



Thermal restraint and fire resistance of columns

I. Cabrita Neves^{a,*}, J.C. Valente^a, J.P. Correia Rodrigues^b

^a *Departamento de Engenharia Civil, Instituto Superior Técnico, Av. Rovisco Pais,
1049-001 Lisboa, Portugal*

^b *Departamento de Engenharia Civil, Faculdade de Ciências e Tecnologia da Universidade de Coimbra,
Polo II - Pinhal de Marrocos, 3030-290 Coimbra, Portugal*

Received 20 November 2001; received in revised form 02 April 2002; accepted 22 May 2002

Abstract

A proposal is made, based on the results of a series of tests and calculations, with the aim of being used as a simple method to correct the value of the critical temperature of steel columns free to elongate, in order to take into account the restraint effect of the structure to which they belong in a practical situation. To better illustrate the possible types of behaviour of heated steel columns with elastic restraint to the thermal elongation, and the reasons why the critical temperature of axially loaded slender steel columns with thermal restraint can sometimes be lower than the critical temperature of the same columns free to elongate, a simple model is presented and used in a qualitative analysis. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

It is well known that the load versus axial displacement relationships obtained at room temperature in a compression test of a slender axially loaded member are different if the test is load-controlled or displacement-controlled (Figs. 1 and 2). In a load-controlled test, a load will be reached where equilibrium can no longer be sustained and buckling occurs giving the maximum load capacity of the member (Fig. 1). In a displacement-controlled test however, after the maximum load capacity has been reached, equilibrium is still possible. In this case, the values of the load measured after the maximum represent the maximum load-carrying capacity of the member for the corresponding deflected shape. In the unloading path the member carries only as much load as permitted by the deformed shape.

*Corresponding author. Tel.: +351-2184-18202; fax: +351-2184-97650.

Nomenclature

A	cross-section area.
E	Young's modulus of steel at room temperature.
K	stiffness.
L	length of an element.
M	bending moment.
$N_{Sd,fi,t=0}$	design value of the axial load in fire situation.
P	load.
R	relative stiffness.
T_{crit}^{free}	critical temperature of a steel column with free elongation.
T_{crit}^{rest}	critical temperature of a steel column with restrained thermal elongation.
U	displacement.
ΔT_{crit}	reduction in the critical temperature of a steel column due to thermal restraint.
λ	slenderness.
η_{fi}	ratio between the initial axial load and the design value of the axial load in fire situation at time $t = 0$.

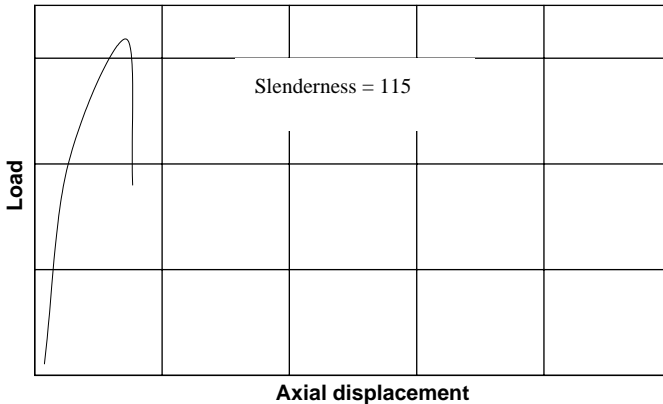


Fig. 1. Characteristic load-displacement diagram for an axially compressed steel member in a load-controlled test.

If a steel column is heated and its axial thermal elongation is fully prevented, restraint forces develop, which increase up to a maximum and then decrease, just like in the displacement-controlled test at room temperature (Fig. 3). In the case of total restraint, the values of the axial force measured at each steel temperature represent the maximum load-carrying capacity of the column for that temperature.

In a displacement-controlled compression test the stress field is generated by the displacement of the extremities of the member towards each other, which causes compression and bending. The heating of an axially fully restrained column will also cause compression and bending. The expression “temperature-controlled compression test under constant displacement” could then be used in the very same sense as the expression “displacement-controlled compression test under constant temperature” is used at room temperature.

When we consider the heating of one single column in a steel frame, the restraint to the thermal elongation of the column depends on the stiffness of the structure, and this may vary from very low (column free to expand) to very high values (column fully restrained). The way the column behaves (the type of diagram load-axial displacement) depends on the stiffness of the structure. The whole process is ruled by

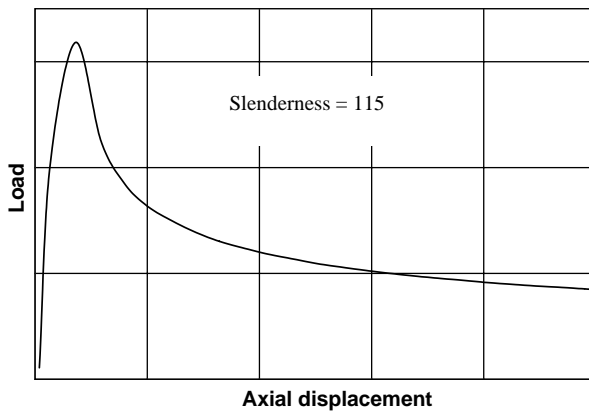


Fig. 2. Characteristic load-displacement diagram for an axially compressed steel member in a displacement-controlled test.

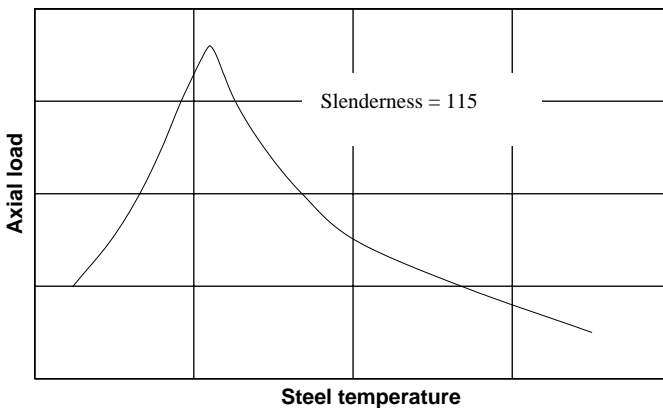


Fig. 3. Characteristic load-temperature diagram for an axially 100% restrained heated steel member.

force equilibrium and displacement compatibility between the column and the structure.

