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Airline Fleet Sizing

Dissertation submitted to obtain the Integrated Master in Civil Engineering in the area of specialization of Urbanism, Transports and Transportation Infrastructures

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RESUMO

Para as companhias aéreas, a composição da frota e seu planeamento são de uma elevada importância devido às implicações económico-financeiras que têm para a empresa.

Esta dissertação tem como objeto o estudo de fatores que envolvem o planeamento de uma frota aérea, nomeadamente as características, desempenho e custos das aeronaves, bem como os modelos de otimização que sustentam as escolhas para a frota durante o seu planeamento.

Será desenvolvido um modelo para otimização de frota que será, posteriormente, aplicado para otimizar à da companhia aérea portuguesa, TAP Portugal.

ABSTRACT

For airlines, the fleet composition and its planning are very important due to the economic and financial implications they have for the company.

This dissertation has as its object the study of factors involving the planning of an air fleet, including the features, performance and costs of aircraft, as well as optimization models that underlie the choices for the fleet during its planning.

A fleet sizing model will be developed that will, later, be applied to optimize the fleet of the Portuguese airline, TAP Portugal.

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SIMBOLOGY

$a_k(t)$ – acquisitions for fleet k in time period t (assumed to occur at the beginning of the period)

c_k – operation cost per seat and kilometer

c_k^O – The ownership cost of an aircraft of type k

c_{ijk} – operating cost of a trip from i to j for a vehicle of type k

$d_{ij}(t)$ – the demand for transportation service between i and j in period t

d_{ij} – distance for a flight between airports i and j

e_{ij} – cost of moving an empty vehicle from i to j

H_i – unit cost of holding a vehicle for one period at location i

l_{ij} – cost of moving a loaded vehicle from i to j

M_{ijk} – set of markets that have origin i, destination j, and are eligible for shipment on vehicles of type k

p – varies between 0,75 and 1,25 and it's used to make the demand vary along the months

P_i – variable which generates a random population for each location between 500 000 and 20.000.000 inhabitants

P_{ij} – penalty cost per period for one unit of unmet demand from i to j

q – cost per vehicle per period to own or lease a vehicle

$q_m(t)$ – the shipments carried for market m in time period t

q_{ijt} – number of daily passengers in month t between airports i and j

$Q_m^{def}(t)$ – total shipments deferred for market m in time period t

$Q_m^{dly}(t)$ – total shipments delayed for market m in time period t

$Q_m(t)$ – number of market m shipments offered for movement in time period t

r_{ij} – revenue per loaded vehicle sent from i to j

$r_k(t)$ – retirements for fleet k in time period t (assumed to occur at the beginning of the period)

r_k – range of an aircraft of type k

s_k – number of seats of an aircraft of type k

t_k – turn time of an aircraft of type k

T – the number of time periods
$U_{ij}(t)$ – unmet demand from i to j in period t
v_k – maximum speed of an aircraft of type k
$V_i(0)$ – number of vehicles initially allocated to location i
$V_i(t)$ – number of vehicles present at location i at the end of period t
$V_k(t)$ – fleet size for type k vehicles in time period t
w_m – allowable window (number of time periods) for shipment of market m shipments
x_k – number of aircraft of type k owned by the airline
$x_{ijk}(t)$ – vehicles of type k moved from i to j in time period t
$X_{ij}(t)$ – number of loaded vehicles dispatched from i to j in period t
$Y_{ij}(t)$ – number of empty vehicles dispatched from i to j in period t
z_{ijkt} – number of daily flights between airports i and j made by an aircraft of type k in month t

α – constant calibration variable
$\alpha_{ij}(\tau,t)$ – proportion of loaded vehicles dispatched from i to j in period τ which arrive in period t
$\beta_{ij}(\tau,t)$ – proportion of empty vehicles dispatched from i to j in period τ which arrive in period t
φ_{ij} – calibration variable
θ_k – cost of owning one vehicle of type k for one time period
λ_k – cost of acquiring one vehicle of type k
ζ_k – cost of retiring one vehicle of type k
ρ_m – per-period penalty for deferring one shipment for market m
γ_m – per-period penalty for delaying one shipment for market m
π_k – Percent of time that a vehicle of type k is available
$\Gamma(t)$ – duration of time period t

ABREVIATIONS

AMS – Amsterdam Schiphol Airport
BCN – Barcelona International Airport
BRU – Brussels Airport
BSB – Brasília International Airport
CASM – Cost per Available Seat Mile
CDG – Charles de Gaulle International Airport
CNF – Tancredo Neves International Airport, Belo Horizonte
ETOPS – Extended Twin Engine Operations
EWR – Newark Liberty International Airport
FAO - Faro Airport
FCO –Leonardo da Vinci International Airport, Rome
FNC – Madeira Airport
FOR – Fortaleza Pinto Martins International Airport
GIG – Rio de Janeiro Galeão International Airport
GRU – S. Paulo, Guarulhos International Airport
GVA – Geneva Cointrin International Airport
LAD – Luanda Quatro de Fevereiro Airport
LIN – Milano Linate Airport
LIS – Lisboa Portela Airport
LGW – London Gatwick Airport
LHR – London Heathrow Airport
LUX – Luxembourg Findel Airport
FRA – Frankfurt International Airport
MAD – Madrid-Barajas Airport
MIP – Mixed-Integer Programming
MPM – Maputo International Airport
MUC – Munich International Airport
MXP – Milano Malpensa Airport
NAT – Greater Natal International Airport

OPO – Francisco Sá Carneiro Airport, Oporto
ORY – Paris Orly Airport
PDL – Ponta Delgada Airport
REC – Recife Airport
SSA – Luís Eduardo Magalhães International Airport, Salvador da Bahia
TAP – Transportes Aéreos Portugueses (Portuguese Air Transports)
ZRH – Zurich International Airport

1. INTRODUCTION

1.1. The Fleet Sizing Problem

Transportation is one of the most vital services in modern society. It makes most of the other functions of society possible. Real transportation systems are so large and complex that in order to build the science of transportation systems it will be necessary to work in many areas, such as: Modeling, Optimization and Simulation. This thesis will focus its attention just for the fleet sizing problem.

The fleet sizing problem consists on calculating the optimal number of vehicles that balances service demands against the cost of purchasing and maintaining them. In other words, the main question that fleet sizing tries to answer is:

What type of vehicle to acquire, when and how many of each?

The capacity of a transportation system is directly related to the number of available vehicles. Owners and operators of transport companies invest in order to provide the capacity to meet the demands. The demand for movements between locations is normally unbalanced which implies the need for redistribution of empty vehicles so they can serve other locations. So, as consequence, the number of vehicles which is available for service at any given time and a certain location depends upon the vehicle redistribution strategy.

Photo 1.1 shows the variety of type of vehicles owned by Fedex transportation.

Photo 1.1 – Example of some vehicles of FedEx's fleet (FedEx@)

Fleet sizing is a very important issue for transportation service companies. The vehicles are expensive to own and keep, so, deciding what vehicles should be acquired is the key for their business to succeed. Fleet sizing is connected to overall service design (Crainic, 2000), and there has been some studies related to trucking (e.g., Hall and Racer, 1995; Du and Hall, 1997; Ozdamar and Yazgac, 1999), multi-level railcar operations (Sherali and Tuncbilek, 1997), material handling systems used for manufacturing operations (e.g., Beamon and Deshpande, 1998; Beamon and Chen, 1998) and airline express package service (Barnhart and Schneur, 1996) that gives importance to these connections.

Normally vehicles are a long-term asset, which means that there is a subjective uncertainty about the demands that they will serve over their lifetime and about the conditions under which they will operate. Besides researchers and operators recognizing the importance of this uncertainty, fleet sizing problems are often quite difficult to solve even under deterministic assumptions and most of the existent studies focus on deterministic models.

1.2. Research Approach and Objectives

Fleet planning is fundamental in order for airlines to be successful in a competitive market such as the commercial air transportation.

This dissertation is divided into six chapters and begins by the introduction where is presented the topic, the objectives, the structure and the content.

The second chapter exposes the criteria to be considered when planning an airline's fleet, such as characteristics and performance of the aircraft, financials issues, and network and hub location.

In the third chapter, two fleet sizing models are described, which were quite important for the development of the proposed optimization model presented in the fourth chapter.

TAP Portugal will be considered in order to illustrate this model, whose fleet is in need of review. In the fifth chapter, the optimal aircraft for TAP's fleet will be chosen that minimizes costs while being able to serve the demand.

In the end, the conclusions and the achieved goals are summarized, as well as the limitations of this dissertation. Also, it is given some suggestions for future studies related to this subject.

2. AIRLINE FLEET PLANNING

2.1. Introduction

"Fleet planning is the process by which an airline acquires and manages appropriate aircraft capacity in order to serve anticipated markets over a variety of defined periods of time with a view to maximizing corporate wealth." ("Buying the Big Jets", P. Clark)

According to "The Global Airline Industry", P. Belobaba and C. Odoni, fleet composition is a very important long-term strategic decision for airlines. An airline's fleet is characterized by the total number of aircraft owned as well its specific aircraft types. Different aircraft models have different features. The most important one is technical performance. It is used to determine its capacity to carry payload over a maximum flight distance, or range.

Figure 2.1 illustrates the hierarchical location, in an airline organization, where fleet planning feets.

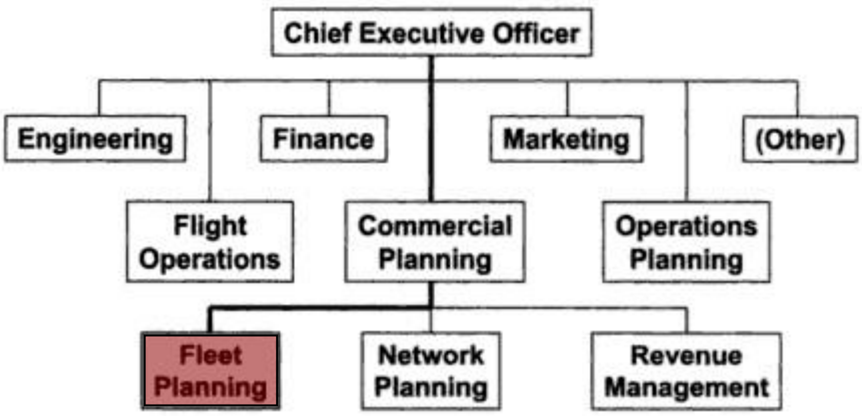

Figure 2.1 – Example of an airline organization (Buying the Big Jets, P. Clark)

Airlines' decisions to acquire new aircraft or retire existing aircraft in its fleet influence the airline's overall financial position, operating costs, as well as the ability to serve specific demands in a profitable manner. Acquiring a new aircraft represents a major investment with a long-term operational and economic horizon.

The fleet planning problem can be considered as an optimal staging problem. For a certain time period there is a fleet composition that changes with every additional aircraft acquired and every existing aircraft that is taken away. Consequently, an airline's fleet plan must reflect a strategy for several periods into the future, such as, the number of aircraft required by aircraft type, the timing of the future deliveries and retirement of existing fleet, as well as backup plans to prevent financial slips when there is uncertainty about future market conditions. Airlines must also recognize that there are constraints imposed by the existing fleet, the ability to dispose of older aircraft, and the availability of future delivery slots from aircraft manufacturers and/or leasing companies. Figure 2.2 shows the life cycle of a typical aircraft program.

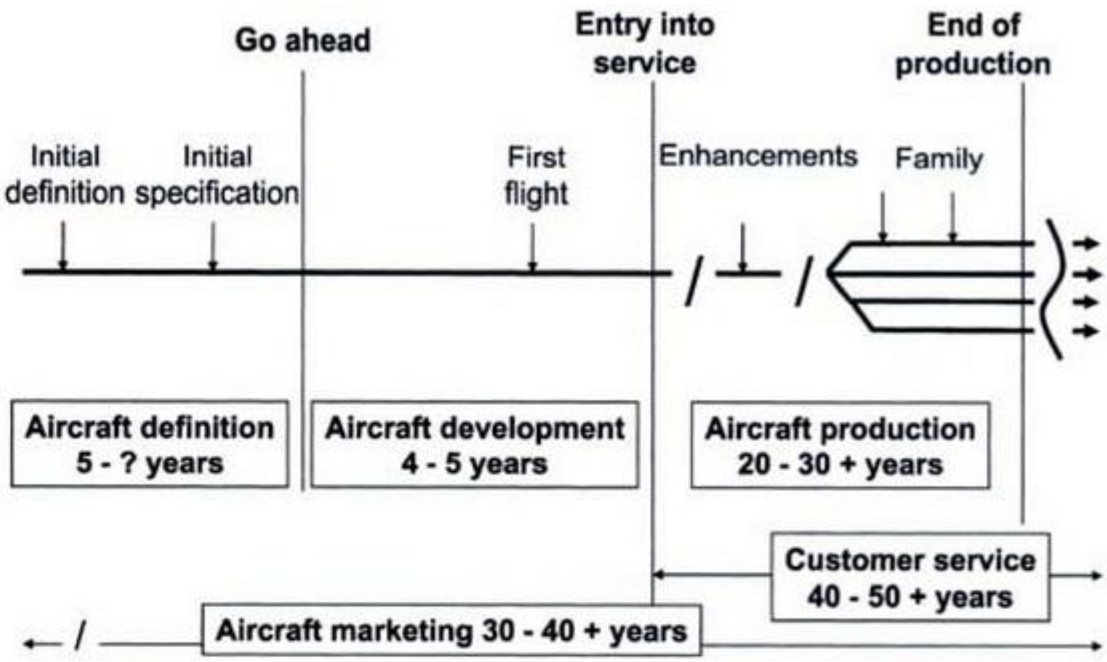

Figure 2.2 – Typical Aircraft Program Life Cycle

2.2. Aircraft Categories and Specifications

Nowadays, aircraft are characterized by two important features: range and size. The range of an aircraft refers to the maximum distance that it can fly without stopping for additional fuel, while still carrying a fair payload of passengers and/or cargo. The size of an aircraft is defined by its weight, seating or cargo capacity, as indicators of the amount of payload that it can carry.

Figure 2.3 represents the size and range characteristics of Boeing and Airbus aircraft. Historically, the largest aircraft were designed for routes with the longest flight distances. The relationship between aircraft size and range in the 1970s was almost linear. So, for example, if an airline wished to serve a very long-haul non-stop route, it had no other choice but to acquire the largest Boeing 747 aircraft type. Throughout the years, manufacturers have been expanding their aircraft product families, providing more variety in order to offer airlines more options to build their fleet.

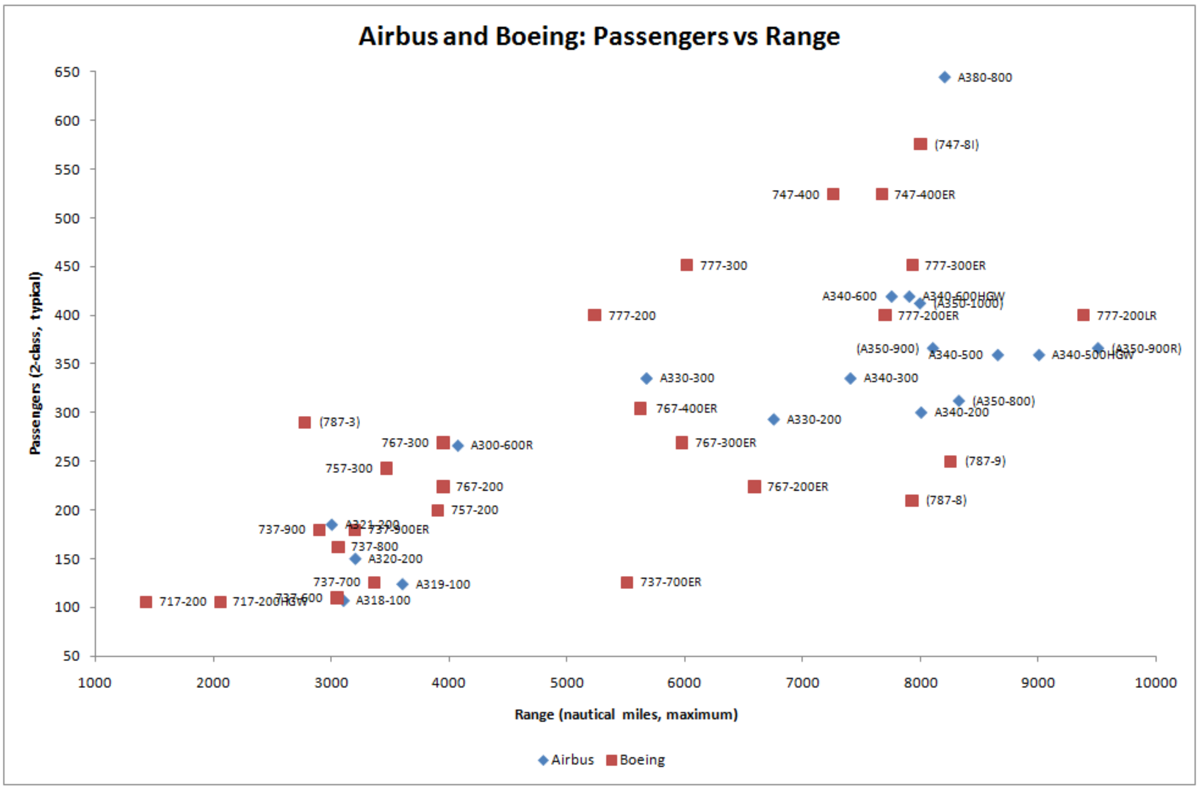

Figure 2.3 – Size and range characteristics of Boeing and Airbus aircraft (Wikipedia@)

According to “Buying the Big Jets, P. Clark”, an aircraft manufacturer, in order to sell its products, has to satisfy the needs of the airline, and also, offer advantages compared to other rival companies. Therefore, the manufacturer has to have an open eye to the airline’s planning, because it is going to declare its needs. The airline’s planners have to be careful and choose wisely what they wish for its fleet, because manufacturers tend to focus the strong points and depreciate the weaknesses of its products, and sometimes the planner finds himself in the middle of two contradictory arguments. To balance these opposing viewpoints, the airline needs find the definition of the assumptions under which analysis is performed.

