COMPARATIVE REVIEW OF POSSIBLE ALTERNATIVES FOR PERFORMING SAFETY ASSESSMENT OF DESIGN RULES FOR STEEL STRUCTURES

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ABSTRACT

Currently, safety assessment is not consistently considered throughout Eurocode 3, mainly due to lack of guidance and existing databanks containing information on distribution of relevant basic variables and steel properties. In addition, some rules are not thoroughly validated, leading to higher uncertainties.

In establishing the partial safety factor - γ_M of a design procedure, scatter related to material and geometrical properties may be isolated from resistance model. Simplified safety assessment procedures are analysed and further tested in which the basic variables are assumed independent from each other, with basis on EN 1990 safety assessment procedure.

In this dissertation, firstly, a theoretical overview is proposed in order to introduce the basic principles in the probability theory; secondly, the basic principles of design codes are presented, focusing on the current European design codes EN 1990 to EN 1999. The design methodologies are listed and their background is discussed, and furthermore focus is given to the procedures applicable to Eurocode 3. The safety assessment procedure, given in Annex D of EN 1990 is also presented and its theoretical background is clarified.

Subsequently, an analytical review of existing methodologies for safety assessment of design rules for steel structures is carried out. The procedures are presented and their field of application is detailed.

Furthermore, numerical validation of simplified procedures, based on assumed distributions of basic variables such as material and cross-section properties, is performed focusing on stability failure modes, in particular flexural buckling of columns.

In addition, a numerical example is presented, which aims at further clarifying latter methodologies. Sensitivity analysis is performed in order to assess the influence of different basic variables. In this example, statistical distributions of the imperfections are also considered.

Finally, the statistical dependence of basic variables is discussed, based on correlation between yield stress and plate thickness. In order to obtain a reasonable correlation coefficient, statistical data from real experiments is used.

This dissertation is part of European research project SAFEBRICTILE RFS-PR-12103 – SEP nº 601596, where safety assessment procedure is developed for brittle to ductile failure modes. Therefore, it is considered a valuable contribution for achieving the project goals and further improvement of the design methodologies in the Eurocodes.

TABLE OF CONTENTS

NOTATIONS

Latin upper case letters

$Cov(.)$	Covariance
$E(.)$	Mean value
E	Event
E_d	Design value of the actions
F_X	Cumulative distribution function for the random variable X
$P(.)$	Probability
P_f	Probability of failure
R_d	Design value of the resistance
S	Certain event
$Var(.)$	Variance
V_X	Coefficient of variation
V_δ	Estimator for the coefficient of variation of the error term
X	Random variable
$\underline{X_m}$	Array of mean values of basic variables
$\underline{X_{nom}}$	Array of nominal values of basic variables

Latin lower case letters

b	Correction factor
$f_X(x)$	Probability density function for the random variable X
$g_{rt}(\underline{X})$	Resistance function used as a design model
$k_{d,n}$	Design fractile factor
n	Number of experiments or numerical results
r_d	Design value of the resistance

r_e	Experimental resistance value
r_n	Nominal value of the resistance
r_t	Theoretical resistance determined with $g_{rt}(\underline{X})$
s	Estimated value for the standard deviation σ
s_Δ	Estimated value for the standard deviation σ_Δ
$\overline{x}$	Estimated value for the mean value

Greek upper case letters

Δ	Logarithm of the error term δ
$\overline{\Delta}$	Estimated value for E(Δ)
Φ	cumulative distribution function (CDF) for the standard normal distribution

Greek lower case letters

α_E	FORM (First Order Reliability Method) sensitivity factor for effects of actions
α_R	FORM (First Order Reliability Method) sensitivity factor for resistance
β	reliability index
γ_F	Partial factor for actions, also accounting for model uncertainties and dimensional variations
γ_f	Partial factor for actions, which takes account of the possibility of unfavorable deviations of the action values from the representative values
γ_M	Partial safety factor for a material property also accounting for model uncertainties and model variations
γ_M^*	Corrected partial safety factor for resistances
γ_m	Partial factor for a material property

γ_{Rd}	Partial factor associated with the uncertainty of the resistance model
γ_{Sd}	Partial factor associated with the uncertainty of the action and/or action effect model
δ	Error term
δ_i	Observed error term for test specimen i
δ_x	Coefficient of variation of the random variable X
λ	Mean value of lnX
μ_x	Mean value of the random variable X
ρ	Correlation coefficient
ξ	Standard deviation of lnX
σ	Standard deviation
σ_{Δ}^2	Variance of the term Δ
σ_X	Standard deviation of the random variable X

1 INTRODUCTION

In real world engineering problems, uncertainties are unavoidable. This issue fully applies to structural engineering. Engineers face problems concerning deviations from their models as well as deviations from the material and geometrical properties on an everyday basis. In this sense, it is important to recognize those problems and to deal with them in an efficient manner. For many years, a way to deal with this problem is using design codes, which incorporate the scatter due to randomness of reality.

In this thesis, it is aimed to compare various methodologies for the safety assessment of stability design rules and the corresponding safety factor γ_M on the basis of EN 1990 [1] and its Annex D. The main objective of this comparison is to support the preparation of clear and unambiguous guidance for the development and assessment of design rules in Eurocode 3 [2].

The structure of this work is the following:

- Firstly, a brief theoretical overview is presented, in order to introduce the reader to the theoretical background of the subject;
- Secondly, an overview of the basis of structural design according to the Eurocodes is presented;
- Subsequently, simplifications for safety assessment of design rules are explained – the methods are briefly summarized and differences between them are emphasized;
- Consequently, a numerical assessment of the possible alternatives is performed and further discussed based on flexural buckling of columns;
- Furthermore, an example is provided in order to illustrate the application of the various alternatives;
- Finally, an attempt to consider statistical dependence between basic variables is provided, focusing on correlation between yield stress and plate thickness;

2 THEORETICAL OVERVIEW

The safety assessment is performed due to the randomness of the relevant basic variables. Therefore, in this section, it is intended to briefly clarify the theoretical background of probability and statistical concepts (as proposed in [3]). The following main topics are included:

- Uncertainties;
- Fundamentals of probability theory;
- Probability distributions;
- Introduction to statistics;
- Covariance and correlation;
- Moments of functions of random variables;
- Regression analysis;

2.1 UNCERTAINTY IN ENGINEERING

Uncertainties are existing in engineering problems and are clearly unavoidable. The available data is often incomplete or insufficient and inevitably contains variability. Moreover, the design methodologies used to estimate and/or predict reality lie on certain assumptions and simplifications, and therefore introduce additional uncertainty. In practice, two broad types of uncertainties are recognized: i) aleatory uncertainty associated with the randomness of the underlying phenomenon that is demonstrated as variability in the observed information; ii) epistemic uncertainty associated with the imperfect models of the real world due to insufficient or imperfect knowledge of the reality;

Many phenomena which are of engineering focus contain randomness, in a sense that measurements or experiments differ one from each other even though they are performed in identical conditions. In other words, there is a range of occurrence, and even certain values may occur more frequently than others. The variability characteristics in such data or information have a statistical nature. For example, aleatory uncertainty can be the variability of the yield stress in steel, deviations of the geometrical properties, etc. Steel profiles are produced in the same conditions; the tests are performed using the same test set up, usually standardized; however, not always the same measure is found, but rather a range of measurements. This problem is of particular interest in this thesis. Another example can be the compression strength of concrete which is highly variable. The parameters which influence the production of concrete are many and some of them are hard to control. Therefore, the resulting strength can be found with a high coefficient of variation.

As already mentioned, the other main type of uncertainty is associated with the imperfect knowledge or the so-called epistemic uncertainty. In engineering problems, the solutions always rely on idealized models of the real world. These models –mathematical, laboratory, numerical etc. – are imperfect representations of the real problem. They are inaccurate with some unknown degree of error and thus are imposing uncertainty. This uncertainty sometimes can be more significant than the aleatory uncertainty. An example of epistemic uncertainty is every design rule that is written in the design code. Design rules have uncertainties incorporated, although these rules are calibrated in a way that the variability of this uncertainty aims to be always safe-sided.

An efficient way to deal with uncertainties is given by the probability theory and statistics. They can be used effectively to "quantify" the uncertainty and therefore to provide aid in decision making process. In the following sections, the essentials of probability theory are briefly summarized.

2.2 FUNDAMENTALS OF PROBABILITY THEORY

The probability can be considered as a numerical measure of the likelihood of occurrence of an event within a set of all possible alternative events. Therefore, an *event* can be identified as a main unit in the formulation. Subsequently, the first requirement in the formulation of the probabilistic problem is the identification of the set of all possibilities i.e. the *probability space* for the event of interest. Probabilities are always associated with specific events within certain probability space and they are only valid for that specific probability space. Hence, each probability space can contain many events, and each event has certain probability of occurrence in that probability space.

In order to formulate the probabilistic problem, elementary set theory is used. Following the set theory, the set of all probabilities in a probabilistic problem is collectively defined as a *sample space*, and each of the individual possibilities is a *sample point*. An *event* is recognized as a subset of the sample space.

The sample spaces can be discrete or continuous, as the discrete space can be finite or infinite. The following special events are recognized within the sample spaces:

- Impossible event – ϕ, it is an even with no sample point. This event can be also referred as an empty set;
- Certain event – S, it is the event containing all sample points in the sample space, therefore the certain event is the space itself;
- Complementary event $\overline{E}$, of the event *E* which contains all sample points which are not in *E*;

The concept is further clarified by *Figure 2.1*, in which the so-called Venn diagram is used to illustrate the sample space S and its events. Moreover, the union or intersection of events may be of interest, as shown in *Figure 2.2*. Those operations are similar to sum and multiplication of numbers, however they are not of special interest here and they will not be further discussed.

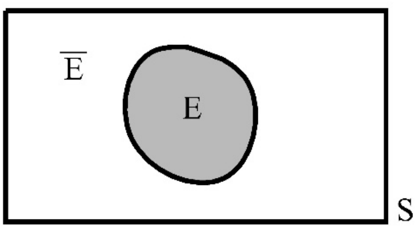

Figure 2.1Venn diagram of sample space S

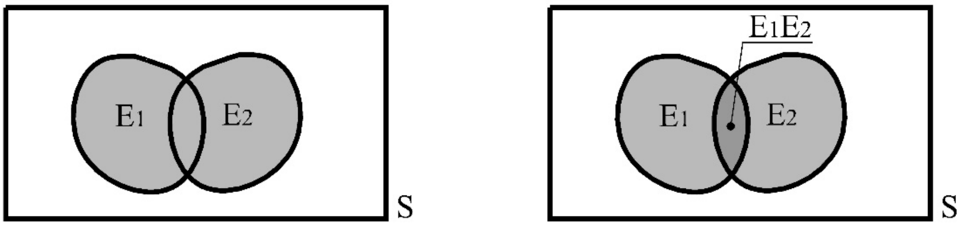

Figure 2.2Examples of union and intersection events

Another type of events is the mutually exclusive events, those are events which cannot occur simultaneously and their corresponding subsets do not have intersections. An example of mutually exclusive events is the steel stress being above and below the yield strength at the same time. The mutually exclusive event should not be confused for statistically independent events. The statistically independent events are those whose occurrence does not exclude the occurrence of the other. They might exist simultaneously; however, there is no dependence, on the contrary in case of mutually exclusive events, the occurrence of both is impossible.

As any other mathematical model, the theory of probability is based on few axioms. The axioms of the probability theory are:

- **Axiom 1**: for every event E in a sample space S, there is a probability:

$$P(E) \geq 0 \tag{2.1}$$

- **Axiom 2**: the probability of the certain event S is:

$$P(S) = 1.0 \tag{2.2}$$

- **Axiom 3**: for two events E1 and E2 which are mutually exclusive, the following can be written:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) \tag{2.3}$$

2.3 PROBABILITY MODELS

When dealing with probabilities, the basic variables are defined as a range of possible values, unlike the deterministic problems where the variables are associated with actual values. Therefore, when using probabilities the random variables are defined as upper case letters. If X is a random variable then $X=x$, $X>x$ or $X<x$ represent different events, where x belonging to (a;b) is the mapping of the event. In *Figure 2.3* the concept of mapping is defined, as the sample space was previously clarified when dealing with sets. In order to apply the probability concepts, it is more convenient to use intervals rather than the sets as explained in the previous section.

An example of two defined events is presented on *Figure 2.3*:

$$E_1 = (a < X < b)$$

$$E_2 = (c < X < d)$$

And their intersection is found as:

$$E_1 \cap E_2 = (c < X < b)$$

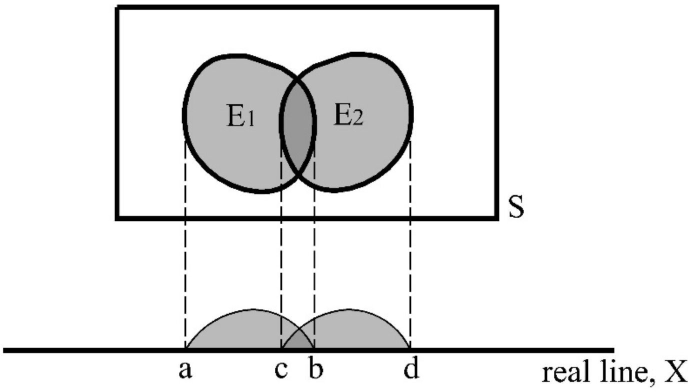

Figure 2.3Mapping through X

The random variables, previously defined, are associated with probability measures called probability distributions or "probability law". For each random variable, its probability distribution can always be described by its cumulative distribution function (CDF):

$$F_X \equiv P(X \leq x) \quad \forall x \tag{2.4}$$

In case that X is a discrete random variable, its probability distribution is described by probability mass function (PMF), where the CDF can be found as:

$$F_X \equiv \sum_{\forall x_i \leq x} P(X = x_i) = \sum_{\forall x_i \leq x} p_X(x_i) \tag{2.5}$$

On the contrary, if X is continuous, it is defined in an interval, therefore for specific value X=x, only the probability density can be obtained, there is no probability. Hence, for continuous random variables the probability law is described by the probability density function, which is denoted as $f_X(x)$ such that:

$$P = (a < X \leq b) = \int_a^b f_X(x)dx = F_X(x) \tag{2.6}$$

$$P = (a < X \leq b) = \int_{-\infty}^{b} f_X(x)dx - \int_{-\infty}^{a} f_X(x)dx \tag{2.7}$$

Every function used to describe the probability distribution of a random variable should satisfy the axioms of the probability theory previously listed, in a way that:

- $F_X(-\infty) = 0$ and $F_X(\infty) = 1.0$
- $F_X(x) \geq 0$ for all x;
- $F_X(x)$ is continuous with x;

Any random variable in practice can be fully described by its probability distribution and each needed parameter can be obtained from the CDF, although in reality it is often difficult to know the exact distribution of a variable. Therefore, assumptions on the distribution can be made based on its main descriptors.