The academic example described in Section 2 uses a simple model that may help clarify, in a qualitative way, how the behaviour of a column changes with the stiffness of the structure. Several simplifications were intentionally introduced in the model, keeping only those features that were considered relevant for the purpose. The fire resistance of slender columns is the most affected by thermal restraint, due to the strong influence of bending. So, lateral deflection by compression was one of the main aspects considered in the model, although using it can give us no quantitative predictions of the fire resistance of restrained steel columns.

The role of a column during a fire must be accepted prior to defining its fire resistance. The definition of fire resistance of a column free to elongate is nowadays clearly established and accepted as the time the column withstands a constant load (the design load in fire situation). Logically, a column with restrained elongation should also be able to withstand the same load. If we accept this, the fire resistance of restrained columns will be defined as the time after which the restraining forces become smaller than the design load in fire situation, and the accurate calculation of the evolution of these restraining forces becomes very important.

Under a practical point of view, these calculations may become very cumbersome and unappealing. The objective of this paper is to provide the reader with a simple method to take axial restraint into account, allowing for the calculation of safer and more realistic values of the fire resistance of steel columns.

2. Representation of the behaviour of a column by means of a simple model

The model shown in Fig. 4 is intended to represent in a simplified manner the interaction between a column and a structure with a given stiffness. In this figure the structure is represented by the upper spring with a stiffness K_s . The two rigid bars AB and BC with length L , connected to each other by the hinge B, represent the column. The bending stiffness of the column is modelled by the spring with stiffness K_1 and the axial stiffness is modelled by the spring connecting the nodes A and C, with stiffness K_2 . This system would not be adequate to model pure compression, but the qualitative compression/bending behaviour is reproduced quite satisfactorily.

The out-of-straightness of the column is represented by an initial value θ_i of the angle θ . Springs 1 and 2 are supposed to behave according to Figs. 5 and 6. For the following qualitative analysis, and for the sake of simplicity, these laws were kept independent of the temperature. The spring representing the structure is taken as behaving in a purely elastic manner.

Ignoring the presence of the restraining structure, the relation between the load applied to the column P_C and the corresponding vertical displacement U_C at the top of the column can be obtained from Eqs. (1)–(4) (Figs. 7 and 8). This displacement results from the deformation between the two ends of the column due to lateral displacement only. The shortening due to direct axial strains has been

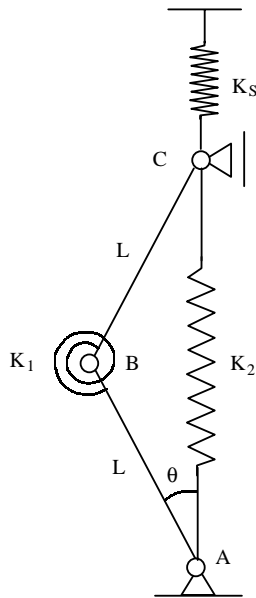


Fig. 4. Simple model to represent the compression/bending behaviour of a column.

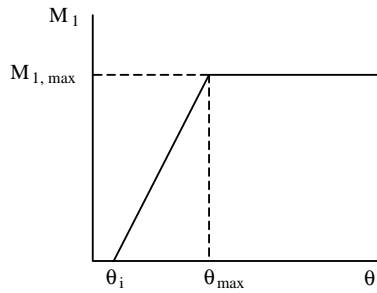


Fig. 5. Diagram moment–angle for spring 1.

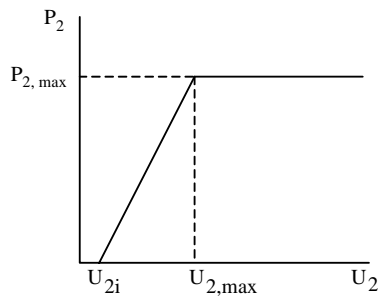


Fig. 6. Diagram force–displacement for spring 2.

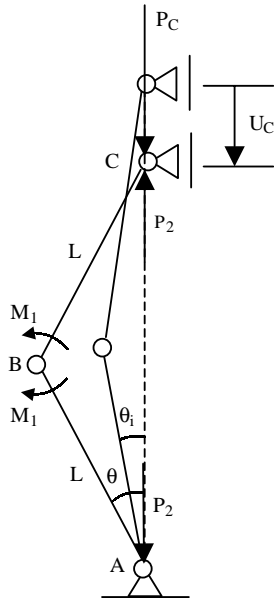


Fig. 7. Deflected column.

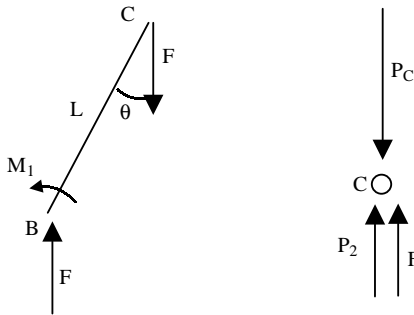


Fig. 8. Equilibrium diagrams of bar BC and node C.

ignored for simplicity.

$$U_C = 2L(\cos \theta_i - \cos \theta), \tag{1}$$

$$P_C = P_2 + \frac{M_1}{L \sin \theta}, \tag{2}$$

$$P_2 = K_2 U_C \quad \text{and} \quad P_2 \leq P_{2,\max}, \tag{3}$$

$$M_1 = 2K_1(\theta - \theta_i) \quad \text{and} \quad \theta \leq \theta_{\max}. \tag{4}$$

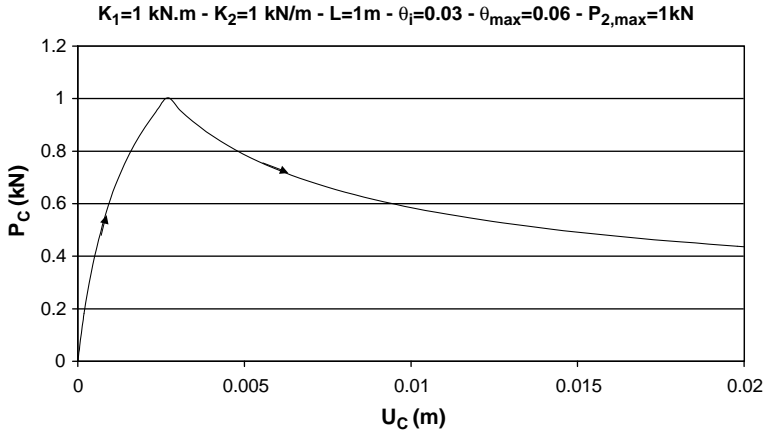


Fig. 9. Load versus vertical displacement.