For experienced Airlines, it is easy for their planners to not be influenced by the manufacturers marketing tricks. Same thing does not happen with start-up airlines, which tend to lack expertise, experience and access to data, in order to conduct a comprehensive analysis. In this case, the manufacturer overpowers the Airline in knowledge, which can result in biased decisions.

2.3. Technical and Performance Characteristics

An important performance characteristic that determines the airline’s choice of aircraft type is the “payload-range curve”. As shown in Figure 2.4, the payload-range curve defines the technical capability of an aircraft type to carry a payload of passengers and/or cargo over a maximum flight distance. Depending on the engine type attached to the airframe as well as the aircraft model, the payload-range curve changes.

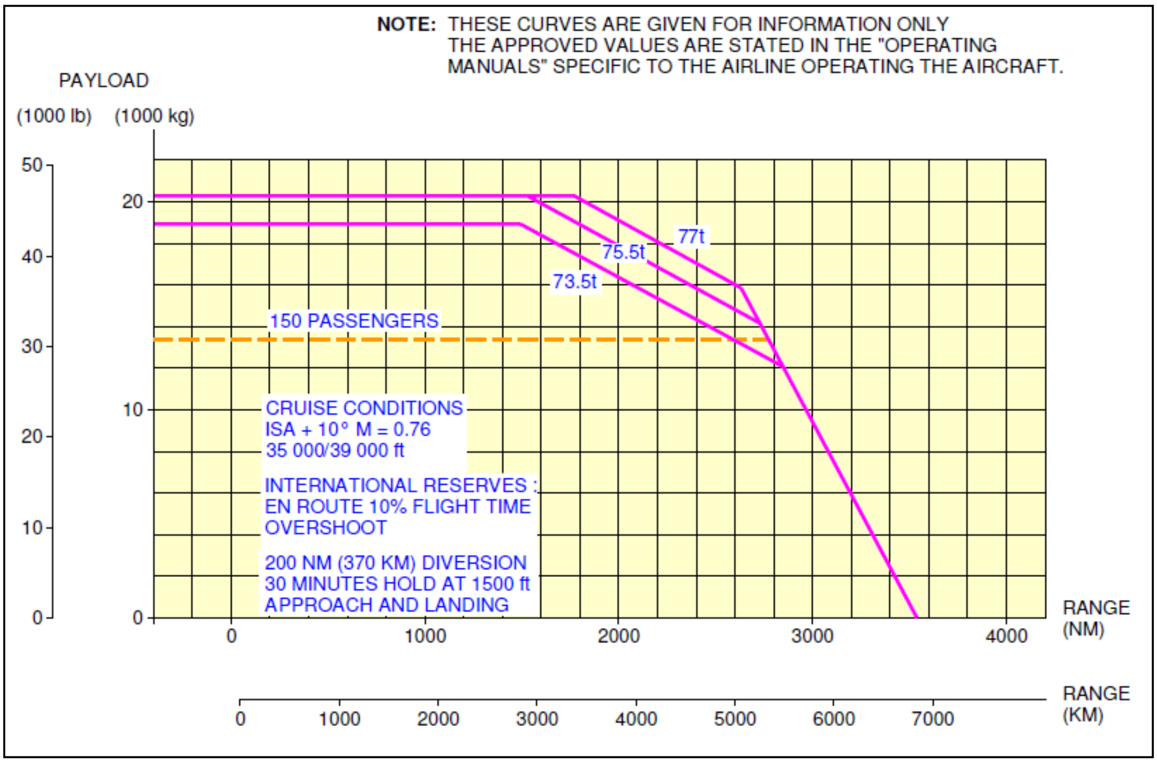

Figure 2.4 – Payload-Range Curve of the Airbus A320-100 and A320-200 ("A320 – Airplane Characteristics For Airport Planning")

Payload-range curves depend on specifications such as aerodynamic design, engine technology, fuel capacity and passenger/cargo configuration. In general, the typical shape of the curve is such that the aircraft is able to carry a maximum payload over a certain distance, while long-haul flights can be executed if the operator is willing to reduce its flight payload in exchange for extra fuel. This trade-off continues until a maximum operational range is reached.

There are other important technical and performance characteristics that include a wide variety of factors related to airline operational and airport constraints. For instance, each aircraft type has its own maximum take-off and landing weights that determine minimum runway length requirements which might not be supported by some airports. Limitations on the taxiways and gate space and even ground equipment at airports can also be a problem and will influence the fleet planning of an airline.

Choosing aircraft with common characteristics is quite a good strategy because it can significantly reduce the costs associated with training of pilots and mechanics, and also the need for new equipment and spare parts inventory for new aircraft types not previously in the airline's fleet. Manufacturers like Boeing and Airbus have at the disposal a couple of aircraft families composed by aircraft types that have similar or identical cockpit layouts, and maintenance and spare parts requirements. For instance, the Airbus A318 (110 seats), A319 (130 seats), A320 (150 seats) and A321 (170 seats) are similar in their physical characteristics except their seating capacity and range. All these mentioned aircraft have the same cockpit crew requirements, which allow crews to easily operate all types in the family, therefore, reducing the airline crew costs.

2.4. Financial and Economic Issues

When acquiring new aircraft, airlines have two forms of payment: full payment or leasing. Full payment is normally required upon aircraft delivery and the payment can be done with cash on hand, retained earnings, debt (loans) or equity (stocks) for aircraft purchases. Leasing might be more expensive in terms of monthly lease payments, but due to its flexibility in allowing frequent fleet renewals and a lower investment required, makes leasing the favorite option for many airlines nowadays as shown in figure 2.5.

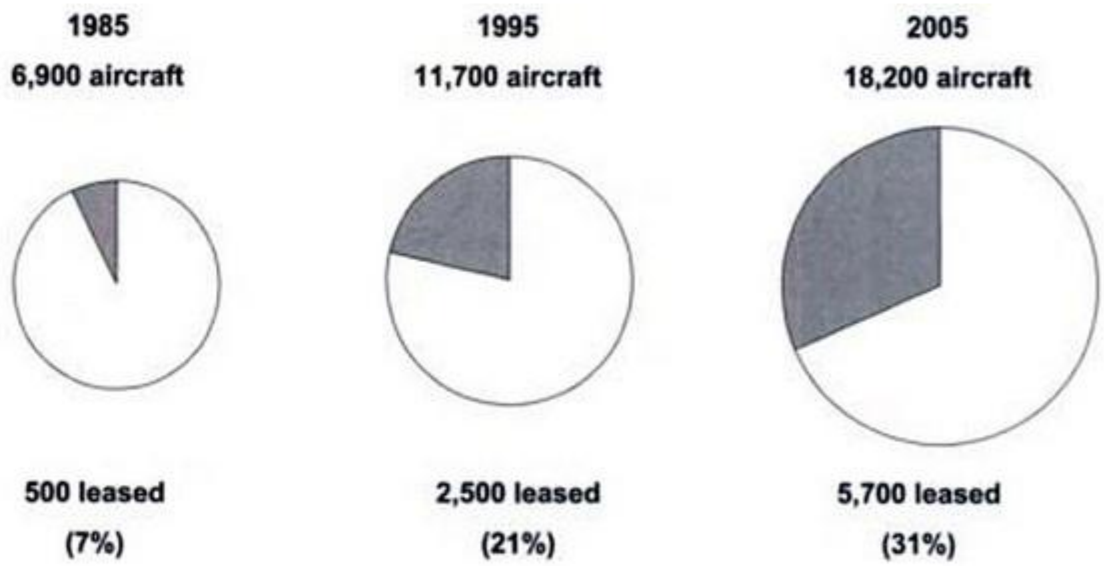

Figure 2.5 – The increase of leasing throughout the years ("Buying the Big Jets", P. Clark)

In order to determine the cost and revenue impacts of each alternative, airlines evaluate all the possibilities with different alternative aircraft, so that in the end, they can make the best decision possible. As mentioned before, if the type of the acquired aircraft is new to the fleet, then there will be costs for spare engines and parts inventory, as well as for new ground equipment and employee training costs. When an airline makes the decision of renewing its fleet, they are concerned about the higher operating costs of the older aircraft and the possible benefits of acquiring new models. Not only do these new aircraft have lower operating costs but they might have greater payload capacity and even marketing appeal of newer aircraft to passengers.

2.5. Hub Location

A hub is an airport that an airline uses as a transfer point to get passengers to their intended destination. Hubs tend to have heavy traffic, which may indeed become their weakness because they have cyclical peaks of high activity. The need to create connections for the large demand that arrives from long-haul flights means that there must be available small sized aircraft which it is not advantageous to the economy because flights should be distributed throughout the time using the less number of aircraft possible and using the maximum amount of time.

On the other side, the larger the hub, the higher is the probability of occurring delays and failed connections. Thus, if an airline chooses to optimize operating efficiencies and passenger satisfaction, it ought to limit the size of a hub. “It would appear that maximum efficiencies occur when around 50-70% of traffic is connecting at a hub” (Buying the Big Jets, Fleet planning for airlines, Paul Clark)

The location of the hubs are an important factor when planning, not just because they determine the network, but they, as well, influence the size and the management of the own hub. For instance, in terms demand peaks and its number which will, consequently, determine the fleet size. On the other hand, the way that the traffic along the spokes is kept, for instance, arriving and departing on the hub at constant level, with small aircraft, determines also the type and size of aircraft needed.

Fleet sizing is also determined by the kind of flights received. For example, if the hub has long-haul flights that are disperse by small flights, the fleet has to be mixture of small and large aircraft. On the other hand, if all the flights have the same distance and the same number of passengers, it requires aircraft of the same size.

The following figure 2.6 shows an example of network composed by two hubs and their respective spokes.

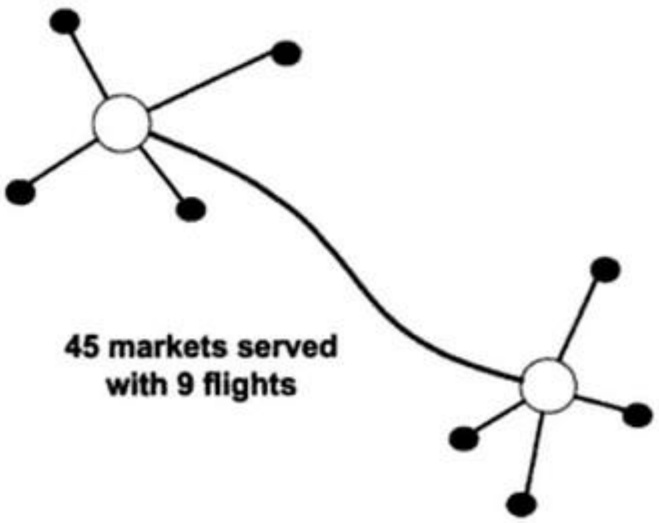

Figure 2.6 – Double hubbing (“Buying the Big Jets”, P. Clark)

Hubs are essential for the distributing traffic, however, they have some problems due to its size and, consequently, the increase of the flight time. Some airlines have found a niche market, and offer clients a by-pass flight that enables passengers to go directly to the destination without going through the hub. This only works if the market in the small city-pairs is large enough.

Curiously, these two types of strategies of development are supported by the two most important manufacturers. Boeing believes that smaller aircraft should link a large number of direct flights “point-to-point flying”, while Airbus is of the opinion that larger aircraft should connect efficiently major centers of population, in other words, hubs would still keep an important role on the network. “A cynic would argue that these views are designed to support the product strategies of the two suppliers. The truth of the matter is that there is more than a grain of truth in both approaches and the situation is certainly not black or white." ("Buying the Big Jets", P. Clark).

2.6. Other Aircraft Selection Criteria

Besides these important characteristics that usually have more weight on an airline's fleet planning decision, there are other aircraft selection criteria that cannot be excluded, such as the environmental impact, marketing and political issues.

All around the world, countries and their governments are imposing regulations to limit the environmental impacts. The population that lives nearby the airports is the most punished by the noise and emissions caused by the aircraft so that now, many airports have regulations and/or curfews that limit or prevent the operation of older aircraft types with engines that exceed specified noise levels. Also, there is a growing trend toward imposition of air pollution regulations designed to cut down the aircraft emissions around airports. These regulations incentive airlines to renew their fleets with modern aircraft that are more environment friendly, but at a higher capital cost to the airlines.

Aircraft manufacturers tend to overstate the marketing advantages of newer aircraft in terms of passenger preference and their impact on generating incremental market share and revenues for the airline. Passengers don't really have aircraft preference. In fact, passengers are less likely to choose (or even be aware of) different aircraft types involved in a given flight. However, it is possible that the first airline to operate the newest aircraft type or the airline with the youngest fleet (with proper advertising of these facts) can generate incremental revenues. When in 2008, Singapore Airlines introduced the new A380 super-jumbo aircraft (Photo 2.1), the company generated a great deal of demand, allowing the airline to charge higher fares on A380 flights than on flights operated with other aircraft types on the same routes.

Photo 2.1 – A380 of Singapore Airlines (Airliners@)

2.7. Conclusion

Economics is, probably, the most important element in fleet planning, followed closely by aircraft performance. Figure 2.7 is an example of the elements involved on a fleet selection process. In short, efficient fleet planning involves a structured and prioritized set of key criteria, relevant to the airline and its position in the market.

Category	*Elements*
Markets and routes	Size, growth, mix, comfort, schedule, Airport compatibility, economics, turn times
Operations	Crewing, aircraft mix, ETOPS, Minimum Equipment List, performance
Finance and contractual	Purchase vs lease, residual value, buy back, insurance, price escalation, guarantees, spares pricing, cost of updates
Engineering	Spares inventory, pooling, commonality, facilities, third party needs
Regulatory and environmental	Certification rules, environment standards special conditions

Figure 2.7 – Example of several elements involved on a fleet selection process ("Buying the Big Jets", P. Clark)

3. OPTIMIZATION MODELS FOR FLEET SIZING

3.1. Introduction

This chapter describes two important fleet sizing optimization models, which were quite important for the development of the proposed optimization model presented in the next chapter.

3.2. Fleet Sizing and Vehicle Allocation Model

George J. Beaujon and Mark A. Turnquist (2001), created a model in which they consider a set of locations denoted by $\boldsymbol{N}$. They assumed that the planning horizon has been divided into discrete "decision periods" – t, and represented the demands for transportation service between points i and j, $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$, in period t, by $d_{ij}(t)$.

$d_{ij}(t)$ has been considered to be a random variable whose mean may depend on t. They consider just full vehicle loads. Demands may change regularly over time and the actual demand observed at any time contains an uncertainty with two components: a stochastic and a deterministic elements. The actual probability distribution of the random component will remain unspecified, except that it should have a mean of zero.

These demands generate loaded vehicle flows which are represented by $X_{ij}(t)$. In the presented model, Beaujon and Turnquist, only contemplate one type of vehicle but if there are several different types of vehicles available to server demands, notation can be expanded to $X_{ijk}(t)$ to represent flows of vehicles of type k. Movement of empty vehicles has been denoted $Y_{ij}(t)$. These movements are necessary because the demand for loaded vehicles that arrive in location i, may be different from the demand of loaded vehicles that depart from i to other locations.

In the formulation of fleet management models, travel time is an important element that, in many systems is uncertain due to equipment failures and/or external interference, and, by consequence, is a random variable. Rather than introduce travel time directly, Beaujon and Turnquist, chose to formulate the problem in terms of the vehicle arrivals. Thus, given that $X_{ij}(\tau)$ vehicles were dispatched from point i in period τ, it is necessary to know how many of these vehicles actually arrive at point j in period t. For that, it was defined two random variables, $\alpha_{ij}(\tau,t)$ and $[\beta_{ij}(\tau,t)]$, which are, respectively, the proportion of loaded and empty vehicles dispatched from i to j in period τ which actually arrive in period t.

In order to offer a security against temporary shortages dues to the dynamic and stochastic fluctuations in demand and uncertain travel times, it is advisable to maintain a pool of vehicles at some locations. That number of vehicles present at location i at the end of period t was represented by $V_i(t)$.

But there is also a possibility that the vehicles available at location i in period t are insufficient to meet all demands, and if so, some of the demands will be either backordered or lost from the system. The quantity of demand from i to j that remains unmet at the end of period t is designated by $U_{ij}(t)$.

Supposing constant revenue per loaded vehicle sent from i to j (r_{ij}), constant costs of moving vehicles from point i to j (l_{ij} for loaded vehicles and e_{ij} for empty vehicles) and constant daily cost of ownership of the vehicle while traveling (q), the optimization is obtained by maximizing the difference between revenues generated by serving demands and costs of vehicle ownership, vehicle movement, and unmet demand.

The holding cost for vehicle pools is considered to be the ownership cost of the vehicles, q, plus additional costs associated with storage and management of the vehicle pool. The cost of holding a vehicle for one period at location i is represented by H, where $H_i \geq q$.

