## FCTUC DEPARTAMENTO DE ENGENHARIA CIVIL FACULDADE DE CIÊNCIAS E TECNOLOGIA UNIVERSIDADE DE COIMBRA

# Support Staff Pick-Up and Drop-Off in a 

## Carsharing System

Dissertation for the attainment of the Master Degree in Civil Engineering in the Urban Development, Transportation and Mobility Infrastructures Specialization

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#### Abstract

Carsharing systems are becoming a new way of traveling in urban areas. Different systems exist and, allowing one-way trips is one of these systems. One-way carsharing systems allow the user to pick a vehicle from one parking area and leave the vehicle in another area, not being obliged to return the vehicle to the original parking area.

This situation creates an imbalanced situation where some areas have a lot of vehicles and others have no vehicles. To balance the system relocation operations are required and those relocations can be held by support staff members that will drive vehicles back and forth from station to station. A truck is used to take the staff members to the areas where they will pickup the vehicle and in the end of the relocation operations take them back to a support operations depot.

This situation constitutes a vehicle routing problem with simultaneous pickup and delivery. This dissertation will introduce a model of the vehicle routing problem with simultaneous pickup and delivery applied to picking up and dropping off support operations staff members of a carsharing system. It will minimize the travel distance for the vehicles picking up and collecting the support operations staff members, ensuring the vehicle's capacity is not exceeded while performing such operations.

The model was developed and it was possible to determine that for an area of 12.25 square kilometers with 40 parking areas and 15 of those sites having nonnegative demand, a three vehicle with capacity for 8 workers each solution constitutes an effective solution for a random period of the workday, considering an urban average speed of $30 \mathrm{~km} / \mathrm{h}$. Some changes were introduced to study how a system would operate throughout a workday. A sensitivity analysis regarding the effects of different variables in the effectiveness of the carsharing system throughout the day was conducted.


## RESUMO

Os sistemas de carsharing estão-se a tornar cada vez mais uma forma de viajar em áreas urbanas. Existem diferentes tipos de sistema de carsharing, sendo que os sistemas one-way permitem que o utilizador use um veículo de uma área de estacionamento e o entregue noutra, não sendo obrigatório o retorno do veículo à sua área de estacionamento de origem.

Esta situação cria um desequilíbrio em que algumas áreas têm demasiados veículos e outras nenhum. Para reequilibrar o sistema são necessários serviços de relocalização que podem ser desempenhados por membros de equipas de operaçães de apoio, que conduzirão os veículos de uma estação para a outra. Um veículo de distribuição é usado para levar os membros da equipa de apoio até à área onde um veículo do sistema irá ser relocalizado, limpo ou será feita a sua manutenção, e no fim dos serviços de apoio recolherá os membros da equipa e irá leválos de volta para o depósito de apoio. Esta situação constitui um Problema de Roteirização de Veículos com Recolha e Distribuição Simultânea (VRPSPD).
Esta dissertação introduz um modelo de um VRPSPD aplicado a equipas de apoio num sistema de carsharing, minimizando a distância percorrida pelos veículos enquanto recolhem e distribuem membros das equipas de apoio, assegurando-se que a capacidade do veículo não é excedida.

O modelo é desenvolvido, constatando-se que para uma área de $12,25 \mathrm{~km}^{2}$ com 40 áreas de estacionamento e 15 dessas áreas com procura, uma solução de 3 veículos com capacidade para 8 trabalhadores cada constitui uma solução eficaz para um período, considerando uma velocidade média urbana de $30 \mathrm{~km} / \mathrm{h}$. Algumas alterações ao modelo são introduzidas para averiguar o funcionamento de um sistema ao longo de um dia de trabalho. Foi levada a cabo uma análise de sensibilidade relativa aos efeitos de diferentes parâmetros na eficácia do sistema de apoio ao carsharing ao longo dia.

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## LIST OF ACRONYMS

VRP- Vehicle Routing Problem
PDP- Pick-up and Delivery Problem
PDPT- Pick-up and Delivery Problem with Transshipment
CVRP- Capacitated Vehicle Routing Problem
ACVRP- Asymmetric Capacitated Vehicle Routing Problem
SCVRP- Symmetric Capacitated Vehicle Routing Problem
TSP- Traveling Salesman Problem
SEC- Subtour Elimination Constraints
DVRP- Distance-Constrained Vehicle Routing Problem
DCVRP- Distance-Constrained Capacitated Vehicle Routing Problem
VRPTW- Vehicle Routing Problem with Time Windows
VRPB- Vehicle Routing Problem with Backhauls
VRPBTW- Vehicle Routing Problem with Backhauls with Time Windows
VRPSPD- Vehicle Routing Problem with Simultaneous Pick-up and Delivery
VRPSPDTW- Vehicle Routing Problem with Simultaneous Pick-up and Delivery with Time Windows

## 1. INTRODUCTION

### 1.1 Carsharing Systems

Although the increasing use of private transports, in industrialized countries, provided at a first stage an increase in accessibility, later it has become a problem, having great impacts in people's lives with negative externalities, such as a great time loss, pollution and stress. This happens mainly in urban areas where demand is concentrated in peak hours.

The suburban areas have developed and spread across large areas due to the increased land prices, and so the personal vehicle became fundamental to get to the workplace for many people. Moreover, the vehicle ownership costs such as fuel, parking and the cost of purchasing and insuring the vehicle itself have increased. These last costs are sunk costs even if the vehicle is not driven anymore, those costs would not be recovered. In the United States, for example, automobiles spend around $90 \%$ of their time parked (U.S. Department of Transportation, 2001). Public transport could be a good alternative, but it has several disadvantages when compared to the automobile. For instance, it does not allow door-to-door service, due to its coverage, even in cities where public transportation has a good network, given the fact that there are schedules where the user may have to wait for considerable time. Moreover it lacks personalization, being impossible to please every customer. Besides, being able to provide enough public transport services in the peak hours implies those extra vehicles remaining idle for the rest of the day, decreasing the efficiency of the service.

Politicians have the difficult task of assuring the combination of mobility growth and being sustainable economically, ecologically and socially. In the year 2004, the transportation sector was responsible for $23 \%$ of the energy-related greenhouse gas emissions (Kahn Ribeiro et al., 2007), which turns the transportation sector into a milestone in the reduction on cities energy consumption. Adding to the fact that we currently face one of the biggest economic crises since 1929, demanding a remarkable effort to reduce costs wherever possible, there is an increased environmental awareness, spreading mainly in the more developed societies, making more and more people feel almost obliged to reduce their ecological footprint in whatever means they can. One way people can maintain their mobility independence, reduce their environmental impact and save money is by using carsharing systems.

Carsharing systems provide the use of an automobile without owning a private vehicle. Instead of owning one or more vehicles, a household or business accesses a fleet of shared-
use automobiles, benefiting from choosing the one that best fits its needs for a specific purpose (Shaheen et al, 1999). One of the first experiments with carsharing systems occured in Zurich, Switzerland, in 1948 by the Sefage cooperative (Shaheen and Cohen, 2007). In the United States, the first steps of carsharing systems were given in 1983 by the Mobility Enterprise program, evolving primarily through field experiments, which then evolved to permanent carsharing services (Shaheen and Cohen, 2007). Unlike European early users, United States first users were concerned mainly by the practicality of the system and less by environmental or social benefits. The users were driven by convenience other than economic advantages, possibly due to lower costs of driving in the United States (Lane, 2005). Recently, there have been carsharing initiatives in Asia, mainly in Japan, where the main focus has been business use, and in Singapore, with household usage being dominant, most likely because of limited vehicle licensing and high car-ownership costs in Singapore (Barth et al., 2006). Today carsharing systems are present in 18 countries, showing enormous growth potential (Shaheen and Cohen, 2007).

There is a wide variety of carsharing services being provided. There can be small community centered systems with one or two vehicles, as well as systems with a covered area as large as an entire country, with many thousands of users in several major cities. Some function as nonprofit organizations and others as commercial ventures run by international companies (Barth and Shaheen, 2002). Concerning depot location, there are systems with depots placed only at transit stations mainly to serve commuter trips, designated as station-car systems, and systems with depots dispersed around a city independently of transit stations.

Finally, and most important to this dissertation, the carsharing systems can be distinguished between one-way systems and round-trip (or two-way) systems. The two-way systems are the most common, being adopted by the three largest organizations that account for $94 \%$ of the North America carsharing membership (Shaheen et al., 2006). Two-way carsharing systems require the return of rented cars to the stations where they were picked up, simplifying the operator's task because they can plan stocks based on demand for each station. However, this is less suitable for the user's needs. This suitability can be found in one-way carsharing systems. In these systems, users can pick up a vehicle from one station, or area, and leave it in another one. If they eventually need a vehicle later on, they can pick up another one. Therefore, theoretically, these systems allow more trips to be captured than two-way systems, which can only be used for a specific purpose, for shopping, leisure and sporadic trips (Barth and Shaheen, 2002).

However, the possibility of not returning the car to the same parking area from where the trip was originated leads to an operational problem of creating imbalanced vehicle stocks in the parking areas due to the uneven nature of the trip pattern in a city (Correia and Jorge, 2013). To even the vehicle stocks, vehicle relocation is necessary, taking vehicles from one parking
area, or location out of the system grid, to more auspicious parking areas. Vehicle relocations require manpower to drive the vehicle from one place to the other. These staff member must be picked-up and delivered in the parking areas and this problem is the object of this dissertation.

### 1.2. InnoVshare Project

This dissertation is integrated in the InnoVshare project. InnoVshare is an FCT (Fundação para a Ciência e Tecnologia - Portuguese Foundation for Science and Technology) funded project, carried out by two associated universities, University of Coimbra and University of Lisbon, particularly the researchers at CIEC and CITTA in Coimbra, and at CESUR and IDMEC in Lisbon.

InnoVshare project's objective is to bring innovation in the methodology for the assessment of the potential of carsharing systems through the use of agent based simulation. A simulation model revealing what are the best carsharing system configurations that could lead to an aggregate net benefit for society, creating a tool usable by government authorities to rigorously estimating carsharing impacts and at the same time help private companies to manage their systems better.

The model will be based on an agent based rationale that provides a detailed characterization of the daily demands for carsharing resulting from competing with other existing transport operators. It aims to access the economic performance of the system from the different stakeholders' perspective: users, carsharing operator, existing transport operators and local government. These performances are a function of several planning and operational decisions which will be included in the model: one-way vs. round-trip; vehicle relocation policies to support one-way carsharing; business models do support the development of the system; station or free parking areas location decision; pricing policies; vehicle technology and electric mobility infrastructure. It will model and contrast two cities: Lisbon and Coimbra, which are different in terms of population, mobility patterns and city mobility management strategies.