Fig. 9 shows, for the given set of parameters, the evolution of the load P_C against the corresponding vertical displacement U_C .

Let us now suppose that the system “column-structure” (Fig. 4) is in an equilibrium state under the action of a vertical load applied at the node C , whereby the load supported by the column under normal temperature conditions (initial state) is P_0 . If a real column in a structure is heated, its elongation will be elastically restrained by the structure and restraint forces will develop. The temperature rise will affect the mechanical properties of the column, which in turn will affect the development of the restraining forces. Apart from this temperature dependency of the mechanical properties of the column, the evolution of the interaction forces between the column and the structure will be qualitatively identical to the evolution that we get if we force the support A to move vertically and continuously upwards by an amount U , keeping the temperature constant. This displacement U is the driving parameter in this simulation, just like the temperature is in a real situation. Eq. (2) and Eqs. (5)–(8) can be used to obtain all variables as a function of the growing angle θ and then the evolution of the differential force ($P_C - P_0$) as a function of the imposed displacement U , where θ_0 is the value of θ when $P_C = P_0$, U_S the displacement of the structure spring as a result of the imposed U , U_{C0} the value of U_C when $P_C = P_0$

$$U_C - U_{C0} = 2L(\cos \theta_0 - \cos \theta), \tag{5}$$

$$P_C = P_0 + P_S, \tag{6}$$

$$P_S = K_S U_S, \tag{7}$$

$$U = (U_C - U_{C0}) + U_S. \tag{8}$$

Fig. 10 shows the evolution of the differential force $P_C - P_0$ as a function of the driving displacement U , for a “high” structural stiffness $K_S = 200 \text{ kN/m}$.

If we choose the initial force P_0 as the minimum required load-carrying capacity of the column, the maximum allowable displacement U will be about 0.017 m in this case. Let us now suppose that the structural stiffness is only $K_s = 50$ kN/m. The calculated corresponding differential force ($P_C - P_0$) is shown in Fig. 11 as a function of U .

Notice the qualitative change in this case. The curve was obtained, as referred above, by letting the angle θ continuously grow. This would imply that the displacement U would have to decrease beyond point A. But if we assume that the driving displacement U is monotonically growing, just as the temperature does in a standard fire resistance test, when point A in the diagram is reached a sudden drop in the force is unavoidable, although the column is the same as in the case of Fig. 10. Nevertheless, the maximum displacement U remains exactly the same as previously (Fig. 11, point C).

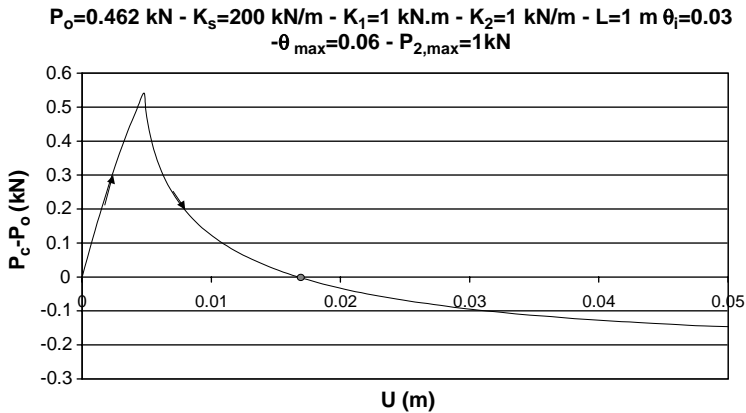


Fig. 10. Differential force ($P_C - P_0$) as a function of the driving displacement U for a stiff structure.

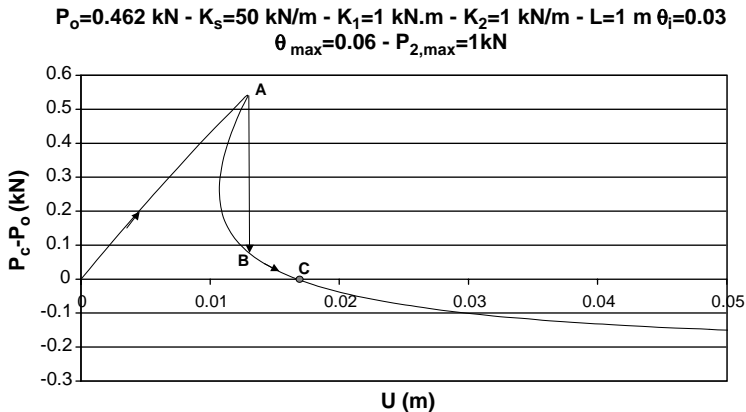


Fig. 11. Differential force ($P_C - P_0$) as a function of the driving displacement U for a flexible structure.

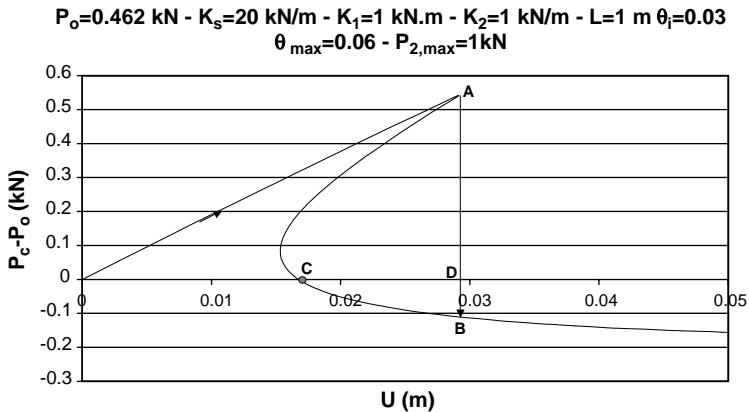


Fig. 12. Differential force ($P_C - P_0$) as a function of the driving displacement U for a very flexible structure.