The random variable is associated with a range of values, thus a central value would be of specific interest. Such value is the so-called mean value which represents the weighted average of the random variable (it is defined as weighted average, since for *x* of *X* there is an associated probability). The mean value is often denoted with *E(X)* and it can also be referred as expected value. It can be found from the following expressions:

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)dx = \mu_X \tag{2.8}$$

$$E(X) = \sum_{\forall x_i} x_i p_X(x_i) = \mu_X \tag{2.9}$$

Another very useful parameter is the degree of the dispersion of the random variable. It is of special interest, since it shows how close/far from the mean value are all the values spreading. Therefore, this descriptor should be defined as a function of the mean value. This measure is known as the variance of the distribution or the second central moment. It can be found using the following expressions for the continuous and discrete variable respectively:

$$Var(X) = \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x)dx \tag{2.10}$$

$$Var(X) = \sum_{\forall x_i} (x - \mu_X)^2 x_i p_X(x_i) \tag{2.11}$$

By expanding the expressions one can find:

$$Var(X) = E(X^2) - {\mu_X}^2 \tag{2.12}$$

For practical purposes, the measure of dispersion is referred to the square root of the variance and it is called *standard deviation*:

$$\sigma_X = \sqrt{Var(X)} \tag{2.13}$$

Since its quantity as a single value might not give significant understanding about the degree of dispersion, it is normalized with regard to the mean is used and it is called *coefficient of variation*:

$$\delta_X = \frac{\sigma_X}{\mu_X} \tag{2.14}$$

Finally, there are several widely known distributions, whose parameters are previously computed and organized in so-called probability tables, which makes them very attractive to use.

2.4 PROBABILITY DISTRIBUTIONS

2.4.1 THE GAUSSIAN (NORMAL) DISTRIBUTION

The Gaussian distribution is probably the most famous and widely used probability distribution. Its PDF for a continuous random variable X is:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \qquad x \in (-\infty;\infty) \tag{2.15}$$

where μ and σ are the parameters of the distribution, in this particular case the mean and the standard deviation.

A Gaussian distribution with parameters $\mu=0$ and $\sigma=1.0$ is called Standard Normal Distribution and its PDF is given by:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}x^2\right] \qquad x \in (-\infty;\infty) \tag{2.16}$$

Its CDF is denoted as Φ, and there are tabulated values of Φ. The probability of an event can be visualized in *Figure 2.4* as the shaded area. In addition, by having the probability, the inverse function of Φ may be used:

$$\Phi(s) = F_S(s)$$

$$s_p = \Phi^{-1}(p)$$

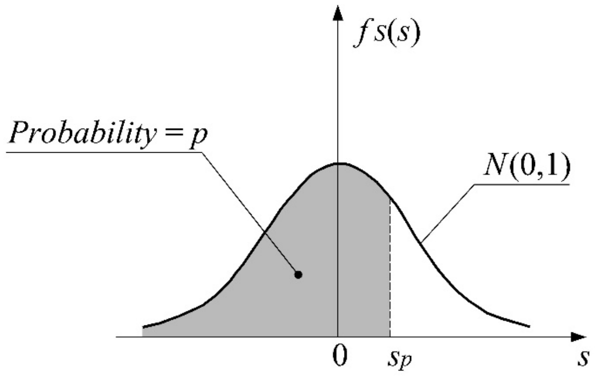

Figure 2.4 The Standard Normal Distribution

Moreover, due to the symmetry of the PDF of the standard normal distribution about zero, then the following can be written:

$$\Phi(-s) = 1 - \Phi(s)$$

A convenient way to use the CDF of the standard normal distribution can be derived from Eq.2.7. The probability can then be found using the expression:

$$P = (a < X \leq b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right) \tag{2.17}$$

2.4.2 THE LOGNORMAL DISTRIBUTION

The lognormal distribution is also a very popular probability distribution. The PDF of the lognormal distribution can be found as:

$$f_X(x) = \frac{1}{\xi x\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda}{\xi}\right)^2\right] \qquad x \in [0;\infty) \tag{2.18}$$

The parameters of the distribution are λ and ξ, which are respectively the mean and standard deviation of *lnX*. It can be easily proven that for a random variable *X* with a lognormal distribution and parameters of the distribution λ and ξ, then *lnX* is normal with mean λ and standard deviation ξ, i.e. $N(\lambda, \xi)$. This is a valuable feature which can be used similarly to Eq.(2.17):

$$P = (a < X \leq b) = \Phi\left(\frac{\ln b - \lambda}{\xi}\right) - \Phi\left(\frac{\ln a - \lambda}{\xi}\right) \tag{2.19}$$

It was seen that both distributions can be connected, and therefore the transformation expressions can be derived from the normal to lognormal parameters and vice-versa, using:

$$\lambda = \ln(\mu_X) - \frac{1}{2}\xi^2 \tag{2.20}$$

$$\xi^2 = \ln\left(1 + \left(\frac{\sigma_X}{\mu_X}\right)^2\right)\lambda = \ln\left(1 + \delta_X{}^2\right) \qquad (2.21)$$

2.5 INTRODUCTION TO STATISTICS

In the previous sections, the probability theory was summarized. In the real life, however, distribution of variables is based on observation of data, i.e. the theoretical distributions are chosen in a way to fit the real behaviour. Moreover, the observed data is usually a test sample or many test samples which are part of the population of the observed parameter, and therefore, additional error due to the limited information should be avoided. The statistics constitutes methods to link the real observations with probability theory. Those principles are based on estimates of the real parameters of the distribution. There are different methods for estimating the parameters, however they all should satisfy certain requirements:

- Unbiasedness – if repeated estimations of the parameter are performed and their mean is equal to the parameter, then the estimator is called unbiased;
- Consistency – if the sample size approaches infinity, then the estimator tends to the value of the parameter;
- Efficiency – one estimator is more efficient than the other, if the variance of the first is smaller than the second;
- Sufficiency – an estimator is considered sufficient, if it can capture all the information in a sample that is relevant for the estimation of the parameter;

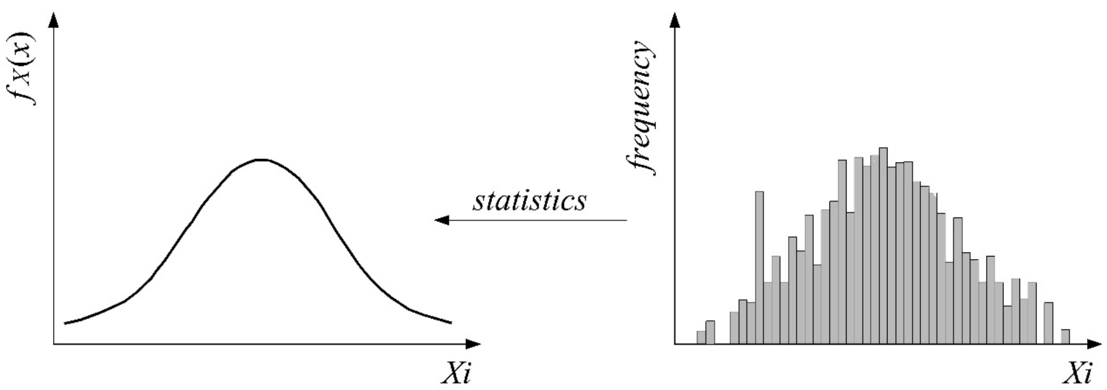

Figure 2.5 Statistics

In reality, however, it is hardly possible to satisfy all of the above parameters of an estimator. Usually, the properties are chosen with respect to the specific needs for estimation.

Previously, it was presented that the mean and standard deviation of a distribution are of specific interest and hereby, the unbiased estimators for the sample mean and the sample variance are presented:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2.22}$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{2.23}$$

2.6 COVARIANCE AND CORRELATION

When there are two random variables *X* and *Y*, there may be a relationship between them. In particular, the presence or absence of a linear statistical relationship is determined as firstly the joint second moment of *X* and *Y* is observed:

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{X,Y}(x,y)dxdy$$

In case, the variables are statistically independent, the equation becomes:

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{X,Y}(x,y)dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_X(x) f_Y(y)dxdy = \int_{-\infty}^{\infty} x f_X(x)dx \int_{-\infty}^{\infty} y f_Y(y)dy = E(X)E(Y)$$

The joint central moment is the covariance of X and Y, i.e.:

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = E(XY) - E(X)E(Y)$$

The significance of the covariance can be studied from the latter equation. If the covariance is large and positive, then the values of *X* and *Y* tend to be both large or both small relatively to their respective means, whereas if the covariance is large and negative, the values of *X* tend to be large when the values of *Y* are small and vice versa, relatively to their means. However, if the covariance is small or zero, there is weak or no (linear) relationship between the values of *X* and *Y*; or the relationship may be non-linear. Therefore, the covariance is a measure of linear relationship between the variables. For practical purposes, a normalized value of the covariance is often used in the literature – so called correlation coefficient, which is defined as:

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \tag{2.24}$$

The correlation coefficient can range between -1 and 1. Its physical representation can be observed in the *Figure 2.6* and *Figure 2.7.*

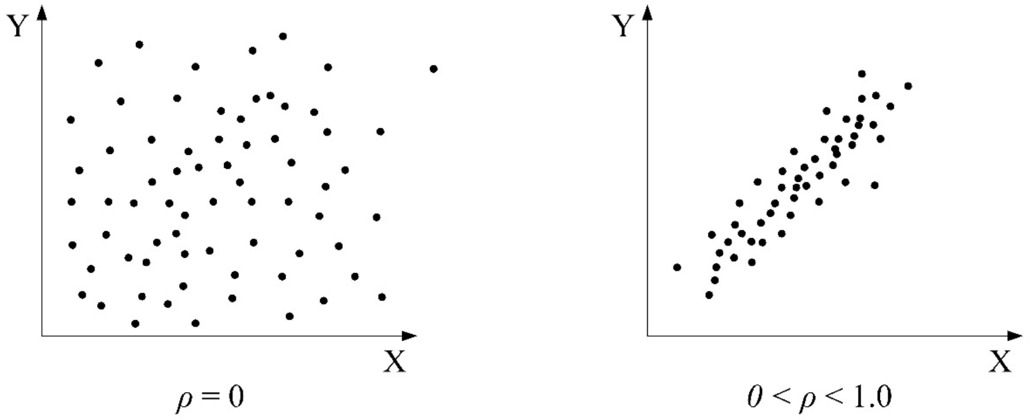

Figure 2.6Correlation coefficient

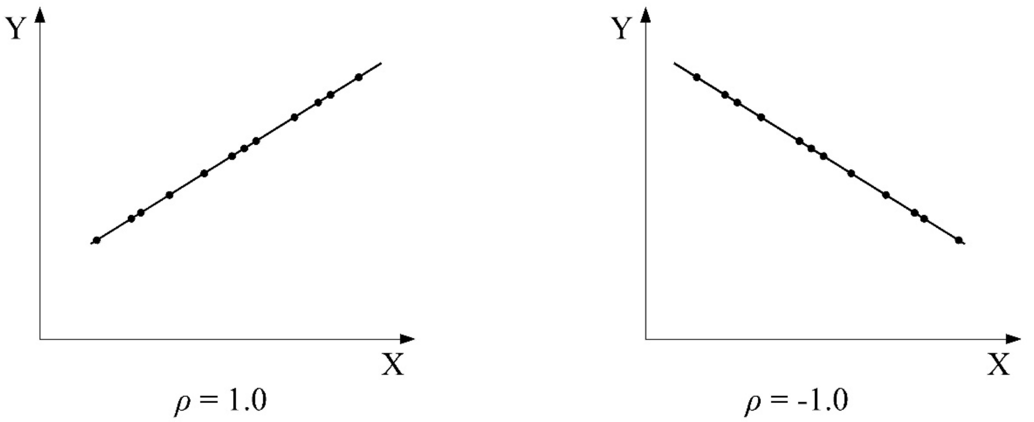

Figure 2.7 Correlation coefficient

Furthermore, an estimation of the correlation coefficient (as proposed in [3]) is found based on a set of n pairs of observations:

$$\rho_{X,Y} = \frac{1}{n-1} \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{s_X s_Y} \tag{2.25}$$

where $\overline{x}$, $\overline{y}$, s_X and s_Y are the sample means and sample standard deviations respectively.

2.7 MOMENTS OF FUNCTIONS OF RANDOM VARIABLES

Usually, in real life engineering problems, functions of basic variables are used. The probability distributions of a random function are usually difficult to derive analytically. In such cases, Monte Carlo simulations can be used. However, it is possible to use the moments of the distribution, particularly the mean and variance, of the function as an approximation of the probability distribution.

This approximation approach is often sufficient for practical purposes, even though the real distribution is left undetermined. Those moments are functionally related to the moments of the individual basic variables and therefore may be derived approximately as functions of the moments of the basic variables.

A function of several random variables Y is considered,

$$Y = g(X_1, X_2, ..., X_n)$$

where the approximate mean and variance of Y can be obtained as follows:

The resistance function can be simplified (as proposed in [3]) by expansion in a Taylor series about the mean values:

$$Y = g(\mu_{X_1}, \mu_{X_2}, ... \mu_{X_n}) + \sum_{i=1}^{n}(X_i - \mu_{X_i})\frac{\partial g}{\partial X_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - \mu_{X_i})(X_j - \mu_{X_j})\frac{\partial^2 g}{\partial X_i \partial X_j} + ...$$

If the series is truncated after the linear terms, i.e.,

$$Y \cong g(\mu_{X_1}, \mu_{X_2}, ... \mu_{X_n}) + \sum_{i=1}^{n}(X_i - \mu_{X_i})\frac{\partial g}{\partial X_i} \tag{2.26}$$

the first order mean and variance are obtained as follows:

$$E(Y) \cong g(\mu_{X_1}, \mu_{X_2}, ... \mu_{X_n}) \tag{2.27}$$

$$Var(Y) \cong \sum_{i=1}^{n}\sigma_i^2\left(\frac{\partial g}{\partial X_i}\right)^2 + \sum_{i,j=1}^{n}\sum_{i \neq j}^{n}\rho_{i,j}\sigma_{X_i}\sigma_{X_j}\frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j} \tag{2.28}$$

In case that the basic variables X_i and X_j are not correlated (i.e. statistically independent), in other words $\rho_{i,j}=0$, then the variance becomes:

$$Var(Y) \cong \sum_{i=1}^{n}\sigma_i^2\left(\frac{\partial g}{\partial X_i}\right)^2 \tag{2.29}$$

The latter equations may also be referred as “propagation of uncertainty”. It is observed that the variance is a function of both the variances of the basic variables and of the sensitivity of the sensitivity coefficients as represented by the partial derivatives.

