{pt}+k_{ec}k_{et}(k_{pc}+k_{pt})]}, \\ q_4 = \frac{1}{k_{ec}+k_{et}} \left[-\frac{z}{2}(k_{ec}-k_{et}) \sin q_1 + 2L_c k_{ec}(1 - \cos q_2) - 2L_t k_{et}(1 - \cos q_3) + N \right]. \end{cases} \quad (10)$$

4. Application of these procedures to a beam-to-column welded joint

4.1. Component characterization

In order to illustrate the application of the equivalent elastic models, one joint configuration was chosen from the database SERICON II (Klein 105.010) [8] corresponding to a welded beam-to-column steel joint, described in Fig. 1, which was tested in pure bending by Klein at the University of Innsbruck in 1985. As mentioned before, the revised Annex J of Eurocode 3 [1] does not cover steel joints subjected to axial force or combined bending and axial force. In the absence of data to characterize the relevant components, (1) column web in shear, (2) column web in compression, (3) column web in tension, the values used by Simões da Silva et al. [6] are adopted for the initial stiffness k_e , post-limit stiffness k_p and resistance F^C reproduced in Table 1 and Fig. 5.

4.2. Welded connection subjected to pure compression

Analysis of Table 1 shows that the compressive zone exhibits the lowest resistance. The simplified model with non-linear springs in the compression zone is thus first applied, giving the force–displacement diagram (N – q_4) of Fig. 6. It is noted that the post-buckling path is unstable, the critical load reaching 594.65 kN, well below the nominal plastic strength of the beam, given as 917.68 kN. It is interesting to note that neglecting the post-limit stiffness of the critical component, as implicitly stated in Eurocode 3, $k_{pc} = 0$ kN/m, the post-buckling path would become even more unstable, with greater sensitivity to imperfections. Also, a flat (zero stiffness) post-buckling response could only be attained for a post-limit stiffness of the critical component of $k_{pc} = 46\,200$ kN/m.

Table 1
Tension and compression zones properties for joint tested by Klein

	Compression zone	Tension zone
k_{ec} (kN/m)	3.82×10^5	1.67×10^6
k_{pc} (kN/m)	3.00×10^3	3.00×10^3
P_C^B (kN)	650.00	795.00
$F^C = P_C^B/2$ (kN)	325.00	397.50

Fig. 5. Comparative graph – calibration of post-limit stiffness and resistance with experimental results (available for pure bending).

Fig. 6. Plot of axial force vs. axial displacement ($N-q_4$) for simplified model with non-linear compression zone.

Fig. 7 illustrates unstable, neutral and stable joint responses for three different values of post-limit equivalent stiffness of the column web in shear, taken in the latter case as $k_{pc} = 65\,000$ kN/m for illustration.

Application of the general non-linear model of Eqs. (7)–(10) to this example yields identical results for critical load and post-buckling path but identifies a secondary bifurcation along the post-buckling path that reflects yielding of the tensile zone of the joint, as seen in Fig. 8. In this particular example, however, this secondary bifurcation occurs for large values of deformation, clearly outside practical relevance.

4.3. Welded joint subjected to bending and axial compression

Testing the same joint subjected to combined bending and axial compression, several moment–rotation curves can be plotted, one for each level of axial force. Using the simplified model of Eqs. (7) and (8), i.e., considering that the

Fig. 7. Comparative plot of axial force vs. axial displacement ($N-q_4$) for different values of the post-limit stiffness of the compression zone.

Fig. 8. Plot of axial force vs. axial displacement ($N-q_4$) for general non-linear model.

tension zone behaves elastically, since the compressive zone exhibits the lowest resistance, the moment–rotation curves ($M-\phi$) of Fig. 9 are obtained, showing a reduction of moment resistance with increasing axial compression, accompanied by a decrease in post-buckling stiffness. Alternatively, fixing the level of bending moment and plotting the resulting force–displacement response yields the results of Fig. 10.

Applying the more general model to the same situation gives the results of Fig. 11, where again secondary bifurcations arise, corresponding to the influence of the non-linear behaviour of the tension zone. It is noted that these secondary bifurcation points become less critical with increasing axial compression, as expected.

Fig. 9. Plot of moment vs. rotation (M – ϕ) for constant levels of axial compression.

Fig. 10. Plot of axial force vs. axial displacement (N – q_4) for constant levels of bending moment.

5. Concluding remarks

The equivalent elastic model generalized in this paper to accommodate compressive forces and combined bending and axial forces provided analytical solutions to an otherwise complex problem, which, in the framework of the component method, would require lengthy non-linear numerical evaluation, as described in Ref. [9].

This model clearly identifies the interaction between axial force and bending moment at any stage along the generalized non-linear force–deformation curve, thus providing a global picture of the behaviour of the joint, unlike previous approaches that could only trace an interaction diagram for the plastic resistance of the joint.

Fig. 11. Plot of moment vs. rotation ($M-\phi$) for constant values of axial force: general model.

Current work on the same model to provide an accurate evaluation of available ductility of a joint and to identify the failure sequence of the various components seems very promising, opening the way to the provision of code clauses that ensure verification that sufficient rotation at a joint location is available for plastic analysis.

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