If we further reduce the value of the structural stiffness to $K_s = 20 \text{ kN/m}$ the evolution shown in Fig. 12 is obtained.

The sudden drop in the differential force when the maximum is reached is still there, but now the next equilibrium point (Fig. 12, point B) lies below the initial force. The maximum allowable displacement U is no longer defined by point C. It has moved to the right (Fig. 12, point D). And it will further move to the right for smaller values of the structural stiffness K_s .

The above-described behaviour of this simple academic model is qualitatively exactly the same as previously observed in calculations [1–4] and in fire resistance tests [5–10] of steel elements with elastic restraint to the thermal elongation. The main difference is that in this model the mechanical properties, for the sake of simplicity in the analysis, were kept independent of the temperature, while in a real column subjected to fire they change with the steel temperature. Curiously, the model is better adapted to describe the behaviour of the column in the compartment immediately above the fire compartment, where no temperature rise occurs and the steel properties remain unchanged. As a matter of fact, the lower node of this column will really be being pushed upwards by the heated column in the lower compartment, just like we have assumed in the model.

3. Reduction in fire resistance by thermal restraint

It was shown in previous works [1,4–6,9,10] that restraining the axial thermal elongation of steel columns causes a reduction in their critical temperature. The critical temperature should be understood as the steel temperature above which the column can no longer support its design load in fire situation $N_{Sd,\bar{t},t=0}$ [11,12]. For an axially restrained column this means the steel temperature above which the force acting on the column becomes smaller than $N_{Sd,\bar{t},t=0}$ [13,14]. Fig. 13 shows the

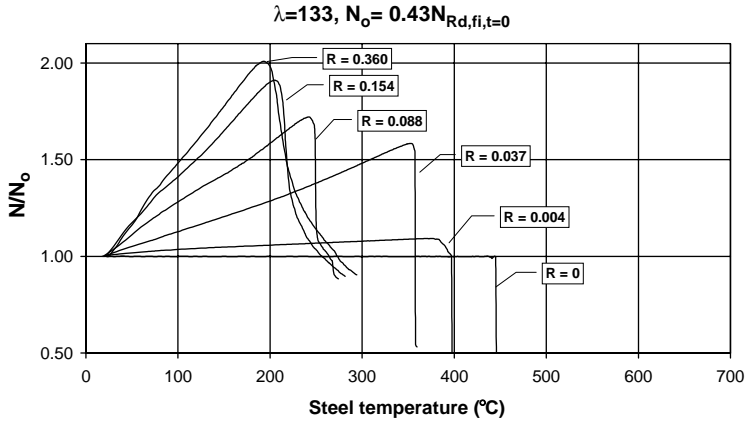


Fig. 13. Evolution of the axial load in axially restrained steel members for several restraint grades—measured values (IST).

evolution of the measured axial load during tests performed in Lisbon at Instituto Superior Técnico (IST) [9,10] on axially restrained steel members with several restraint grades and bending about the weak axis. In this figure, the non-dimensional stiffness ratio R is defined by

$$R = \frac{K_s L}{EA}, \quad (9)$$

where L is the length of the column, A the area of the cross section of the steel member, E the Young's modulus of steel at room temperature.

The qualitative behaviour of the simple model described above in Section 2 can clearly be recognized here.

Why must then a 100% axially restrained slender column have a critical temperature lower than the same column free to expand? In other words, why is the load-carrying capacity of a 100% axially restrained slender column, heated to a temperature T , smaller than the load-carrying capacity of the same column free to expand at the same temperature? The answer can be found in Figs. 14 and 15.

During the whole heating process the deflections of the restrained column are much greater than the deflections of the unrestrained column. At every temperature, the more deflected column is always much closer to its load-carrying capacity as a consequence of the geometrical 2nd-order effects. When the temperature T_{crit}^{rest} is reached (Fig. 14), the load-carrying capacity of the restrained column has decreased to the level $N_{Sd,fi,t=0}$. For the unrestrained column however, this will happen only at a later stage, when the steel temperature has reached a higher value T_{crit}^{free} .

If the stiffness K_s of the restraining surrounding structure is small, the deflections of the column during heating are also smaller. Therefore, the difference $(T_{crit}^{free} - T_{crit}^{rest})$ becomes smaller too and vanishes when $K_s = 0$.

As it was pointed out above, this reduction in the critical temperature of a slender axially restrained column is a direct consequence of the geometrical 2nd-order

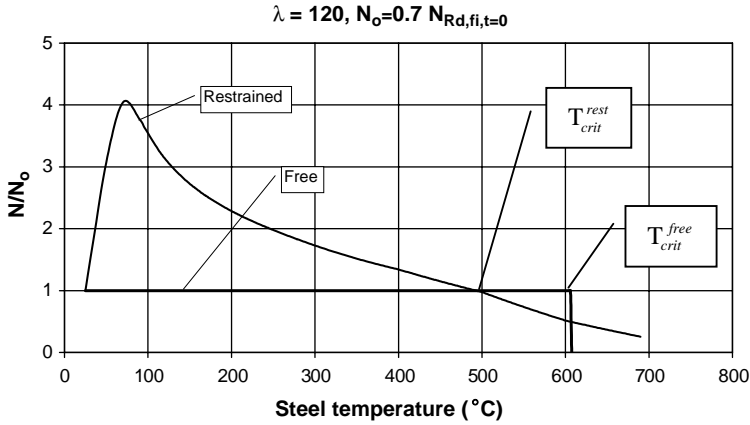


Fig. 14. Calculated load evolution in steel members, free to expand and 100% restrained (FINEFIRE).