2.8 REGRESSION ANALYSIS

If two random variables are considered, there may be a relationship between them. Moreover, the presence of randomness makes the relationship not unique and thus it leads to scatter. The two variables can be plotted together in a scattergram type graph, see *Figure 2.8.*

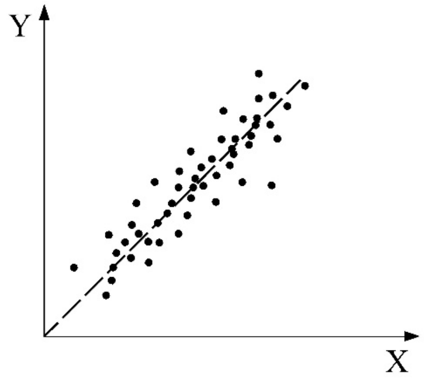

Figure 2.8 Scattergram

where the equation of a straight line passing through the origin is given by:

$$Y = bX \tag{2.30}$$

There is a trend for the values of Y to increase with increasing *X*. However, no sample point of X will be accurate and representative enough to give absolutely perfect information on *Y*. A line through the origin (regression line) may be used to approximate the trend, however many such lines exist. In order to minimize the cumulative error and assuming that the variance of the residual *ε* is constant, a least square calculation is usually performed, based on the following quadrature of the residual:

$$\varepsilon^2 = \sum (y_i - bx_i)^2 \tag{2.31}$$

Minimization of the error is obtained by setting the derivative of ε^2 with respect to *b* to zero. The linear regression coefficient *b* is then found:

$$b = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} \tag{2.32}$$

However, the variance of -*ε* may not be constant. In such cases equations (2.31) and (2.32) become:

$$\varepsilon^2 = \sum w_x (y_i - bx_i)^2 \tag{2.33}$$

$$b = \frac{\sum_{i=1}^{n} w_x x_i y_i}{\sum_{i=1}^{n} w_x x_i^2} \tag{2.34}$$

where:

$$w_x = \frac{1}{Var(\varepsilon)} \tag{2.35}$$

If *w* is proportional to *X* then expression (2.34) can be rewritten as:

$$b = \frac{\sum_{i=1}^{n} (k/x_i)\, y_i x_i}{\sum_{i=1}^{n} (k/x_i) x_i^{\ 2}} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} = \frac{\bar{y}}{\bar{x}} \tag{2.36}$$

In case that w_x is proportional to r_t^2 then expression (2.34) becomes:

$$b = \frac{\sum_{i=1}^{n} (k/x_i^{\ 2})\, y_i x_i}{\sum_{i=1}^{n} (k/x_i^{\ 2})\, x_i^{\ 2}} = \frac{\sum_{i=1}^{n} y_i/x_i}{n} = \frac{1}{n}\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \tag{2.37}$$

This last assumption is correct if *X* is proportional to Y, excluding the sampling error. Furthermore, the three estimators for b (2.32), (2.36) and (2.37) are all unbiased. As it is explained in [4], the choice of approximation is a question of precision: as the first one should be used when the standard deviation of the residual is constant (2.32); the second one when the standard deviation of the residual is proportional to *X* (2.36); and the third one when the standard deviation of the residual is proportional to X^2 (2.37).

3 BASIS OF DESIGN

The structural reliability may be verified using fully probabilistic approach or partial factor method. In practice, the partial factor method is often applied, since it incorporates the variability in the design code and offers a clear guidance to the engineer. The fully probabilistic methods are not as frequently used because usually, there is not sufficient information for their application, moreover the outcome depends on the person who performs the analysis and therefore the level of safety between different analyses, would be left undetermined. A model application of design code is given by [5] where the fully probabilistic approach is adopted.

However, here focus is given to the current structural design codes in Europe are EN 1990 to EN 1999. The basic document of this family of codes is EN 1990 – *Basis of structural design* [1]. It establishes principles and requirements for the safety assessment of structures; it describes the basis of their design and provides guidelines for structural reliability.

In this chapter, a brief introduction to the basis of design as well as its background is presented.

A procedure for design assisted by testing is provided in the scope of EN 1990 – given in its Annex D. The latter procedure is based on a semi – probabilistic approach. Its theoretical background is also summarized in the following subsections.

3.1 GENERAL OVERVIEW OF EN 1990

All parts of the Eurocodes are based on the partial safety factor method. EN 1990 states the basis of the method. The partial safety factor method recognizes relevant design situations. The safety factors are used on load and on the resistance sides and the design is considered adequate whenever the appropriate limit states are verified:

$$E_d \leq R_d \tag{3.1}$$

where:

E_d – is the design value of the actions;

R_d – is the design value of the resistance;

The safety factors are established based on a statistical evaluation of experimental data; or based on a calibration to experience derived from a long building tradition. The partial safety factors should be calibrated such that the reliability level is as close as possible to the target reliability. The calibration of safety factors can be performed based on full probabilistic methods, or on First Order Reliability Methods. The full probabilistic approach is often not possible to use due to the lack of sufficient statistical data.

However, in [6] it is reported that the analysis can be based on the Bayesian interpretation of probabilities, where the probabilities are evaluated using available data and previous knowledge. It is believed that if the analysis is carried out carefully and based on large number of data points, the results would be correct.

Figure 3.1 illustrates the various possible reliability methods according to EN 1990 [1].

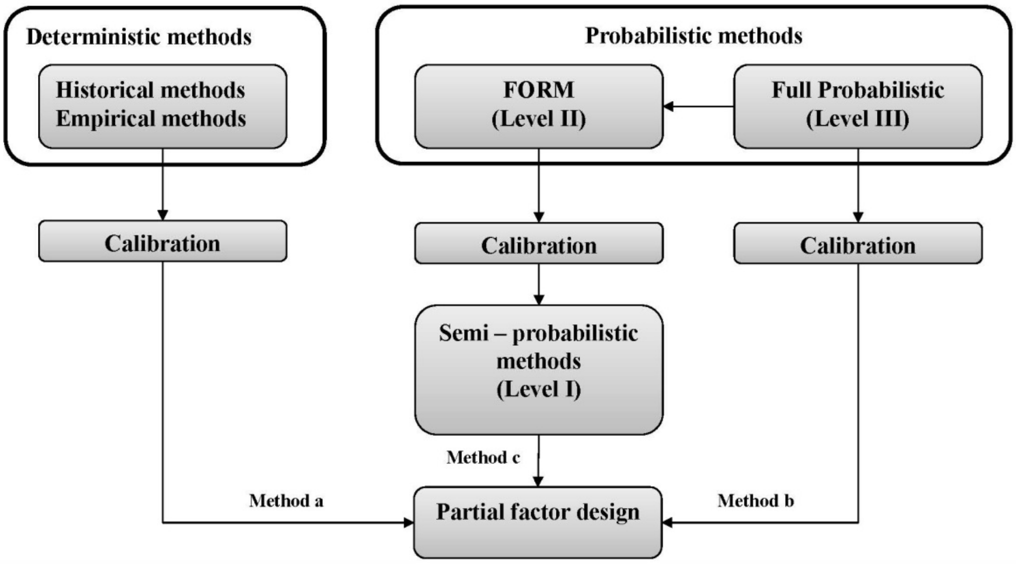

Figure 3.1 Possible Reliability methods [1]

The level of safety in EN 1990 is chosen according to Consequence classes (CC) defined in Annex B. The consequence classes establish the reliability differentiation of the code by considering the consequence of failure or malfunction of the structure. The Consequence Classes (CC) correspond to Reliability classes (RC), which define the target reliability level though the reliability index β. This index defines the probability of failure, given by:

$$P_f = \Phi(-\beta) \tag{3.2}$$

where Φ is the cumulative distribution function (CDF) for the standard normal distribution.

The reliability index covers the scatter on both resistance and action sides. It can be expressed in terms of number of standard deviations as shown on *Figure 3.2.*

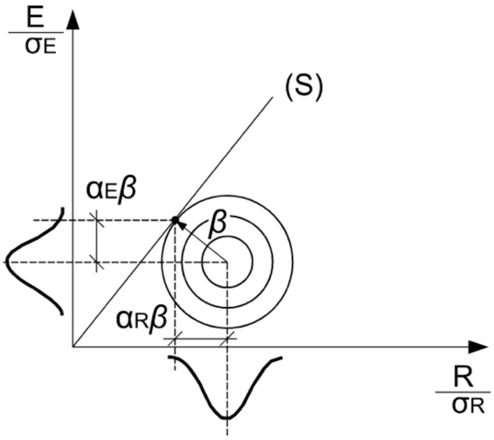

Figure 3.2 Reliability index β [1]

According to [7], "*the target reliability index or the target failure probability is the minimum requirement for human safety from the individual or societal point of view when the expected number of fatalities is taken into account. It starts from an accepted lethal accident rate of 10^{-6} per year, corresponding to a reliability index β_l = 4.7*". The reference period (the design life) depends on the Reliability class, i.e. for most of the structures it is 50 years which leads to *β=3.8*.

The probability of failure as expressed in Eq. (3.2) includes the loading and the resistance parts. However, EN 1990 allows one to separate the scatter due to loading and resistance in terms of coefficients α_E and α_R, respectively (see *Figure 3.2*), where:

$$\sqrt{\alpha_R{}^2 + \alpha_E{}^2} \approx 1.0 \qquad (3.3)$$

The partial safety factors related to the resistance are determined based on the following expression:

$$P(r \le r_d) = \Phi(-\alpha_r \beta) \qquad (3.4)$$

where *r* stands for resistance and r_d is the design resistance. The factor α_R may be assumed to have a fixed value of 0.8 in case the standard deviation of the load effect and the resistance do not deviate very much ($0.16<\sigma_E/\sigma_R<7.6$) [1]. This simplification is *crucial* for a standardized determination of the partial safety factors for the resistance side without the need to simultaneously consider the action side.

3.2 METHODOLOGICAL ASSUMPTIONS FOR DESIGN RESISTANCE

3.2.1 DESIGN RESISTANCE

In Section 6 of EN 1990, three different alternatives for the evaluation of the design resistance are proposed, as follows:

METHOD 1 (clause 6.3.5(1)):

On the resistance side, the general format is given in expressions (3.5) to (3.8).

$$R_d = \frac{1}{\gamma_{Rd}} R\left\{\eta_i \frac{X_{k,i}}{\gamma_{m,i}}; a_d\right\} \quad i \geq 1 \tag{3.5}$$

where:

γ_{Rd} –partial safety factor covering uncertainty in the resistance model, plus geometric deviations if these are not modeled explicitly;

$X_{k,i}$ – characteristic value of material property i;

η_i – conversion factor, which can alternatively be incorporated in γ_M (see expression (3.7));

a_d – design value of geometrical data, it can be represented by nominal values in cases not severely affected by geometrical shape deviations.

$$a_d = a_{nom} \tag{3.6}$$

Expression (3.5) may be simplified as follows:

$$R_d = R\left\{\eta_i \frac{X_{k,i}}{\gamma_{M,i}}; a_d\right\} \quad i \geq 1 \tag{3.7}$$

$$\gamma_{M,i} = \gamma_{Rd} \times \gamma_{m,i} \tag{3.8}$$

Further simplifications may be given for different structural materials but they should not reduce the level of reliability [1].

METHOD 2 (clause 6.3.5(3)):

Alternatively to (3.7), the design resistance may be obtained directly from the characteristic value of product or material resistance, without explicit determination of the design values for individual basic variables:

$$R_d = \frac{R_k}{\gamma_M} \tag{3.9}$$

The latter is applicable to products or members made of a single material and it is also used in connection with Annex D of EN 1990. It is noted that this simplified approach is used for the evaluation of the design resistance of most failure modes in Eurocode 3 [2].

METHOD 3 (clause 6.3.5(4)):

Alternatively to expressions (3.7) and (3.9), for structures or structural members that are analysed by non-linear methods and comprise more than one material acting together, the following expression can be used:

$$R_d = \frac{1}{\gamma_{M,i}} R\left\{\eta_i X_{k,i}; \eta_i X_{k,i} \frac{\gamma_{m,1}}{\gamma_{m,i}}; a_d\right\} \quad i \geq 1 \tag{3.10}$$

In section 2.3.4 of EN 1993-1-1 [2], it is stated that the evaluation of design resistance should be based on equations (3.9) or (3.10).

The method proposed by expression (3.10) will not be addressed in this study.

3.2.2 PARTIAL FACTORS IN EN 1990

The following partial factors are defined in EN 1990:

- γ_F – Partial factor for actions, also accounting for model uncertainties and dimensional variations;
- γ_f – Partial factor for actions, which takes account of the possibility of unfavorable deviations of the action values from the representative values;
- γ_{Sd} - Partial factor associated with the uncertainty of the action and/or action effect model;
- γ_M – Partial safety factor for a material property also accounting for model uncertainties and model variations;

- γ_m – Partial factor for a material property;
- γ_{Rd} – Partial factor associated with the uncertainty of the resistance model;

The relation between individual partial factors in the Eurocodes is schematically shown in *Figure 3.3*:

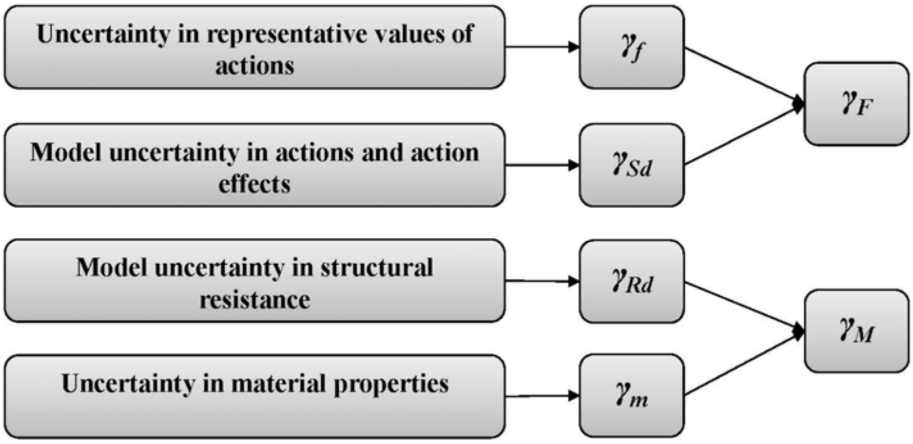

Figure 3.3Relation between individual partial factors [1]

In accordance with *Figure 3.3* and following [8],

$$\gamma_F = \gamma_f \gamma_{Sd} \tag{3.11}$$

$$\gamma_M = \gamma_m \gamma_{Rd} \tag{3.12}$$

Expression (3.12) is equal to the one given in (3.8) and may be used in conjunction with (3.7).

3.3 Safety assessment procedure – Annex D Design assisted by testing

Annex D of EN 1990 gives a semi-probabilistic procedure for the safety assessment of design methods. According to [8], Annex D distinguishes several types of tests depending on their purpose that may be classified into the two following categories: (i) results used directly in design; (ii) control or acceptance tests. According to the objectives here, the procedures for the statistical determination of resistance models and procedures for deriving design values from tests of type (i) are detailed in the following.

Various types of uncertainties are present in a resistance model and they are unavoidable. As it is explained in [3], they can be associated with inaccuracies of the prediction of the reality or with the natural randomness. In order to quantify the uncertainties testing is used. It can be done by numerical or experimental testing.

The procedure proposed in Annex D is used to evaluate the safety on the resistance side. The resistance function r_t is the theoretical value of the strength.

$$r_t = g_{rt}(\underline{X}) \tag{3.13}$$

The theoretical estimate r_t is compared with the experimental one r_e, which is based on numerical or experimental test results. The procedure considers both types of possible errors, due to epistemic and aleatory uncertainties, and it is presented in the subsequent subsections.

3.3.1 ERROR RELATED TO THE DESIGN MODEL

Design models or "resistance functions" are usually theoretical expressions which include as many relevant physical parameters (i.e. "basic variables") as possible and reasonable. As the design model is introduced in terms of r_t, Eq. (3.13), it should be further verified via numerical or ***e***xperimental tests - r_e. The plot in *Figure 3.4* is similar to *Figure 2.8,* yet using the notations adopted in Annex D of EN 1990.