### 1.3. Dissertation Objectives

This dissertation will focus on the problem concerning the routing of the support staff members responsible for vehicle relocation needed in one-way systems, whose tasks may also include repairing and cleaning of vehicles that do not fulfill customer's needs. Depending on the operation area, the vehicle stations' location and number of available vehicles for staff distribution the same vehicles will have to go longer or shorter distances. In this dissertation a mathematical programming model is developed to ensure the minimization of the travelled distance, taking into account factors from previous instances, or not, using Mosel language and the FICO Xpress-MP software. It will also be investigated if every request for service is
satisfied or not and a study concerning the effectiveness of systems under different conditions will be held.

### 1.4. Dissertation Organization

This dissertation is organized into 9 chapters, concerning the different steps in the development of the model and obtaining and showing results. This chapter, Introduction, respects to a literature review concerning carsharing systems and their evolution; the connection between this dissertation and its objectives to the InnoVshare Project; dissertation objectives and organization.

The second chapter will attempt to clarify the reader regarding problems inherent to carsharing systems, such as vehicle relocation in one-way carsharing systems, and some of the solutions presented in the literature.

The third chapter is a literature review of the different models concerning vehicle routing, their evolution and interrelation. It aims to show the different type of problems present in the literature so the most similar to the problem described previously can be used as a reference for the development of our model.

In the fourth chapter the mathematical formulation of the model developed will be presented, and clarifications regarding some constraints will be provided.

For the fifth chapter an application methodology will be presented. In this chapter some situations will be revealed on how the model can be used. An overview of the relocation problem will be presented, followed by an explanation on the methodology used in every period of the workday and an explanation on how the model can be used to assess the effectiveness of the system throughout the workday.

Chapter six presents an example of a situation where the model can be used regarding a fictional city and fictional demand, showing how the system works. A sensitivity analysis will be held to ensure the operation of the model in different scenarios, using different variables and studying their effect on the results.

In chapter seven conclusions regarding this dissertation will take place. An overview of the work developed, and reference to possible future work is presented.

## 2. PROBLEM DESCRIPTION

To implement a carsharing system, the company must first figure out the best solution for several optimization problems, for instance to determine the optimal fleet size (Barth and Todd, 1999) or the location of parking areas (Correia and Antunes, 2011). Barth and Todd (1999) concluded that a sufficient fleet size for satisfying customers is 3-6 vehicles for every 100 trips but that $18-24$ vehicles per 100 trips are required to minimize relocation costs, confirmed for the case study of Lisbon by Correia and Antunes (2011) with 22.7 vehicles per 100 trips. In two-way systems, the most common, users return the vehicle to its original parking area. However, in one-way carsharing, users are allowed to leave the vehicle in a different station. This creates an imbalance between the demand and availability of vehicles. For example, in the morning peak hour a carsharing parking area near a train station will have several users requesting available cars but will not be a probable destination for other users to leave their vehicle, reducing the number of available vehicles. In the evening, the opposite will probably occur. There may be a lot of vehicles being dropped near the station and not enough parking spaces for each one. A carsharing system provider can make decisions concerning the fleet size and relocation policies. The fleet can be reduced by increasing the number of vehicle relocations, or even increasing the required reservation time, if it exists, to allow more time to relocate the vehicles (Nourinejad and Roorda, 2014). When this happens, the service provider must develop strategies to relocate some vehicles and optimize the distribution of available vehicles, depending on the available data and main relocation goal. Barth and Todd (1999) propose the following classification:

- Static Relocation- based on the immediate needs of a parking area, maximum and minimum limits for the number of vehicles present at each area are imposed, in order to activate the mechanism for relocation. When the maximum is reach the system will prompt the operator to move a vehicle to another area. If the minimum is reached, the operator will be prompted to bring in another vehicle;
- Historical Predictive Relocation- on an estimation of the requests made using historical data of the service or techniques of travel demand estimation. Estimates what the deficit or excess of vehicles will be at each area, activating in advance the relocation mechanism.
- Exact Predictive Relocation- this occurs if there is a perfect knowledge of the requests. This is the case of a carsharing service on reservation, when relocation can be organized in an optimal way, minimizing customer's waiting time.

Not meeting user demands means a loss in potential revenue for the operator and increased frustration for the customers, making carsharing a less attractive mode of transportation. Not having parking stalls for users to return vehicles to is effectively forcing them to rent the vehicles longer than necessary, thus increasing their usage costs (Kek et al., 2009).

The need for vehicle relocations can be reduced when the user is incentivized to choose another location or reservation time, driving the vehicle to a location with shortage of vehicles; when that situation does not happen, the vehicles are physically transported using trucks or personnel. There are different approaches to these problems in the literature. Chauvet et al. (1999) propose algorithms to optimize the use of a fleet of trucks to move the cars between the stations. Duron et al. (2000) present an heuristic approach based on the immediate needs of the stations, i.e., the next station to be visited by the vehicles transport truck is chosen according to the current state of the system, creating an algorithm which will give priority to visit the stations that are most likely to run out of vehicles in the fleet of truck's route.

Relocations, however, are not the only reason demand may not be satisfied. There is also the need for maintenance, refueling and cleaning operations, making sure the vehicle is in proper usage conditions, keeping the user service opinion positive. The staff may be based near the largest parking areas in the carsharing system, providing for relocation and other services inloco. This, however, can be expensive due to the infrastructure needed in each of those parking areas. The depot's optimal location will not be considered in this dissertation.

In this particular case, it will be considered that all the operations tasks, such as relocations, maintenance, refueling and cleaning, can be performed by the same staff, with origin in a single support operations depot.

Considering that all the staff members leave from the same place, the operations system will be in a single place, receiving the alert for when vehicle relocation is necessary, when a vehicle needs to be refueled, when a vehicle does not work and needs to be towed and when a vehicle needs to be cleaned amounting to appropriate levels of quality and user appreciation towards the service.

In these conditions, the support staff would have to be transported to the station where they are requested, for any of the situations described previously, and after the maintenance operation is concluded, picked up back to the maintenance depot, either from the same place or, in case of a vehicle relocation, from a station different from where they were left.

Considering a system operating at a city wide range, if a vehicle with support staff was dispatched for every single situation, the fleet required for this service could be enormous hence expensive. Therefore, it is assumed vehicles with the support staff will be dispatched with a certain interval between them, trying to comply with every emergency in that interval. So, for each period, a route will be calculated and delivered to the personnel distribution vehicle which will leave the maintenance depot with the number of workers needed to satisfy the operational requests, picking up workers that finished their previous tasks, relocating them into a different station to perform another service, if necessary, and then, returning to the maintenance depot for the next dispatch. If a situation cannot be resolved in the period between dispatches it will remain unsolved until the next dispatch. Vehicles which cannot return to the maintenance depot in time for the next dispatch will not be available for it, considering an average urban speed.

A model will be created to minimize the distance travelled by distribution vehicles, therefore, minimizing the maintenance costs in one-way carsharing systems, with an imbalanced stock of vehicles.

## 3. EXISTING MODELS

This dissertation focuses on the routing problem concerning support staffs in carsharing systems. Being a problem that concerns the distribution of workers (which can be identified with goods in the literature) between depots and carsharing parking areas (customers), this problem can be generally addressed as a Vehicle Routing Problem. The Vehicle Routing Problem can, according to Kulkarni and Bhave (1985), in its most general sense, be defined as a set of customers each with a known location and a known requirement for some commodity, which should be supplied from a set of depots by a set of delivery vehicles of known capacity. Toth and Vigo (2001) also consider the road network, given the fact that this is where the vehicles perform their movements. The road network can be described through a graph, whose arcs represent the road segments and nodes correspond to the road junctions and to the depot and customer locations. The arcs can be directed or undirected, depending on whether they can be traversed in only one direction (one-way streets, for instance) or in the two directions, respectively. A number of consulting firms and other organizations have compiled road network models. For example, the Swedish Postal Service has developed a network model of all roads in Sweden (Fisher, 1995). In cases where the Euclidean distance is considered the distance is often scaled up by a factor to compensate for roads that deviate from a straight-line path between customers (Fisher, 1995). The objective of the problem in this dissertation will be to minimize the distance travelled by the delivery vehicles.

According to Toth and Vigo (2001) the typical characteristics of the customers are the nodes of the road graph in which the customer is located; amount of goods (demand), possibly of different types, which must be delivered or collected at the customer; periods of the day (time windows; times required to deliver, or collect, the goods at the customer location; subset of the available vehicles that can be used to serve the customer (for instance, because of possible access limitations or loading and unloading requirements).

The vehicles used for the distribution are also defined by their home depot, where they start their route and where they will finish it, or if they can finish in another depot; their capacity, expressed in this dissertation by the amount of workers it can legally transport; their possible subdivision in compartments, each characterized by its capacity and by the goods it can carry, such as number of workers and number of equipment it can carry; the subset of arcs of the road graph which they can transverse and the cost associated with the utilization of the vehicle. The vehicle routing problem has the following constraints:
I. All customers' requirements are met and each costumer appears on exactly one route;
II. The maximum number of customers serviced by a vehicle is $L$;
III. The total requirement of customers appearing in each vehicle route cannot exceed the vehicle's capacity;
IV. The total distance travelled by each vehicle cannot exceed $T$.

Dantzig and Ramser did the first formulation of the general vehicle routing problem, concerning the delivery of gasoline to gas stations by the Atlanta Refining Company in 1959. Kulkarni and Bhave (1985) developed an integer formulation, but they had an error concerning the maximum distance constraint. Brodie and Waters (1988) pointed out the error and Achuthan and Cacetta (1991) present a solution for the previous error by converting the problem of finding a vehicle route (cycle) into a path problem in an equivalent model.

Laporte (1987) states that all known models for the VRP can be classified into one of the following categories: (i) direct tree search method; (ii) dynamic programming (DP) and (iii) integer linear programming (ILP). The latter category is very broad and accounts for most of the research effort. This category is subdivided by Magnanti (1981) into (iiia) set partitioning formulations, (iiib) vehicle flow formulations (by far the most widely used) and (iiic) commodity flow formulations. Our model will have a vehicle flow formulation, using binary variables to indicate whether a vehicle travels between two given sites in the optimal solution. Vehicle flow formulations can also be subdivided into two categories, three-index or twoindex formulation. Two-index formulation, has two indices to each flow variable: the origin and the destination node. In the three-index case, the vehicle making the trip is also identified with an index.