The penalty cost for unmet demand (backordered vehicles) or the unit penalty cost per period for vehicle loads waiting at i to be transported to j is denoted by P_{ij}.

The decision variable notation summarized:

Quickly summarizing the notation, we have as decision variable:

$X_{ij}(t)$ = number of loaded vehicles dispatched from i to j in period t

$Y_{ij}(t)$ = number of empty vehicles dispatched from i to j in period t

$V_i(0)$ = number of vehicles initially allocated to location i.

Because demands and travel times are uncertain, the optimal values for vehicle dispatching decisions, $X_{ij}(t)$ and $Y_{ij}(t)$ will depend on the realization of $d_{ij}(\,)$'s, $\alpha_{ij}(\,)$'s and $\beta_{ij}(\,)$'s.

The state of the system at any time t is given by:

$V_i(t)$ = number of vehicles present at location i at the end of period t

$U_{ij}(t)$ = unmet demand from i to j in period t.

The revenues and costs associated with operating the system are:

r_{ij} = revenue per loaded vehicle sent from i to j

l_{ij} = cost of moving a loaded vehicle from i to j

e_{ij} = cost of moving an empty vehicle from i to j

q = cost per vehicle per period to own or lease a vehicle

H_i = unit cost of holding a vehicle for one period at location i

P_{ij} = penalty cost per period for one unit of unmet demand from i to j.

In addition, because travel times are uncertain, the following random variables are needed to describe vehicle movements:

$\alpha_{ij}(\tau,t)$=proportion of loaded vehicles dispatched from i to j in period τ which arrive in period t

$\beta_{ij}(\tau,t)$=proportion of empty vehicles dispatched from i to j in period τ which arrive in period t.

Finally, the demand for vehicles is given by:

$d_{ij}(t)$ = the demand for transportation service between i and j in period t.

The model is formulated as follows:

$$\text{Max } \pi = \sum_t \sum_i \sum_j r_{ij} X_{ij}(t)$$

$$- \sum_t \sum_i \sum_j \left[l_{ij} X_{ij}(t) + e_{ij} Y_{ij}(t) \right]$$

$$- \sum_i \sum_j \sum_\tau \left\{ \left[X_{ij}(\tau) \sum_{t>\tau} (t-\tau) q \alpha_{ij}(\tau,t) \right] + \left[Y_{ij}(\tau) \sum_{t>\tau} (t-\tau) q \beta_{ij}(\tau,t) \right] \right\} \quad (1)$$

$$- \sum_t \sum_i H_i V_i(t)$$

$$- \sum_t \sum_i \sum_j P_{ij} U_{ij}(t)$$

Subject to:

$$U_{ij}(t) = U_{ij}(t-1) + d_{ij}(t) - X_{ij}(t) \quad \forall \; i,j,t \quad (2)$$

$$V_i(t) = V_i(t-1)$$

$$+ \sum_j \sum_{\tau<t} \left[X_{ji}(\tau) \alpha_{ji}(\tau,t) + Y_{ji}(\tau) \beta(\tau,t) \right] \quad (3)$$

$$- \sum_j \left[X_{ij}(t) + Y_{ij}(t) \right] \quad \forall \; i,t$$

$$X_{ij}(t), Y_{ij}(t), U_{ij}(t), V_i(t) \geq 0 \quad (4)$$

$$\textit{and integer} \quad \forall \; i,j,t$$

“The objective function (1) includes terms for revenues, direct transportation cost, ownership cost for vehicles en route, holding costs for idle equipment, and penalty costs for unmet demand. Constrains (2) ensure that all demand is accounted for; unmet demand in period t must equal unmet demand from the previous period plus new demand minus loaded movements. Constrains (3) are conservation of flow constrains for vehicles at each location in each time period which include the effects of stochastic travel times for vehicle movements through the α and β terms, representing the uncertain arrival times of vehicles at their destinations. Constrains (4) ensure that $X_{ij}(t)$, $Y_{ij}(t)$, $U_{ij}(t)$, and $V_i(t)$ are always non-negative and integer.”

The optimization problem represented by equations (1) to (4) can be viewed from the three different perspectives explained subsequently:

1 – As a stochastic programming problem. The demands, $d_{ij}(t)$, and the travel times, reflected in the α and β terms, are random, variables, so the problem is a stochastic programming one, with random variables both in the objective function and in the constraints. This and the size of the problem, makes it unattractive to solve with standard stochastic programming techniques.

2 - As a stochastic control problem. "The $V_i(t)$ and $U_{ij}(t)$ represent state variables for the system, whose values are affected by control actions, $X_{ij}(t)$ and $Y_{ij}(t)$, and uncontrollable inputs to the system, $d_{ij}(t)$. The major difficulties with this view of the problem are: 1) the effects of the control actions on the state variables are lagged because it takes time to reposition vehicles, 2) the length of the delay (travel time) is uncertain, and 3) the state variables and control actions are all bounded. While some work has been done on modeling distributed delays in control problems (e.g., MANETSCH), their presence greatly complicates solution procedures because the state space must be expanded. For the problems of interest here, the state space is already very large, and the expansion required appears to make the problem computationally intractable".

3 - Taking advantage of its implicit structure. "The problem could be viewed from two perspectives, one emphasizing the inventory-like "vehicle pool" aspects of fleet sizing, and the other emphasizing the "routing" aspects of vehicle allocation on a network. The formulation in (1)-(4) shows both of these elements intertwined. The number of vehicles available at each location and time period, together with the number of backorders and the expected demands, determines what vehicle flows are feasible as well as desirable. Conversely, the vehicle flows over time affect the available vehicle supply (or pool) at each location".

3.3. Fleet Sizing Under Uncertainty

G. F. List, B. Wood, L. K. Nozick, M. A. Turnquist, D. A. Jones, E. D. Kjeldgaard and C. R. Lawton (2002) developed a model intended for systems that transport freight where "three major inputs are involved: the demands to be served, the network over which operations are conducted, and cost parameters associated with various investments and operating decisions". This model could also be applied to systems that transport passengers.

In systems for transport freight the term shipment will be used to denote demand served. Figure 3.1 emphasizes that tradeoffs are made among postponed shipments, shipments carried, vehicle flows (loaded and empty movements across the network) and vehicle fleet size(s) to optimize a combination of one or more objectives (e.g., service quality and cost).

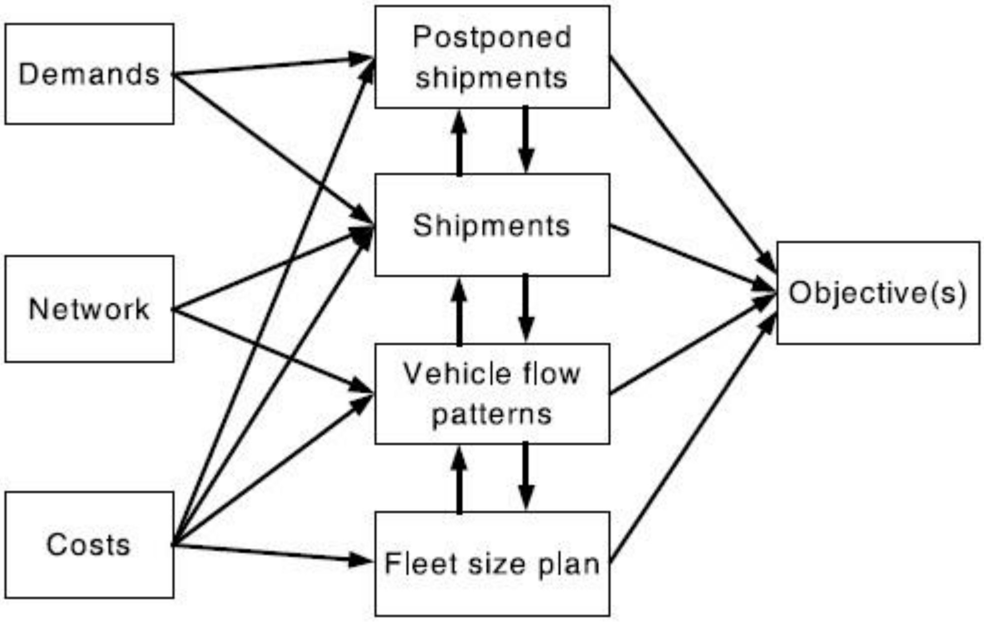

Figure 3.1 – Fleet sizing problem relationships

Authors considered three types of costs: fleet ownership costs, fleet operating costs, and service quality penalties for not meeting demands at the requested time. The postponed shipments are separated into two sub-categories depending on the allowable time window for moving each shipment:

1) delayed shipments - they are carried within their allowable time window, but at later times than requested;

2) deferred shipments – they are not served within the allowable time window.

The penalty cost for deferring a demand is higher than for simply delaying it.

The willingness to transform quality penalties in monetary terms, thus combining the service quality and cost objectives, has the intent to easily compare and determine the optimal tradeoff between the costs of fleet ownership and operation on the one hand, and the costs of service quality on the other. For example, “purchasing too small a fleet often results in large penalty costs for demand that is served late or not at all, while purchasing too large a fleet results in excessive ownership (and perhaps operating) costs".

Here, demands are named *markets* and designed by the index *m* that simplifies the subscripting on the variables. It means a particular commodity (or class of commodities) being transported from an origin node *i* to a destination node *j*. In order to fit these demands (*markets*) it is necessary to acquire and use one or more fleets (vehicle types) over time. The problem is to determine the desired fleet size, $V_k(t)$, for all vehicle types *k* and time periods *t*.

The model center on ways to consider uncertainty in fleet sizing problems, and uses a generic statement of the fleet sizing problem, referred to as problem ***P1***.

$$z_1 = \sum_{k,t} \theta_k(t) V_k(t) + \sum_{k,t} \lambda_k a_k(t) + \sum_{k,t} \zeta_k r_k(t) + \sum_{i,j,k,t} c_{ijk} x_{ijk}(t) \quad (5)$$

$$z_2 = \sum_{m,t} \rho_m Q_m^{def}(t) + \sum_{m,t} \gamma_m Q_m^{del}(t) \quad (6)$$

subject to:

$$\sum_{\tau=1}^{t} q_m(\tau) + Q_m^{def}(\tau) \geq \sum_{\tau=1}^{t-w_m} Q_m(\tau) \quad \forall\, m, t \quad (7)$$

$$\sum_{\tau=1}^{t} q_m(\tau) + Q_m^{dly}(\tau) \geq \sum_{\tau=1}^{t} Q_m(\tau) \quad \forall\, m, t \quad (8)$$

$$\sum_{m \in M_{ijk}} q_m(t) \leq x_{ijk}(t) \quad \forall\, i, j, k, m, t \quad (9)$$

$$\sum_{j \neq i} x_{jik}(t) = \sum_{j \neq i} x_{ijk}(t) \quad \forall\, i, k, t \quad (10)$$

$$\sum_{i,j} x_{ijk}(t) d_{ij} \leq \pi_k \Gamma(t) V_k(t) \quad \forall\, k, t \quad (11)$$

$$V_k(t) = V_k(t-1) + a_k(t) - r_k(t) \quad \forall\, k, t \quad (12)$$

The choice variables are:

$q_m(t)$	the shipments carried for market *m* in time period *t*
$Q_m^{def}(t)$	total shipments deferred for market *m* in time period *t*
$Q_m^{dly}(t)$	total shipments delayed for market *m* in time period *t*

$x_{ijk}(t)$ vehicles of type k moved from *i* to *j* in time period *t*

$V_k(t)$ fleet size for type *k* vehicles in time period *t*

$a_k(t)$ acquisitions for fleet *k* in time period *t* (assumed to occur at the beginning of the period)

$r_k(t)$ retirements for fleet *k* in time period *t* (assumed to occur at the beginning of the period)

And the inputs are:

θ_k cost of owning one vehicle of type *k* for one time period

λ_k cost of acquiring one vehicle of type *k*

ζ_k cost of retiring one vehicle of type *k*

c_{ijk} operating cost of a trip from *i* to *j* for a vehicle of type *k*

ρ_m per-period penalty for deferring one shipment for market *m*

γ_m per-period penalty for delaying one shipment for market *m*

M_{ijk} set of markets that have origin *i*, destination *j*, and are eligible for shipment on vehicles of type *k*

$Q_m(t)$ number of market *m* shipments offered for movement in time period *t*

w_m allowable window (number of time periods) for shipment of market *m* shipments

π_k the percent of time that a vehicle of type *k* is available

$\Gamma(t)$ the duration of time period *t*

d_{ij} travel time from *i* to *j*

T the number of time periods

"The model contains two objectives: z_1 (total cost) and z_2 (penalties related to service quality). The equation for z_1 has four terms: (a) the ownership cost of the active vehicle fleet, (b) the cost of additions to that fleet, (c) the cost of deletions from that fleet, and (d) the operating cost of using the fleet. The equation for z_2 captures the penalty cost for deferring and delaying shipments. The concepts of deferred and delayed shipments are based on a premise that each shipment has a time *t* at which the shipper desires it to be moved, and an acceptable window of time within which it should be moved, [t , t + w_m]. If the shipment is not moved at the earliest time available (the shipper's desired movement time), it is considered *delayed*, and the model includes a penalty cost for this reduction in service quality. If the shipment is not moved within the allowable time window, it is considered *deferred*, and a (larger) penalty is assessed on this more severe reduction in service. The implementation of the time windows is through constraints (7) and (8).

The concept of a service window for shipments in each market, combined with penalty parameters for delay and deferral, γ_m and ρ_m, allow for a very flexible representation of the workload requirements in the system. For example, at one extreme, if the window is zero for some market *m*, the right-hand-sides of (7) and (8) will be equal for all values of *t*, and any demand that is not met on time will be considered deferred. Alternatively, if a wide window is specified and γ_m = 0, the system is free to carry that demand anytime within the window without penalty. This allows for much greater operating efficiencies, as well as load balancing that can reduce required fleet size. Thus, ***P1*** can reflect demand conditions in a wide variety of application situations.

Constraint (9) ties the shipments $q_m(t)$ passing over the arc from i to j in vehicle type *k* during time period *t* to the vehicle flows on that same arc in time period *t*, $x_{ijk}(t)$. The inequality in (9) allows for empty movements that may be required to balance vehicle flows, as specified in constraint (10). Constraint (10) specifies that the vehicle flows must balance at each node within each period, not just across the entire planning horizon.

Constraint (11) measures the total availability of a fleet of $V_k(t)$ vehicles. That availability is reflected in vehicle-hours, and (11) ensures that the vehicle-hours of use for each vehicle type *k* accruing during time period *t* is less than or equal to the total number of useful vehicle-hours that can be provided by the fleet in time period *t*. An equivalent resource constraint could be written in terms of vehicle-miles (or vehicle-km), if desired. This is a crucial constraint in the model, because it links the decisions on vehicle fleet size to the operational requirements of meeting demand. Constraint (12) tracks the vehicle fleet sizes across time, as adjusted by additions and deletions, in response to the demands derived from constraint (11). For long term planning and relatively large fleet sizes, it is reasonable to allow the vehicle acquisition and retirement variables (and hence the fleet size) to take on any real values, and thus problem ***P1*** is a linear programming problem. For some situations, it may be difficult to interpret non-integer vehicle variables, and thus ***P1*** would have to be solved as an integer programming problem.

The formulation of ***P1*** draws on ideas represented in earlier models from several different authors. Simpson (1969) suggested the use of a constraint like (11) to represent fleet availability in airline models. The concept of service windows has been used by several previous authors (e.g., Crainic et al., 1993; Cheung and Powell, 1996), although the implementation of the concept in ***P1*** is slightly different.

P1 is based on an assumption that the time periods used are relatively long (as compared with node-to-node travel times), so that in general vehicles that depart from node i in time period t will arrive at another node j in the same time period. It also means that it is possible for a single vehicle to make more than one trip (i to j and then to k or back to i) within a single period. The intensity of use of individual vehicles is constrained by the quantity $\pi_k \, \Gamma(t)$ in constraint (11), rather than by assuming (for example) that they can make only one movement per time period. This is a reasonable assumption for fleet planning studies that may use an overall planning horizon of multiple years, but it differs from more operationally oriented models that are focused on allocation of available vehicles over very short time periods. There is clearly a modeling issue in implementation of ***P1***, to choose time periods that are long enough so that (10) reasonably reflects flow balance in the network, but short enough that the time windows on service requirements are meaningful. If an appropriate choice for a given situation proves difficult, the flow balance constraints can be modified to reflect travel times across multiple periods. However, that detail is not the primary focus, so it will be used the simpler version represented in (10).

Uncertainty is important in at least two areas of this model. The spatial and temporal aspects of future demands are uncertain, and this is reflected in uncertainty in the $Q_m(t)$ values. Both travel times and fleet productivity (the π_k parameter) are also subject to uncertainty. Either of these uncertainties affects the amount of work that a fleet of a given size can accomplish, as specified through constraint (11).”

4. PROPOSED FLEET SIZING MODEL

4.1. Introduction

The optimization model described in this chapter was created during the development of this thesis. After several attempts to create a model similar to those described in the previous chapter, it was decided to adopt an easier optimization model without resorting to the aircraft tracking.

The model was tested using the program Xpress-IVE, and three examples were created to calibrate the model according to the reality. In chapter five, this same model was applied to determine the optimal fleet for TAP Portugal.

The computer used to run this model has an Intel Core i7 CPU Quad 720 @ 1.60GHz, with 6,00GB of RAM.