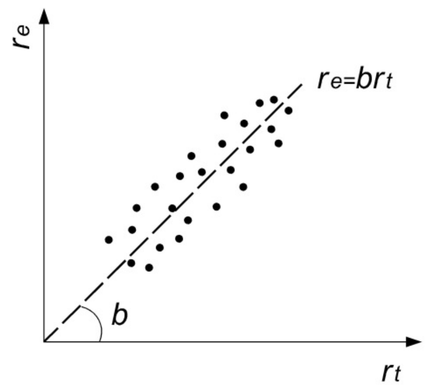

Figure 3.4 Scatter due to epistemic uncertainty

where the equation of the regression line passing through the origin is given by:

$$r_e = br_t \tag{3.14}$$

In Annex D, the assumption that the variance of the residual is constant is adopted, and therefore the regression coefficient b is found from (3.15) (which is coming from expression (2.32)) as previously discussed in section 2.8, equation (3.15) can be considered unbiased estimator for the regression analysis.

$$b = \frac{\sum_{i=1}^{n} r_{t,i} r_{e,i}}{\sum_{i=1}^{n} {r_{t,i}}^2} \tag{3.15}$$

The scatter in *Figure 3.4* represents the epistemic uncertainty which is related to the differences that arise between the adopted design model and the reality. This variation is caused by the simplifications of every design model when compared to reality.

The differences are considered in terms of the error δ_i:

$$\delta_i = \frac{r_{e,i}}{b\,r_{t,i}} = \frac{b\,r_{t,i} + \varepsilon_i}{b\,r_{t,i}} \xrightarrow{\varepsilon_i \to 0} \delta_i \approx 1 \tag{3.16}$$

Assuming that the resistance distribution follows a lognormal distribution, the logarithm of the error δ_i is given by:

$$\Delta_i = \ln(\delta_i) \tag{3.17}$$

The mean value of Δ is found from:

$$\overline{\Delta} = \frac{1}{n}\sum_{i=1}^{n}\Delta_i \tag{3.18}$$

The estimate of the error variance is:

$${s_\Delta}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\Delta_i - \overline{\Delta}\right)^2 \tag{3.19}$$

Finally, the estimator for the coefficient of variation of the error term δ_i is given by:

$$V_\delta = \sqrt{\exp({s_\Delta}^2) - 1} \tag{3.20}$$

3.3.2 ERROR RELATED TO THE BASIC INPUT VARIABLES

The aleatory uncertainty, accounting for the natural randomness, is associated with the basic input variables X_i – yield strength, ultimate strain, geometrical properties, etc.

Considering the correction factor for the model variance, the resistance function (3.13) may be rewritten as:

$$r = bg_{rt}(X_1, X_2 ... X_n) \tag{3.21}$$

Furthermore, as previously presented in section 2.7, approximations about the moments of functions of random variables, can be found using expressions (2.27) and (2.29) (as proposed in [3]) by expansion in a Taylor series about the mean values. Hence the following expressions can be found, using the notations adopted in EN 1990;

If the series is truncated after the linear terms, i.e.,

$$r \cong bg_{rt}(\underline{X_m}) + \sum_{i=1}^{n} b(X_i - X_{m,i}) \frac{\partial g_{rt}}{\partial X_i} \tag{3.22}$$

the first order mean and variance are obtained as follows:

$$E(r) \cong bg_{rt}(\underline{X_m}) \tag{3.23}$$

$$Var(r) \cong \sum_{i=1}^{n} \sigma_i^2 \left(\frac{\partial g_{rt}}{\partial X_i} \right)^2 \tag{3.24}$$

Expression (3.24) is based on the assumption that the basic variables X_i and X_j are statistically independent and is obtained from the truncated series of Eq. (2.29) as given in Eq. (3.22), otherwise more terms shall be included from Eq.(2.28).

The sensitivity of the resistance function to the variability of the basic input parameters is considered through the coefficient of variation V_{rt}. In case that the resistance function is not very complex such as a simple product function, V_{rt} may be obtained as follows:

$$Var(\ln r) = \sum_{i=1}^{n} Var(\ln X_i) \tag{3.25}$$

$$\ln(V_r + 1) = \sum_{i=1}^{n} \ln(V_{x,i} + 1) \tag{3.26}$$

$$V_{rt}^2 = \sum_{j=1}^{k} V_x^2 \tag{3.27a}$$

However, if the resistance function is expressed by a more complex function, then V_{rt} should be based on Eq. (3.24), leading to:

$$V_{rt}^{\ 2} = \frac{1}{g(\underline{X_m})^2} \sum_{j=1}^{k} \left(\frac{\partial g(X_j)}{\partial X_j} \sigma_j \right)^2 \tag{3.27b}$$

3.3.3 THE PARTIAL SAFETY FACTOR

Finally, both uncertainties are combined in order to obtain the partial safety factor – γ_M.

Expression (3.28) combines the effect of scatter due to design model and scatter due to the basic random variables, is obtained similarly to Eq. (3.26):

$$(V_r^{\ 2} + 1) = (V_\delta^{\ 2} + 1)(V_{rt}^{\ 2} + 1) \tag{3.28}$$

The second order terms may be ignored if the coefficients of variation are small, leading to:

$$V_r^{\ 2} \cong V_\delta^{\ 2} + V_{rt}^{\ 2} \tag{3.29}$$

The standard deviations of the lognormal variables are given by:

$$Q_\delta = \sqrt{\ln(V_\delta^{\ 2} + 1)} \tag{3.30}$$

$$Q_{rt} = \sqrt{\ln(V_{rt}^{\ 2} + 1)} \tag{3.31}$$

$$Q = \sqrt{\ln(V_r^{\ 2} + 1)} \tag{3.32}$$

From a probabilistic stand point, the design value of the resistance should satisfy the following relation, in case of large number of tests (n>30):

$$P(r \le r_d) = P\left(r \le \frac{r_k}{\gamma_M} \right) = \Phi\left(\frac{\ln\left(\frac{r_k}{\gamma_M}\right) - \lambda_r}{Q} \right) = \Phi(-\alpha_R \beta) \tag{3.33}$$

λ_r is the lognormal mean and it can be found using the following relationship:

$$\lambda_r = \ln(r_m) - \frac{1}{2} Q^2 \tag{3.34}$$

so that

$$\frac{r_k}{\gamma_M} = r_m e^{-\alpha_R \beta Q - 0.5 Q^2} = b g_{rt}(\underline{X_m}) e^{-\alpha_R \beta Q - 0.5 Q^2} \tag{3.35}$$

leading finally to:

$$\gamma_M = \frac{r_k}{b g_{rt}(\underline{X_m}) e^{-\alpha_R \beta Q - 0.5Q^2}} \tag{3.36}$$

In this way, the design value of the resistance considers both uncertainties – the one due to the scatter of the basic input variables and the one related with simplifications introduced in the design model. It also corresponds to the selected reliability level β according to Reliability Classes and the corresponding Consequence Classes.

The characteristic value of the resistance is the resistance function evaluated at nominal values of the basic input variables as follows:

$$r_k = r_{t,nom} = g_{rt}(\underline{X_{nom}}) \tag{3.37}$$

In case of a limited number tests (say n<30) allowance should be made in the distribution of Δ for statistical uncertainties. The distribution should be considered as a central t-distribution leading to (3.38a).

The design value of the resistance function based on the mean values of the input parameters is calculated, depending on the sample size used:

$$r_d = \begin{bmatrix} b g_{rt}(\underline{X_m}) \exp\left(-k_{d,\infty} \frac{Q_{rt}^{\ 2}}{Q} - k_{d,n} \frac{Q_\delta^{\ 2}}{Q} - 0.5Q^2 \right) & n \le 30 \\ b g_{rt}(\underline{X_m}) \exp\left(-k_{d,\infty} Q - 0.5Q^2 \right) & n \to \infty \end{bmatrix} \quad \begin{matrix} (3.38a) \\ (3.38b) \end{matrix}$$

Coefficients $k_{d,\infty}$ and $k_{d,n}$ are design fractile factors, which can be obtained from *Table 3.1*. However; it should be noted that Table 2.1 is based on $\alpha_R\beta$=3.04.

Table 3.1Values for $k_{d,n}$

n	1	2	3	4	5	6	8	10	20	30	∞
V_X known	4.36	3.77	3.56	3.44	3.37	3.33	3.27	3.23	3.16	3.13	3.04
V_X unknown	-	-	-	11.40	7.85	6.36	5.07	4.51	3.64	3.44	3.04

The partial safety factor is then found:

$$\gamma_M^* = \frac{r_{t,nom}}{r_d} \tag{3.39}$$

The procedure proposed in Annex D of EN 1990 is summarized in *Figure 3.5*:

Step 1: Resistance function

$$r_t = g_{rt}(X_1, X_2 ... X_n)$$

Step 2: Compare experimental and theoretical values – ($r_{t,i}$; $r_{e,i}$)

Step 3: Estimate the mean value correction factor – b

$$b = \frac{\sum_{i=1}^{n} r_{e,i} r_{t,i}}{\sum_{i=1}^{n} r_{t,i}^2}$$

Step 4: Estimate the c.o.v of the errors

$$\delta_i = r_{e,i} / b\, r_{t,i} \qquad \Delta_i = \ln(\delta_i) \qquad \overline{\Delta} = \frac{1}{n}\sum_{i=1}^{n} \Delta_i$$

$$s_\Delta^2 = \sum^{n} (\Delta_i - \overline{\Delta})^2 \Big/ (n-1) \qquad V_\delta = \sqrt{\exp(s_\Delta^2) - 1}$$

Step 5: Analyse compatibility – if the resistance function is acceptable

Step 6: Determine c.o.v V_{Xi} of the basic variables

Step 7: Determine the design value r_d of the resistance

$$V_r^2 \cong V_\delta^2 + V_{rt}^2$$

$$Q_\delta = \sqrt{\ln(V_\delta^2 + 1)} \qquad Q_{rt} = \sqrt{\ln(V_{rt}^2 + 1)} \qquad Q = \sqrt{\ln(V_r^2 + 1)}$$

$$r_d = \begin{cases} b g_{rt}(\underline{X_m}) \exp\left(-k_{d,\infty} \frac{Q_{rt}^2}{Q} - k_{d,n} \frac{Q_\delta^2}{Q} - 0.5Q^2\right) & n \leq 30 \\ b g_{rt}(\underline{X_m}) \exp\left(-k_{d,\infty} Q - 0.5Q^2\right) & n \to \infty \end{cases}$$

Figure 3.5Flow chart – Annex D

4 ALTERNATIVES FOR SAFETY ASSESSMENT

4.1 ASSUMPTIONS

The procedures described in section 3.3 are of general application. Consequently, since they have to cover different types of materials and different types of problems, they provide many alternative possibilities, leading to several possible implementations for a given problem. Furthermore, the framework is also necessarily complex, leading to difficult implementation in certain cases. These constitute a difficulty whenever it is necessary to compare two alternative design rules because different implementations will lead to different failure probabilities even when the basic data is the same. Also, very often, the available data is not sufficient to characterize statistically all the relevant basic variables. Finally, carrying out a probabilistic analysis that includes all the relevant variability's may be impossible because of the size of the required sample [9].

In this chapter, various procedures for the implementation of the methodology of Annex D are presented. They include a number of simplifications, starting from the simplest to the most complex and are proposed in the context of design models for the evaluation of the buckling resistance of steel members.

Without loss of generality, in the following, it is assumed that a large number of experiments is available (at least 100 results). Also, whenever it is necessary to exemplify some detail using a specific stability phenomenon, the well-known example of flexural buckling of columns is used.

In the context of the stability of steel structures, the following basic variables should be considered for the evaluation of the error related to the basic input variables (V_{rt}):

- mechanical properties of steel
- cross section and member dimensions
- geometrical imperfections
- residual stresses
- load eccentricity.

In steel structures, the variability of these variables contributes differently to the buckling resistance of steel members:

- the variability of the mechanical properties of steel has a significant relevance, in particular the yield stress. They should be considered as random variables, bearing in mind that the nominal properties of steel are guaranteed values.

- the variability of the cross section dimensions and member lengths may be considered small or negligible in most cases [8]. However, if a systematic deviation from nominal values is identified, they should also be considered as random variables.
- The remaining basic variables listed above (geometric imperfections, residual stresses and load eccentricities) have a crucial effect on the buckling resistance. They could be considered explicitly as random variables in the design model but, because of the complexity of the stability design models, they do not appear explicitly in the stability design expressions. Consequently, they are usually considered implicitly in the models. Whenever relevant (e.g. advanced numerical models), these imperfections should be represented directly by their design values (clause 4.3(1) of EN 1990) or by values corresponding to some prescribed fractile of the available statistical distributions (clause 4.3(3) of EN 1990).

The different alternatives presented in this chapter consider the variability of the (geometrical, material and loading) imperfections implicitly in the models. Hence, the first level of simplifications presented below consists of neglecting the error related to the first two basic variables, i.e., material and geometry. *Table 4.1* summarizes the assumptions for each proposed simplified procedure.

Table 4.1 Simplified procedures

	P0	***P1***	***P2***
Mech. Properties of steel (yield stress)	X (approx.)	X	X
Geometry			X

Finally, it is noted that as a reference simplification, PROCEDURE 0 only considers model uncertainty and material (yield stress) uncertainties and calculates independently the partial safety factors for each of them according to expression (3.12) while the other two follow strictly the methodology of Annex D of EN 1990.

4.2 PROCEDURE 0 (P0)

Figure 4.1 summarizes procedure P0. It only considers model uncertainty and material (yield stress) uncertainty and calculates independently (in an approximate way) the partial safety factors for each of them [9],[10]. Hence, (3.38b) is used with nominal values of material and geometrical properties leading to:

$$\gamma_{Rd} = \frac{r_{t,nom}}{r_d} = \frac{1}{R_m \exp\left(-k_{d,n}Q - 0.5Q^2\right)} \tag{4.1}$$

The material variability is considered independently from the design model and it is evaluated separately as follows**:**

$$\gamma_m = \frac{f_{y,nom}}{f_{y,m}(1-1.64V_{fy})} \tag{4.2}$$

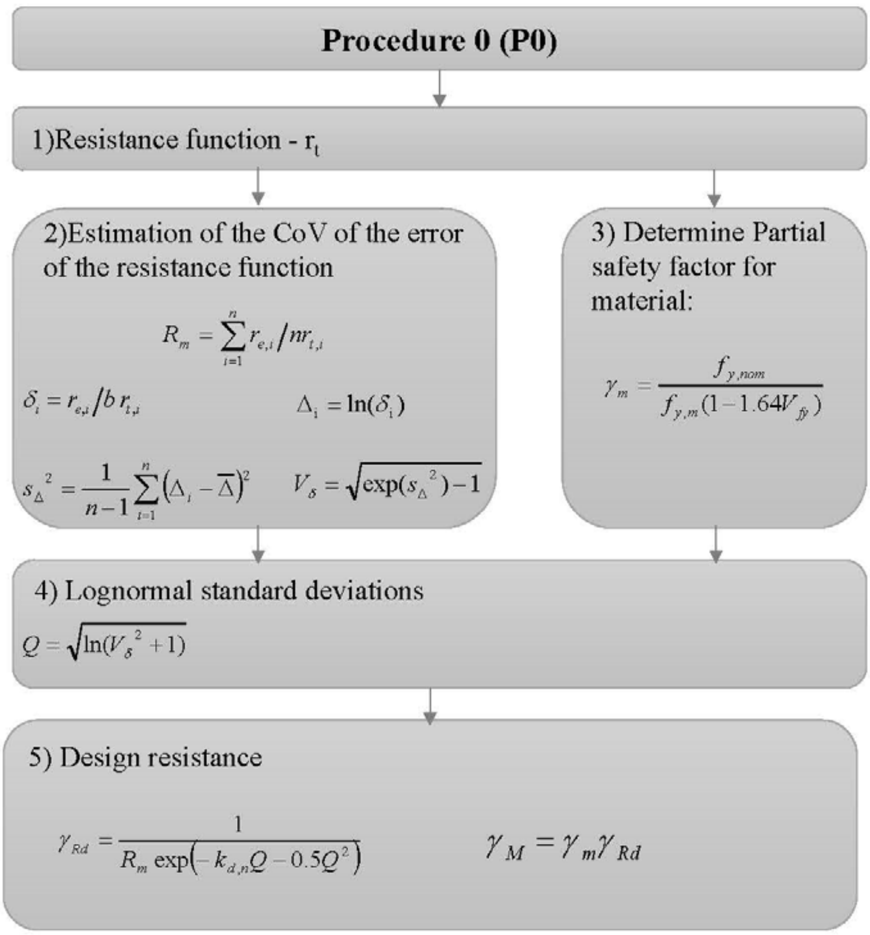

Figure 4.1 - Procedure 0

Geometry is assumed with nominal values.