Besides the typical constraints present in VRP's, more constraints can be imposed. Precedence constraints can be imposed on the order in which the customers served in a route are visited. One type of precedence constraint requires a given customer to be served in the same route serving a subset of other customers and that the customer must be visited before (or after) the customers belonging to the subset. This is the case, for instance, of the so-called Pickup and Delivery Problems (PDP) (Toth and Vigo, 2001). PDP can be static or dynamic. In static problems, all information is assumed to be deterministic and known a priori. In dynamic problems, information is gradually revealed over time (Berbeglia et al, 2007) and a solution strategy must be devised adjusting the current solution accordingly to the new information. Besides precedence, PDP also imposes pairing, which states that both pickup and delivery should be performed by the same vehicle. The Pickup and Delivery Problem with Transfer, or Transshipment, (PDPT) is a PDP variant that eliminates the pairing constraint. In PDPT, cargo or passengers can be transferred, adjusted or swapped between vehicles in specified transfer nodes (Cortés et al., 2010 and Rais et al., 2011). In PDP, three
kinds of nodes can be specified: depots are the nodes where vehicles start and finish their route with no passengers, origins are the pickup nodes and destination nodes are the delivery ones. Origin or destination nodes have only one associated operation, loading (origin) or unloading (destination), but not both. In PDPT, transfer nodes allow vehicles to load or unload cargo or passengers (Cortés et al. 2009).

The Capacitated Vehicle Routing Problem (CVRP), the basic version of VRP, imposes the capacity constraints for the vehicles. It can be an asymmetric or symmetric problem (ACVRP or SCVRP) whether the complete graph in which the problem is based is a directed graph or not, respectively. Heuristic approaches have been held in the attempt to achieve either better efficiency or better effectiveness, such as Vigo's (1994) and Doerner's (2004).

A problem similar to the VRP is the Traveling Salesman Problem (TSP). This problem is similar to the VRP problem but with less constraints, such as capacity, time windows and with a single vehicle. This problem can be described as determining the shortest distance or lowest cost for one vehicle to visit a predetermined number of sites once and only once. Its name derives from the most common interpretation: a single salesman seeks the shortest tour through a number of cities to visit his clients. It is similar to the CRVP when the vehicle's capacity greatly exceeds the amount of goods to be collected and, therefore, is not a constraint (Toth and Vigo, 2001). The TSP is one of the most widely studied combinatorial optimization problems. Among all the authors and studies concerning the TSP, the work of Bektas (2006) and Bektas and Kara (2006) must be pointed due to the fact that a particular formulation for Subtour Elimination Constraints (SEC's) developed by these authors was used in the model developed in this dissertation.

A variant of the CVRP is the Distance-Constrained VRP (DVRP), where, for each route, the capacity constraint is replaced by a maximum length (or time) constraint. This can be applied to situations where the vehicles have limited range concerning the total route length or when drivers must not exceed a specific number of consecutive driving hours. The case in which both the vehicle capacity and the maximum distance constraints are present is called Distance-Constrained CVRP (DCVRP).

The VRP with Time Windows (VRPTW) is the case of CVRP in which a time interval for service is associated to each customer, along with the capacity constraint. The service for each customer must start within the specified time windows and is considered that the vehicle is stopped at the customer location for a certain amount of time. This can be applied to an appliances delivery service that also does the proper installation, for instance, in which the client is expecting them at a certain hour and they must also remain in the customer location for the duration of the installation. Potvin (2009) addressed this problem concerning multiple vehicles. The time windows constraint present in this type of problem can be considered in other vehicle routing problems.

The VRP with Backhauls (VRPB), also known as the linehaul-backhaul problem, is an extension of the VRP involving both delivery and pickup points (Goetschalckx and JacobsBlecha, 1989). Linehaul (delivery) points, if they exist, must be served prior to the Backhaul (pickup) points, if any exist, considering it's not feasible to rearrange the loads in the delivery points. For instance, the vehicles housed in a Distribution Center must deliver all goods, then pickup all the cargo and transport it back to the Distribution Center. This problem was addressed by Mingozzi and Giorgi (1999), Ropke and Pisinger (2004), Salhi and Wade (2002), Toth and Vigo (1997) among others.

The VRP with Simultaneous Pickup and Deliveries (VRPSPD) was first addressed by Min (1989) concerning a distribution problem regarding the library-material distributor routing of the public library of Columbus and Franklin County in Ohio. In the VRPSPD each customer requires a delivery of a certain amount of goods, a pickup of a given amount of waste, or both (Dell'Amico et al., 2006). Each client visited by a capacitated vehicle may require pickup and delivery services simultaneously. A separated service for the delivery and pickup may cause extra handling effort. This effort can be reduced considerably by having a single stop on the customer's location (Dethloff, 2001). Dethloff (2001) gives the example of the soft drink industry where empty bottles have to be returned. The VRPSPD can be described as follows: A set of vehicles with limited capacity must visit a set of customers located on a transportation network; each customer requires a delivery of a certain amount of goods, a pickup of another kind of goods, or both; all vehicles start and finish their route in a common depot; all the goods to be delivered are transported from the depot and the pickup goods are transported back to the depot and the goal is to minimize the overall vehicle route length (Dell'Amico et al., 2006).

Image 3.1 shows the different types of VRP and their interconnections, for better understanding.


Image 3.1- The basic problems of the VRP class and their interconnections (Toth and Vigo, 2001)

## 4. MODEL

### 4.1 Introduction

As seen before, in one-way carsharing systems there is need for relocations when the amount of vehicles in the parking areas is imbalanced. This model considers the relocation is made by support operation staff members that also carry out the needs for cleaning and maintenance.

The problem in hands is the need for the support staff members to get to the specified intervention site, the carsharing parking area where the maintenance is needed, from the support operations depot and to get back after the job is done, i.e. the routing of the support staff.

The model presented in the next chapter will determine the shortest route, given the staff needs in each site, making a balance between the pickup and delivery of personnel and the van capacity. Being a Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD) several constraints must be defined, such as the vans capacity, the number of vans available and the number of workers available at the moment of dispatch so that the model can have more resemblances with an operating situation in a carsharing system support staff.

### 4.2 Model Formulation

The VRPSPD problem will be defined by the following data:

- A set $\boldsymbol{S}$ of sites to be visited, numbered $2, \ldots, \boldsymbol{S}$;
- A depot, numbered as site 1 ;
- A directed graph $G=(\boldsymbol{V}, \boldsymbol{A})$, where $\boldsymbol{V}$ is the set of $n$ nodes (vertices), $\boldsymbol{A}$ is the set of arcs and $C=\left(c_{i j}\right)$ is the cost (distance between nodes) matrix associated with each arc (i,j) $\in \boldsymbol{A}$;
- A set of $m$ vehicles;
- A capacity $q$ for each vehicle;
- A set of workers available in the maintenance depot, $N_{\text {workers }}$;
- $b_{i}$, which represents the number of picked up or delivered personnel in each site (positive for pickup and negative for delivery).

Decision variables:

- A binary variable $x_{i j}$, that takes value 1 if $\operatorname{arc}(i, j) \in \boldsymbol{A}$ belongs to the optimal route, 0 otherwise;
- Variable $u_{i}$, which represents the order in which the site is visited in each route;
- $w_{i j}$, which represents the load carried by the van in the $\operatorname{arc}(i, j)$;

Min.
$C=\sum_{(i, j) \in A} c_{i j} x_{i j}$
s.t.
$\sum_{j=2 \in S}^{n} x_{1 j}=m$,
$\sum_{j=2 \in S}^{n} x_{j 1}=m$,

$$
\sum_{i \in S} x_{i j}=1, j=2, \ldots, n \text { with } j \neq i
$$

$\sum_{j \in S} x_{i j}=1, \quad i=2, \ldots, n$ with $i \neq j$,

$$
\begin{equation*}
\sum_{j \in S} x_{j i}+\sum_{j \in S} x_{i j}=2, \quad i=2, \ldots, n \tag{6}
\end{equation*}
$$

$u_{i}+(L-2) x_{1 i}-x_{i 1} \leq(L-1), \quad i=2, . ., n$,
$u_{i}+x_{1 i}+(2-K) x_{i 1} \geq 2, \quad i=2, . ., n$,
$x_{1 i}+x_{i 1} \leq 1, \quad i=2, . ., n$,

$$
\begin{align*}
& u_{i}+u_{j}+L x_{i j}+(L-2) x_{j i} \leq L-1, \quad 2 \leq i \neq j \leq n, \\
& \sum_{j \in S} w_{i j}-\sum_{j \in S} w_{j i}=b_{i}, \quad i=2, \ldots, n, \\
& w_{i j} \leq Q x_{i j}, \quad i, j \in S, \quad i \neq j, \\
& w_{i j} \geq 0, \quad i, j \in S, \quad i \neq j, \\
& \sum_{j \in S} w_{1 j} \leq N_{\text {workers }}, \tag{14}
\end{align*}
$$

$x_{i j} \in\{0,1\}, \forall(i, j) \in A$
The objective function is the sum of the cost of all the arcs travelled by $m$ vehicles and this sum must be minimized.

This formulation is valid when $2 \leq K \leq[(n-1) / m]$ and $L \geq K$. When $K \geq 4$, constraints (7) and (8) do not allow the situation $x_{1 i}=x_{i 1}=1$, constraint (10) becomes redundant when $K \geq 4$. Thus, we need constraint (10) only for cases $K=3$ or $K=2$.

In this formulation, constraints (2) and (3) ensure that exactly $m$ vans leave from and return to the depot.

Constraints (4) and (5) and (6) are the degree constraints, if a site is to be served it will be served only once.

Constraint (7) and (8) serve as upper and lower bound constraints on the number of nodes visited by each van, and initialize the value of $u_{i}$ to 1 if $i$ is the first node on the tour for any van. We call constraints (7) and (8) the bounding constraints. Since $x_{1 i}=x_{i 1}=1$ is not allowed, there are three remaining cases:
i. If $x_{1 i}=x_{i 1}=0$, then (7) and (8) imply $2 \leq u_{i} \leq L-1$.
ii. If $x_{1 i}=1$ and $x_{i 1}=0$, then, from (7) and (8) it can be obtained that $u_{i} \leq 1$ and $u_{i} \geq 1$, which implies $u_{i}=1$.
iii. If $x_{1 i}=0$ and $x_{i 1}=1$, then $K \leq u_{i} \leq L$.

Inequality (9) forbids a vehicle from visiting only a single node. Given the fact that $K$ must be larger than 1 , for the formulation to be valid, a vehicle cannot visit one single node.

The inequalities given in (10) ensure that $u_{j}=u_{i}+1$ if and only if $x_{i j}=1$. Thus, they prohibit the formation of any subtour between nodes in $\boldsymbol{V} \backslash\{1\}$, so constraints (7) to (10) are the subtour elimination constraints (SECs) of the formulation.