4.2. Problem Description

An airline needs to carry q_{ijt} daily passengers in month t between airports i and j of airport set $\boldsymbol{N} = \{1, \dots, N\}$. The distance for a flight between airports i and j is d_{ij}. The set of aircraft types the airline can use for the flights is $\boldsymbol{K} = \{1, \dots, K\}$. The number of seats of an aircraft of type k is s_k, the range is r_k, the maximum speed is v_k and the turn time is t_k. The ownership cost of an aircraft of type k is c_k^O, and the operation cost is c_k per (seat×kilometer). The objective is to determine how many aircraft of each type should the airline own, so that the total costs in n years are minimized. Since it is expected a rapid progress in aircraft technology in the next few years, it is assumed $n = 10$.

4.2.1. Decision variables

x_k: number of aircraft of type *k* owned by the airline.
z_{ijkt}: number of daily flights between airports *i* and *j* made by an aircraft of type *k* in month *t*.

4.2.2. Objective function

$$\min C = \sum_{k \in K} c_k^O x_k + n \times \sum_{i,j \in N} \sum_{k \in K} \sum_{t \in T} 30{,}5 \times c_k \times \frac{d_{ij}}{v_k} \times z_{ijkt} \text{ [M\$]} \tag{13}$$

4.2.3. Constraints

<u>Seat capacity</u>: The number of seats offered in flights between airports *i* and *j* in month *t* is enough to accommodate the demand.

$$\sum_{k \in K} s_k z_{ijkt} \geq q_{ijt} \ , \ \forall\, i,j \in N \, , t \in T \tag{14}$$

<u>Time capacity</u>: The total time spent by each type of aircraft with the flights and turn around operations does not exceed the maximum operation time of the available fleet in each month (assuming that the aircraft are available 16 hours per day).

$$\sum_{i,j \in N} \left(\frac{d_{ij}}{v_k} + t_k \right) z_{ijkt} \leq 16\, x_k \ , \ \forall\, k \in K \, , t \in T \tag{15}$$

<u>Continuity</u>: The number of flights made in an aircraft of type *k* arriving to airport *j* in month *t*, is the same as the number of flights that take-off from that airport.

$$\sum_j z_{jikt} = \sum_j z_{ijkt} \ , \ \forall\, i \in N \, , k \in K \, , t \in T \tag{16}$$

<u>Range</u>: The type of aircraft used to fly from *i* to *j* has to have a larger range than the flight distance.

$$z_{ijkt} \leq a_k \times G \ , \ \forall\, i,j \in N \, , k \in K \, , t \in T \tag{17}$$

$$a_k = \begin{cases} 1 \leftarrow d_{ij} \leq r_k \\ 0 \leftarrow d_{ij} > r_k \end{cases} \ , \ G \text{ is a large number}$$

4.3. Data Generation

In order to calibrate the optimization model, a group of random locations given by the program was considered. It was assumed that these locations have to be inside a 10.000×10.000 kilometers area.

To test the model, it was generated three examples. The first example is tested with ten locations, where two of them are considered the airline's hubs. On the second example, it is generated twenty locations and three of them will be the hubs. Finally, on the last example, it is created forty locations and, this time, there are four hubs. The program chooses an airport for a hub according to the distance between locations but this will be explained later on each example.

To determine the demand between locations, it was created the variable P_i , which generates a random population for each location between 500.000 and 20.000.000 inhabitants. This variable will be used on the expression that determines the demand q_{ijt} :

$$q_{ijt} = p \times \alpha \times \varphi_{ij} \times P_i \frac{P_j}{D_{ij}^{1,5}} \quad (18)$$

p varies between 0,75 and 1,25 and it's used to make the demand vary throughout the months

α constant calibration variable

φ_{ij} Assumes value 0 if the distance between *i* and *j* is less than 200 kilometers, and 1 if the that distance is larger than 2000 kilometers. Between those distances, the value of φ_{ij} grows proportionally to the distance.

The program does not generate demand for all the possible destinations and it will be explained on each example what assumptions were made.

The data of the available aircraft to buy is provided by the following table:

Aircraft	**Key Data**				**Costs**		
	Seats	Range	Cruise Speed	Turn Time	C_{ow}	C_{op}	
A318	107	5950	828	30	67,7	0,0470	
A319	124	6850	828	35	80,7	0,0437	
A320	150	6150	828	40	88,3	0,0401	
A321	185	5950	828	45	103,6	0,0385	*
A330-200	253	13430	871	60	208,6	0,0398	
A330-300	295	10830	871	70	231,1	0,0350	
A340-300	295	13700	871	70	238	0,0390	
A340-500	313	17000	881	75	261,8	0,0368	*
A340-600	380	14600	881	85	275,4	0,0303	*
A350-800	270	15700	903	65	245,5	0,0350	**
A350-900	314	15000	903	75	277,7	0,0301	**
A350-1000	350	15600	903	80	320,6	0,0270	**
Bombardier CS100	100	4074	828	30	58,28	0,0396	
Bombardier CS300	120	4074	828	35	66,57	0,0364	
Embraer ERJ145 LR	50	3706	851	20	19,5	0,0516	*
Embraer E-170	70	3892	851	25	28,5	0,0497	*
		[Km]	[Km/h]	[min]	M$	[$/seat.km]	

* Estimated values after comparing similar values of aircraft of the same type but different engine or size difference
** Predicted values found after some researching through several websites for predictions of the new A350's CASM

Table 4.1 – Key data and costs of each type of aircraft (Airliners@, Wikipedia@, Airbus@, AirInSight@, "Airline Economic Analysis", Oliver Wyman)

4.4. Example 1

As mentioned earlier, in this first example there are ten airports where two of them will be the hubs of the airline. The program chooses randomly the first hub between all the ten locations, and the second one is the airport closest to it.

As for the generated demand, if the population of a destination city is less than the average of the population of the cities where the hubs are located, then the demand of the smallest hub is transferred to the most important one. It is considered that the least important hub does not generate demand for long-haul flights. Also, it is ignored all long-haul flights that do not have a demand larger than 100 passengers per day.

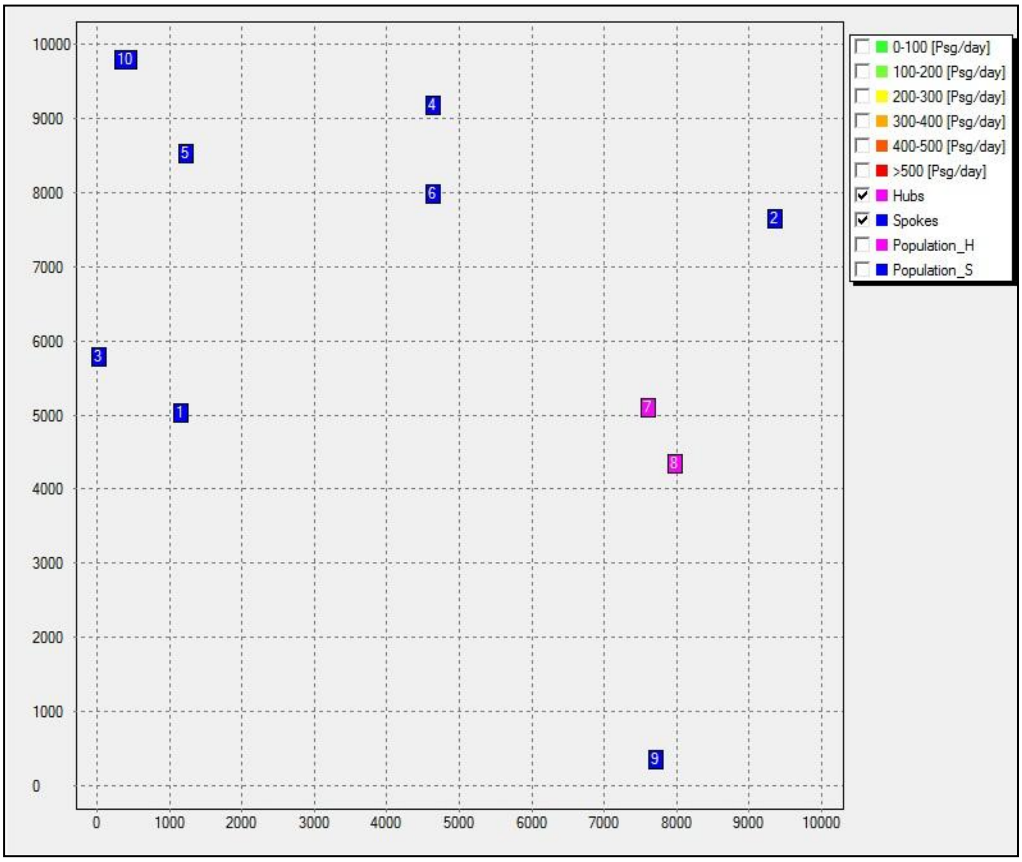

Figure 4.1 – Locations of the airports

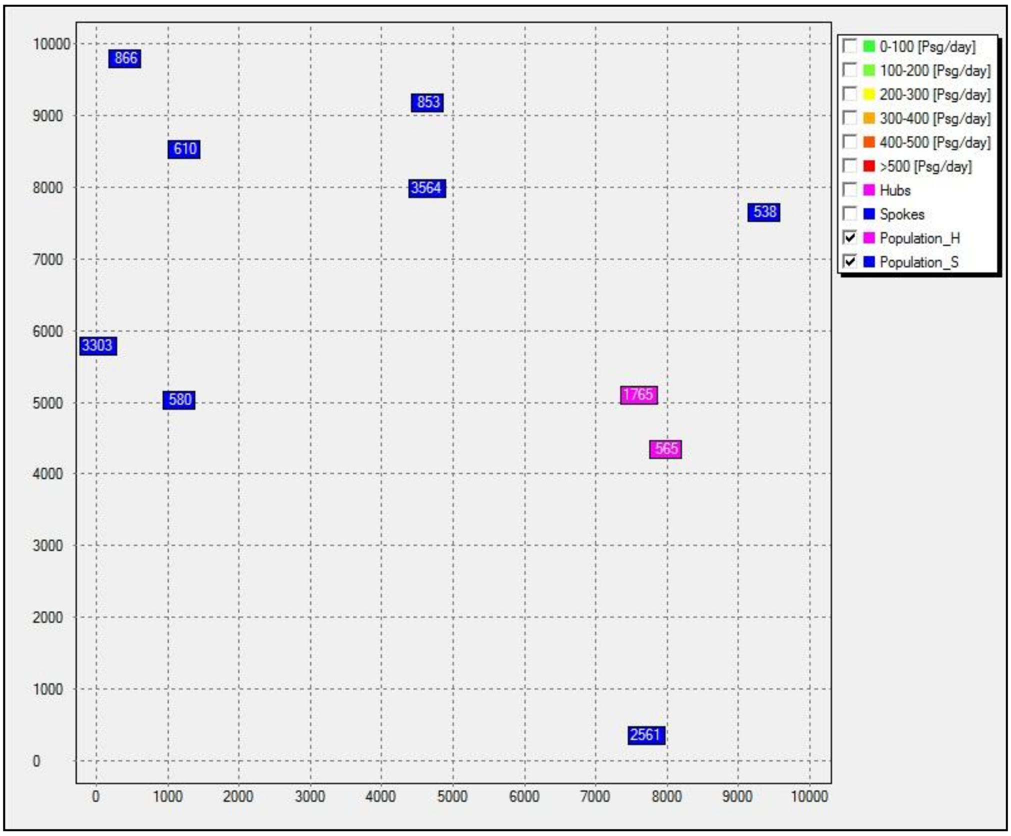

Figure 4.2 – Population of each city

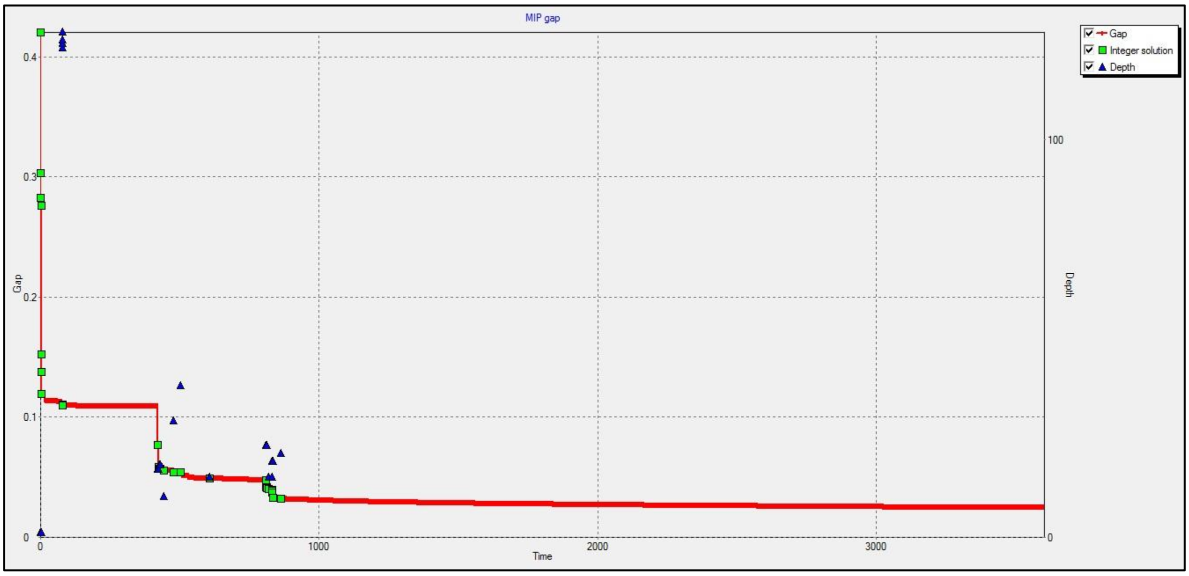

Figure 4.3 – MIP Gap

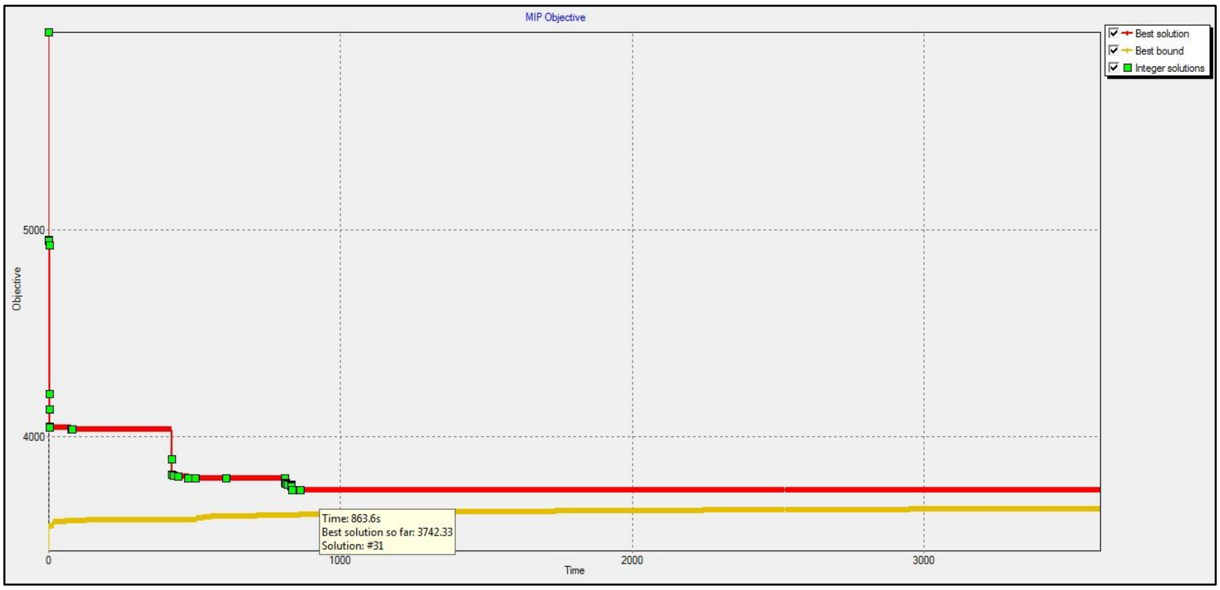

Figure 4.4 – MIP Objective

Results for the month of September:

	Aircraft Types																
O/D	A318	A319	A320	A321	A330-200	A330-300	A340-300	A340-500	A340-600	A350-800	A350-900	A350-1000	CS100	CS300	ERJ145 LR	E-170	Total
7-1		1															1
7-2			1														1
7-3										1							1
7-4	1																1
7-5																	0
7-6		1	3														4
7-8				1										1			2
7-9										1							1
7-10																	0
8-1																	0
8-2																	0
8-3																	0
8-4																	0
8-5																	0
8-6	1																1
8-7				1										1			2
8-9														1			1
8-10																	0
Total	2	2	4	2	0	0	0	0	0	2	0	0	0	3	0	0	15

Table 4.2 – Number of flights per type of aircraft on each leg

This table shows the number of departure flights from the hubs to each location as well as the type of aircraft used. The results for the arrival flights are exactly the same, because the demand is the same on both departure and arrival flights.