According to *Figure 4.1*, Step 2, the following assumption is adopted in the construction of the regression line:

$$R_m = \frac{1}{n}\sum_{i=1}^{n}\frac{r_{e,i}}{r_{t,i}} \tag{4.3}$$

This was applied in [9],[10], as previously proposed by [11] and [12]. It is noted that the adoption of different estimators for the regression line is a matter of precision. However, as the sample size tends to infinity, the difference between estimators becomes negligible [4].

It is further noted that *Procedure 0* presents some drawbacks for the evaluation of partial safety factors for stability problems, since it considers γ_{Rd} and γ_m with the same weight, which is not true in cases with high slenderness. Nevertheless, besides its simplicity, it may be considered for other failure modes, e.g. ductile failure modes driven by plasticity. Finally, P0 does not ensure strict compliance with a predefined target failure probability.

4.3 PROCEDURE 1 (P1)

4.3.1 DESCRIPTION

This procedure was also applied in [9],[10]. It constitutes a simplification of Annex D and also disregards the variability of geometry ($a_d = a_{nom}$). It is performed on the basis of expression (3.9). The procedure is summarized in *Figure 4.2.*

A further simplification is introduced with regard to the coefficient of variation of the (error of the) resistance function, V_{rt}, using expression (3.27a). It is simply assumed to be equal to the coefficient of variation of the yield strength. Annex D explicitly allows such assumption in cases where the resistance function is described by a simple product function, as explained before.

$$V_{rt}^{\ 2} = V_{fy}^{\ 2} = \frac{\sigma_{fy}^2}{f_{ym}^2} \tag{4.4}$$

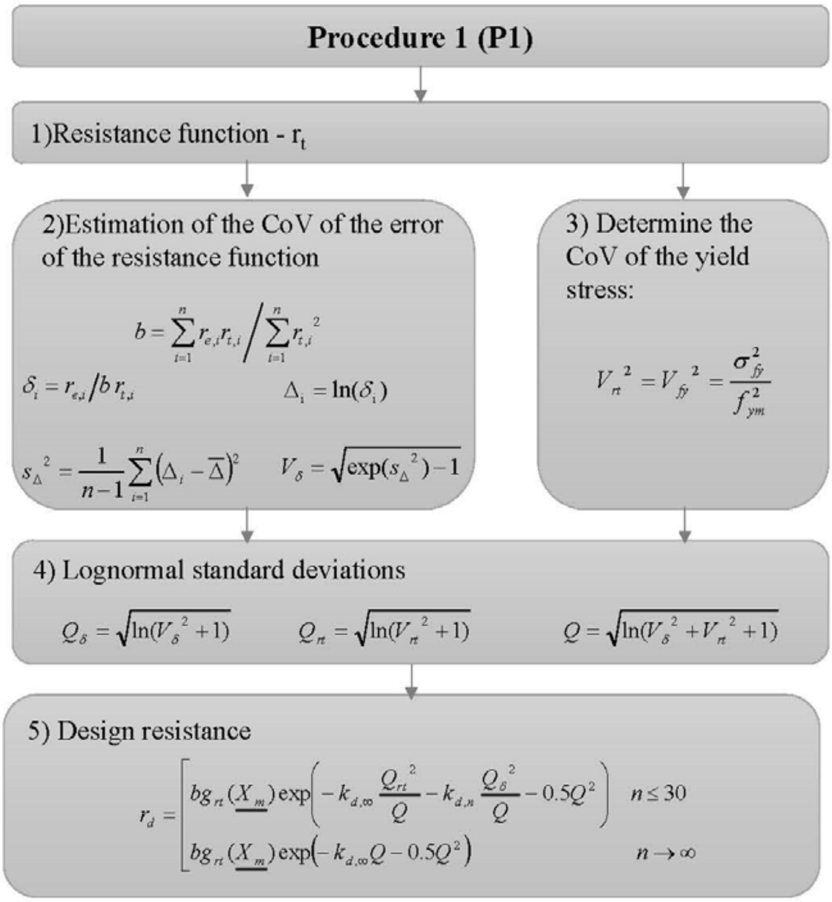

Figure 4.2 - Procedure 1

The assumption of f_y as the only basic variable (P1) is expected to be more inaccurate within the high slenderness range for stability problems, due to the fact that yield strength has no significant importance in that range. However, it is considered to be more adequate than P0 since it accounts for the propagation of the variability of the yield strength in the resistance function.

4.3.2 ASSESSMENT OF THE CONSERVATIVE NATURE OF PROCEDURE 1

Comparing P1 with the Annex D procedure and assuming that only the model and the yield stress variabilities are explicitly considered, the only difference consists then on the evaluation of V_{rt}. It would be useful if it could be proven that P1 is a safe-sided approach with regard to Annex D and also that the differences between the two methods are not significant. This is analytically proven in this subsection for a realistic range of yield stress distribution and assumed realistic values of the model variability.

Consider expression (3.36) that yields the partial safety factor γ_M :

$$\gamma_M^* = \frac{g_{rt}(\underline{X}_{nom})}{b g_{rt}(\underline{X}_m)\exp\left(-k_{d,\infty}Q - 0.5Q^2\right)} \tag{4.5}$$

Comparing P1 with Annex D shows that he terms b, $g_{rt}(\underline{X}_m)$, $g_{rt}(\underline{X}_{nom})$ are the same regardless of the method used. Therefore, any differences between the two procedures are solely related to the following expression:

$$\frac{1}{\exp\left(-k_{d,\infty}Q - 0.5Q^2\right)} \tag{4.6}$$

In addition, the procedure of Annex D evaluates partial safety factors $\gamma_{M,i}^*$ for each specimen, while P1 computes directly a total value for the sample. However, here, each value i, will be compared to the total value of P1.

The following assumption is considered: large number of test results – more than 100.

The following notation is adopted:

- The coefficient of variation V_{rt} calculated using the procedure of Annex D with partial derivatives is henceforth denoted as $V_{rt,D}$;
- The coefficient of variation V_{rt} calculated using Procedure 1 is henceforth denoted as $V_{rt,1;}$
-

$$Q_D = \sqrt{\ln(V_\delta^2 + V_{rt,D}^2 + 1)} \tag{4.7}$$

$$Q_1 = \sqrt{\ln(V_\delta^2 + V_{rt,1}^2 + 1)} \tag{4.8}$$

The derivative of Q with respect to V_{rt} is:

$$\frac{\partial Q}{\partial V_{rt}} = \frac{V_{rt}}{(V_\delta^{\ 2} + V_{rt}^{\ 2} + 1)\sqrt{\ln(V_\delta^{\ 2} + V_{rt}^{\ 2} + 1)}} > 0 \quad \forall V_{rt} > 0 \tag{4.9}$$

Since it is larger or equal to zero for any V_{rt}, Q is a monotonically increasing function and the following conclusion may be drawn:

$$If \quad V_{rt,D} < V_{rt,1} \xrightarrow{then} Q(V_{rt,D}) < Q(V_{rt,2}) \tag{4.10}$$

Differentiating expression (4.6) with respect to Q yields:

$$\frac{1}{\exp\left(-\mathrm{k}_{\mathrm{d},\infty}\mathrm{Q} - 0.5\mathrm{Q}^2\right)} = \exp\left(\mathrm{k}_{\mathrm{d},\infty}\mathrm{Q} + 0.5\mathrm{Q}^2\right)$$

$$\frac{d\exp\left(k_{d,\infty}Q + 0.5Q^2\right)}{dQ} = \exp\left(k_{d,\infty}Q + 0.5Q^2\right) \times (Q + k_{d,\infty}) > 0 \ \ \forall Q > 0$$

Since it is larger or equal to zero for all Q, the function is also monotonically increasing, then

$$If \quad Q(V_{rt,D}) < Q(V_{rt,1}) \xrightarrow{then} \frac{1}{\exp\left(-k_{d,\infty}Q_D - 0.5Q_D^{\ 2}\right)} < \frac{1}{\exp\left(-k_{d,\infty}Q_1 - 0.5Q_1^{\ 2}\right)} \tag{4.11}$$

Combining (4.10) and (4.11), it can be concluded that if it can be proven that $V_{rt,D} < V_{rt,1}$ then the values of γ_M obtained with P1 would be a safe-sided estimate of the partial safety factor.

In order to compare $V_{rt,D}$ and $V_{rt,1}$, a resistance function needs to be assumed. Considering the formulation of EN 1993 [2] for the flexural buckling of columns leads to:

$$g_{rt}(X_i) = g_{rt}(f_{fy}) = \chi(f_y)f_yA \tag{4.12}$$

Differentiating the resistance function with respect to the yield stress f_y, gives:

$$\frac{dg_{rt}(f_y)}{df_y} = \frac{d\chi(f_y)}{df_y} f_yA + \chi(f_y).A \tag{4.13}$$

The coefficient of variation V_{rt} is evaluated by:

$$V_{rt}^{\ 2} = \frac{1}{g_{rt}^{\ 2}(f_{ym})}\left[\frac{dg_{rt}(f_y)}{df_y}\right]^2 \sigma_{fy}^{\ 2} = \frac{\sigma_{fy}^{\ 2}}{f_{ym}^{\ 2}} \frac{\left[\frac{d\chi(f_y)}{df_y} f_y + \chi(f_y)\right]^2}{\chi^2(f_{ym})} \tag{4.14a}$$

$$V_{rt,1}{}^{2} = \frac{\sigma_{fy}{}^{2}}{f_{ym}{}^{2}} \tag{4.14b}$$

Dividing (3.13a) by (3.13b) yields:

$$C = \frac{\left[\dfrac{d\chi(f_y)}{df_y} f_y + \chi(f_y)\right]^2}{\chi^2(f_{ym})} \tag{4.15}$$

The derivative of χ with respect to fy is found:

$$\frac{d\chi(f_y)}{df_y} = -\left[\phi(f_y) + \sqrt{\phi(f_y)^2 - \bar{\lambda}(f_y)^2}\right]^{-2} \times \frac{d\bar{\lambda}(f_y)}{df_y}\left[\bar{\lambda}(f_y) + 0.5\alpha + \frac{\phi(f_y)\left[\bar{\lambda}(f_y) + 0.5\alpha\right] - \bar{\lambda}(f_y)}{\sqrt{\phi(f_y)^2 - \bar{\lambda}(f_y)^2}}\right]$$

It can be seen that it is lower than zero for any value of the yield stress, because it is a decreasing function.

$$\frac{d\chi(f_y)}{df_y} f_y \leq 0$$

The numerator and the denominator of the ratio C from expression (4.15) are plotted on *Figure 4.3*. In order to be able to analyse the value of *C*, these quantities have to be measured separately, since different values of the yield stress enter the equation: the specimen *i* value of $f_{y,}$, $f_{y,i}$, is present in the numerator, while the mean value of f_y, $f_{y,m}$, is present in the denominator.

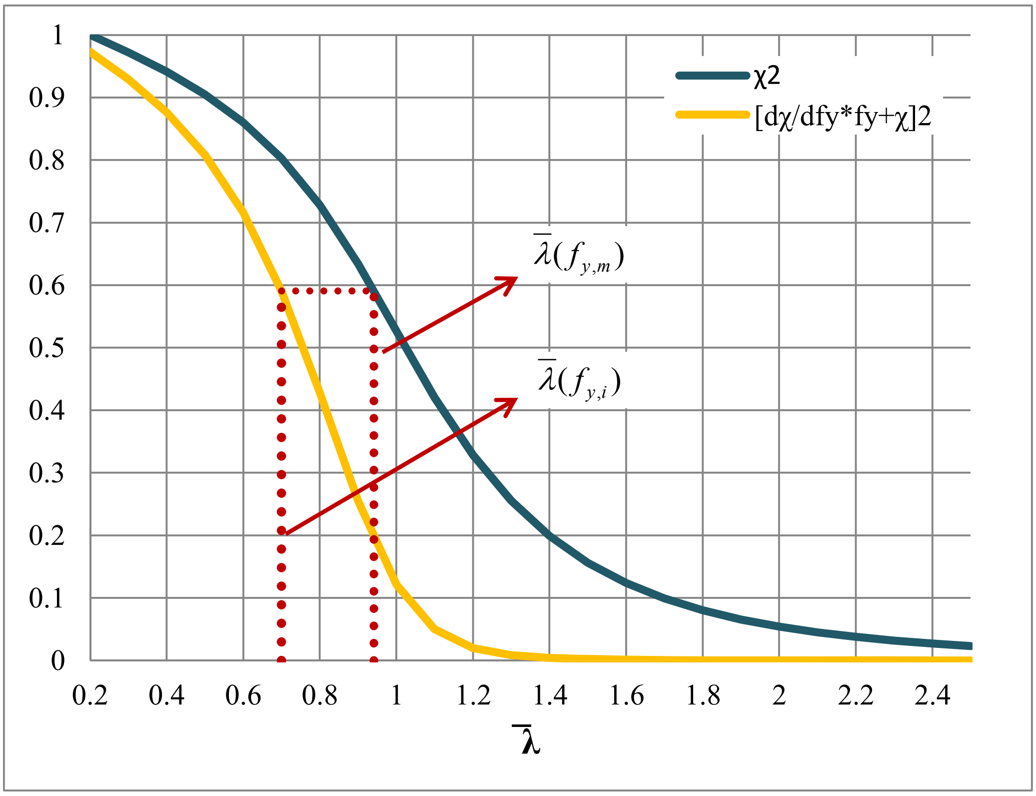

Figure 4.3 - Analysis of denominator (yellow curve) and numerator (blue curve) of Ratio C

A fully analytical proof of the conservatism of ratio C is not possible, since different values of the yield stress have to be assumed – i.e. $f_{y,m} \neq f_{y,i}$. However, it can be proven that only an unrealistic distribution of the yield stress will lead to C higher than unity. Firstly, let us notice that the ratio C is smaller than unity whenever $f_{y,i}=f_{ym}$, or $f_{y,i}>f_{ym}$. However, when the mean value of the yield stress is higher than the one for a certain specimen, this does not necessarily occur: when $f_{y,i}<f_{ym}$, this leads to a lower value of the normalized slenderness and consequently a higher χ for this specimen. In such case it should be checked if the ratio remains lower than unity. Nevertheless, by observing *Figure 4.3*, it can be seen that this is unlikely to occur and only a distribution with an unrealistic high scatter of the yield strength would lead to such a situation. For example, if an extreme case is considered when the minimum value of $f_{yi}=f_{ynom}$, since the absolute minimum of the yield stress is nominal (guaranteed value), then both terms of the ratio C become equal:

$$\left[\frac{d\chi(f_{y,i})}{df_{y,i}} f_{y,i} + \chi(f_{y,i})\right]^2 = \chi^2(f_{ym}) \tag{4.16}$$

By going through all slenderness ranges of $\bar{\lambda}(f_{y,i})$, one can obtain $\bar{\lambda}(f_{y,m})$ by combining equations (4.16) and (4.17) and consequently can extract the value of $f_{y,m}$ that leads to such normalized slenderness. This was carried out considering all imperfection factors α from EC3-1-1. The minimum difference between $f_{y,i}$ and $f_{y,m}$ was observed for $\bar{\lambda}(f_{y,i}) = 0.7$ and $\bar{\lambda}(f_{y,m}) \approx 0.941$, buckling curve a_0. This leads to $f_{y,m}$= 424.52 *MPa* when $f_{y,i}$=235 *MPa* (guaranteed value). Because the yield stress follows a log-normal distribution, the consideration of such parameters leads to a coefficient of variation of CoV=14.6% which is unlikely to occur. The CoV. value is determined as follows:

1) Fistly, $f_{y,i}$ = 235 MPa is assumed as the value for which the probability of finding a lower $f_{y,i}$ approaches zero. With this, the mean and standard deviation of the correspondent normal distribution can be found:

$$P(X \leq f_{y,i}) = \Phi\left(\frac{\ln(f_{y,i}) - \lambda}{\xi}\right) \approx 0 \rightarrow \Phi(-4) \approx 0$$
$$\Rightarrow -4 = \frac{\ln(f_{y,i}) - \lambda}{\xi} \tag{4.17}$$

where the value of the x=-4 such that the CDF $\Phi(x) \approx 0$ can be retrieved from standard normal distribution tables.