Constraint (11) is the flow conservation constraint on the amount of load.
Constraints (12) and (13) ensure that the vehicle's capacity is not exceed and the load is nonnegative.

Constraint (14) sets the limit for the number of workers leaving the depot as $N_{\text {workers }}$, the number of workers available.

Proposition 1. The constraints
$u_{i}+u_{j}+L x_{i j}+(L-2) x_{j i} \leq L-1, \quad 2 \leq i \neq j \leq n$,
with $u_{i}, u_{j} \in[1, L-1]$, are lifted Kulkarni-Bhave (1985) SECs.
Proof. Similar to the proof of Proposition 1 in Desrocher and Laporte (1991). Consider the constraints
$u_{i}+u_{j}+L x_{i j}+\alpha_{j i} x_{j i} \leq L-1$,
where currently $\alpha_{j i}=0$. The lifting process computes the largest possible value for $\alpha_{j i}$ so that (17) remains a valid inequality. There are two cases, $x_{j i}=0$ and $x_{j i}=1$.

Case 1. $x_{j i}=0$. Then (17) is valid for any $\alpha_{j i}$.
Case 2. . $x_{j i}=1$. This implies $x_{i j}=0$ and $u_{j}+1=u_{i}$, so that $\alpha_{j i} \leq L-2$.

## 5. APPLICATION METHODOLOGY

### 5.1. Geographic Characterization

The different areas within the city generate and attract trips all over the day, for example, residential areas generate departure travelers and business areas attract those same travelers during the morning peak hour. This generation and attraction relation suffers alterations along the day, due to the different population needs along the day. For each period the city will be split into different areas, some of them will generate more trips then attract and the opposite will occur in other areas. Some of these trips will occur using carsharing vehicles. To travel through areas the user must travel in the existing road system, subject to a speed, that is intertwined with the traffic density present along the road system throughout the different periods of the day. The existence of a road system also implies that the distance between two areas will not be the length of a straight line between two points, representing the origin/destination sites of each area, i.e. the Euclidean distance between those two areas. The distance between areas will, therefore, be greater than their Euclidean distance. Which means that, beside the fact that the distance between two areas is greater than their Euclidean distance, the speed in the route connecting the two areas will vary along the day.

### 5.2. Relocation Problem Characterization

Among all the generated trips in a city some of them will be made using carsharing vehicles. Given the fact that the carsharing system considered in this dissertation is one-way carsharing, vehicles can be picked in one area and left in another one. This generates an unbalance between the vehicles available in each area which must be moved by a group of workers that will relocate the vehicles from one area to another. The relocated vehicles will then be taken to the areas that generate more trips and therefore, have more need for vehicles along the day. The relocations can occur throughout the day, in a specific interval; only when in immediate need of vehicles or at the end of day when the system is closed for users. The latter cases require a bigger set of vehicles available. Within these options an Optimal Relocation Plan can be devised, regarding the expected vehicle flow within the system. Another situation that deems the usage of a vehicle impossible is a vehicle malfunction or a vehicle that is utterly unclean and improper to use. This situation cannot be foreseen and is dealt with as soon as possible by the staff.

### 5.3. Workday Methodology

Every 30 minutes of the workday a support team will be dispatched to do the necessary vehicle relocations and maintenance in the system. These workers will leave the support operations depot in the available number of vehicles, be dropped off (delivered) at the area where their services are needed, perform the solicited action, relocation or maintenance, and signal they are available for the next service or to be picked up back to the support operations depot. In the next period, 30 minutes, the control system in the support operations depot will have access to the new system requirements, i.e., how many new services are needed in that new period, how many workers are ready to be picked up, and therefore, available for a new service and are going to be relocated, how many other workers have to be taken from the support operations depot to respond to the demand existing in the system, how many vehicles are available to do the workers delivery, pickup or relocation. To the pickup/delivery vehicle's driver is given a route going through the sites where a service is requested or a worker is to be picked up. The latter route will be obtained using the model presented before to minimize the distance travelled by the vehicle. Given the fact that before the service hours there is no support staff in the system, the first route will only drop off, deliver, the support staff elements. In the last route of the day only pickup services will be taken into account. If there are still support staff members in the system after the last route of the day an extra route must be considered.

A situation that should be taken into account is the vehicles available for the support staff distribution. If a vehicle's route takes longer, considering the average urban speed mentioned before, than the period between periods, this vehicle will not be able to run in the next period, because it still has not come back to the depot. This way, the number of vehicles available for a certain period may vary during the day and a certain period may not even have routes due to the lack of vehicles available.

### 5.4. Cycle

Along the workday the model must run on the information available for the period it is meant and regard some information from the previous period. To represent this situation, the model in this dissertation can be run in a cycle, gathering some information from one period and using it in the next. For this to work some alterations in the previous models are required.

For this type of problem it will be needed:

- A Time Limit for the run to be deemed as acceptable, and the vehicle available for the next period, will have to be established;
- The travel duration of each arc $\left(t_{i j}\right)$, obtained considering a certain traveling speed and the "cost" $\left(c_{i j}\right)$ of each arc;
- A set $\boldsymbol{T}$ of the periods we are considering, numbered $1, \ldots, \mathrm{P}$.

It will also be necessary to include parameters that will be derived from one period to the following one:

- The number of vehicles that can finish their route within the Time Limit established before, $A$, i.e. the number of available vehicles for the next period. The duration of each route will be determined as the sum of the traveling duration of its arcs.
- For the first period $A$ will be equal to the number of vehicles established in the beginning;
- For the rest of the periods, $A \times K \leq R$, with R as the number of sites that should be visited. Using an iterative procedure reducing the number of available vehicles, $A$, each time $A \times K>R$;
- The formulation can only be valid if $A>0$.
- Constraints (2) and (3) will become:

$$
\begin{equation*}
\sum_{j=2 \in S}^{n} x_{1 j}=A \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{j=2 \in S}^{n} x_{j 1}=A \tag{18}
\end{equation*}
$$

- If the number of available vehicles for a certain period is zero, $A=0$, then the model will not be applicable and there will be no circuit. Given the fact that demand still exists, there will be an unsatisfied demand that should be satisfied in the next period. Therefore, it will be included a parameter $d_{j}$, representing unsatisfied demand.
- $d_{j}=b_{j}$, if a site isn't served and zero otherwise;
- For the next period, $d_{j}$ should be added to the new demand, $b_{j}$.
- The demand for every cycle should also be modified for different periods:
- For the first period $b_{i}$ should only be negative;
- For the last period, we should add, if it exists, $d_{i}$ from the previous period, and the new demand should only be positive;
- For the remainder periods the new demand $b_{i}$ can have any value.

Working as a cycle there can also be a model where $K$ takes the maximum value possible, enabling for different vehicles to have routes with similar distance travelled. This time it will be considered the number of available vehicles for each period, $A$.
$K=B / m$ will now become $K=B / A$.
When $A=1$ it is recommended the minimum number of sites visited has the minimum value, $K=2$. With only one vehicle there is no need to balance the number of sites between vehicles. If this condition is not imposed and there is only one vehicle available, the value of $K$ will be the same as the value of the number of sites that have requests. Given the fact that in the first and last cycle only delivery/pickup services occur, respectively, it may not be possible for one single vehicle to deliver/collect passengers from all the sites that require services and the model will not find any feasible solutions, because under those parameters, there will be none. If $K=2$, the vehicle will deliver/collect as many support staff members as its capacity allows and then return to the depot. Although not every demand is fulfilled in the immediate period the request occurs, we have the information of how many weren't delivered/picked up on time and that may serve as indicator of the system efficiency.

## 6. APPLICATION EXAMPLE

A carsharing system working from 8:00 to 22:00 is going to be considered for this application example. This system will have 28 support staff pickup and drop off instances related to the 28 periods of 30 minutes each throughout the day, the first occurring at 8:30 and the last at 22:00. The first only occurs 30 minutes after the system is working and the need for relocation or maintenance may exist and the last occurs when the system is closed to prepare the system for the next day.

It is also considered that the number of support staff workers is not a constraint and so, let us consider a number high enough to not restrain the model. Considering that there are two existing vehicles for the support staff member's transport in the system with a capacity to transport 8 workers each, typical 9 seat van, the number of $N_{\text {workers }}$ will be 16 .

The system considered will have 40 areas, represented by a single site. The sites will be randomly distributed in an area of 12,25 square kilometers, 3,5 per $3,5 \mathrm{~km}$.

It was defined that an average of $37,5 \%$ of the sites would have nonzero demand, so, an average of 15 of the sites will be served every period, with demands from 1 to 2 situations per site, deliver or pickup. This sums up to around 400 demands per day.

The time spent travelling from one site to the other will be calculated dividing the distance for the average urban speed of $30 \mathrm{~km} / \mathrm{h}$, the time spent in getting on and off the vehicle will be considered negligible.

### 6.1. Vehicle Routing Problem with Simultaneous Pick-up and Drop-off

Considering the previous information on the model proposed in this dissertation, the following results will be obtained:

- Aggregate travelled distance: $248,55 \mathrm{hm}$;
- Route for each available vehicle:
- Vehicle 1: Site Site 1 - Site 14 - Site 38 - Site 16 - Site 12 - Site 24 - Site 3 - Site 39- Site 28 - Site 10 - Site 31 - Site 1 ;
- Vehicle 2: Site 1-Site 34 - Site 18 - Site 5 - Site 25 - Site 1 ;
- Length for each route:
- Vehicle 1: 171,09 hm;
- Vehicle 2: 77,46 hm;
- Duration of each route, rounded to minutes:
- Vehicle 1: 35 minutes;
- Vehicle 2: 15 minutes;
- How many workers are required to be taken from the depot at the beginning of the route: 6 workers;
- Number of vehicles available for the next period: 1 vehicle;
- A graph, representing the routes, and the amount of workers dropped off or picked up at each site (negative means dropping off and positive means picking up):


Image 6.1- Graphic display of the Sites and Routes for the VRPSPD


Image 6.2-Graphic display of the Sites, Routes and Demand for the VRPSPD
The number of working staff members in the vehicles will vary throughout the routes, as shown in table 6.1.

| Vehicle 1 |  | Vehicle 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Segment <br> From-To | Workers <br> in van | Segment <br> From-To | Workers <br> in van |  |  |
| $1-14$ | 4 | $1-34$ | 2 |  |  |
| $14-38$ | 5 | $34-18$ | 0 |  |  |
| $38-16$ | 3 | $18-5$ | 2 |  |  |
| $16-12$ | 4 | $5-25$ | 1 |  |  |
| $12-24$ | 6 | $25-1$ | 2 |  |  |
| $24-3$ | 5 |  |  |  |  |
| $3-39$ | 6 |  |  |  |  |
| $39-28$ | 4 |  |  |  |  |
| $28-10$ | 3 |  |  |  |  |
| $10-31$ | 1 |  |  |  |  |
| $31-1$ | 2 |  |  |  |  |
|  |  |  |  |  |  |

Table 6.1- Number of working staff members in the vehicles throughout the period in the VRPSPD

As can be seen from previous information all demand in the period is satisfied. It can also be seen that none of the vehicles left the support operations depot with all staff seats occupied.