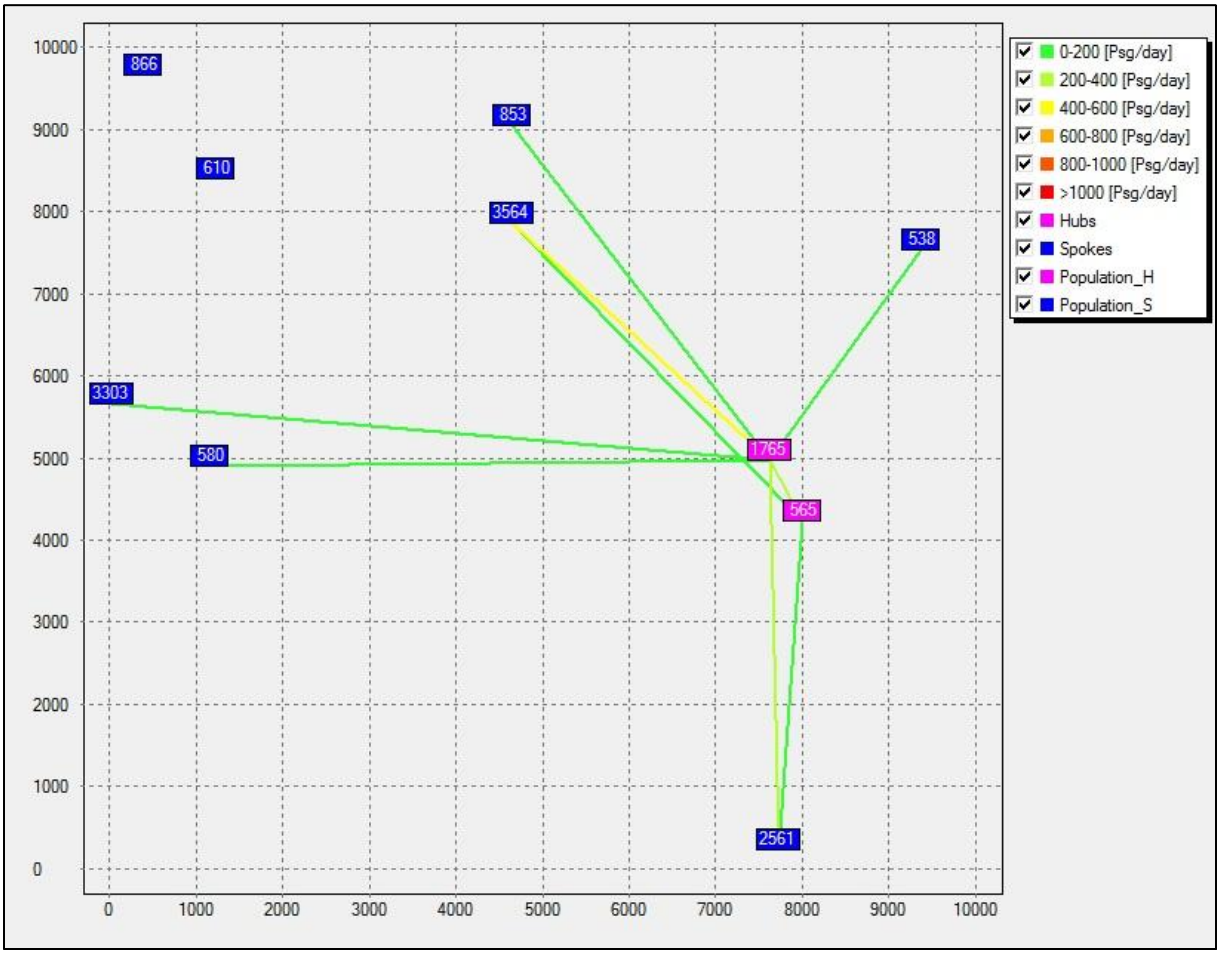

Figure 4.3 – Average Passenger Flow

O/D	Average Daily Demand per Month
7-1	46
7-2	135
7-3	172
7-4	104
7-5	0
7-6	492
7-8	287
7-9	263
7-10	0
8-1	0
8-2	0
8-3	0
8-4	0
8-5	0
8-6	117
8-7	0
8-9	110
8-10	0

Month	Hub 7	Hub 8
1	11	4
2	11	4
3	10	4
4	11	5
5	12	5
6	11	4
7	10	4
8	11	4
9	11	4
10	11	4
11	10	4
12	11	4

Aircraft Types	#
A318	2
A319	2
A320	3
A321	1
A330-200	0
A330-300	0
A340-300	0
A340-500	0
A340-600	0
A350-800	2
A350-900	0
A350-1000	0
CS100	0
CS300	1
ERJ145 LR	0
E-170	0
Total	**11**

Table 4.3 – Average daily demand per month; Number of daily flights (departures and arrivals) on each hub; Optimal number of aircraft to acquire

The program spent, approximately, 1 hour (3601,5 seconds) running the model and it did not reach the end. After finding 31 solutions, the best one was found after 834 seconds, with a gap of 2,42768 % and a total cost of 3741,82 million dollars.

4.5. Example 2

In this second example there are twenty airports where three of them will be the hubs of the airline. The program chooses the first two hubs the same way as explained in Example 1. As for the third hub, it has to distance at least 3000 kilometers from the other two and the sum of the flight distance to other destinations has to be minimum.

The demand for the first two hubs is generated the same way as the in the Example 1. As for the third hub, it is considered as an important hub, so the demand will be generated the same way as the biggest hub from the first two hubs. To avoid having situations where there are flights from one hub to destinations that are very close to the other distant hub, it was assumed that there will not be demand if it verifies the following expression:

$$d(i,ii) + d(ii,j) - d(i,j) < d(i,j)/2 \quad (19)$$

Where *i* and *ii* are the two most important hubs that distance at least 3000 km from each other, and *j* is the destination airport.

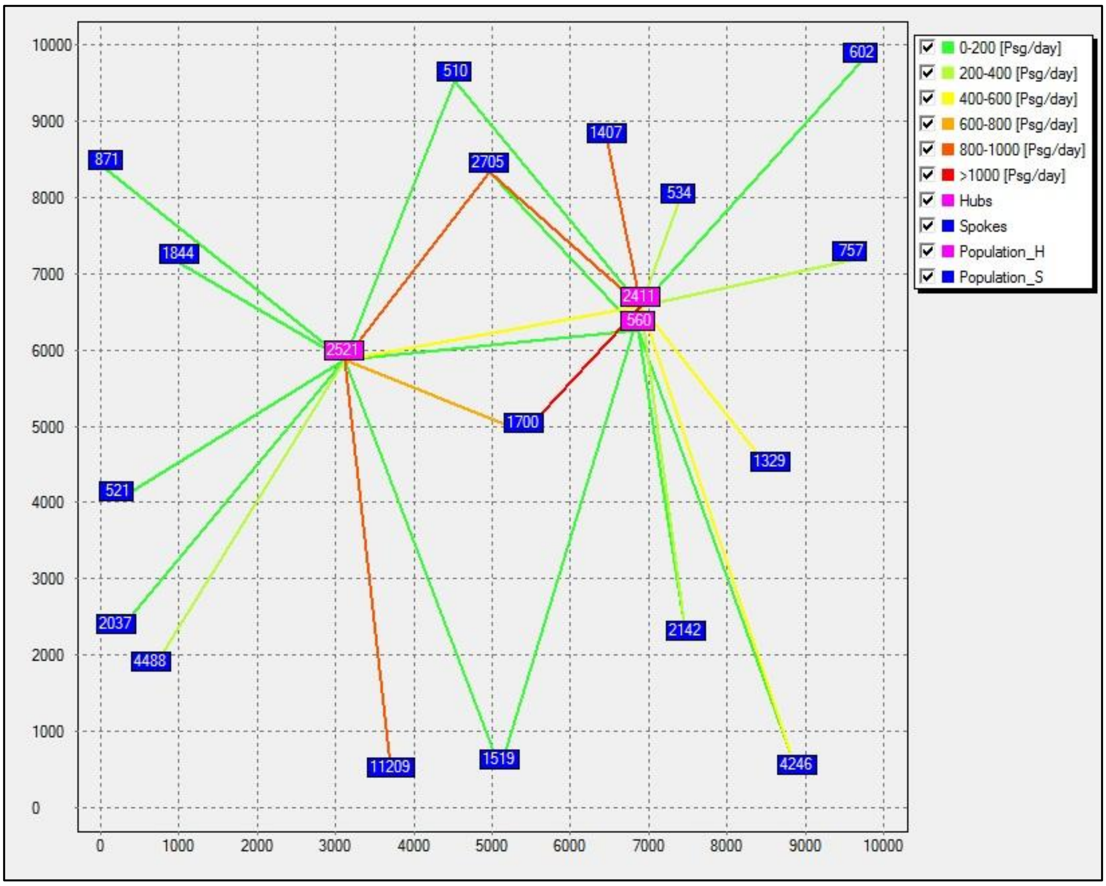

Figure 4.4 – Average Passenger Flow

Month	Hub 4 (Pop. 2411)	Hub 7 (Pop. 560)	Hub 8 (Pop. 2521)
1	54	13	32
2	48	10	25
3	45	7	23
4	43	8	28
5	47	7	29
6	48	9	28
7	46	8	27
8	42	8	25
9	54	10	31
10	54	11	33
11	58	9	26
12	47	7	22

Aircraft Types	#
A318	3
A319	6
A320	3
A321	3
A330-200	0
A330-300	0
A340-300	0
A340-500	0
A340-600	3
A350-800	0
A350-900	1
A350-1000	3
CS100	2
CS300	21
ERJ145 LR	3
E-170	3
Total	51

Table 4.4 – Number of daily flights (departures/arrivals) on each hub; Optimal number of aircraft to acquire

The program spent, approximately, 1 hour (3602,0 seconds) running the model and it did not reach the end. After finding 18 solutions, the best one was found after 62 seconds, with a gap of 6,08486 % and a total cost of 14559,6 million dollars.

4.5. Example 3

Forty locations were considered in this last example and now there will be total of four hubs. The program chooses the first three hubs like in the Example 2, and the last one will be the airport closest to the third one.

The demand is generated the same way as in the last example and for the forth hub it will be applied the same considerations that were made for the least important hub in the Example 1.

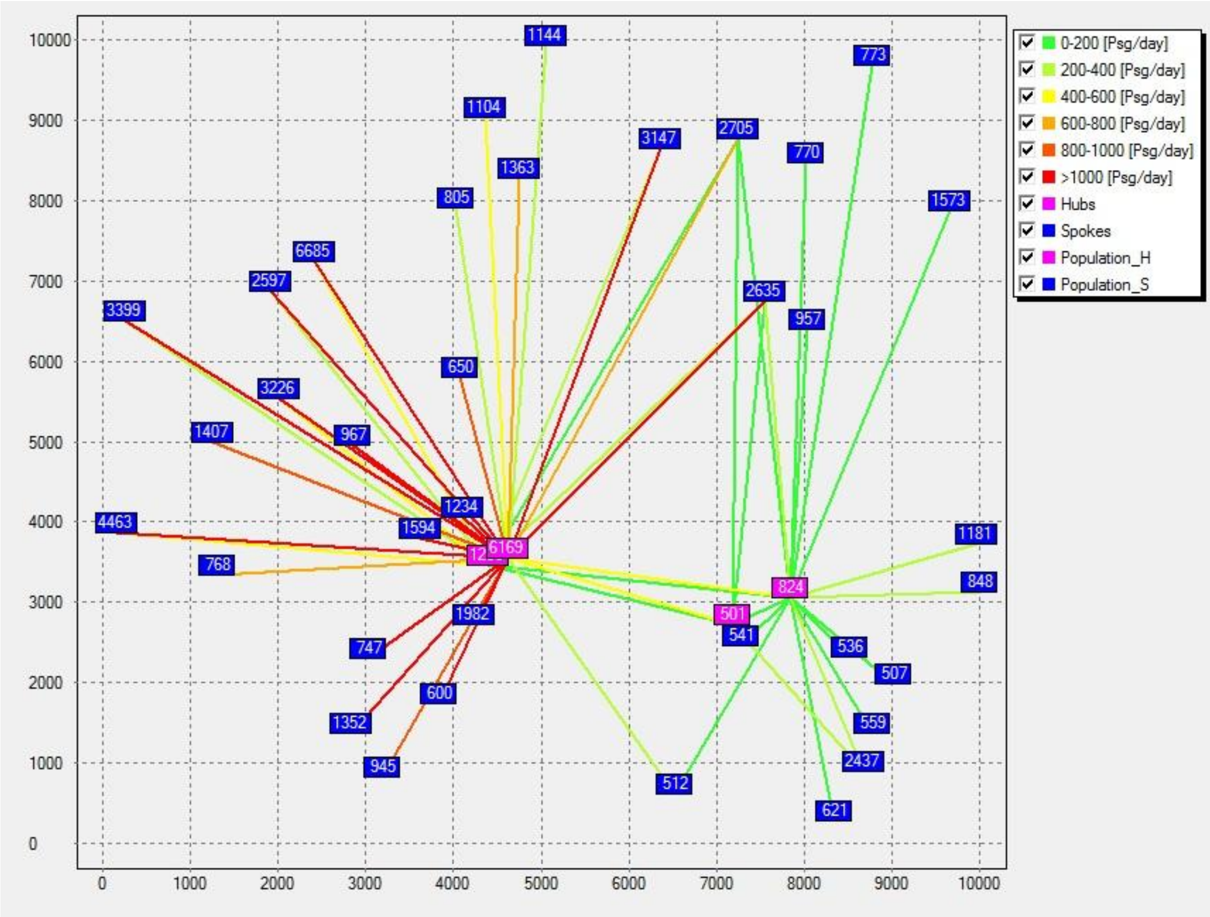

Figure 4.5 – Average Passenger Flow

Month	Hub 16 (Pop. 1286)	Hub 23 (Pop. 6169)	Hub 25 (Pop. 824)	Hub 30 (Pop. 501)
1	19	104	29	10
2	26	102	30	11
3	21	106	34	9
4	25	103	39	11
5	21	105	28	8
6	25	108	32	12
7	21	108	30	9
8	21	98	34	9
9	17	112	30	11
10	18	108	28	10
11	23	107	36	12
12	20	113	30	10

Aircraft Types	#
A318	9
A319	8
A320	7
A321	4
A330-200	0
A330-300	0
A340-300	0
A340-500	0
A340-600	15
A350-800	0
A350-900	3
A350-1000	13
CS100	15
CS300	16
ERJ145 LR	6
E-170	2
Total	98

Table 4.5 – Number of daily flights (departures/arrivals) on each hub; Optimal number of aircraft to acquire

The program spent, approximately, 1 hour (3602,5 seconds) running the model and it did not reach the end. After finding 27 solutions, the best one was found after 641,5 seconds, with a gap of 5,99798 % and a total cost of 45340,8 million dollars.

5. EXAMPLE FOR TAP PORTUGAL AIRLINES

5.1. TAP Portugal

TAP Portugal possesses one of the most modern and youthful fleets in Europe with an average age of 8 years old. However, the company is focused on innovation and since the nineties decade, they have been committed to a total renewal of their fleet.

TAP is considered an "All-Airbus" company (Photo 5.1). The main fleet is composed by fifty five Airbus airplanes. Later, in 2007, with the acquisition of Portugália Airlines, TAP's fleet grew to seventy one airplanes.

Photo 5.1 – An Airbus A330-200 from TAP Portugal (Airliners@)

The Portuguese airline is currently renewing the old A340 long range airplanes with the most recent and technological advanced, the A350 model.

	Airplane	Model	#	Passenger Capacity	Fuel Capacity (L)	Range (Km)
Main Fleet	Airbus	A319	19	132	23859	5700
	Airbus	A320	17	162	23859	5500
	Airbus	A321	3	201	23700	4600
	Airbus	A330	12	263	139090	12000
	Airbus	A340	4	274	139605	13300
Portugália Fleet	Fokker	100	6	97	12800	3600
	Embraer	145 Private	8	49	5200	2400
	Beechcraft	1900 D	2	19	2500	1300

Table 5.1 – TAP Current Fleet

5.2. Applying the Proposed Optimization Model

In this example, it will be considered that TAP Portugal does not have a fleet, so, the objective is to determine how many and which aircraft should this airline buy, in order to minimize the costs.

The data relatively to demand, monthly evolution, coordinates and distances between airports, is in the Appendix. Unfortunately, the data available is from 2009 and it is missing some flights, specially, those operated by Portugália Airlines.

It will be assumed that TAP can only buy Airbus aircraft, due to its close partnership between these two companies, and also, two small Embraer aircraft. Embraer is a Brazilian aircraft manufacturer, with some infrastructures in Portugal, which means it is a likely possible business partner for TAP. The data of the available aircraft to buy is provided by the following table:

Aircraft	Key Data				Costs		
	Seats	Range	Cruise Speed	Turn Time	C_{ow}	C_{op}	
A318	107	5950	828	30	67,7	,0470	
A319	124	6850	828	35	80,7	,0437	
A320	150	6150	828	40	88,3	,0401	
A321	185	5950	828	45	103,6	,0385	*
A330-200	253	13430	871	60	208,6	,0398	
A330-300	295	10830	871	70	231,1	,0350	
A340-300	295	13700	871	70	238	,0390	
A340-500	313	17000	881	75	261,8	,0368	*
A340-600	380	14600	881	85	275,4	,0303	*
A350-800	270	15700	903	65	245,5	,0350	**
A350-900	314	15000	903	75	277,7	,0301	**
A350-1000	350	15600	903	80	320,6	,0270	**
Embraer ERJ145 LR	50	3706	851	20	19,5	,0516	*
Embraer E-170	70	3892	851	25	28,5	,0497	*
		[Km]	[Km/h]	[min]	M$	[$/seat.km]	

* Estimated values after comparing aircraft of the same type but different engine or size
** Predicted values after some researching throughout several websites for predictions of the new A350's CASM

Table 5.2 - Key data and costs of each type of aircraft

The optimization model used in this example is the same used earlier on chapter 4.4, but this time, it is not necessary to have the formulation to generate random cities and demand. It is considered that the Lisboa Portela Airport is the main hub, since it has the most demand, then the second most important is the Francisco Sá Carneiro Airport and, finally, the least important, the Madeira Airport.