2) Secondly, the following set of equations to transform a lognormal distribution $LogN(f_{ym};\sigma_{fy})$ in a normal distribution $N(\lambda;\xi)$ is used:

$$\begin{cases} \xi^2 = \ln\left(1 + \left(\frac{\sigma_{fy}}{f_{ym}}\right)^2\right) \\ \lambda = \ln f_{ym} - 0.5\xi^2 \end{cases} \tag{4.18}$$

3) Finally, considering $f_{y,i}$=235 MPa, and replacing Eq. (4.17) in Eq (4.18), the $CoV = \sigma_{fy} / f_{ym}$ can be found.

4.4 PROCEDURE 2 (P2)

This procedure also constitutes a simplification of Annex D that can be considered as an extension of Procedure 1 as it now considers explicitly the variability of the geometry (see *Figure 4.4*). It is based on expression (3.9). It considers both variability of material and cross-section, so that the coefficient of variation V_{rt} is now calculated as:

$$V_{rt}^2 = V_{fy}^2 + V_{cs}^2 \tag{4.19}$$

Steel profiles are available in multiple cross sectional shapes ranging from tubular profiles or I and H cross sections to arbitrary cross section shapes. The geometry of a generic cross section is defined by the shape and pairs of values for thickness and width (t_i, b_i). Although the statistical characterization of the variability of the cross section geometry must be done for each individual family of profiles, the buckling resistance formulas are written as a function of cross sectional properties (area, elastic or plastic moduli, moments of inertia). Consequently, it is possible to consider either the basic dimensions or directly the relevant geometrical properties as the random variables. The latter option is certainly better from a practical point of view. In both cases, the coefficient of variation V_{cs} may be written in a more general way as:

$$V_{cs}^{2} = \sum_{j=1}^{k} V_{x}^{2} \qquad (4.20)$$

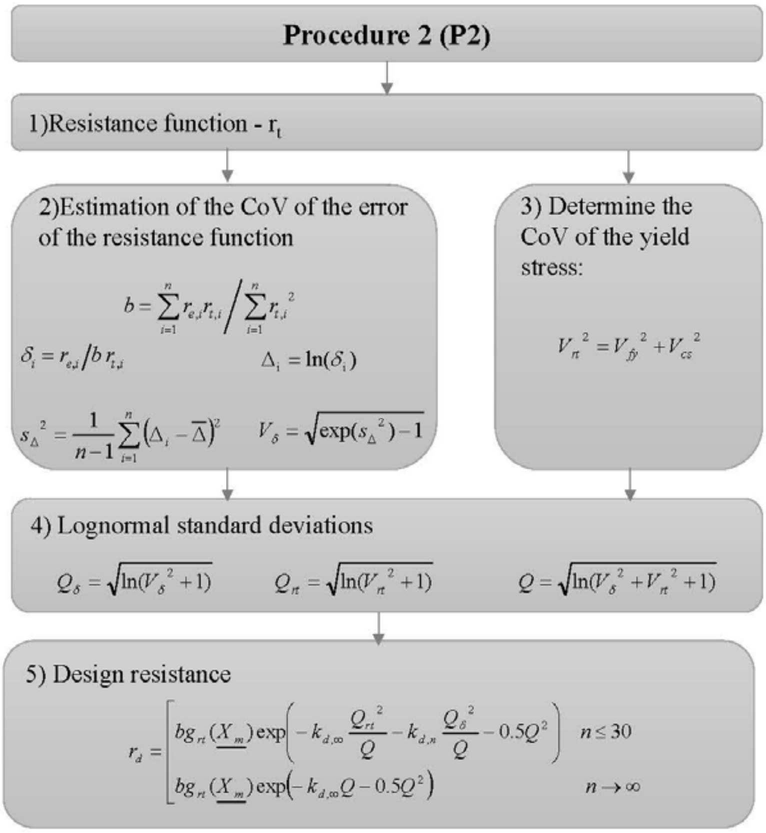

Figure 4.4 - Procedure 2

4.5 SUMMARY

All procedures are summarized in *Table 4.2* to *Table 4.5*:

Table 4.2 - Comparison of different procedures

Method	Regression line	$\Delta_i = \ln(\delta_i)$	$\overline{\Delta}$	$s_\Delta^{\ 2}$
Annex D	$b = \frac{\sum_{i=1}^{n} r_{e,i} r_{t,i}}{\sum_{i=1}^{n} r_{t,i}^{\ 2}}$	$\Delta_i = \ln\left(\frac{r_{e,i}}{b\, r_{t,i}}\right)$	$\frac{1}{n}\sum_{i=1}^{n}\Delta_i$	$\frac{1}{n-1}\sum_{i=1}^{n}\left(\Delta_i - \overline{\Delta}\right)^2$
P0	$R_m = \frac{1}{n}\sum_{i=1}^{n}\frac{r_{e,i}}{r_{t,i}}$	$\Delta_i = \ln\left(\frac{r_{e,i}}{R_m\, r_{t,i}}\right)$		
P1,P2	$b = \frac{\sum_{i=1}^{n} r_{e,i} r_{t,i}}{\sum_{i=1}^{n} r_{t,i}^{\ 2}}$	$\Delta_i = \ln\left(\frac{r_{e,i}}{b\, r_{t,i}}\right)$		

Table 4.3 Comparison of different procedures, continued

Method	V_δ	$V_{rt}^{\ 2}$	$V_r^{\ 2}$
Annex D	$\sqrt{\exp(s_\Delta^{\ 2})-1}$	$\frac{1}{g(X_m)^2}\sum_{j=1}^{k}\left(\frac{\partial g(X_j)}{\partial X_j}\sigma_j\right)^2$	$V_\delta^{\ 2} + V_{rt}^{\ 2}$
P0		-	
P1		σ_{fy}^2 / f_{ym}^2	
P2		$\sigma_{fy}^2 / f_{ym}^2 + \sigma_{cs}^2 / CS_m^{\ 2}$	

Table 4.4 - Comparison of different procedures, continued

Method	Q_δ	Q_{rt}	Q
Annex D	$\sqrt{\ln(V_\delta^2+1)}$	$\sqrt{\ln(V_{rt}^2+1)}$	$\sqrt{\ln(V_r^2+1)}$
P0		-	
P1, P2		$\sqrt{\ln(V_{rt}^2+1)}$	

Table 4.5 - Comparison of different procedures, continued

Method	$r_{t,nom}$	r_d	γ_M^*
Annex D	$g_{rt}(\underline{X_{nom}})$	$\begin{cases} b g_{rt}(\underline{X_m})\exp\left(-k_{d,\infty}\frac{Q_{rt}^2}{Q}-k_{d,n}\frac{Q_\delta^2}{Q}-0.5Q^2\right) & n\le 30 \\ b g_{rt}(\underline{X_m})\exp\left(-k_{d,\infty}Q-0.5Q^2\right) & n\to\infty \end{cases}$	$\frac{r_{t,nom}}{r_d}$
P0		$\begin{cases} R_m g_{rt}(\underline{X_{nom}})\exp\left(-k_{d,\infty}\frac{Q_{rt}^2}{Q}-k_{d,n}\frac{Q_\delta^2}{Q}-0.5Q^2\right) & n\le 30 \\ R_m g_{rt}(\underline{X_{nom}})\exp\left(-k_{d,\infty}Q-0.5Q^2\right) & n\to\infty \end{cases}$	$\gamma_{M,i}=\gamma_{Rd}\times\gamma_{m,i}$ $\frac{r_{t,nom}}{r_d}\ \frac{f_{y,nom}}{f_{y,m}(1-3.04V_{fy})}$
P1, P2		$\begin{cases} b g_{rt}(\underline{X_m})\exp\left(-k_{d,\infty}\frac{Q_{rt}^2}{Q}-k_{d,n}\frac{Q_\delta^2}{Q}-0.5Q^2\right) & n\le 30 \\ b g_{rt}(\underline{X_m})\exp\left(-k_{d,\infty}Q-0.5Q^2\right) & n\to\infty \end{cases}$	$\frac{r_{t,nom}}{r_d}$

5 NUMERICAL VALIDATION OF POSSIBLE ALTERNATIVES

5.1 INTRODUCTION

In this chapter, the various procedures are assessed in the context of the flexural buckling resistance of steel columns. Because the buckling resistance of steel members presents distinctive behaviours for the various slenderness ranges, all comparisons are carried out for the slenderness interval [0.2 – 2.5].

The following methodology is adopted to implement the comparative assessment:

(i) assumed statistical distributions for the basic input variables are adopted, which are plausible representations of the reality;

(ii) the coefficient of variation of the design model V_δ is assumed and varied from 0% to 10%;

(iii) in order to assess the compatibility of the test population, it is split into several sub-sets according to slenderness intervals;

(iv) the "resistance function" formulation for flexural bucking of columns, section 6.3.1 of EC3-1-1[2] is considered;

(v) a large number of experiments are available (at least 100).

In this chapter the following basic input variables are used:

- Yield strength – f_y;
- Cross-section area – A;
- Second moment of area – I;
- Modulus of elasticity – E;

The following cases are analysed and compared:

- Procedure 1 – f_y (regarding the basic input variables, only f_y is considered as a random variable);
- Procedure 2 – f_y + A (in accordance with the discussion in section 3.4, besides f_y, only the area is considered as a random variable since for flexural buckling of columns it is a primary variable);
- Procedure 2 – f_y+A+I (also I (implicit variable within reduction factor χ) is considered as a random variable);

- Procedure 2 – f_y+A+I+E (also I and E (implicit variables within reduction factor χ) are considered as a random variables);

The above notation is henceforth used to distinguish the various cases.

The results of the application of the simplified procedures P1 and P2 are compared with the Annex D results in two ways:

- comparison with Annex D results assuming the same random variables: this comparison is aimed at establishing the conservative nature of the simplification of using expression (3.27a) instead of (3.27b).
- comparison with Annex D results using all the relevant basic input variables as random variables: this comparison is aimed at establishing the level of safety of the simplified procedures.

5.2 GENERATION OF SAMPLES

5.2.1 YIELD STRENGTH

The population of the yield stress can be represented by a lognormal distribution [10].

Three fictitious samples are generated. They are aimed at studying the influence of the scatter of yield strength on the resistance function. Therefore, all samples have nominal yield strength of 235MPa, the same mean value of 297.3 MPa, but different standard deviations. The assumptions are summarized in *Table 5.1*.

In order to simulate a lognormal distribution, the following transformation was again performed:

$$\left[\begin{array}{l} \xi^2 = \ln\left(1+\left(\dfrac{\sigma_{fy}}{f_{ym}}\right)^2\right) \\ \lambda = \ln f_{ym} - 0.5\xi^2 \end{array}\right. \tag{5.1}$$

Table 5.1- Assumed descriptors of the yield strength distributions

Steel	f_{ym}	σ_{fy}	λ	ξ
	MPa	MPa	-	-
X1 (n=100)	297.3	25	5.691219	0.083942
X2 (n=100)	297.3	17.1	5.69309	0.05747
X3 (n=100)	297.3	9	5.694284	0.030266

Each sample consists of 100 specimens. *Figure 5.1* to *Figure 5.3* show the generated values of f_y in the form of histograms.