### 6.2. Daily Cycle

It is possible to perform an analysis of the system's efficiency throughout the day, observing the following indicators:

- The number of periods with no service for lack of vehicles, because they were not able to perform their route in 30 minutes;
- The number of sites that were not visited due to capacity limitations;
- The number of workers in those sites that were not picked up or dropped off;
- The total aggregated distance travelled by all vehicles throughout the day.

Considering 28 periods, of 30 minutes each, in a day; a constant average urban speed; and the condition that if a vehicle's route takes longer than 30 minutes it will not be available for the next period the following results were obtained,:

- There are 3 periods without service, when no vehicle is able to respond to the demands;
- There are 58 unserved sites throughout the day, due to lack of vehicles or lack of capacity of the vehicles performing the route;
- On the unserved sites there are 89 workers who are not picked up/delivered for at least one period;
- The total aggregate travel distance in the day was $6900,49 \mathrm{hm}$.

As can be seen from the previous information, there were 3 periods where no vehicle was available to respond to demand in the system. Therefore, all the sites in those periods were left unserved and no demand was satisfied. It should be taken into account that not every unserved site occurs in a period with no service, there may be sites left unserved due to lack of capacity in the support staff distribution vehicles.

### 6.3. Changing the number of vehicles

Considering the same conditions as the previous example, except for the number of vehicles, which will be raised to 3 , and the number of workers $N_{\text {workers, }}$, which will be raised to 24 , a new application example will be shown as an attempt to improve the previous system.

Considering only one period in the day, with the same demand, the following results are obtained:

- Aggregate travelled distance: $290,76 \mathrm{hm}$;
- Route for each available vehicle:
- Vehicle 1: Site Site 1- Site 10- Site 31- Site 1;
- Vehicle 2: Site 1- Site 28- Site 39- Site 3- Site 24- Site 12- Site 16- Site 38Site 14-Site 1;
- Vehicle 3: Site 1-Site 34- Site 18- Site 5- Site 25- Site 1;
- Length for each route:
- Vehicle 1: $56,15 \mathrm{hm}$;
- Vehicle 2: $157,15 \mathrm{hm}$;
- Vehicle 3: 77,46 hm;
- Duration of each route, rounded to minutes:
- Vehicle 1: 11 minutes;
- Vehicle 2: 32 minutes;
- Vehicle 3: 15 minutes;
- Number of workers required to be taken from the depot at the beginning of the route: 8 workers;
- Number of vehicles available for the next period: 2 vehicles;
- A graph, representing the routes, and the amount of workers dropped off or picked up at each site (negative means dropping off and positive means picking up):


Image 6.3- Graphic display of the Sites and Routes for the VRPSPD in the new example


Image 6.4-Graphic display of the Sites, Routes and Demand for the VRPSPD in the new example

The number of working staff members in the vehicles will vary throughout the routes, as shown by the following table 6.2:

| Vehicle 1 |  | Vehicle 2 |  | Vehicle 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Segment From-To | Workers in van | Segment From-To | Workers in van | Segment From-To | Workers in van |
| 1-10 | 3 | 1-28 | 3 | 1-34 | 2 |
| 10-31 | 1 | 28-39 | 2 | 34-18 | 0 |
| 31-1 | 0 | 39-3 | 0 | 18-5 | 2 |
|  |  | 3-24 | 1 | 5-25 | 1 |
|  |  | 24-12 | 0 | 25-1 | 2 |
|  |  | 12-16 | 2 |  |  |
|  |  | 16-38 | 3 |  |  |
|  |  | 38-14 | 1 |  |  |
|  |  | 14-1 | 2 |  |  |

Table 6.2- Number of working staff members in the vehicles throughout the period in the VRPSPD in the new example

Considering the alterations indicated before it can be seen that the system would function better for a single period. Considering an analysis of how it would work throughout the day, the following results would be obtained:

- There will be no periods without service, all the periods will have, at least, one vehicle available to perform the route;
- There will be 4 unserved sites throughout the day, due to lack of capacity of the vehicles performing the route;
- On the unserved sites there were 8 workers who were not picked up/delivered for at least one period;
- The total aggregate travel distance in the day was $934,287 \mathrm{~km}$.

As can be seen from the results, there were no periods where no vehicle was available to respond to demands in the system. Therefore, all the sites which were left unserved were caused by lack of capacity in the support vehicles available in that specific period. It should be taken into account that although the total aggregate travel distance was higher than the previous example, all the periods had routes. For example, if every period had a total travel distance of 300 hm , the system with more periods where service was possible would have a higher aggregate travel distance. This, however, is a possibility and not necessarily true for all situations.

In this new example two variables were changed, the number of vehicles and the number of available workers in the depot at every period. To understand if the results obtained in the new example occurred due to changing the number of vehicles, the number of available workers, or both, a sensitivity analysis must be performed.

### 6.4. Sensitivity Analysis

Several factors have influence in the effectiveness of the support operations staff in a carsharing system, concerning the aggregate travel distance, the number of periods with no service, number of unserved sites and number of support workers not dropped off or picked up in the right period, i.e. demand not satisfied.

The following analysis will vary:

- The demand, it will vary from a normal demand, with probability of no demand equal to $62,5 \%(p(0)=62,5 \%)$; sparse demand with $p(0)=72,5 \%$ and concentrated demand with $\mathrm{p}(0)=52,5 \%$;
- The travelling speed, different cities may have different average urban speeds and so three speeds will be tested, $20 \mathrm{~km} / \mathrm{h}, 30 \mathrm{~km} / \mathrm{h}$ and $40 \mathrm{~km} / \mathrm{h}$;
- The number of workers available at the depot for every period, $N_{\text {workers; }}$
- The vehicle capacity, instead of 9 seat vans, 5 seat vehicles will be considered with capacity for 4 support staff members.

The variables were modified until a completely efficient system was obtained. The results of such variations are presented in the tables in the Appendix. A few graphic displays of the effect of the variable variation in some indicators of the system's efficiency are shown, based on the tables in the Appendix.

The following figures 6.5 and 6.6 will show the changes that the number $N_{\text {workers }}$ variation imposes to the system's effectiveness. This variation can be seen in the tables presented in the Appendix, showing the variation of $N_{\text {workers }}$ to a system working with 4 vehicles, a normal demand, an average urban speed of $30 \mathrm{~km} / \mathrm{h}$ and a capacity of 8 workers per vehicle.


Figure 6.5- Effect of the $N_{\text {workers }}$ variation on the aggregate distance and periods with no service I


Figure 6.6- Effect of the $N_{\text {workers }}$ variation on the number of unserved sites and requests not satisfied I

It can be seen in the previous figures that the number of workers available in the depot at each period has influence on the number of unserved sites and requests not satisfied throughout the workday. Although the relation between these factors is not linear it can be seen that the more workers available in the depot the fewer sites are left unserved and less requests are left unsatisfied. The main objective in the model developed in this dissertation is to minimize the distance travelled by the vehicle when performing their route. It can be seen in the charts above that for lesser $N_{\text {workers }}$, the total aggregate distance is shorter. This, however, may occur because there are lesser sites visited and so the vehicles travel less. However, the opposite may happen because the model developed in this dissertation was designed to minimize the travel distance for each period and this may add up to a system less effective than others throughout the workday.

Figures 6.7 and 6.8 show a system with 4 vehicles with capacity for 8 workers each, a concentrated demand and an urban average speed of $20 \mathrm{~km} / \mathrm{h}$. This example was chosen to show a situation where the number of vehicles was not sufficient and so, although the value $N_{\text {workers }}$ becomes higher, the lack of vehicles prevent the number of unserved sites and requests not satisfied to be equal to zero. This happens because the number of vehicles relates to the number of periods with no service. If there are no vehicles available, there is no service in that period.


Figure 6.7- Effect of the $N_{\text {workers }}$ variation on the aggregate distance and periods with no service II


Figure 6.8- Effect of the $N_{\text {workers }}$ variation on the number of unserved sites and requests not satisfied II

Figures 6.9 and 6.10 show the results of changing the average speed of the vehicles in a system with 4 vehicles, normal demand and a capacity of 8 workers per vehicle will be shown. It will go:


Figure 6.9- Effect of the urban average speed variation on the aggregate distance and periods with no service I


Figure 6.10- Effect of the urban average speed variation on the number of unserved sites and requests not satisfied I

As can be seen the impact of urban average speed in the system is mostly in the number of periods with no service, which will raise the number of unserved sites and the number of requests not satisfied.

Figures 6.11 and 6.12 show a situation similar to the previous but in a case with a concentrated demand:


Figure 6.11- Effect of the urban average speed variation on the aggregate distance and periods with no service II


Figure 6.12- Effect of the urban average speed variation on the number of unserved sites and requests not satisfied II

As can be seen the relation between urban average speed and the effectiveness of the system follows the same pattern. However, with a concentrated demand a system similar to the
previous would be less effective with more periods without service, more unserved sites and a greater number of requests not satisfied.

Figures 6.13 and 6.14 show the variation in systems due to different types of demand, these systems have 4 vehicles with a capacity for 8 workers each, the average urban speed is $30 \mathrm{~km} / \mathrm{h}$ and $N_{\text {workers }}$ is 15 .


Figure 6.13- Effect of demand variation on the aggregate travelled distance and periods with no service


Figure 6.14- Effect of the demand variation on the number of unserved sites and requests not satisfied

The previous figures show that a system with less demand is more effective with the same amount of vehicles and capacity. If there is less demand, there will be lesser sites left unserved, lesser requests not satisfied and the routes will be shorter.

The following figures 6.15 and 6.16 will show the variation in the system effectiveness due to the variation in the number of vehicles for a situation of normal demand, an average urban speed of $20 \mathrm{~km} / \mathrm{h}$, a $N_{\text {workers }}$ value of equal to 15 and a capacity of 8 workers per vehicle:


Figure 6.15- Effect of the number of vehicles variation on the aggregate distance and number of periods with no service


Figure 6.16- Effect of the number of vehicles variation on the number of unserved sites and requests not satisfied

It can be seen that the number of available vehicles reduces the number of unserved sites in the system only to a certain situation. From that point forward the number of vehicles may not have impact in the number periods with no service, number of unserved sites and requests not satisfied, with the final optimization of the system's effectiveness depending of the value of
$N_{\text {workers }}$. It is also shown that, despite the other effectiveness control items are the same, the aggregate distance from the situation with 5 vehicles and 6 vehicles is not similar, the system with 6 vehicles is less effective. A situation with more vehicles may be less effective due to the minimum number of sites visited per vehicle. The same situation explains the situation for the figures 6.9 and 6.11 where for a greater average urban speed the aggregate distance is higher, in a certain period there may be more vehicles available for the route and that situation will generate a greater travel distance for that specific period and, therefore, for the aggregate travel distance in the workday.