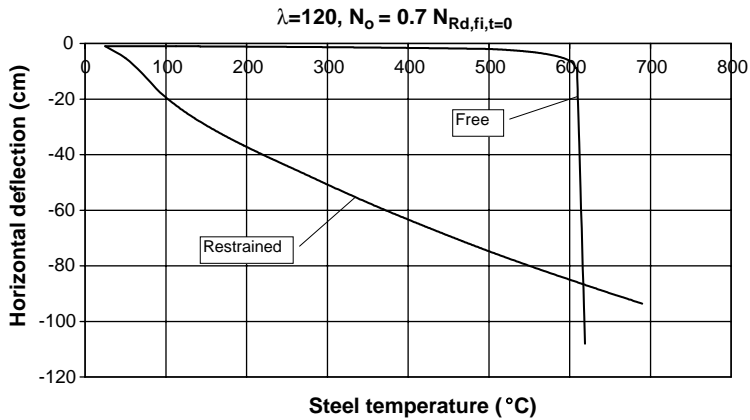


Fig. 15. Calculated horizontal deflections of steel members, free to expand and 100% restrained (FINEFIRE).

effects. Hence, it is expected to affect slender steel, reinforced concrete or composite steel–concrete columns. The geometrical 2nd-order effects depend upon the length of the column and on the mass distribution within the cross section (bending about the weak axis or about the strong axis).

It has been observed both numerically and experimentally [1,2,4,6–10] that below certain values of the structural stiffness K_s axially restrained columns begin to show sudden buckling (Fig. 13). It was also observed that these columns reach a second equilibrium state after buckling, for a lower value of the restraining load (Figs. 11–13 and 16). Some of them reach this equilibrium state for a value of the restraining load

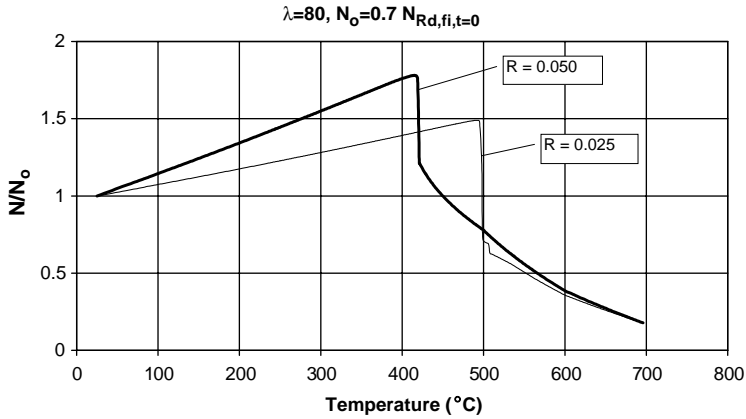


Fig. 16. Buckling of axially restrained steel members at high temperatures—calculated values (FINE-FIRE).

$N > N_{Sd,fi,t=0}$ (Figs. 11,13,16), others for $N < N_{Sd,fi,t=0}$ (Figs. 12,16). This type of buckling deserves to be analysed in some detail.

Buckling of these columns occurs when the increase in the steel temperature has forced the restraining load N to reach the maximum that the column can support at that temperature. Further equilibrium states will only be possible if the load acting on the column is allowed to decrease according to the descending path of the diagram load—axial displacement, characteristic for the column behaviour at the corresponding steel temperature. If elastic behaviour of the surrounding structure is assumed, the characteristic load-displacement diagram of the structure is obviously a straight line. As both equilibrium and displacement compatibility between heated column and structure must exist, the next equilibrium state can only be found for a much lower load at costs of a sudden deflection (buckling) of the column (Figs. 11–13 and 16). If it would be possible to decrease instantaneously the load just after buckling has occurred in a load-controlled compression test, the column would reach a stable condition as well. This reduction in the load is automatically done by the structural system during a fire test or during a real fire. Again, the phenomenon is qualitatively independent of the structural material. So, it shall be present in reinforced concrete and composite steel–concrete slender columns as well.

It would be useful to have some simple guidance on how to correct the fire resistance of an unrestrained column to obtain the fire resistance of the same column when its thermal elongation is restrained by a real structure. Based on numerical and experimental results, the authors make in the following section a proposal for the correction of the critical temperature of steel columns. It is the authors' belief that in the future similar corrections will probably have to be applied to reinforced concrete and composite steel–concrete slender columns as well, but the necessary supporting data is not yet available.

4. Proposal for the correction of the critical temperature of steel columns

Using the finite element program FINEFIRE [15] the authors have calculated the critical temperature of several steel columns with pinned ends and fixed ends, free to expand and with restrained thermal elongation, for several stiffness values K_s of the restraining structure, different initial load levels $\eta_{fi} = N_0/N_{Rd,fi,t=0}$, different cross-section profiles, and three values of the column slenderness λ , as shown in Table 1. Figs. 17–20 summarize the results obtained.

The programme FINEFIRE uses an isoparametric Euler–Bernoulli beam finite element. The element was developed taking into account geometrical non-linearity and using an approximate updated Lagrangian formulation. This type of element has two nodes in the plane x, y and 3 degrees of freedom at each node.

Structural elements under fire conditions usually have non-homogeneous temperatures along the longitudinal axis and in the cross sections. So, at each point, the material has different mechanical and thermal properties. These conditions are introduced into the stiffness matrix by numerical Gauss integration points.

The program uses the temperature dependent relationships of Eurocode 3 for the thermal elongation of steel [14].