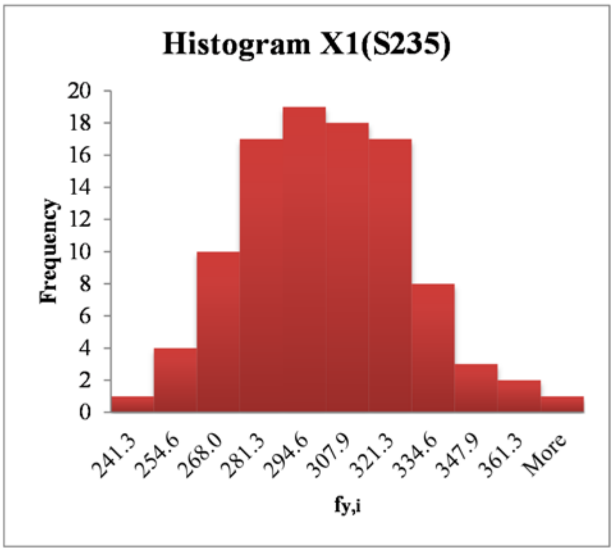

Figure 5.1 - Histogram X1(S235), f_{ym}*=297.3MPa,* σ_{fy}*=25MPa*

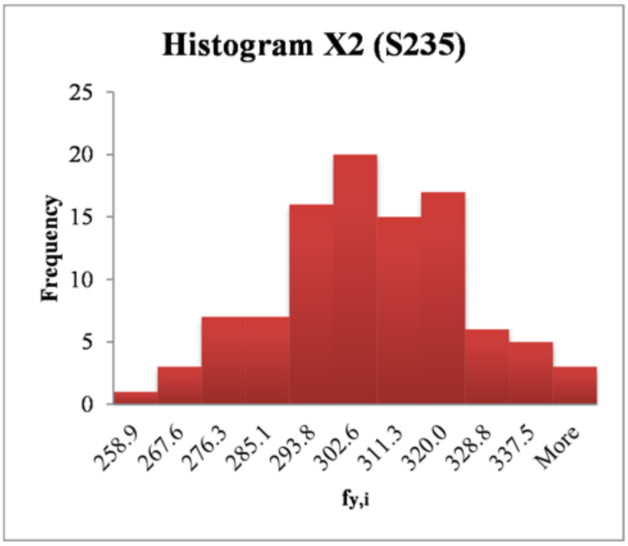

Figure 5.2 - Histogram X2(S235), f_{ym}*=297.3MPa,* σ_{fy}*=17.1MPa*

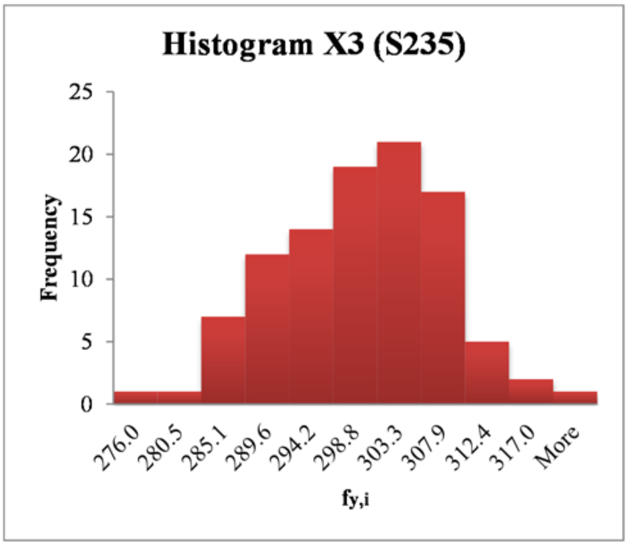

Figure 5.3 - Histogram X3(S235) f_{ym}=297.3MPa, σ_{fy}=9MPa

5.2.2 GEOMETRICAL PROPERTIES

Hypothetical samples are simulated for the cross-sectional area and moment of inertia. The distributions are normal and the mean and standard deviation are assumed to follow expressions (5.2) [13]. Each distribution consists of 100 specimens. The cross section used in this example is an *IPE 200*.

$$\begin{bmatrix} \mu_x = 0.99X_{nom} \\ \sigma_x \in [0.01;0.04\mu_x] \end{bmatrix} \tag{5.2}$$

Table 5.2 summarizes the assumptions.

Table 5.2 - Assumed descriptors of cross-section properties distributions

Param.	X_{nom}	X_m	σ_X
	$mm^2(mm^4)$	$mm^2(mm^4)$	$mm^2(mm^4)$
A (n=100)	2848	2819.52	112.7808
I (n=100)	1424000	1409760	56390.4

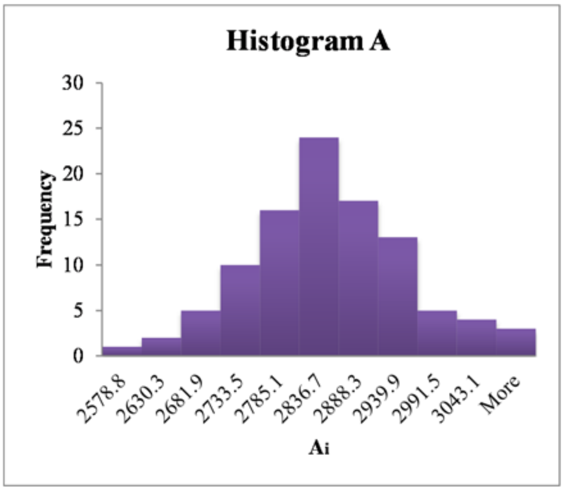

Figure 3.1 - Normal distribution of Area, n=100

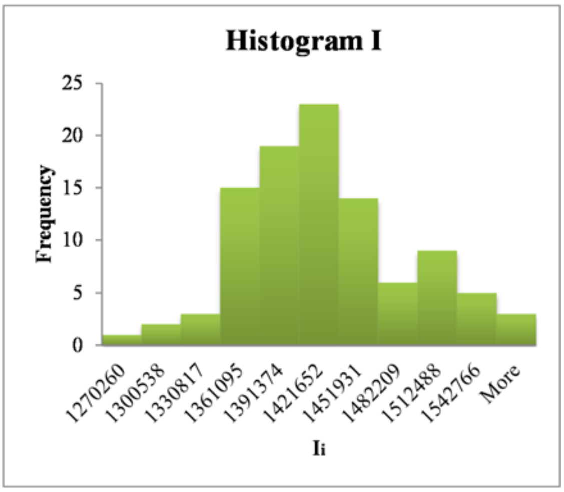

Figure 3.2 - Normal distribution of moment of inertia, n=100

5.2.3 MODULUS OF ELASTICITY

In order to study the influence of the modulus of elasticity, a sample is generated with 100 specimens. The distribution is assumed as normal Gaussian with c.o.v. of 5% [7].

Table 5.3 - Assumed descriptors of modulus of elasticity distribution

Param.	X_{nom}	X_m	σ_X
	MPa	MPa	MPa
E (n=100)	210000	210000	10500

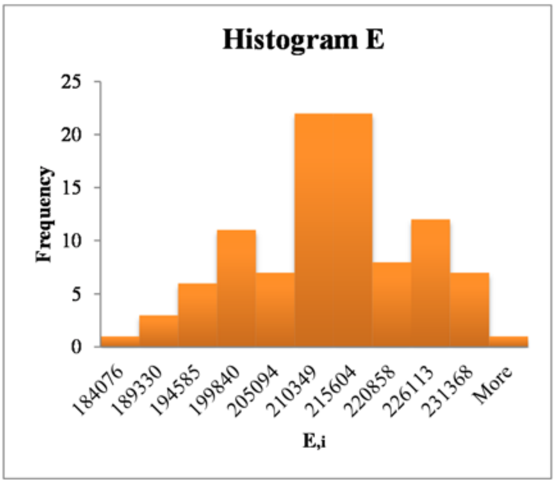

Figure 3. 3 - Normal distribution of modulus of elasticity

5.3 Results

5.3.1 Methodology

In this section the result of the numerical assessment of the simplified procedures is presented. The assessment is performed on the basis of the previously generated distributions. For all cases the sample size is 100 specimens so that the partial safety factor γ_M can be obtained from expression (3.36):

$$\gamma_M{}^* = \frac{g_{rt}(\underline{X_{nom}})}{b g_{rt}(\underline{X_m}) \exp\left(-k_{d,\infty} Q - 0.5 Q^2\right)} \quad (5.3)$$

As already mentioned in chapter 4, b, $g_{rt}(\underline{X_m})$, $g_{rt}(\underline{X_{nom}})$ are the same regardless of the method used. Therefore, the following term can be isolated and used for comparison of the resulting partial safety factors:

$$\frac{1}{\exp\left(-k_{d,\infty} Q - 0.5 Q^2\right)} \quad (5.4)$$

Using expression (3.27b) coefficients of variation V_{rt} are calculated for each data point i, yet simplified procedures propose a single value for the sample.

Finally, the safety factor given by each approach is determined as follows:

(i) When procedures P1 and P2 are used, a global value of γ_M from expression (5.3) is obtained for the whole sample;

(ii) When Annex D is used with Eq. (3.27b), different values of γ_M from expression (5.3) are obtained for each specimen due to the variation of the basic input variables. As a result, in order to get a comparable result from expression (5.3) for both alternatives, the mean value of the single (specimen) results from expression (5.3) is determined.

In the subsequent sub-sections the ratio (5.4) is used to compare different values of the partial safety factor.

5.3.2 NUMERICAL ASSESSMENT OF P1

In this sub-section, the behavior of P1 in case of flexural column buckling is studied.

In order to compare P1 with the procedure of Annex D when all basic input variables are taken into account, it is assumed:

- Distribution of the yield strength f_y – X2(S235), presented in sub-section 5.2.1;
- Distributions of the area *A* and moment of inertia *I* – presented in sub-section 5.2.2;
- Distribution of the modulus of elasticity *E* – presented in sub-section 5.2.3;

The coefficient of variation of the model is assumed – V_δ=0.05; The results are plotted in *Figure 5.4*. The Annex D procedure is presented in terms of each specimen *i* and the corresponding mean value (in orange). Additionally, results are given in case *fy* is assumed as the only random variable in the Annex D procedure (in red). It can be observed that P1 presents some *unsafe* results when compared to the “full” Annex D. On the contrary, when compared to Annex D (*fy*), it is always safe.

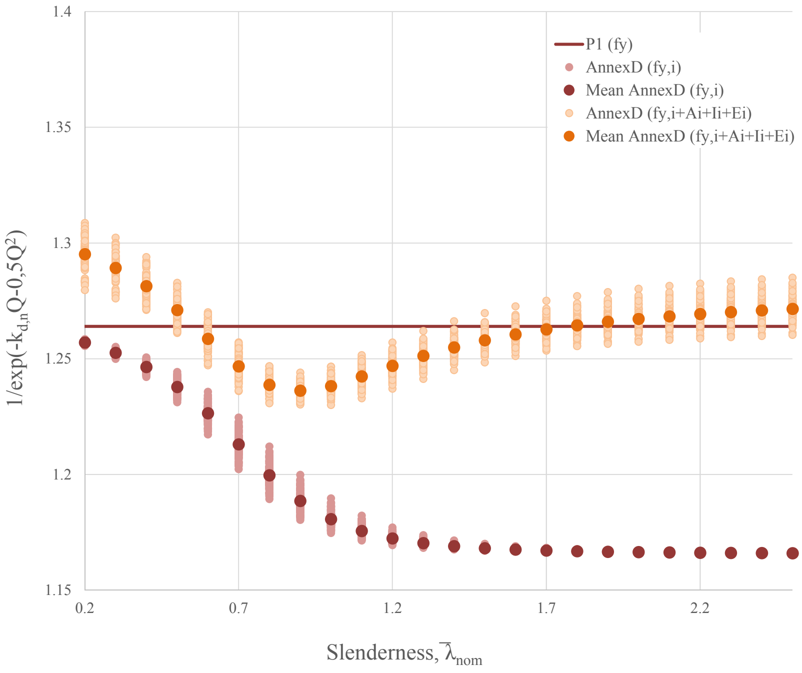

Figure 5.4 - Results for distribution X2(S235), V_δ =0.05

A further comparison is performed on the basis of different values of the variability of the model, V_δ. The results are plotted in terms of percentage difference between partial safety factors evaluated using P1 and the "full" Annex D (when basic input variables are *fy+A+I+E)*. The "unsafe" trend is maintained for different V_δ as shown in *Figure 5.5*.

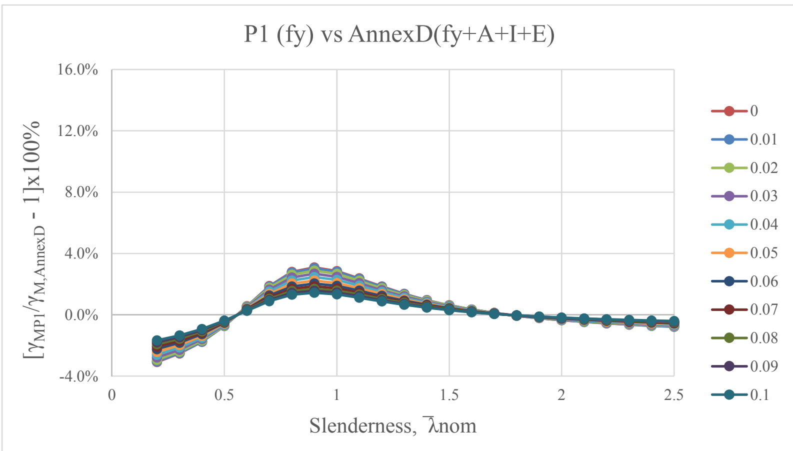

Figure 5.5 - P1(f_y) for various V_δ

It should be mentioned that the percentage difference strongly depends on the distributions that are being used and *Figure 5.5* cannot be taken as a reference case for flexural buckling of columns. It aims to be illustrative for the problem.

The influence of the standard deviation of the yield strength is additionally analysed. The samples presented in sub-section 5.2.1 are used. Procedure P1 is compared with the procedure of Annex D, when partial derivatives are evaluated only for the yield strength *fy,* so that the impact of the other basic variables is omitted. The comparison is performed on the basis of expression (5.4). The coefficient of variation V_δ is assumed equal to 0.05 for all three cases.

It can be noticed that the variation of the yield stress has more significant influence within the low to intermediate slenderness range. It is also remarked that P1 would always give higher results when compared to Annex D if the yield strength were indeed the only variable of the problem, thus confirming the analytical derivation proposed in sub-section 4.3.2.