In the next figures, figure 6.17 to 6.24 , a comparison between two systems working with vehicles with different capacities will be shown. This comparison will be made in three parts, the first part will show the variation for a system with the same capacity in a system with normal demand, 3 vehicles with 8 workers each and 6 vehicles with 4 workers each; the second part will show the variation for a system with the same number of vehicles in a system with normal demand; the third will repeat the previous parts but for a system with a concentrated demand.

In figures 6.17 and 6.18 a situation with 3 vehicles with capacity for 8 workers each will be compared to a situation with 6 vehicles with capacity for 4 workers each. This will occur in a system with normal demand, average urban speed of $20 \mathrm{~km} / \mathrm{h}$ and value of $N_{\text {workers }}$ equal to 15 .


Figure 6.17-Comparison of the variation imposed by different vehicle capacity on aggregate distance and periods with no service I


Figure 6.18- Comparison of the variation imposed by different vehicle capacity unserved sites and requests not satisfied I

In this situation it can be seen that, when comparing two systems with the same total capacity, with one using more vehicles for that purpose than the other, the situation with more vehicles is more effective. This situation occurs because there are no periods without service using more vehicles.

It will now be compared the previous situation to one similar, except the number of available vehicles, with both systems using 6 vehicles:


Figure 6.19- Comparison of the variation imposed by different vehicle capacity on aggregate distance and periods with no service II


Figure 6.20- Comparison of the variation imposed by different vehicle capacity unserved sites and requests not satisfied II

In a situation where two systems with the same capacity and different number of vehicles are compared, the situation with more capacity is more effective than the other. This occurs
because the vehicles will not have to perform greater distances in their route and the number of unvisited sites due to lack of capacity is reduced.

The same variations were introduced in a system similar to the previous but for a concentrated demand. Figures 6.21 and 6.22 will compare two systems with the same total capacity and figures 6.23 and 6.24 will compare two systems with the same number of vehicles:


Figure 6.21- Comparison Comparison of the variation imposed by different vehicle capacity on aggregate distance and periods with no service III


Figure 6.22- Comparison of the variation imposed by different vehicle capacity unserved sites and requests not satisfied III


Figure 6.23-Comparison of the variation imposed by different vehicle capacity on aggregate distance and periods with no service IV


Figure 6.24-Comparison of the variation imposed by different vehicle capacity unserved sites and requests not satisfied IV

It can be seen in the previous figures that the same conclusions can be made for the systems with a concentrated demand as for the systems with a normal demand. The situation with more vehicles is more effective in systems with the same total capacity and the situation with more capacity is more effective in systems with the same number of vehicles.

Summarizing the effect of the experiments:

- The value of $N_{\text {workers }}$ reduces the number of unserved sites and requests unsatisfied;
- The type of demand may reduce or increase the stress on the system, whether it is sparse or concentrated, respectively;
- The number of vehicles available reduces the number of periods with no service, and therefore the number of unserved sites and requests unsatisfied. However, they only reduce the latter until a certain point, depending on the value of $N_{\text {workers }}$;
- A number of vehicles higher than a certain value will lead to longer aggregate distances in the system;
- A slower average urban speed reduces the system's effectiveness, leading to fewer vehicles available and more unserved sites and requests unsatisfied. A higher average speed will lead to longer aggregate distances in the system;
- In systems with the a determined capacity, more smaller vehicles will be more effective than a few bigger vehicles;
- For a given number of vehicles in the fleet, the usage of larger vehicles will lead to a more effective system.


## 7. CONCLUSIONS

This dissertation focused on the problem concerning the routing of staff members responsible for support operations in a carsharing system. A literature review was done on carsharing systems operation and on vehicle routing problems in general. Vehicle routing problems seek to minimize travel distances for a set of vehicles, originating in a set of depots, while supplying a set of customers in a specific location of some commodity or service. The information gathered in the literature concerning vehicle routing problems and related problems was used to develop a model. The model developed minimizes the distance travelled by a set of vehicles originating in a support operations depot that pick-up and/or drop-off support staff members in areas throughout the city, returning to the depot at the end of their route. The existing demands are required assistance in any vehicle or required pick-up of staff members that performed some type of assistance.

Using the developed model it was investigated how a carsharing system support staff using the model would operate and if it would be effective for different situations. To investigate the systems effectiveness a sensitivity analysis was performed under different experimental parameters. The model found solutions for all different tested experimental parameters and so it can be concluded that it functions properly.

Moreover we found through those experiments that more available workers at a depot implies less unserved sites and requests not satisfied; if more distribution vehicles are available more periods will have vehicles available, however this may mean more aggregate distance; cities with a low average urban speed may have less effective support systems; in systems where the total capacity is predetermined a solution with more vehicles of less capacity outperforms one with less vehicles of more capacity and a system with predetermined number of vehicles performs better the more capacity those vehicles have.

In this model the main objective was to minimize travel distances for the vehicles dropping off or picking up support staff members. A similar model could be formulated in which the objective function would be the minimization of the number of available workers, the number of vehicles, or even to combine these objectives.

In this dissertation it was considered that the value of $K$, minimum number of sites visited, would be equal to 2 . A study concerning the variation in the value of $K$ and the comparison of the results obtained in this dissertation could also be done.

It was considered that a single support operations depot would exist in the system and every vehicle would originate in that depot. A study concerning a multi-depot system and comparison of the results obtained could also take place in the future.

This model was developed to minimize the travel distance in each individual period and that information was compiled to a workday, 28 periods. That information was used to verify how a system would work under different conditions. A new model developed specifically as a cycle could be developed to verify the effectiveness of the support system under those different conditions.

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## APPENDIX

## A- Tables of system effectiveness for vehicles with capacity for 8 workers

| Normal Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 8366,92 | 9098,05 | 9249,26 | 9233,9 | 9342,87 |
| Periods with no service | 2 | 0 | 0 | 1 | 0 |
| Unserved Sites | 67 | 15 | 9 | 16 | 4 |
| Requests not satisfied | 97 | 29 | 18 | 23 | 8 |

Table A.1- System effectiveness for vehicles with capacity for 8 workers I

| Normal Demand-30km/h | Norkers |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 9894,85 | 10142,2 | 10240,6 | 10181,2 | 10292,4 | 10292,4 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 35 | 11 | 5 | 2 | 0 | 0 |  |
| Requests not satisfied | 57 | 21 | 10 | 4 | 0 | 0 |  |

Table A.2- System effectiveness for vehicles with capacity for 8 workers II

| Normal Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 10526,7 | 11311,3 | 11219,6 | 11178,7 | 11226,6 | 11262,8 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 47 | 11 | 5 | 2 | 0 | 0 |  |
| Requests not satisfied | 76 | 21 | 10 | 4 | 0 | 0 |  |

Table A.3- System effectiveness for vehicles with capacity for 8 workers III

| Normal Demand-40km/h | Nworkers |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 9298,58 | 9631,09 | 9720,35 | 9833,46 | 9978,97 |
| Periods with no service | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | 39 | 15 | 9 | 6 | 4 |
| Requests not satisfied | 66 | 29 | 18 | 12 | 8 |

Table A.4- System effectiveness for vehicles with capacity for 8 workers IV

| Normal Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 10260,3 | 10462,1 | 10560,5 | 10639,8 | 10845,8 | 10845,8 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 35 | 11 | 5 | 2 | 0 | 0 |  |
| Requests not satisfied | 57 | 21 | 10 | 4 | 0 | 0 |  |

Table A.5- System effectiveness for vehicles with capacity for 8 workers V

| Normal Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 11518,5 | 12213,6 | 12198,9 | 12275 | 12325,4 | 12361,7 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 47 | 11 | 5 | 2 | 0 | 0 |  |
| Requests not satisfied | 76 | 21 | 10 | 4 | 0 | 0 |  |

Table A.6- System effectiveness for vehicles with capacity for 8 workers VI

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 7006,89 | 7003,49 | 7342,45 | 7069,84 | 7140,95 |
| Periods with no service | 8 | 8 | 7 | 8 | 7 |
| Unserved Sites | 165 | 139 | 124 | 139 | 120 |
| Requests not satisfied | 228 | 187 | 173 | 182 | 156 |

Table A.7- System effectiveness for vehicles with capacity for 8 workers VII

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 8713,36 | 9125,1 | 9110,58 | 9189,12 | 9175,05 | 9175,05 |  |
| Periods with no service | 0 | 2 | 2 | 2 | 3 | 3 |  |
| Unserved Sites | 39 | 27 | 21 | 20 | 27 | 27 |  |
| Requests not satisfied | 66 | 41 | 30 | 27 | 33 | 33 |  |

Table A.8- System effectiveness for vehicles with capacity for 8 workers VIII

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 9978,37 | 10506,7 | 10458,6 | 10591,8 | 10606,6 | 10642,9 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 47 | 11 | 5 | 2 | 0 | 0 |  |
| Requests not satisfied | 76 | 21 | 10 | 4 | 0 | 0 |  |

Table A.9- System effectiveness for vehicles with capacity for 8 workers IX

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 10805,9 | 11122,5 | 11535,6 | 11546,6 | 11642,3 | 11696,1 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 47 | 26 | 5 | 2 | 0 | 0 |  |
| Requests not satisfied | 76 | 41 | 10 | 4 | 0 | 0 |  |

Table A.10- System effectiveness for vehicles with capacity for 8 workers X

| Sparse Demand-30km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 2 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 6390,35 | 6487,08 | 6316,28 | 6299,25 |
| Periods with no service | 4 | 4 | 6 | 6 |
| Unserved Sites | 61 | 51 | 63 | 63 |
| Requests not satisfied | 84 | 76 | 92 | 92 |

Table A.11- System effectiveness for vehicles with capacity for 8 workers XI

| Sparse Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 7382,73 | 7733,6 | 7714,91 | 7805,36 | 7754,09 |
| Periods with no service | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | 21 | 5 | 2 | 1 | 1 |
| Requests not satisfied | 33 | 9 | 4 | 2 | 2 |

Table A.12- System effectiveness for vehicles with capacity for 8 workers XII

| Sparse Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 8940,42 | 9508,02 | 9534,32 | 9629,93 | 9637,33 | 9637,33 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 20 | 4 | 1 | 0 | 0 | 0 |  |
| Requests not satisfied | 31 | 7 | 2 | 0 | 0 | 0 |  |