Just like before, it was used the program Xpress-IVE to run the optimization model, and the results are the following:

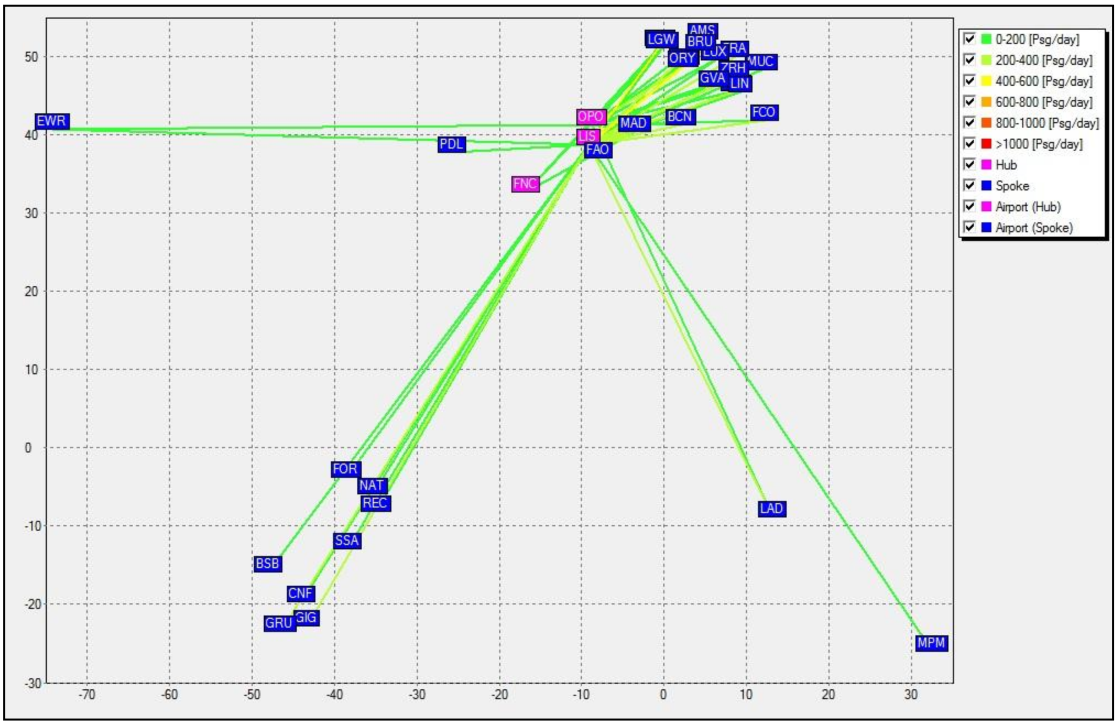

Figure 5.1 – Average flow of passengers on each leg

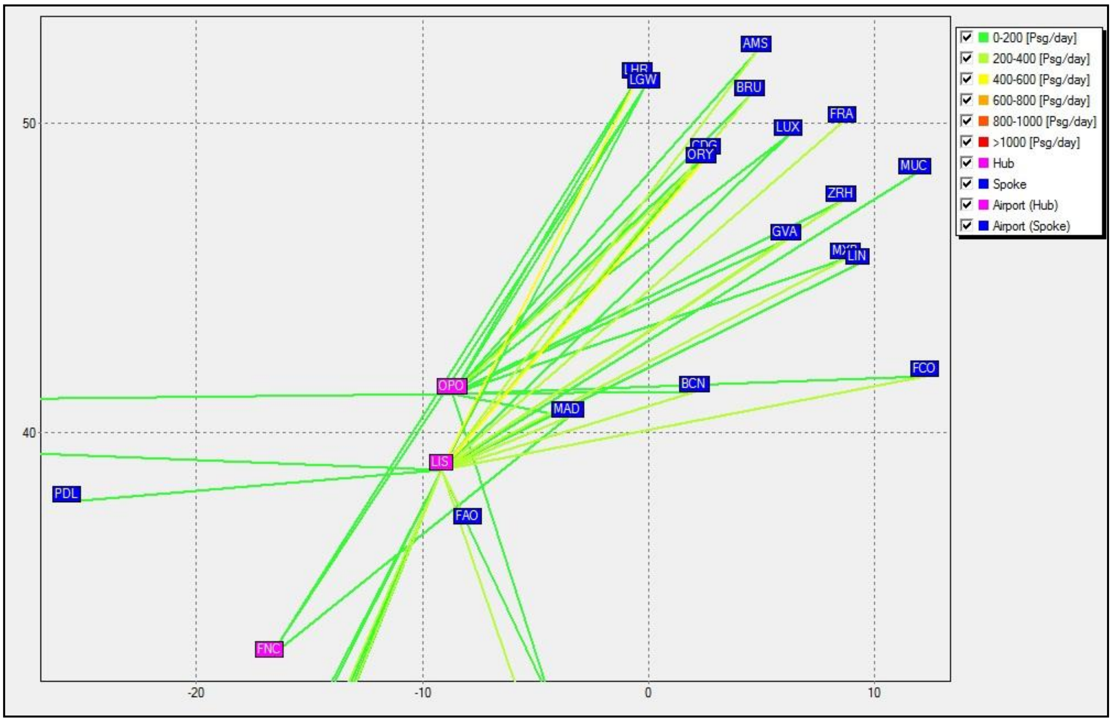

Figure 5.2 – Average flow of passengers on each leg (Europe Close-Up)

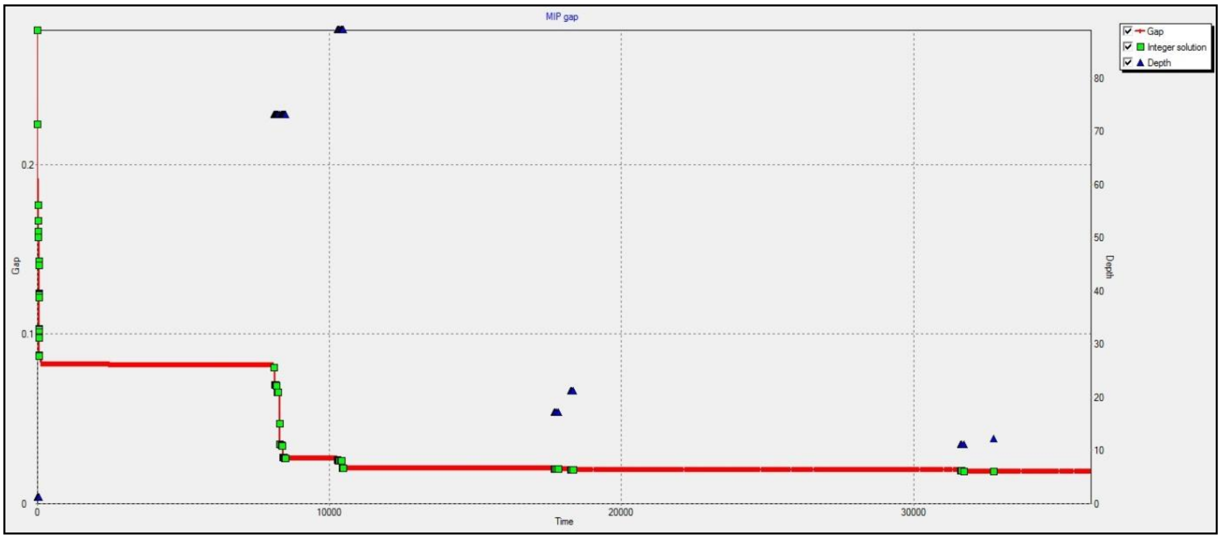

Figure 5.3 – MIP Gap

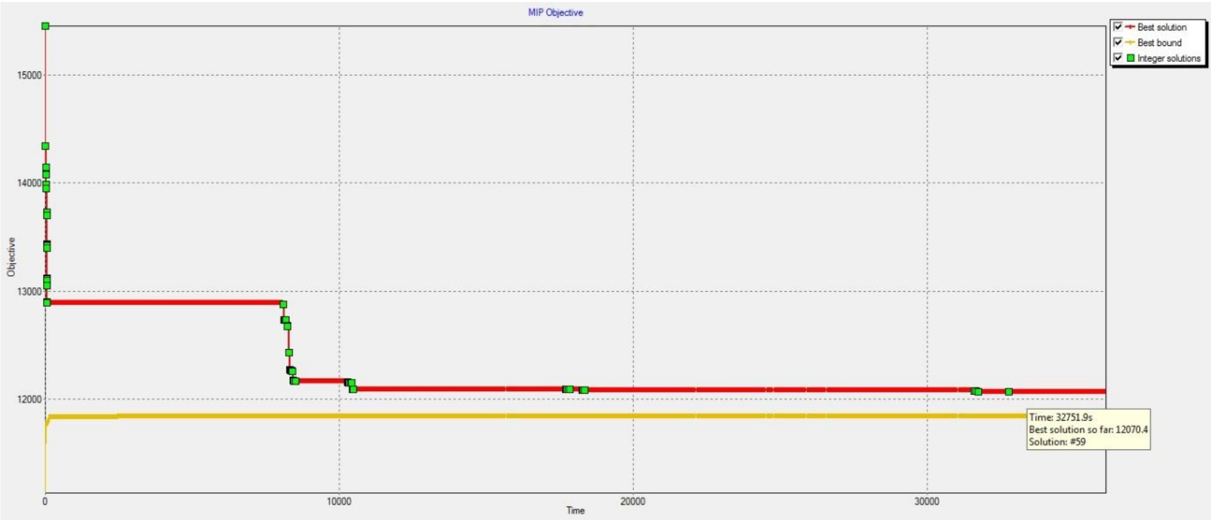

Figure 5.4 – MIP Objective

The program spent, approximately, 10 hours running the model and it did not reach the end. After finding 59 solutions, the best one was found after 32752 seconds, with a gap of 1,86413% and a total cost of 12070,4 million dollars.

Aircraft Types	x_k
A318	12
A319	6
A320	5
A321	6
A330-200	4
A330-300	0
A340-300	0
A340-500	0
A340-600	3
A350-800	0
A350-900	2
A350-1000	0
ERJ145 LR	0
E-170	0
Total	**38**

Table 5.3 – Number of each type of aircraft that TAP should buy, in order to minimize the costs, according to the results of the optimization model

5.3. Conclusion

These results, unfortunately, cannot be compared with TAP Portugal's current fleet since the available demand data of this airline is incomplete.

Instead, perhaps it would be good to compare the percentage of acquired aircraft between the fleet proposed by the optimization model results and the original TAP's fleet (excluding Portugália Airlines') in terms of aircraft category (short-range, medium-range and long-range).

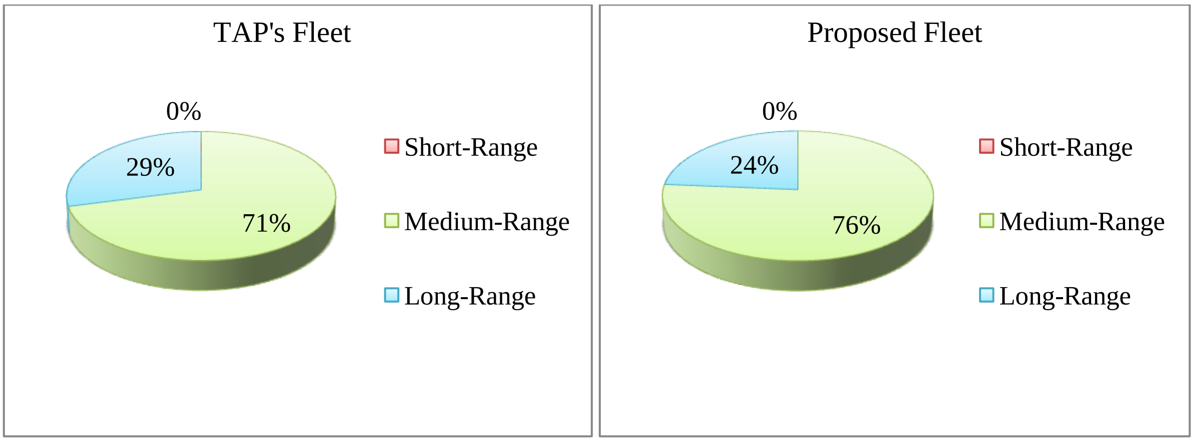

Figure 5.5 – TAP Portugal’s Fleet versus Proposed Fleet

For this airline and taking into account that the destination network is not complete, it can be said that its TAP’s original fleet does not need short-range aircraft. As for the other two aircraft categories, the results from figure 5.5 show that the fleet proposed by the optimization model achieved reasonable results.

Perhaps this model could be more accurate in the future, since it is very basic and simple, and some elements related to aircraft costs could be more detailed and treated separately.

6. CONCLUSION AND FUTURE STUDIES

In this dissertation, it was described the factors that are important when an airline needs to plan its fleet. Not only do the airlines need to consider aircraft performance and costs, but also some other elements like hub location or aircraft characteristics.

The proposed fleet sizing model presented in this dissertation, is very basic and it has room for further improvements. Despite the fact that the model would be more accurate if it used aircraft tracking, it would be good, in future studies, to approach the costs parameters minutely. For instance, instead of having just ownership costs, it could be added leasing costs. As for operating costs, they could be divided in three categories: crew, fuel and operating; which would possibly create a much more detailed and accurate model.

As already mentioned in chapter five, it is a shame that the collected data of TAP’s demand is not fully complete, otherwise, the example could be approached from a different perspective. For instance, instead of determining a complete new fleet for TAP, it could be considered its current fleet or its fleet in 2009, and determined which aircraft this airline should buy. For future work, perhaps it would be good if TAP Portugal could directly cooperate with data because, after all, this airline may receive benefits if these studies are successful.

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APPENDIX

APPENDIX A – DEMAND OF PASSENGERS FOR TAP PORTUGAL (2009)

Airport	**Code**	LIS	OPO	FAO	FNC	PDL
Lisboa Portela Airport	LIS	0	411 346	180 910	551 344	49 972
Francisco de Sá Carneiro Airport	OPO	411 346	0	0	195 220	0
Faro Airport	FAO	180 910	0	0	0	0
Madeira Airport	FNC	551 344	195 220	0	0	0
Ponta Delgada João Paulo II Airport	PDL	49 972	0	0	0	0
Madrid-Barajas Airport	MAD	269 975	53 404	0	3 434	0
Barcelona Int. Airport	BCN	276 755	76 416	0	0	0
Charles de Gaulle Int. Airport	CDG	95 346	35 431	0	0	0
Paris Orly Airport	ORY	360 345	195 421	0	0	0
London Heathrow Airport	LHR	381 078	64 955	0	4 466	0
London Gatwick Airport	LGW	33 753	76 028	0	65 800	0
Munich Int. Airport	MUC	123 158	0	0	0	0
Frankfurt Int. Airport	FRA	188 370	0	0	0	0
Leonardo da Vinci Int. Airport	FCO	267 720	38 874	0	0	0
Milano Malpensa Airport	MXP	149 385	39 577	0	0	0
Milano Linate Airport	LIN	54 453	0	0	0	0
Zürich Int. Airport	ZRH	171 103	65 713	0	0	0
Geneva Cointrin Int. Airport	GVA	163 075	83 345	0	0	0
Amsterdam Schiphol Airport	AMS	185 875	60 093	0	0	0
Luxembourg – Findel Airport	LUX	43 550	32 637	0	0	0
Brussels Airport	BRU	227 855	23 030	0	0	0
Newark Liberty Int. Airport	EWR	111 360	46 743	0	0	0
Quatro de Fevereiro Airport	LAD	179 466	948	0	0	0
Maputo Int. Airport	MPM	57 047	0	0	0	0
Rio de Janeiro-Galeão Int. Airport	GIG	200 242	43 371	0	0	0
São Paulo-Guarulhos Int. Airport	GRU	211 345	39 778	0	0	0
Tancredo Neves Int. Airport	CNF	82 798	0	0	0	0
Brasília Int. Airport	BSB	121 993	0	0	0	0
Pinto Martins Int. Airport	FOR	129 661	0	0	0	0
Recife Airport	REC	112 828	0	0	0	0
Greater Natal Int. Airport	NAT	69 818	0	0	0	0
Luís Eduardo Magalhães Int. Airport	SSA	128 817	0	0	0	0

Table A.1 – Passengers on both flights of each leg. (e.g., the 411 346 passengers on LIS-OPO includes the passengers that fly LIS-OPO and OPO-LIS)