Table 1
Summary of the studied parameters

Cross-section profile	HEB120, HEB260, HEB300, HEB400
K_s (kN/cm)	0, 25, 50, 100, 250, 500, 1000, 2500, 5000
λ	40, 80, 120
η_{fi}	0.3–0.5–0.7
End nodes	Pin-ended, built-in
Bending about	Strong axis, weak axis

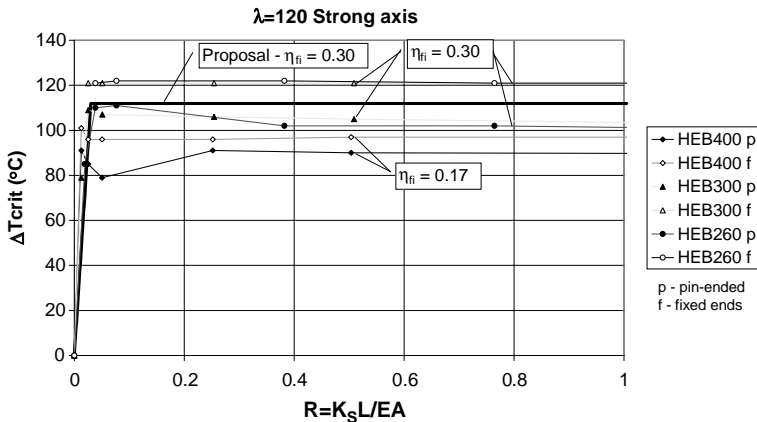


Fig. 17. Comparison of calculation results with the proposal $\lambda = 120$.

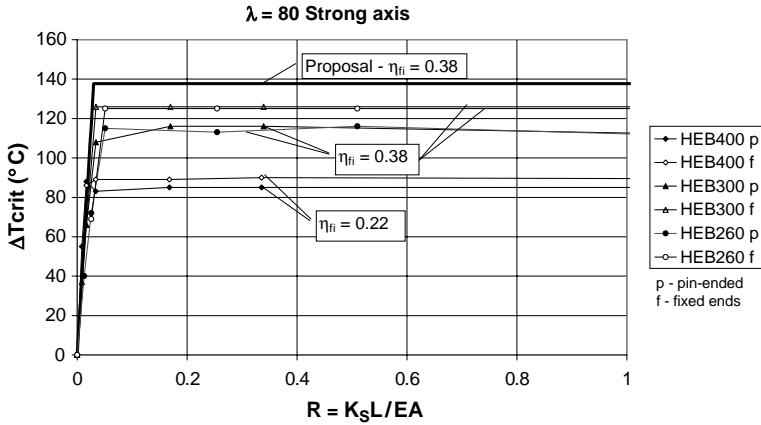


Fig. 18. Comparison of calculation results with the proposal $\lambda = 80$.

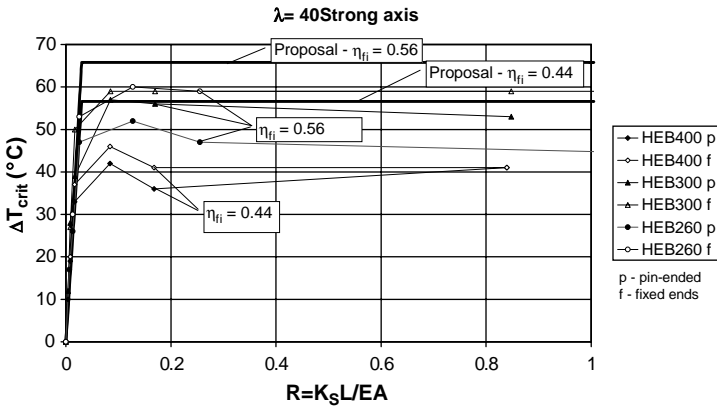


Fig. 19. Comparison of calculation results with the proposal $\lambda = 40$.

The non-linear equation relating the stiffness matrix $[K(\sigma)]$ of the structure, the displacement vector $\{\Delta u\}$ and the force vector $\{\Delta P\}$

$$[K(\sigma)]\{\Delta u\} = \{\Delta P\} \tag{10}$$

is solved using the Newton–Raphson method. The stiffness matrix is updated in each iteration and it also takes into account the out-of-balance forces as applied forces in the next iteration. The equilibrium criterion is based on the out-of-balance forces.

The stress–strain relationships for steel of Eurocode 3 were used, with the strain hardening option.

The graphs represent the reduction in the critical temperature $\Delta T_{crit} = T_{crit}^{free} - T_{crit}^{rest}$ as a function of the non-dimensional parameter R , where T_{crit}^{free} is the critical

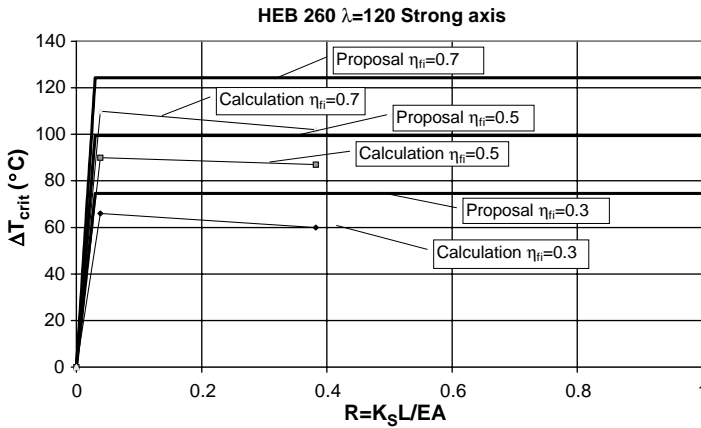


Fig. 20. Comparison of calculation results with the proposal—load level influence.

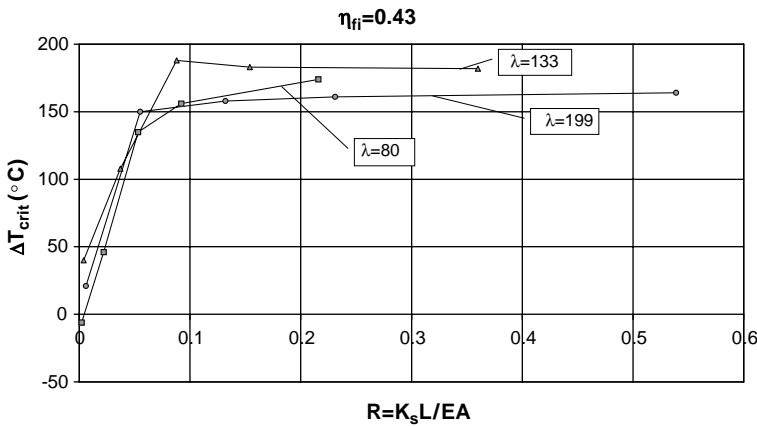


Fig. 21. Reduction in the critical temperature caused by restraint to the thermal elongation—measured values (IST).

temperature of the column free to expand, T_{crit}^{rest} the critical temperature of the column with restrained thermal elongation.