Table A.13- System effectiveness for vehicles with capacity for 8 workers XIII

| Sparse Demand-40km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 2 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 7140,83 | 7347,05 | 7447,22 | 7430,19 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 13 | 5 | 2 | 2 |
| Requests not satisfied | 22 | 9 | 4 | 4 |

Table A.14- System effectiveness for vehicles with capacity for 8 workers XIV

| Sparse Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 7781,43 | 8167,77 | 8275,47 | 8310,11 | 8340,97 |
| Periods with no service | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | 20 | 4 | 1 | 0 | 0 |
| Requests not satisfied | 31 | 7 | 2 | 0 | 0 |

Table A.15- System effectiveness for vehicles with capacity for 8 workers XV

| Sparse Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 9652 | 10050 | 10060,6 | 10060,6 | 10060,6 | 10060,6 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 20 | 4 | 0 | 0 | 0 | 0 |  |
| Requests not satisfied | 30 | 7 | 0 | 0 | 0 | 0 |  |

Table A.16- System effectiveness for vehicles with capacity for 8 workers XVI

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 2 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 4752,47 | 4879,04 | 4925,37 | 4908,34 |
| Periods with no service | 13 | 12 | 12 | 12 |
| Unserved Sites | 182 | 144 | 139 | 139 |
| Requests not satisfied | 246 | 198 | 190 | 190 |

Table A.17- System effectiveness for vehicles with capacity for 8 workers XVII

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 6473,96 | 6792,88 | 6772,28 | 6797,97 | 6828,83 |
| Periods with no service | 2 | 3 | 3 | 3 | 3 |
| Unserved Sites | 49 | 34 | 31 | 30 | 30 |
| Requests not satisfied | 74 | 51 | 46 | 44 | 44 |

Table A.18- System effectiveness for vehicles with capacity for 8 workers XVIII

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 7673,54 | 8266,47 | 8246,62 | 8172,34 | 8172,34 | 8147,25 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 20 | 4 | 1 | 0 | 0 | 0 |  |
| Requests not satisfied | 31 | 7 | 2 | 0 | 0 | 0 |  |

Table A.19- System effectiveness for vehicles with capacity for 8 workers XIX

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |
| Agreggate Distance [hm] | 9038,18 | 9104,77 | 9382,54 | 9382,54 | 9382,54 | 9382,54 |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | 19 | 15 | 0 | 0 | 0 | 0 |
| Requests not satisfied | 28 | 21 | 0 | 0 | 0 | 0 |

Table A.20- System effectiveness for vehicles with capacity for 8 workers XX

| Concentrated Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 8815,93 | 9502,27 | 9710,83 | 9398,71 | 9491,92 |
| Periods with no service | 2 | 0 | 1 | 2 | 1 |
| Unserved Sites | 115 | 42 | 49 | 63 | 42 |
| Requests not satisfied | 173 | 73 | 74 | 89 | 62 |

Table A.21-System effectiveness for vehicles with capacity for 8 workers XXI

| Concentrated Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 32 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 10258 | 10462 | 10506 | 10650,7 | 10557,9 | 10787,7 | 10766,9 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 63 | 23 | 11 | 6 | 4 | 1 | 0 |  |
| Requests not satisfied | 110 | 42 | 21 | 12 | 8 | 2 | 0 |  |

Table A.22- System effectiveness for vehicles with capacity for 8 workers XXII

| Concentrated Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 11155,7 | 11780,8 | 11802,8 | 11923,5 | 11791,3 | 11995 | 12011,4 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 75 | 23 | 11 | 6 | 4 | 1 | 0 |  |
| Requests not satisfied | 130 | 43 | 21 | 12 | 8 | 2 | 0 |  |

Table A.23- System effectiveness for vehicles with capacity for 8 workers XXIII

| Concentrated Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 9820,52 | 9523,99 | 10070,9 | 10029,6 | 10011,1 |
| Periods with no service | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | 67 | 38 | 14 | 9 | 7 |
| Requests not satisfied | 119 | 65 | 27 | 18 | 14 |

Table A.24- System effectiveness for vehicles with capacity for 8 workers XXIV

| Concentrated Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 32 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 11212,1 | 11204,9 | 11166,4 | 11297,6 | 11191,7 | 11440 | 11400,8 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 60 | 23 | 11 | 6 | 4 | 1 | 0 |  |
| Requests not satisfied | 105 | 42 | 21 | 12 | 8 | 2 | 0 |  |

Table A.25- System effectiveness for vehicles with capacity for 8 workers XXV

| Concentrated Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 11631,4 | 12525,7 | 12492,3 | 12671,6 | 12531,2 | 12832,5 | 12911,2 |  |
| Periods with no service | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Unserved Sites | 75 | 23 | 11 | 6 | 4 | 1 | 0 |  |
| Requests not satisfied | 130 | 43 | 21 | 12 | 8 | 2 | 0 |  |

Table A.26- System effectiveness for vehicles with capacity for 8 workers XXVI

| Concentrated Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | 6955,95 | 7700 | 7635,19 | 7639,56 | 7207,72 |
| Periods with no service | 10 | 9 | 8 | 10 | 9 |
| Unserved Sites | 262 | 197 | 173 | 180 | 197 |
| Requests not satisfied | 379 | 286 | 249 | 256 | 280 |

Table A.27- System effectiveness for vehicles with capacity for 8 workers XXVII

| Concentrated Demand-20km/h | Nworkers |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 32 |
| Agreggate Distance [hm] | 8738,68 | 9489,92 | 9197,04 | 9304,98 | 9427,07 | 9321,82 | 9669,04 |
| Periods with no service | 2 | 2 | 3 | 3 | 2 | 3 | 2 |
| Unserved Sites | 99 | 75 | 80 | 76 | 56 | 72 | 45 |
| Requests not satisfied | 157 | 118 | 112 | 109 | 84 | 101 | 71 |

Table A.28- System effectiveness for vehicles with capacity for 8 workers XXVIII

| Concentrated Demand-20km/h | Nworkers |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |  |
| Agreggate Distance $[\mathrm{hm}]$ | 10575,6 | 10508,6 | 10520,9 | 10679,8 | 10594,5 | 10851,3 | 10681,9 |  |
| Periods with no service | 1 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| Unserved Sites | 78 | 30 | 14 | 9 | 7 | 4 | 17 |  |
| Requests not satisfied | 135 | 57 | 27 | 18 | 14 | 8 | 26 |  |

Table A.29- System effectiveness for vehicles with capacity for 8 workers XXIX

| Concentrated Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 11733,5 | 12349,4 | 12460 | 12284,8 | 12391 | 12474,5 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 46 | 11 | 6 | 4 | 1 | 0 |
| Requests not satisfied | - | 77 | 21 | 12 | 8 | 2 | 0 |

Table A.30- System effectiveness for vehicles with capacity for 8 workers XXX

| Concentrated Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 13061,3 | 13278,6 | 13226,5 | 13239,2 | 13316,7 | 13332,1 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 43 | 11 | 6 | 4 | 1 | 0 |
| Requests not satisfied | - | 71 | 21 | 12 | 8 | 2 | 0 |

Table A.31- System effectiveness for vehicles with capacity for 8 workers XXXI

B- Tables of system effectiveness for vehicles with capacity for 4 workers

| Normal Demand-30km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 9432,47 | 9519,3 | 9681,67 | 9670,86 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 45 | 10 | 3 | 2 |
| Requests not satisfied | 100 | 20 | 5 | 4 |

Table B.1- System effectiveness for vehicles with capacity for 4 workers I

| Normal Demand-30km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance $[\mathrm{hm}]$ | 11181,5 | 11043,1 | 11361,7 | 11401,3 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 46 | 22 | 3 | 0 |
| Requests not satisfied | 100 | 40 | 6 | 0 |

Table B.2- System effectiveness for vehicles with capacity for 4 workers II

| Normal Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 13219,9 | 13235,7 | 13840,9 | 13645 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 22 | 15 | 0 | 0 |
| Requests not satisfied | - | 37 | 24 | 0 | 0 |

Table B.3-System effectiveness for vehicles with capacity for 4 workers III

| Normal Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 28 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 14537,3 | 14702,4 | 15497,8 | 15421,8 | 15345,5 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 35 | 15 | 0 | 0 | 0 |
| Requests not satisfied | - | 56 | 24 | 0 | 0 | 0 |

Table B.4- System effectiveness for vehicles with capacity for 4 workers IV

| Normal Demand-40km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance [hm] | 10270,2 | 10439 | 10418,6 | 10521,6 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 45 | 10 | 3 | 2 |
| Requests not satisfied | 99 | 20 | 5 | 4 |

Table B.5- System effectiveness for vehicles with capacity for 4 workers $V$

| Normal Demand-40km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance [hm] | 12017,6 | 11912,3 | 12350 | 12315,7 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 46 | 22 | 3 | 0 |
| Requests not satisfied | 99 | 40 | 6 | 0 |

Table B.6-System effectiveness for vehicles with capacity for 4 workers VI

| Normal Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance [hm] | - | 13831,2 | 13890,3 | 14241,7 | 14341,8 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 22 | 15 | 0 | 0 |
| Requests not satisfied | - | 37 | 24 | 0 | 0 |

Table B.7-System effectiveness for vehicles with capacity for 4 workers VII

| Normal Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 28 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 14495,5 | 14911,3 | 15615,6 | 15539,6 | 15554,4 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 35 | 15 | 0 | 0 | 0 |
| Requests not satisfied | - | 56 | 24 | 0 | 0 | 0 |

Table B.8- System effectiveness for vehicles with capacity for 4 workers VIII

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 8415,45 | 8497,53 | 8532,24 | 8606,97 |
| Periods with no service | 1 | 0 | 1 | 0 |
| Unserved Sites | 45 | 14 | 19 | 6 |
| Requests not satisfied | 102 | 27 | 29 | 12 |

Table B.9- System effectiveness for vehicles with capacity for 4 workers IX

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance $[\mathrm{hm}]$ | 10049,5 | 10049,5 | 9995,54 | 10005,6 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 44 | 44 | 3 | 0 |
| Requests not satisfied | 96 | 96 | 6 | 0 |

Table B.10- System effectiveness for vehicles with capacity for 4 workers X

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 11205,1 | 11383,5 | 11650,3 | 11664,3 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 22 | 15 | 0 | 0 |
| Requests not satisfied | - | 38 | 24 | 0 | 0 |