APPENDIX B – MONTHLY EVOLUTION OF PASSENGERS

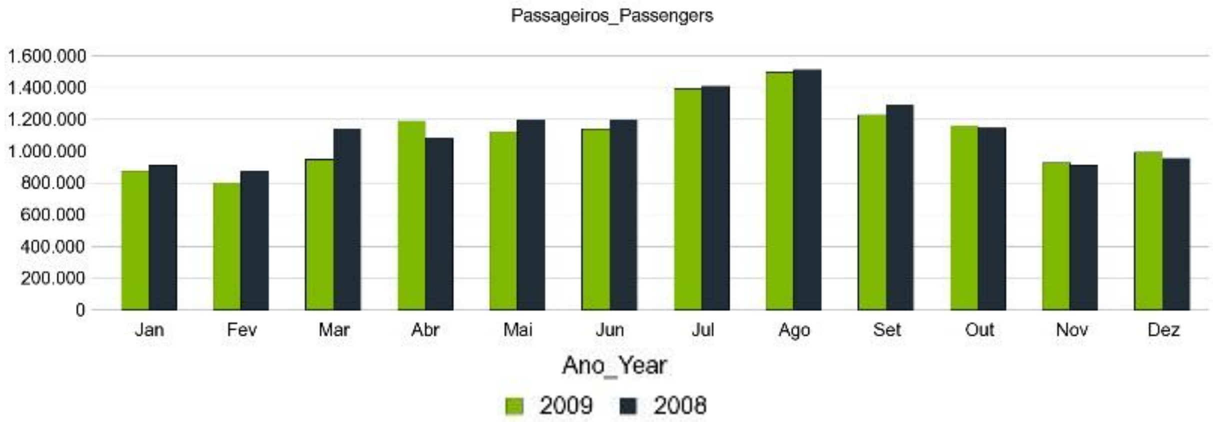

Figure B.1 – Monthly evolution of passengers in Lisboa Portela Airport

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
Pax ($\times 10^3$)	900	800	950	1 200	1 125	1 125	1 400	1 500	1 225	1 175	925	1 000	13 325
%	6,75%	6,00%	7,13%	9,01%	8,44%	8,44%	10,51%	11,26%	9,19%	8,82%	6,94%	7,50%	100%

Table B.1 – Passengers per month and its respective percentage relatively to 2009

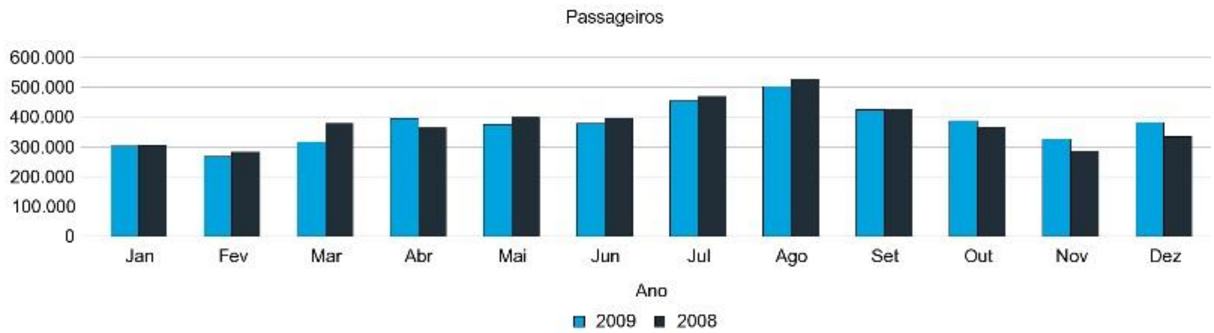

Figure B.2 – Monthly evolution of passengers in Francisco de Sá Carneiro Airport

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
Pax ($\times 10^3$)	300	275	325	400	375	375	450	500	425	400	325	400	4 550
%	6,59%	6,04%	7,14%	8,79%	8,24%	8,24%	9,89%	10,99%	9,34%	8,79%	7,14%	8,79%	100%

Table B.2 – Passengers per month and its respective percentage relatively to 2009

APPENDIX C – AIRPORT COORDINATES

Airport	Code	Geographic Coordinates	Coordinates on Xpress	
			XC	YC
Lisboa Portela Airport	LIS	38,7814° N ; 9,1358° W	-9,1358	38,7814
Francisco de Sá Carneiro Airport	OPO	41,2356° N ; 8,6781° W	-8,6781	41,2356
Faro Airport	FAO	37,0144° N ; 7,9658° W	-7,9658	37,0144
Madeira Airport	FNC	32,6978° N ; 16,7744° W	-16,7744	32,6978
Ponta Delgada Joao Paulo II Airport	PDL	37,7419° N ; 25,6978° W	-25,6978	37,7419
Madrid-Barajas Airport	MAD	40,4722° N ; 3,5608° W	-3,5608	40,4722
Barcelona Int. Airport	BCN	41,2969° N ; 2,0672° E	2,0672	41,2969
Charles de Gaulle Int. Airport	CDG	49,0128° N ; 2,5500° E	2,5500	49,0128
Paris Orly Airport	ORY	48,7233° N ; 2,3628° E	2,3628	48,7233
London Heathrow Airport	LHR	51,4775° N ; 0,4614° W	-0,4614	51,4775
London Gatwick Airport	LGW	51,1481° N ; 0,1903° W	-0,1903	51,1481
Munich Int. Airport	MUC	48,3539° N ; 11,7861° E	11,7861	48,3539
Frankfurt Int. Airport	FRA	50,0333° N ; 8,5706° E	8,5706	50,0333
Leonardo da Vinci Int. Airport	FCO	41,8044° N ; 12,2508° E	12,2508	41,8044
Milano Malpensa Airport	MXP	45,6300° N ; 8,7231° E	8,7231	45,6300
Linate Airport	LIN	45,4494° N ; 9,2783° E	9,2783	45,4494
Zürich Int. Airport	ZRH	47,4647° N ; 8,5492° E	8,5492	47,4647
Geneva Cointrin Int. Airport	GVA	46,2369° N ; 6,1089° E	6,1089	46,2369
Amsterdam Schiphol Airport	AMS	52,3086° N ; 4,7639° E	4,7639	52,3086
Luxembourg – Findel Airport	LUX	49,6233° N ; 6,2044° E	6,2044	49,6233
Brussels Airport	BRU	50,9014° N ; 4,4844° E	4,4844	50,9014
Newark Liberty Int. Airport	EWR	40,6925° N ; 74,1686° W	-74,1686	40,6925
Quatro de Fevereiro Airport	LAD	8,8583° S ; 13,2311° E	13,2311	-8,8583
Maputo Int. Airport	MPM	25,9208° S ; 32,5725° E	32,5725	-25,9208
Rio de Janeiro-Galeão Int. Airport	GIG	22,8100° S ; 43,2506° W	-43,2506	-22,8100
São Paulo-Guarulhos Int. Airport	GRU	23,4356° S ; 46,4731° W	-46,4731	-23,4356
Tancredo Neves Int. Airport	CNF	19,6239° S ; 43,9714° W	-43,9714	-19,6239
Brasília Int. Airport	BSB	15,8692° S ; 47,9208° W	-47,9208	-15,8692
Pinto Martins Int. Airport	FOR	3,7764° S ; 38,5325° W	-38,5325	-3,7764
Recife Airport	REC	8,1264° S ; 34,9228° W	-34,9228	-8,1264
Greater Natal Int. Airport	NAT	5,9114° S ; 35,2477° W	-35,2477	-5,9114
Luís Eduardo Magalhães Int. Airport	SSA	12,9086° S ; 38,3225° W	-38,3225	-12,9086

Table C.1

APPENDIX D – AIRPORT DISTANCE MATRIX

	LIS	OPO	FAO	FNC	PDL	MAD	BCN	CDG	ORY	LHR	LGW	MUC	FRA	FCO	MXP	LIN
LIS	0	276	222	965	1449	513	993	1470	1437	1565	1542	1984	1874	1840	1651	1685
OPO	276	0	473	1190	1509	439	898	1232	1201	1300	1279	1791	1653	1739	1484	1522
FAO	222	473	0	936	1567	542	987	1581	1547	1714	1688	2038	1959	1812	1686	1715
FNC	965	1190	936	0	985	1460	1921	2421	2390	2472	2456	2948	2836	2745	2609	2640
PDL	1449	1509	1567	985	0	1929	2404	2583	2560	2493	2493	3228	3032	3248	2964	3006
MAD	513	439	542	1460	1929	0	482	1064	1030	1247	1215	1497	1422	1330	1149	1180
BCN	993	898	987	1921	2404	482	0	859	826	1148	1109	1095	1094	849	721	743
CDG	1470	1232	1581	2421	2583	1064	859	0	35	348	308	682	449	1101	598	644
ORY	1437	1201	1547	2390	2560	1030	826	35	0	366	326	695	472	1090	591	637
LHR	1565	1300	1714	2472	2493	1247	1148	348	366	0	41	942	655	1445	937	981
LGW	1542	1279	1688	2456	2493	1215	1109	308	326	41	0	914	630	1406	899	943
MUC	1984	1791	2038	2948	3228	1497	1095	682	695	942	914	0	299	729	382	375
FRA	1874	1653	1959	2836	3032	1422	1094	449	472	655	630	299	0	958	490	512
FCO	1840	1739	1812	2745	3248	1330	849	1101	1090	1445	1406	729	958	0	511	471
MXP	1651	1484	1686	2609	2964	1149	721	598	591	937	899	382	490	511	0	48
LIN	1685	1522	1715	2640	3006	1180	743	644	637	981	943	375	512	471	48	0
ZRH	1723	1530	1782	2688	2975	1240	857	476	480	788	754	261	286	694	204	231
GVA	1496	1309	1552	2460	2771	1010	638	408	394	755	715	488	460	695	213	261
AMS	1846	1596	1970	2784	2857	1461	1241	398	433	370	365	664	367	1297	797	831
LUX	1711	1485	1805	2671	2856	1272	980	273	297	514	484	431	176	987	482	518
BRU	1717	1474	1832	2664	2783	1316	1084	251	286	351	328	597	305	1173	665	702
EWR	5433	5362	5609	5103	4136	5790	6176	5857	5856	5561	5591	6503	6211	6891	6436	6484
LAD	5781	6004	5559	5609	6571	5750	5693	6519	6491	6837	6796	6363	6564	5634	6076	6052
MPM	8400	8592	8182	8352	9328	8273	8108	8849	8827	9192	9151	8515	8772	7815	8314	8279
GIG	7715	7964	7610	6782	6979	8147	8522	9184	9151	9254	9237	9617	9568	9172	9241	9260
GRU	7935	8179	7839	6992	7141	8378	8764	9405	9373	9461	9446	9857	9798	9434	9485	9505
CNF	7439	7683	7342	6497	6659	7881	8268	8909	8876	8966	8951	9361	9302	8942	8989	9009
BSB	7294	7524	7217	6335	6398	7759	8172	8756	8725	8782	8771	9253	9167	8891	8891	8915
FOR	5612	5849	5530	4659	4803	6071	6481	7079	7047	7121	7107	7564	7484	7201	7200	7224
REC	5857	6108	5751	4930	5190	6287	6666	7326	7293	7401	7384	7760	7709	7334	7387	7406
NAT	5651	5899	5551	4716	4954	6089	6476	7121	7088	7188	7172	7569	7510	7160	7197	7217
SSA	6498	6746	6395	5564	5784	6932	7314	7967	7934	8036	8020	8408	8354	7982	8034	8054

Table D.1 – Distance Matrix [Kilometers]

	ZRH	GVA	AMS	LUX	BRU	EWR	LAD	MPM	GIG	GRU	CNF	BSB	FOR	REC	NAT	SSA
LIS	1723	1496	1846	1711	1717	5433	5781	8400	7715	7935	7439	7294	5612	5857	5651	6498
OPO	1530	1309	1596	1485	1474	5362	6004	8592	7964	8179	7683	7524	5849	6108	5899	6746
FAO	1782	1552	1970	1805	1832	5609	5559	8182	7610	7839	7342	7217	5530	5751	5551	6395
FNC	2688	2460	2784	2671	2664	5103	5609	8352	6782	6992	6497	6335	4659	4930	4716	5564
PDL	2975	2771	2857	2856	2783	4136	6571	9328	6979	7141	6659	6398	4803	5190	4954	5784
MAD	1240	1010	1461	1272	1316	5790	5750	8273	8147	8378	7881	7759	6071	6287	6089	6932
BCN	857	638	1241	980	1084	6176	5693	8108	8522	8764	8268	8172	6481	6666	6476	7314
CDG	476	408	398	273	251	5857	6519	8849	9184	9405	8909	8756	7079	7326	7121	7967
ORY	480	394	433	297	286	5856	6491	8827	9151	9373	8876	8725	7047	7293	7088	7934
LHR	788	755	370	514	351	5561	6837	9192	9254	9461	8966	8782	7121	7401	7188	8036
LGW	754	715	365	484	328	5591	6796	9151	9237	9446	8951	8771	7107	7384	7172	8020
MUC	261	488	664	431	597	6503	6363	8515	9617	9857	9361	9253	7564	7760	7569	8408
FRA	286	460	367	176	305	6211	6564	8772	9568	9798	9302	9167	7484	7709	7510	8354
FCO	694	695	1297	987	1173	6891	5634	7815	9172	9434	8942	8891	7201	7334	7160	7982
MXP	204	213	797	482	665	6436	6076	8314	9241	9485	8989	8891	7200	7387	7197	8034
LIN	231	261	831	518	702	6484	6052	8279	9260	9505	9009	8915	7224	7406	7217	8054
ZRH	0	230	603	296	483	6332	6280	8508	9371	9609	9113	8999	7310	7514	7320	8160
GVA	230	0	682	377	532	6225	6167	8455	9143	9380	8883	8768	7080	7285	7090	7932
AMS	603	682	0	315	158	5868	6849	9110	9560	9775	9279	9112	7442	7703	7495	8342
LUX	296	377	315	0	187	6075	6539	8796	9415	9642	9145	9004	7323	7556	7354	8200
BRU	483	532	158	187	0	5907	6698	8976	9433	9652	9156	8997	7323	7574	7368	8215
EWR	6332	6225	5868	6075	5907	0	10431	13208	7751	7684	7398	6848	6128	6751	6526	7009
LAD	6280	6167	6849	6539	6698	10431	0	2787	6200	6534	6253	6667	5744	5293	5352	5639
MPM	8508	8455	9110	8796	8976	13208	2787	0	7576	7860	7771	8316	7936	7368	7505	7493
GIG	9371	9143	9560	9415	9433	7751	6200	7576	0	337	362	914	2177	1859	2066	1218
GRU	9609	9380	9775	9642	9652	7684	6534	7860	337	0	497	855	2347	2101	2289	1452
CNF	9113	8883	9279	9145	9156	7398	6253	7771	362	497	0	591	1858	1608	1793	959
BSB	8999	8768	9112	9004	8997	6848	6667	8316	914	855	591	0	1692	1654	1771	1085
FOR	7310	7080	7442	7323	7323	6128	5744	7936	2177	2347	1858	1692	0	627	435	1016
REC	7514	7285	7703	7556	7574	6751	5293	7368	1859	2101	1608	1654	627	0	249	649
NAT	7320	7090	7495	7354	7368	6526	5352	7505	2066	2289	1793	1771	435	249	0	848
SSA	8160	7932	8342	8200	8215	7009	5639	7493	1218	1452	959	1085	1016	649	848	0

Table D.2 – Distance Matrix [Kilometers]

APPENDIX E – DAILY PASSENGERS IN EACH MONTH

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
OPO	448	441	473	617	560	579	697	747	630	585	476	498	**411 346**
FAO	197	194	208	272	246	255	307	328	277	257	209	219	**180 910**
FNC	601	591	634	828	751	776	934	1001	845	784	638	667	**551 344**
PDL	54	54	57	75	68	70	85	91	77	71	58	60	**49 972**
MAD	294	289	310	405	368	380	458	490	414	384	312	327	**269 975**
BCN	301	297	318	415	377	389	469	502	424	394	320	335	**276 755**
CDG	104	102	110	143	130	134	162	173	146	136	110	115	**95 346**
ORY	393	386	414	541	491	507	611	654	552	513	417	436	**360 345**
LHR	415	409	438	572	519	536	646	692	584	542	441	461	**381 078**
LGW	37	36	39	51	46	47	57	61	52	48	39	41	**33 753**
MUC	134	132	142	185	168	173	209	224	189	175	142	149	**123 158**
FRA	205	202	217	283	257	265	319	342	289	268	218	228	**188 370**
FCO	292	287	308	402	365	377	454	486	410	381	310	324	**267 720**
MXP	163	160	172	224	203	210	253	271	229	212	173	181	**149 385**
LIN	59	58	63	82	74	77	92	99	83	77	63	66	**54 453**
ZRH	186	183	197	257	233	241	290	311	262	243	198	207	**171 103**
GVA	178	175	188	245	222	229	276	296	250	232	189	197	**163 075**
AMS	202	199	214	279	253	262	315	337	285	264	215	225	**185 875**
LUX	47	47	50	65	59	61	74	79	67	62	50	53	**43 550**
BRU	248	244	262	342	310	321	386	414	349	324	264	276	**227 855**
EWR	121	119	128	167	152	157	189	202	171	158	129	135	**111 360**
LAD	196	192	206	269	244	253	304	326	275	255	208	217	**179 466**
MPM	62	61	66	86	78	80	97	104	87	81	66	69	**57 047**
GIG	218	215	230	301	273	282	339	364	307	285	232	242	**200 242**
GRU	230	227	243	317	288	297	358	384	324	301	245	256	**211 345**
CNF	90	89	95	124	113	117	140	150	127	118	96	100	**82 798**
BSB	133	131	140	183	166	172	207	221	187	174	141	148	**121 993**
FOR	141	139	149	195	177	182	220	235	199	184	150	157	**129 661**
REC	123	121	130	169	154	159	191	205	173	160	131	137	**112 828**
NAT	76	75	80	105	95	98	118	127	107	99	81	85	**69 818**
SSA	140	138	148	193	175	181	218	234	197	183	149	156	**128 817**