Fig. 21 summarizes the results obtained in a series of experimental tests performed at IST on small steel elements with elastic restraint to the thermal elongation and bending about the weak axis, reported in [9,10] (Fig. 13). The patterns are identical to those in Figs. 17–20.

The above diagrams show the following general aspects:

1. The critical temperature decreases with increasing values of the non-dimensional stiffness ratio R .
2. There is a value of R , above which no further reduction in T_{crit} occurs.

Table 2

Proposal for the calculation of the reduction in critical temperature

$T_{\text{crit}} = T_{\text{crit}}^{\text{free}} - C_b \Delta T_{\text{crit}}$		$R = \frac{K_s L}{EA}$
$C_b =$	0.9 1.25	Bending about strong axis Bending about weak axis
$\Delta T_{\text{crit}} =$	$\frac{\Delta T}{0.03} R$ ΔT	$R \leq 0.03$ $R > 0.03$
$\Delta T =$	0 $85 C_\eta \frac{\lambda - 20}{20}$ $\left[85 + \frac{140}{40} (\lambda - 40) \right] C_\eta$ $(260 - 0.44 \lambda) C_\eta$	$\lambda < 20$ $20 \leq \lambda \leq 40$ $40 < \lambda \leq 80$ $80 < \lambda \leq 200$
$C_\eta =$	$0.3 + \eta_{fi}$	$0.3 \leq \eta_{fi} \leq 0.7$

3. The decrease in the critical temperature depends on the slenderness value λ of the column.
4. The decrease in the critical temperature depends on the initial load level η_{fi} .
5. When bending is about the weak axis the decrease in critical temperature is greater than when it is about the strong axis.

The proposal forwarded in Table 2 is based on both calculation and test results. This proposal takes the above conclusions into consideration and is intended to be used, within the frame of single element analysis, as a simple method to correct the value of the critical temperature of a steel column free to expand $T_{\text{crit}}^{\text{free}}$ in order to take into account the restraint effect of the structure to which the column belongs.

Figs. 22–24 make a comparison between experimental and calculation results with the values obtained when using the proposal.

A further comparison between the proposal and calculation results can also be found in Figs. 17–20.

The use of the proposal requires an estimation of the stiffness K_s of the structure, but this can easily be done either on the basis of the 3D computer code used to design the structure at room temperature, or by using simplified methods like the one described in [7]. In this estimation, consideration should be given to certain factors, which are not yet well known. Some of them, like cracking in the concrete, may have a positive effect, because they lead to smaller values of the structural stiffness. Others may increase the stiffness of the structure, like the presence of shear walls not considered as such in the cold design.

Of course, there is always the possibility of considering full restraint [2], but this might represent a too safe assumption in many cases. If this is done however, full restraint should be applied to the initially compressed column (initial load level must

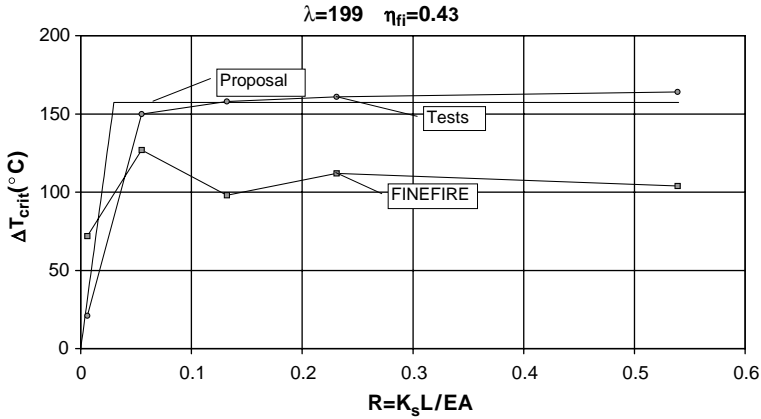


Fig. 22. Comparison between test results, calculation and proposal.

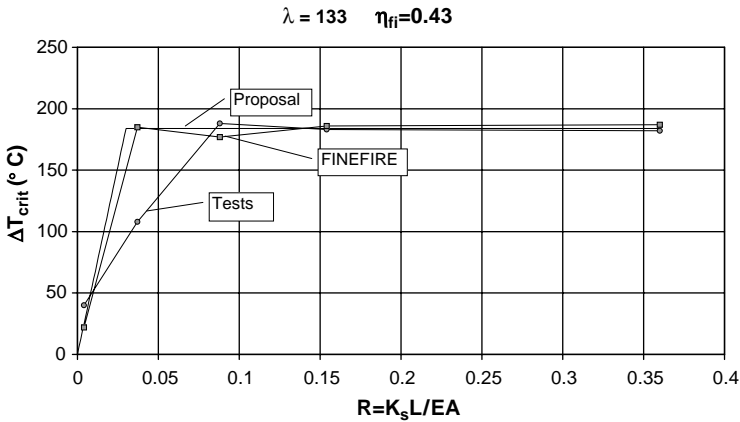


Fig. 23. Comparison between test results, calculation and proposal.

be considered), and not to an unstressed element. To consider the critical temperature as the steel temperature when the buckling of the column occurs (temperature for maximum restraint load) would be an even more conservative assumption in many practical situations [16], unless it will be proved that the associated dynamic effects play a decisive role.