Table B.11- System effectiveness for vehicles with capacity for 4 workers XI

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 28 |
| Agreggate Distance [hm] | - | 12540,6 | 12463,8 | 12909,3 | 12889,9 | 12901,3 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 22 | 15 | 0 | 0 | 0 |
| Requests not satisfied | - | 39 | 24 | 0 | 0 | 0 |

Table B.12- System effectiveness for vehicles with capacity for 4 workers XII

| Normal Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 vehicles | 5 | 10 | 15 | 20 | 25 | 30 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 13941,2 | 14135,4 | 14362,1 | 14458,6 | 14470 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 22 | 15 | 0 | 0 | 0 |
| Requests not satisfied | - | 40 | 24 | 0 | 0 | 0 |

Table B.13- System effectiveness for vehicles with capacity for 4 workers XIII

| Sparse Demand-30km/h | $N_{\text {workers }}$ |  |  |
| :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 12 |
| Agreggate Distance $[\mathrm{hm}]$ | 8127,79 | 8736,95 | 8658,9 |
| Periods with no service | 0 | 0 | 0 |
| Unserved Sites | 27 | 9 | 8 |
| Requests not satisfied | 43 | 17 | 15 |

Table B.14- System effectiveness for vehicles with capacity for 4 workers XIV

| Sparse Demand-30km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 9310,46 | 9773,62 | 9857,13 | 9778,46 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 21 | 5 | 2 | 2 |
| Requests not satisfied | 32 | 9 | 4 | 4 |

Table B.15- System effectiveness for vehicles with capacity for 4 workers XV

| Sparse Demand-30km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance $[\mathrm{hm}]$ | 10930,9 | 10869,8 | 11186,2 | 11319,3 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 20 | 15 | 1 | 0 |
| Requests not satisfied | 30 | 21 | 2 | 0 |

Table B.16-System effectiveness for vehicles with capacity for 4 workers XVI

| Sparse Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 12110 | 12110 | 12199,9 | 12083,4 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 1 | 1 | 0 | 0 |
| Requests not satisfied | - | 2 | 2 | 0 | 0 |

Table B.17- System effectiveness for vehicles with capacity for 4 workers XVII

| Sparse Demand-40km/h | $N_{\text {workers }}$ |  |  |
| :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 12 |
| Agreggate Distance $[\mathrm{hm}]$ | 8127,79 | 8736,95 | 8658,9 |
| Periods with no service | 0 | 0 | 0 |
| Unserved Sites | 27 | 9 | 8 |
| Requests not satisfied | 43 | 17 | 15 |

Table B.18- System effectiveness for vehicles with capacity for 4 workers XVIII

| Sparse Demand-40km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 9310,46 | 9773,62 | 9857,13 | 9778,46 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 21 | 5 | 2 | 2 |
| Requests not satisfied | 32 | 9 | 4 | 4 |

Table B.19- System effectiveness for vehicles with capacity for 4 workers XIX

| Sparse Demand-40km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance $[\mathrm{hm}]$ | 10930,9 | 10869,8 | 11186,2 | 11319,3 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 20 | 15 | 1 | 0 |
| Requests not satisfied | 30 | 21 | 2 | 0 |

Table B.20- System effectiveness for vehicles with capacity for 4 workers XX

| Sparse Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 11818,8 | 12110 | 12199,9 | 12083,4 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 15 | 1 | 0 | 0 |
| Requests not satisfied | - | 21 | 2 | 0 | 0 |

Table B.21- System effectiveness for vehicles with capacity for 4 workers XXI

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |
| :---: | :---: | :---: | :---: |
| 3 vehicles | 5 | 10 | 12 |
| Agreggate Distance $[\mathrm{hm}]$ | 7052,66 | 7251,68 | 7146,29 |
| Periods with no service | 2 | 3 | 2 |
| Unserved Sites | 60 | 53 | 50 |
| Requests not satisfied | 125 | 85 | 87 |

Table B.22- System effectiveness for vehicles with capacity for 4 workers XXII

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 vehicles | 5 | 10 | 15 | 16 |
| Agreggate Distance $[\mathrm{hm}]$ | 8126,64 | 8283,38 | 8188,14 | 8326,48 |
| Periods with no service | 1 | 2 | 3 | 2 |
| Unserved Sites | 37 | 35 | 43 | 34 |
| Requests not satisfied | 58 | 49 | 60 | 47 |

Table B.23- System effectiveness for vehicles with capacity for 4 workers XXIII

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance $[\mathrm{hm}]$ | 9207 | 9563,16 | 9901,28 | 9900,31 |
| Periods with no service | 0 | 0 | 0 | 1 |
| Unserved Sites | 21 | 16 | 2 | 10 |
| Requests not satisfied | 33 | 23 | 4 | 15 |

Table B.24- System effectiveness for vehicles with capacity for 4 workers XXIV

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 10492,5 | 10801,7 | 10832,2 | 10723,3 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 15 | 1 | 0 | 0 |
| Requests not satisfied | - | 21 | 2 | 0 | 0 |

Table B.25- System effectiveness for vehicles with capacity for 4 workers XXV

| Sparse Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 28 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 11438,1 | 11564,5 | 11667,5 | 11653 | 11578,7 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 15 | 1 | 0 | 0 | 0 |
| Requests not satisfied | - | 21 | 2 | 0 | 0 | 0 |

Table B.26- System effectiveness for vehicles with capacity for 4 workers XXVI

| Concentrated Demand-30km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance $[\mathrm{hm}]$ | 11756,9 | 12688,3 | 12829,6 | 12810 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 85 | 43 | 22 | 17 |
| Requests not satisfied | 169 | 103 | 39 | 30 |

Table B.27-System effectiveness for vehicles with capacity for 4 workers XXVII

| Concentrated Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 13632,5 | 13739,7 | 13991,2 | 14047,7 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 55 | 16 | 11 | 9 |
| Requests not satisfied | - | 121 | 31 | 22 | 18 |

Table B.28- System effectiveness for vehicles with capacity for 4 workers XXVIII

| Concentrated Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 28 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 15508,1 | 16241,9 | 16349 | 16244,9 | 16383,5 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 50 | 14 | 9 | 6 | 5 |
| Requests not satisfied | - | 111 | 27 | 18 | 12 | 10 |

Table B.29- System effectiveness for vehicles with capacity for 4 workers XXIX

| Concentrated Demand-30km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 32 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 16218 | 16859,9 | 18077,4 | 17957,9 | 18154,5 | 18095,4 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 85 | 57 | 6 | 4 | 1 | 0 |
| Requests not satisfied | - | 160 | 120 | 12 | 8 | 2 | 0 |

Table B.30- System effectiveness for vehicles with capacity for 4 workers XXX

| Concentrated Demand-30km/h | Nworkers |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 17987,2 | 18366,1 | 19446,5 | 19287,9 | 19479,6 | 19509,2 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 85 | 57 | 6 | 4 | 1 | 0 |
| Requests not satisfied | - | 160 | 119 | 12 | 7 | 2 | 0 |

Table B.30- System effectiveness for vehicles with capacity for 4 workers XXXI

| Concentrated Demand-40km/h | $N_{\text {workers }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 vehicles | 5 | 10 | 15 | 20 |
| Agreggate Distance $[\mathrm{hm}]$ | 12794,8 | 13357,3 | 13406,6 | 13366,8 |
| Periods with no service | 0 | 0 | 0 | 0 |
| Unserved Sites | 85 | 35 | 16 | 11 |
| Requests not satisfied | 169 | 91 | 31 | 22 |

Table B.31- System effectiveness for vehicles with capacity for 4 workers XXXII

| Concentrated Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 14755,5 | 14481,4 | 14700,9 | 14908,6 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 53 | 14 | 9 | 7 |
| Requests not satisfied | - | 117 | 27 | 18 | 14 |

Table B.33- System effectiveness for vehicles with capacity for 4 workers XXXIII

| Concentrated Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 28 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 16186,1 | 16534,3 | 16665,8 | 16609,7 | 16748,2 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 50 | 11 | 6 | 4 | 3 |
| Requests not satisfied | - | 111 | 21 | 12 | 8 | 6 |

Table B.34- System effectiveness for vehicles with capacity for 4 workers XXXIV

| Concentrated Demand-40km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 32 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 16344,1 | 17093,5 | 18228,9 | 18109,4 | 18306 | 18246,9 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 85 | 57 | 6 | 4 | 1 | 0 |
| Requests not satisfied | - | 159 | 120 | 12 | 8 | 2 | 0 |

Table B.35-System effectiveness for vehicles with capacity for 4 workers XXXV

| Concentrated Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 vehicles | 5 | 10 | 15 | 20 | 24 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 11891,7 | 12196,2 | 12111,8 | 12307 |
| Periods with no service | - | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 62 | 27 | 23 | 18 |
| Requests not satisfied | - | 131 | 48 | 41 | 31 |

Table B.36- System effectiveness for vehicles with capacity for 4 workers XXXVI

| Concentrated Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 vehicles | 5 | 10 | 15 | 20 | 25 | 28 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 13847,2 | 13964,9 | 13984,3 | 13974,2 | 14044 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 54 | 15 | 10 | 7 | 6 |
| Requests not satisfied | - | 119 | 29 | 20 | 14 | 12 |

Table B.37- System effectiveness for vehicles with capacity for 4 workers XXXVII

| Concentrated Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 32 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 14553,4 | 14690 | 15402,3 | 15361 | 15414,7 | 15366,1 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 70 | 42 | 6 | 5 | 2 | 1 |
| Requests not satisfied | - | 138 | 98 | 12 | 10 | 4 | 2 |

Table B.38-System effectiveness for vehicles with capacity for 4 workers XXXVIII

| Concentrated Demand-20km/h | Nworkers |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| Agreggate Distance [hm] | - | 15588 | 16370,1 | 16826,9 | 16705,3 | 16762,4 | 16869,7 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 70 | 42 | 6 | 4 | 1 | 0 |
| Requests not satisfied | - | 138 | 98 | 12 | 7 | 2 | 0 |

Table B.39- System effectiveness for vehicles with capacity for 4 workers XXXIX

| Concentrated Demand-20km/h | $N_{\text {workers }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 vehicles | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| Agreggate Distance $[\mathrm{hm}]$ | - | 16757,1 | 16248,6 | 17299,1 | 17737,8 | 17795,7 | 17791,6 |
| Periods with no service | - | 0 | 0 | 0 | 0 | 0 | 0 |
| Unserved Sites | - | 70 | 82 | 31 | 4 | 1 | 0 |
| Requests not satisfied | - | 138 | 152 | 46 | 7 | 2 | 0 |

Table B.40- System effectiveness for vehicles with capacity for 4 workers XXXX


[^0]:    The author is entirely responsible for this dissertation, not having been corrected after it's public defense. The Department of Civil Engineering from FCTUC denies any responsability for the use of the information presented.