Table E.1 – Daily passengers on each month for the flights operated by TAP Portugal that depart or arrive at Lisboa Portela Airport

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
OPO	0	0	0	0	0	0	0	0	0	0	0	0	0
FAO	0	0	0	0	0	0	0	0	0	0	0	0	0
FNC	208	211	225	286	260	268	311	346	304	277	232	277	195 220
PDL	0	0	0	0	0	0	0	0	0	0	0	0	0
MAD	57	58	62	78	71	73	85	95	83	76	64	76	53 404
BCN	81	82	88	112	102	105	122	135	119	108	91	108	76 416
CDG	38	38	41	52	47	49	57	63	55	50	42	50	35 431
ORY	208	211	225	286	260	268	312	346	304	277	233	277	195 421
LHR	69	70	75	95	86	89	104	115	101	92	77	92	64 955
LGW	81	82	88	111	101	104	121	135	118	108	91	108	76 028
MUC	0	0	0	0	0	0	0	0	0	0	0	0	0
FRA	0	0	0	0	0	0	0	0	0	0	0	0	0
FCO	41	42	45	57	52	53	62	69	61	55	46	55	38 874
MXP	42	43	46	58	53	54	63	70	62	56	47	56	39 577
LIN	0	0	0	0	0	0	0	0	0	0	0	0	0
ZRH	70	71	76	96	87	90	105	116	102	93	78	93	65 713
GVA	89	90	96	122	111	114	133	148	130	118	99	118	83 345
AMS	64	65	69	88	80	83	96	107	94	85	72	85	60 093
LUX	35	35	38	48	43	45	52	58	51	46	39	46	32 637
BRU	24	25	27	34	31	32	37	41	36	33	27	33	23 030
EWR	50	50	54	68	62	64	75	83	73	66	56	66	46 743
LAD	1	1	1	1	1	1	2	2	1	1	1	1	948
MPM	0	0	0	0	0	0	0	0	0	0	0	0	0
GIG	46	47	50	64	58	60	69	77	68	61	52	61	43 371
GRU	42	43	46	58	53	55	63	71	62	56	47	56	39 778
CNF	0	0	0	0	0	0	0	0	0	0	0	0	0
BSB	0	0	0	0	0	0	0	0	0	0	0	0	0
FOR	0	0	0	0	0	0	0	0	0	0	0	0	0
REC	0	0	0	0	0	0	0	0	0	0	0	0	0
NAT	0	0	0	0	0	0	0	0	0	0	0	0	0
SSA	0	0	0	0	0	0	0	0	0	0	0	0	0

Table E.2 – Daily passengers on each month for the flights operated by TAP Portugal that depart or arrive at Francisco de Sá Carneiro Airport

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
OPO	0	0	0	0	0	0	0	0	0	0	0	0	0
FAO	0	0	0	0	0	0	0	0	0	0	0	0	0
FNC	0	0	0	0	0	0	0	0	0	0	0	0	0
PDL	0	0	0	0	0	0	0	0	0	0	0	0	0
MAD	4	4	4	5	5	5	6	6	5	5	4	4	3 434
BCN	0	0	0	0	0	0	0	0	0	0	0	0	0
CDG	0	0	0	0	0	0	0	0	0	0	0	0	0
ORY	0	0	0	0	0	0	0	0	0	0	0	0	0
LHR	5	5	5	7	6	6	8	8	7	6	5	5	4 466
LGW	72	71	76	99	90	93	112	119	101	94	76	80	65 800
MUC	0	0	0	0	0	0	0	0	0	0	0	0	0
FRA	0	0	0	0	0	0	0	0	0	0	0	0	0
FCO	0	0	0	0	0	0	0	0	0	0	0	0	0
MXP	0	0	0	0	0	0	0	0	0	0	0	0	0
LIN	0	0	0	0	0	0	0	0	0	0	0	0	0
ZRH	0	0	0	0	0	0	0	0	0	0	0	0	0
GVA	0	0	0	0	0	0	0	0	0	0	0	0	0
AMS	0	0	0	0	0	0	0	0	0	0	0	0	0
LUX	0	0	0	0	0	0	0	0	0	0	0	0	0
BRU	0	0	0	0	0	0	0	0	0	0	0	0	0
EWR	0	0	0	0	0	0	0	0	0	0	0	0	0
LAD	0	0	0	0	0	0	0	0	0	0	0	0	0
MPM	0	0	0	0	0	0	0	0	0	0	0	0	0
GIG	0	0	0	0	0	0	0	0	0	0	0	0	0
GRU	0	0	0	0	0	0	0	0	0	0	0	0	0
CNF	0	0	0	0	0	0	0	0	0	0	0	0	0
BSB	0	0	0	0	0	0	0	0	0	0	0	0	0
FOR	0	0	0	0	0	0	0	0	0	0	0	0	0
REC	0	0	0	0	0	0	0	0	0	0	0	0	0
NAT	0	0	0	0	0	0	0	0	0	0	0	0	0
SSA	0	0	0	0	0	0	0	0	0	0	0	0	0

Table E.2 – Daily passengers on each month for the flights operated by TAP Portugal that depart or arrive at Madeira Airport

APPENDIX F – PROPOSED FLEET SIZING MODEL – XPRESS FORMULATION

model Fleet_Sizing_Model

uses "mmxprs","mmive"

declarations

N = 20	!20 Locations
K = 16	!A318 A319 A320 A321 A330-200 A330-300 A340-300 A340-500 A340-600 A350-800 A350-900 A350-1000 CS100 CS300 ERJ145-LR E-170
NN = 1..N	
KK = 1..K	
TT = 1..12	!12 Months

X: array(KK) of mpvar !number of owned aircraft

Z: array(NN,NN,KK,TT) of mpvar !number of daily flights per aircraft of type k in month t

Cow: array(KK)		of real	!Ownership cost of an aircraft of type k [m$]
Cop: array(KK)		of real	!Operating cost of an aircraft of type k [$/(seat*km)]
D:	array(NN,NN)	of real	!Distances between airports [km]
S:	array(KK)	of real	!Number of seats of an aircraft of type k
R:	array(KK)	of real	!Range of an aircraft of type k [km]

```
V:      array(KK)          of real     !Maximum speed of an aircraft of type k [Km/h]

T:      array(KK)          of real     !Turn time of an aircraft of type k [min]

Q:      array(NN,NN,TT)    of real     !Daily passengers in month t between airports i
                                        and j

phi: array(NN,NN)    of real

MinD: real

MinP: real

Dist: real

XC: array(NN) of real                  !Coordinates of each location

YC: array(NN) of real

end-declarations


setrandseed(28)

forall(i in NN) do
        XC(i):= round(10000*random)
        YC(i):= round(10000*random)
        Value:=random
        if Value <= 0.25 then
                Type(i):=1
                elif Value <= 0.75 then
                        Type(i):=2
                else
                Type(i):=3
        end-if
        P(i) := round(19500*random*random*random*random)+ 500
end-do
```

```
forall(i in NN, j in NN | i<=j) do
        D(i,j):= round(((XC(i)-XC(j))^2+(YC(i)-YC(j))^2)^0.5)
        D(j,i):=D(i,j)
end-do

MinD:=1000000

forall(i in NN, j in NN | i<j) do
        if D(i,j) < MinD then
                MinD:=D(i,j)
                Fix1:=i
                Fix2:=j
        end-if
end-do

Dist:=10000000

forall(j in NN)
        if sum(i in NN) D(i,j) < Dist and D(Fix1,j)>3000 and D(Fix2,j)>3000 then
                Dist:= sum(i in NN) D(i,j)
                Fix3:=j
        end-if

forall(i in NN)
H(Fix1):=1
H(Fix2):=1
H(Fix3):=1

HubC(Fix1):=1
HubC(Fix2):=1
HubF(Fix3):=1

Dmin:= 200
Dmax:= 2000

forall(i in NN, j in NN)
 if D(i,j)<=Dmin then
        phi(i,j):=0
```

```
    elif D(i,j)>=Dmax then
        phi(i,j):=1
    else
        phi(i,j):= (D(i,j)-Dmin)/(Dmax-Dmin)
    end-if

forall(i in NN, t in TT)Q(i,i,t):=0

forall(i in NN, ii in NN, iii in NN) do
    if H(i)=1 and H(ii)=1 and H(iii)=1 and P(i)<P(ii) and P(ii)<P(iii) then
        HubS(i):=1
        HubM(ii):=1
        HubL(iii):=1
    end-if
end-do

forall(i in NN, ii in NN, iii in NN, j in NN)
    if HubS(i)=1 and HubM(ii)=1 and HubL(iii)=1 and P(j)<(P(i)+P(ii)+P(iii))/3 then
        Small(j):=1
    end-if

alpha:= 20

forall(i in NN, ii in NN, iii in NN, j in NN, t in TT | i<>j and ii<>j and iii<>j) do
    if HubC(i)=1 and HubC(ii)=1 and HubF(iii)=1 and P(i)<P(ii) and Small(j)=1 and
    D(ii,iii)+D(iii,j)-D(ii,j)>D(ii,j)/2 then
        Q(ii,j,t):=
        round((0.75+0.5*random)*alpha*phi(ii,j)*P(ii)*P(j)/D(ii,j)^1.5)+round(((0.75
        +0.5*random)*alpha*phi(i,j)*P(i)*P(j)/D(i,j)^1.5))
        Q(j,ii,t):= Q(ii,j,t)
    end-if

    if HubC(i)=1 and HubC(ii)=1 and HubF(iii)=1 and Small(j)=0 and D(ii,iii)+D(iii,j)-
    D(ii,j)>D(ii,j)/2 then
            Q(i,j,t):= round((0.75+0.5*random)*alpha*phi(i,j)*P(i)*P(j)/D(i,j)^1.5)
            Q(j,i,t):= Q(i,j,t)
            Q(ii,j,t):=
```

```
                        round((0.75+0.5*random)*alpha*phi(ii,j)*P(ii)*P(j)/D(ii,j)^1.5)
                        Q(j,ii,t):= Q(ii,j,t)
        end-if

        if HubC(i)=1 and HubC(ii)=1 and HubF(iii)=1 and P(i)<P(ii) and D(iii,ii)+D(ii,j)-
        D(iii,j)>D(iii,j)/2 then
                Q(iii,j,t):= round((0.75+0.5*random)*alpha*phi(ii,j)*P(ii)*P(j)/D(ii,j)^1.5)
                Q(j,ii,t):= Q(ii,j,t)
        end-if
end-do

forall(i in NN, ii in NN, j in NN, t in TT | i<>j and ii<>j) do
        if HubL(i)=1 and D(i,j)>6850 and Q(i,j,t)<100 then
                Q(i,j,t):=0

                Q(j,i,t):=Q(i,j,t)
        end-if

        if HubM(ii)=1 and D(ii,j)>6850 and Q(ii,j,t)<100 then
                Q(ii,j,t):=0
                Q(j,ii,t):=Q(ii,j,t)
        end-if
end-do

forall(i in NN, j in NN, t in TT | i<>j) do
        if HubS(i)=1 and D(i,j)>6850 then
                Q(i,j,t):=0
                Q(j,i,t):=Q(i,j,t)
        end-if
end-do

forall(i in NN, j in NN, t in TT | i<>j) do
        if H(i)=1 and H(j)=1 then
                Q(i,j,t):= round((0.75+0.5*random)*alpha*phi(i,j)*P(i)*P(j)/D(i,j)^1.5)
                Q(j,i,t):= Q(i,j,t)
        end-if
end-do
```

```
Cow::  [67.7 , 80.7 , 88.3 , 103.6 , 208.6 , 231.1 , 238 , 261.8 , 275.4 , 245.5 , 277.7 , 320.6 ,
       58.28 , 66.57 , 19.5 , 28.5]          !m$

Cop::  [0.0470 , 0.0437 , 0.0401 , 0.0385 , 0.0398 , 0.0350 , 0.0390 , 0.0368 , 0.0303 , 0.0350
       , 0.0301 , 0.0270 , 0.0396 , 0.0364 , 0.0516 , 0.0497]          !$/(Seat.Km)

S::    [107 , 124 , 150 , 185 , 253 , 295 , 295 , 313 , 380 , 270 , 314 , 350 , 100 , 120 , 50 ,
       70]          !Seats

R::    [5950 , 6850 , 6150 , 5950 , 13430 , 10830 , 13700 , 17000 , 14600 , 15700 , 15000 ,
       15600 , 4074 , 4074 , 3706 , 3892]          !Km

forall (i in NN, j in NN, k in KK) do
       if D(i,j)<=R(k) then
              A(i,j,k):=1
       else
              A(i,j,k):=0
       end-if
end-do

V::    [828 , 828 , 828 , 828 , 871 , 871 , 871 , 881 , 881 , 903 , 903 , 903 , 828 , 828 , 851 ,
       851]          !Km/h

T::    [30 , 35 , 40 , 45 , 60 , 70 , 70 , 75 , 85 , 65 , 75 , 80 , 30 , 35 , 20 , 25]          !min

n:=10


!objective-function

Cost:= sum(k in KK) Cow(k)*X(k) +          !Ownership cost

       sum(i in NN, j in NN, k in KK, t in TT) n*30.5*Cop(k)/(10^6)*D(i,j)*S(k)*Z(i,j,k,t)
                                                  !Operating costs


!constraints
```

```
forall(i in NN, j in NN, t in TT) sum(k in KK) S(k)*Z(i,j,k,t) >= Q(i,j,t)            !Seat capacity

forall(k in KK, t in TT) sum(i in NN, j in NN) ((D(i,j)/V(k)+T(k)/60)*Z(i,j,k,t)) <= 16*X(k)
                                                                                   !Time Capacity

forall(i in NN, k in KK, t in TT) sum(j in NN) Z(j,i,k,t) = sum(j in NN) Z(i,j,k,t)    !Continuity

forall(i in NN, j in NN, k in KK, t in TT) Z(i,j,k,t) <= A(i,j,k)*100                  !Range


!variable domain

forall(k in KK) X(k) is_integer

forall(i in NN, j in NN, k in KK, t in TT) Z(i,j,k,t) is_integer


!objective

minimize(Cost)


forall (i in NN, t in TT)        TZ(i,t):=sum(j in NN,k in KK) getsol (Z(i,j,k,t))
        !Number of daily flights that depart/arrive at the airport i on each month

forall (i in NN, j in NN,t in TT)        TLZ(i,j,t):=sum(k in KK) getsol (Z(i,j,k,t))
        !Number of daily flights that depart at the airport i and arrive at the airport j on each
        month

forall (i in NN, j in NN)        QM(i,j):= sum(t in TT) Q(i,j,t)/12
        !Average daily demand per month
```

```
!User graph

PlotFlw6:= IVEaddplot("0-200 [Psg/day]",IVE_RGB(50,255,50))
PlotFlw5:= IVEaddplot("200-400 [Psg/day]",IVE_RGB(180,255,50))
PlotFlw4:= IVEaddplot("400-600 [Psg/day]",IVE_RGB(255,255,0))
PlotFlw3:= IVEaddplot("600-800 [Psg/day]",IVE_RGB(255,170,0))
PlotFlw2:= IVEaddplot("800-1000 [Psg/day]",IVE_RGB(255,85,0))
PlotFlw1:= IVEaddplot(">1000 [Psg/day]",IVE_RGB(255,0,0))

PlotHub:= IVEaddplot("Hubs",IVE_MAGENTA)
PlotSpo:= IVEaddplot("Spokes",IVE_BLUE)
PlotPopHub:= IVEaddplot("Population_H",IVE_MAGENTA)
PlotPopSpo:= IVEaddplot("Population_S",IVE_BLUE)

IVEzoom(-300,-300,10300,10300)

forall(i in NN, j in NN, t in TT)
	if QM(i,j)>=1000 then
		IVEdrawline(PlotFlw1,XC(i),YC(i),XC(j),YC(j))
	end-if

forall(i in NN, j in NN, t in TT)
	if QM(i,j)<1000 and QM(i,j)>=800 then
		IVEdrawline(PlotFlw2,XC(i),YC(i),XC(j),YC(j))
	end-if

forall(i in NN, j in NN, t in TT)
	if QM(i,j)<800 and QM(i,j)>=600 then
		IVEdrawline(PlotFlw3,XC(i),YC(i),XC(j),YC(j))
	end-if

forall(i in NN, j in NN, t in TT)
	if QM(i,j)<600 and QM(i,j)>=400 then
		IVEdrawline(PlotFlw4,XC(i),YC(i),XC(j),YC(j))
	end-if


forall(i in NN, j in NN, t in TT)
```

```
	if QM(i,j)<400 and QM(i,j)>=200 then
		IVEdrawline(PlotFlw5,XC(i),YC(i),XC(j),YC(j))
	end-if

forall(i in NN, j in NN, t in TT)
	if QM(i,j)<200 and QM(i,j)>0.001 then
		IVEdrawline(PlotFlw6,XC(i),YC(i),XC(j),YC(j))
	end-if

forall(i in NN)
	if H(i) = 1 then
		IVEdrawlabel(PlotPopHub,XC(i),YC(i),strfmt(P(i),4))
	else
		IVEdrawlabel(PlotPopSpo,XC(i),YC(i),strfmt(P(i),4))
	end-if

forall(i in NN)
	if H(i) = 1 then
		IVEdrawlabel(PlotHub,XC(i),YC(i),strfmt(i,1))
	else
		IVEdrawlabel(PlotSpo,XC(i),YC(i),strfmt(i,1))
	end-if


end-model
```