5. Conclusions

To illustrate the possible types of behaviour of heated steel columns with elastic restraint to the thermal elongation, and the reasons why the critical temperature of

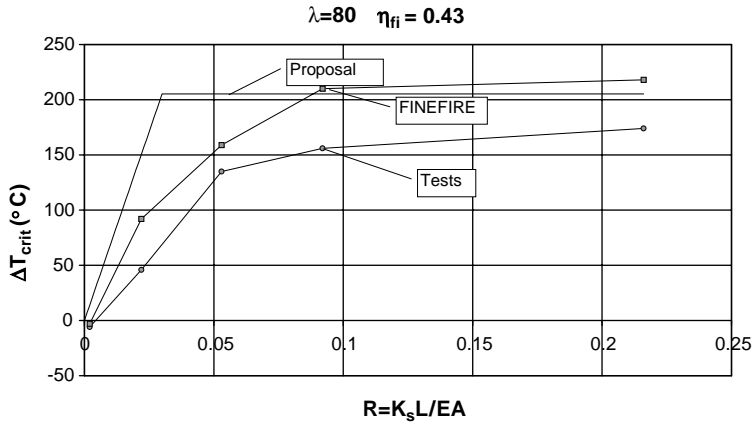


Fig. 24. Comparison between test results, calculation and proposal.

axially loaded slender steel columns with thermal restraint can be lower than the critical temperature of the same columns free to elongate, a simple model was presented and used in a qualitative analysis. This model showed the same type of behaviour already known from tests and FEM calculations on steel columns with elastic restraint to the thermal elongation. The simplicity of the model leaves no doubt about the reasons why thermal restraint causes a reduction in the critical temperature of axially loaded slender columns.

Based on test and calculation results a proposal was made, with the aim of being used, within the frame of single element analysis, as a simple method to correct the value of the critical temperature of axially loaded steel columns free to elongate, in order to take into account the restraint effect of the structure to which they belong in a practical situation. The proposal has been prepared to give safe results. The test results were obtained from restrained small elements with buckling about the weak axis and the calculations were performed on several steel cross-section profiles, considering either bending about the weak axis or the strong axis. Some tests on real-scale steel columns with elastic restraint are foreseen in a near future. Among other aspects, it is intended to use these tests to assess the margin of safety of the proposal.

As a final remark, it should be remembered that slender columns ($\lambda \geq 80$) are the most affected by thermal restraint. So, it is expected that the fire resistance of steel columns will not be much penalized by the thermal restraint in most of the practical situations, where the slenderness is low. Moreover, it is obvious that in a global analysis of structures having redundant connections, due to load transfer, the decrease of the load-bearing capacity of a column to values below its design value in fire situation $N_{Sd,fi,t=0}$ can be tolerated, as long as this does not cause the overall collapse of the structure, or in any way entrains the performance of other elements with a fire compartmentation function [17]. However, if this decrease happens in a sudden way (sudden buckling of the column), the load transfer is accompanied by a sudden movement of part of the structure, mainly in the zone located above the

buckled column. The dynamic effects of this mass movement on the structure are worth being studied, mainly taking in mind that more than one column will be heated at the same time in the fire compartment and will also have a reduced load-bearing capacity. On the other hand, it has been shown that the decrease in critical temperature loses importance as the eccentricity of the load increases, and this load eccentricity is most of the time present in real structures, either due to non-uniform load distribution or to the location of the column inside the structure.

References

- [1] Cabrita Neves I. The critical temperature of steel columns with restrained thermal elongation. *Fire Saf J* 1995;24(3):211–27.
- [2] Franssen J-M. Failure temperature of a system comprising a restrained column submitted to fire. *Fire Saf J* 2000;34:191–207.
- [3] Poh KW, Bennetts ID. Behaviour of steel columns at elevated temperatures. *J Struct Eng, Am Soc Civ Eng (ASCE)*, 1995;121(4):676–84.
- [4] Valente JC, Cabrita Neves I. Fire resistance of steel columns with elastically restrained axial elongation and bending. *J Constr Steel Res*, 1999;52(3):319–31.
- [5] Ali F, O'Connor D. Structural performance of rotationally restrained steel columns in fire. *Fire Saf J* 2001;36(7):679–91.
- [6] Ali F, Shepherd A, Randall M, Simms I, O'Connor, D, Burgess I. The effect of axial restraint on the fire resistance of steel columns. *J Constr Steel Res* 1998;46(1–3): Paper no. 117, England.
- [7] Cabrita Neves I. The column–structure interaction on the behaviour of steel columns under local fire. Ph.D. thesis, Instituto Superior Técnico, Lisboa, 1983.
- [8] Cabrita Neves I, Rodrigues JPC. Resistência ao Fogo de Pilares de Aço, I Encontro Nacional de Construção Metálica e Mista, Porto, Portugal, November 1997, pp. 543–550 (in Portuguese).
- [9] Rodrigues JPC. Fire resistance of steel columns with restrained thermal elongation. Ph.D. thesis, Instituto Superior Técnico, Lisboa, 2001.
- [10] Rodrigues JPC, Cabrita Neves I, Valente JC. Experimental research on the critical temperature of compressed steel elements with restrained thermal elongation. *Fire Saf J* 2000;35(2):77–98.
- [11] Comité Européen de Normalisation (CEN/TC250/SC1). Eurocode 1: basis of design and actions on structures. Part 1: basis of design, ENV 1991-1, 1993.
- [12] Comité Européen de Normalisation (CEN/TC250/SC1). Eurocode 1: basis of design and actions on structures. Part 2.2: actions on structures exposed to fire, ENV 1991-2-2, 1995.
- [13] Comité Européen de Normalisation (CEN/TC250/SC3). Eurocode 3: design of steel structures. Part 1.1: general rules and rules for buildings, ENV 1993-1-1, 1992.
- [14] Comité Européen de Normalisation (CEN/TC250/SC3), Eurocode 3: design of steel structures. Part 1.2: structural fire design, ENV 1993-1-2, 1995.
- [15] Valente JC. Simulação do Comportamento das Estruturas Metálicas Sujeitas a Altas Temperaturas, Ph.D. thesis, Instituto Superior Técnico, Lisboa, 1988 (in Portuguese).
- [16] Centre Technique Industriel de la Construction Métallique (CTICM). Méthode de Prévision par le Calcul du Comportement au Feu des Structures en Acier. *Construction Métallique*, no. 3, Puteaux, September, 1982, pp. 39–79 (in French).
- [17] British Steel–Swinden Technology Centre. The behaviour of multi-storey steel framed buildings in fire, Rotherham, UK, 1999.