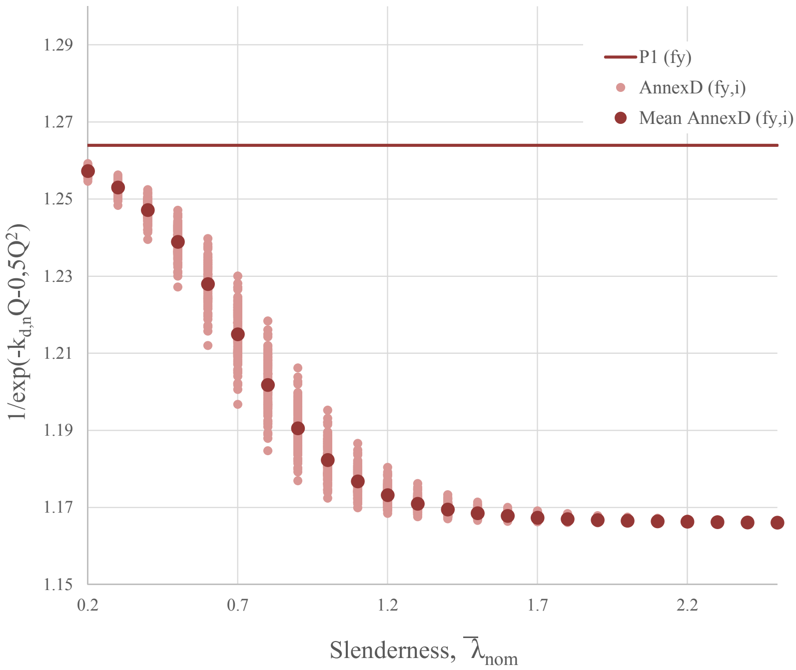

Figure 5.6 - Results for distribution X1(S235), V_δ =0.05

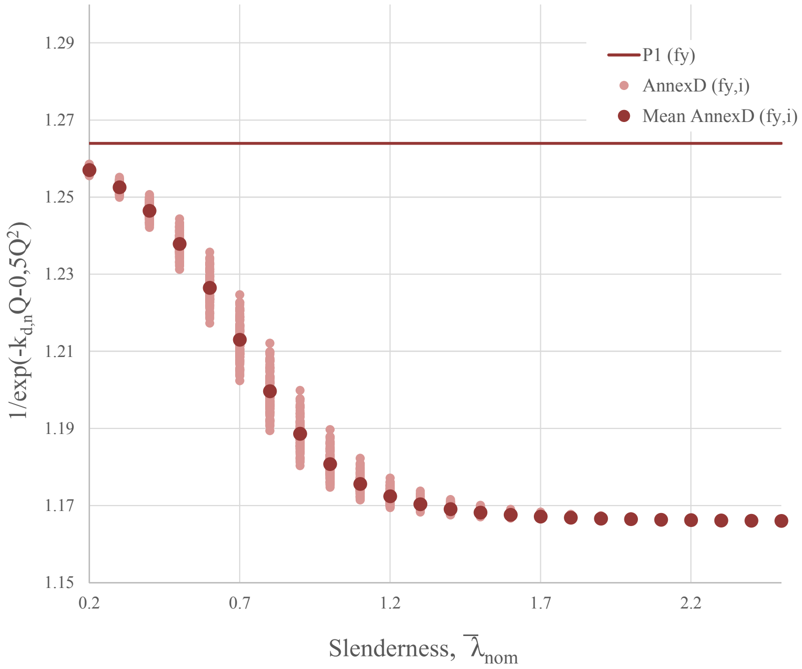

Figure 5.7 - Results for distribution X2(S235), V_δ =0.05

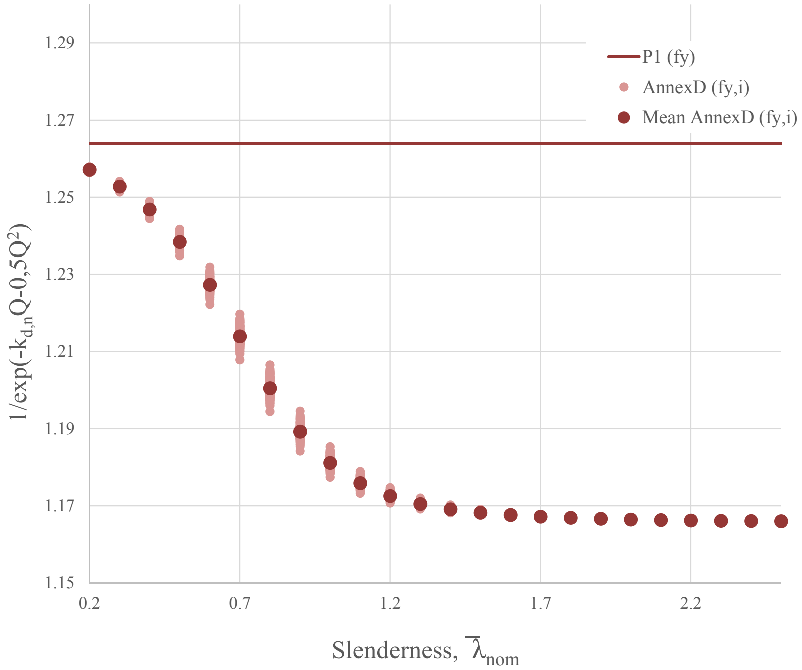

Figure 5.8 - Results for distribution X3(S235), V_δ =0.05

5.3.3 NUMERICAL ASSESSMENT OF P2

For the numerical assessment of P2 in case of the flexural buckling of columns the influence of the consideration of several random variables related to geometry or related to properties are considered, as already described in section 5.1. Specifically, the following basic variables are included: cross-section area, moment of inertia and Young's modulus. The following cases are analysed:

- P2 – f_y + A versus Annex D with variability of f_y+A; (*Figure 5.9*)
- P2 – f_y+A+I versus Annex D with variability of f_y+A+I; (*Figure 5.10*)
- P2 – f_y+A+I+E versus Annex D with variability of f_y+A+I+E; (*Figure 5.11*)

Assumptions:

- Distribution of the yield strength f_y – X2(S235), presented in sub-section 5.2.1;
- Distributions of the area A and second moment of area I – presented in sub-section 5.2.2;
- Distribution of the modulus of elasticity E – presented in sub-section 5.2.3;
- All alternatives are compared with the corresponding assumption for Annex D for V_δ=0.05;

Firstly, the alternatives are compared to the full Annex D procedure using expression (5.4) similarly to the previous section, in order to see the deviations between the different assumptions. Unlike P1, the presence of geometrical properties and Young's modulus leads to results that are always on the safe-side.

However, it is observed that as more variables are included, the differences with Annex D become higher. This issue is associated to the fact that simplified procedures are only adding the variability of each parameter. On the contrary, the Annex D procedure using partial derivatives takes into account the variability of each parameter as they appear in the "resistance function".

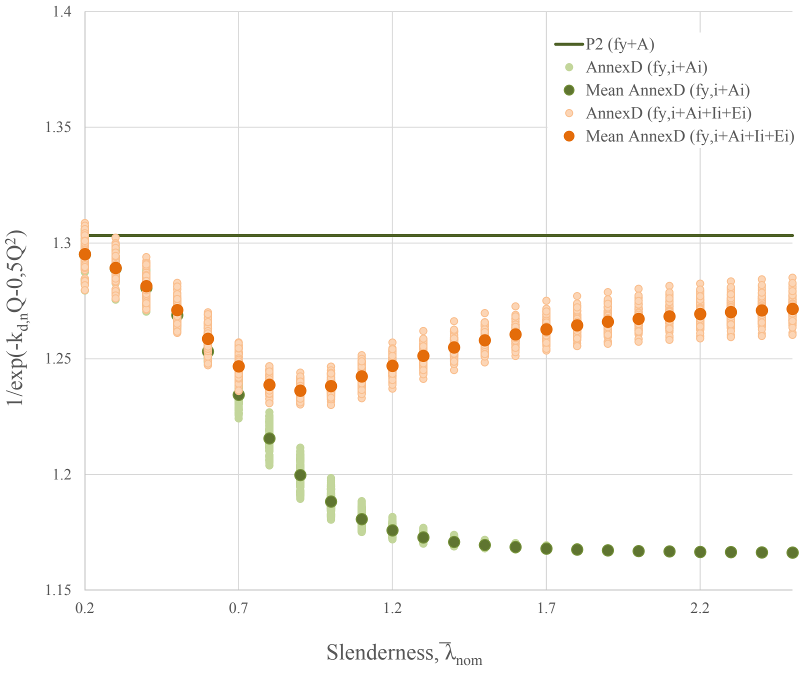

Figure 5.9 - P2(fy+A) vs Annex D, absolute values

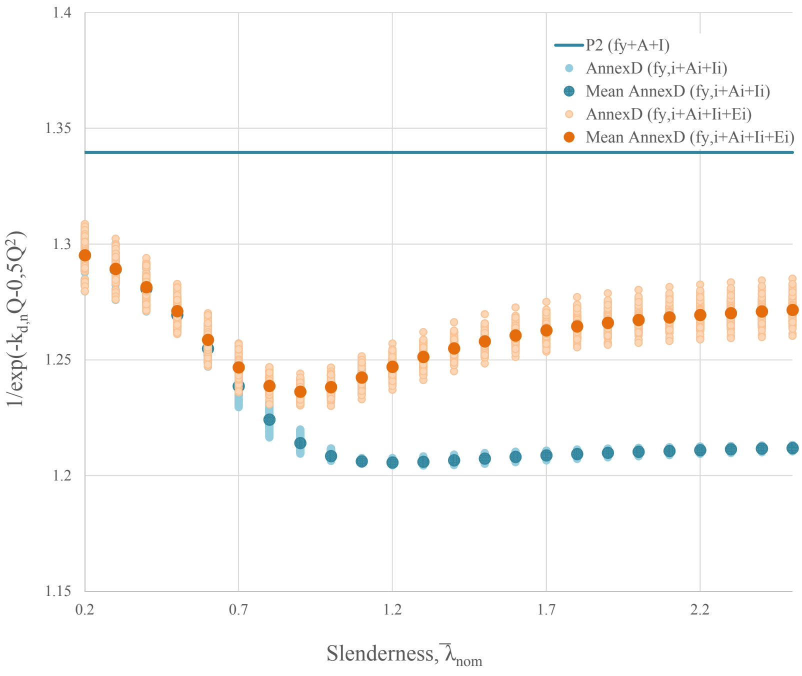

Figure 5.10 - P2(fy+A+I) vs Annex D, absolute values

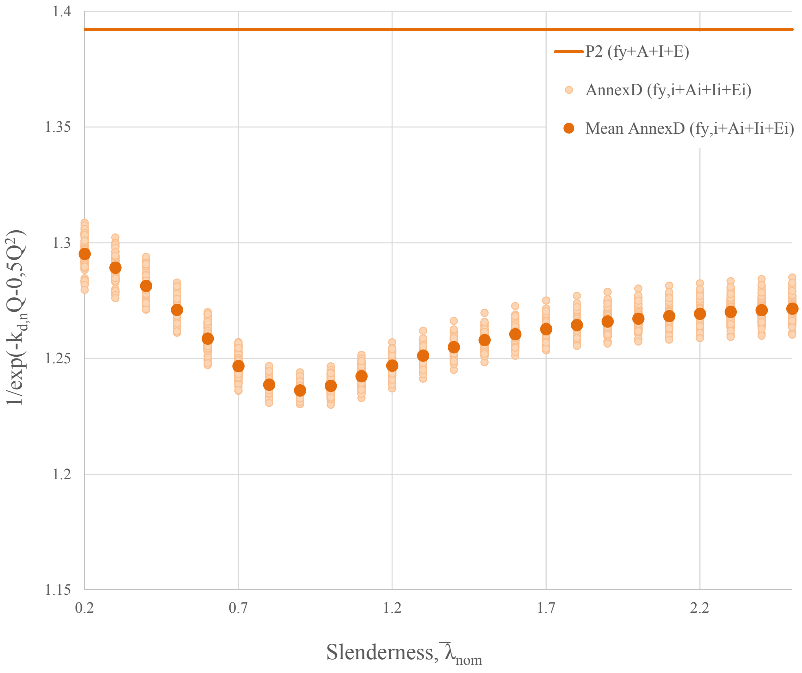

Figure 5.11 - P2(fy+A+I+E) vs Annex D, absolute values

Since no experimental tests were performed, the influence of the coefficient of variation V_δ is worth studying as in the previous section. The difference is given in percentage with respect to the Annex D using partial derivatives and including the variability of all basic variables assumed in this work (f_y+A+I+E) for various coefficients of variation V_δ, since, in reality, it is impossible to differentiate between the random variables as previously mentioned.

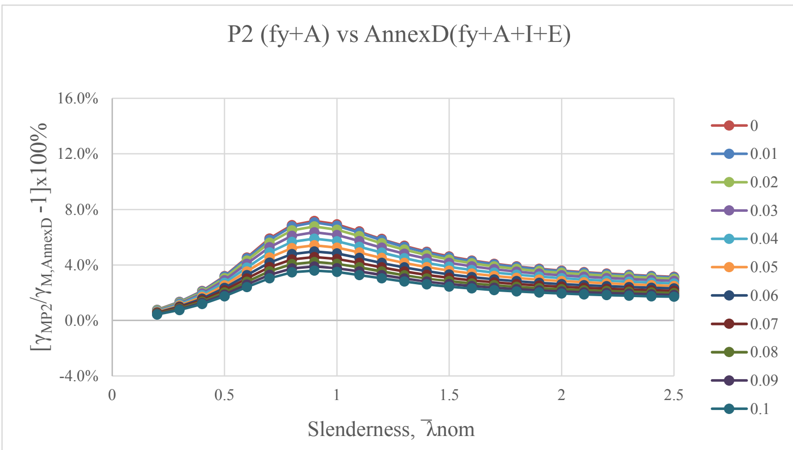

Figure 5.12 - P2(f_y+A) for various V_δ

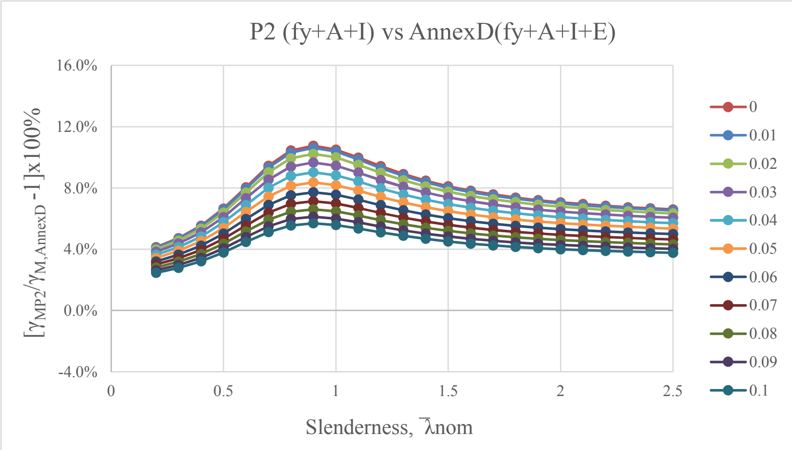

Figure 5.13 - P2(f_y+A+I) for various V_δ

The trend of increased error with increased number of basic input variables considered in P2 remains. It can be seen as a positive trend, in a way that the more variables are used the more uniform the difference becomes. It can be favourable, in case adjusting function or correction coefficient are introduced in the simplified procedure, as proposed in [14].

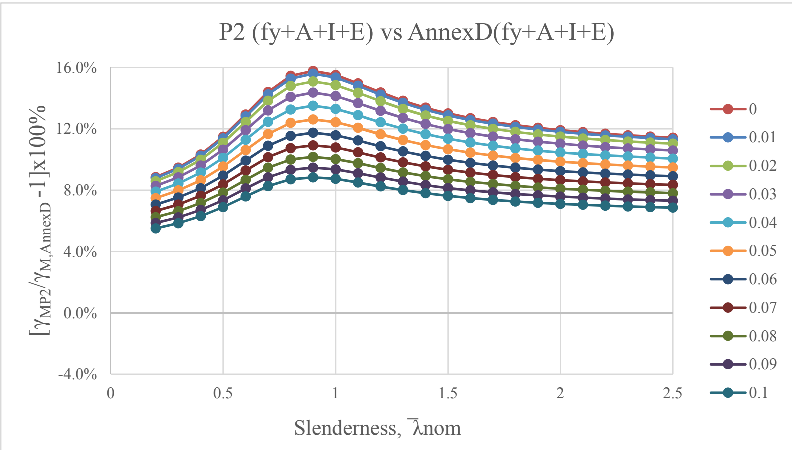

Figure 5.14 - P2(f_y+A+I+E) for various V_δ

6 NUMERICAL EXAMPLE

6.1 DEFINITION AND PURPOSE

In this chapter a numerical example is presented. The example aims at further clarifying the previously discussed issues in terms of partial safety factors γ_M. In addition, the example intends to compare the impact of different basic variables on the resistance function using the method of Annex D.

The example is based on flexural buckling formulae of EN 1993-1-1. Here, a profile IPE 200 is chosen for the column cross-section. The steel grade is S355. The specimens are tested in pure compression for nominal values of the normalized slenderness $\bar{\lambda}$ =0.3, 0.6, 1.0, 1.4, 1.8 and 2.2. For each specimen, the following parameters are measured (i.e. randomly generated for this fictitious "test campaign"):

- Yield strength – f_y;
- Modulus of elasticity – E;
- Cross-section (CS);
  - Cross-section width – b;
  - Cross-section height – h;
  - Flange thickness – t_f;
  - Web thickness – t_w;
- Residual stresses - RS;
- Geometrical imperfections – GI (member imperfection);

The buckling strength of the columns is obtained using numerical finite element simulations. Each case is modelled using the respective geometrical and material properties.

Moreover, here variability of residual stresses and geometrical imperfections is also considered, in order to be able to achieve understanding about their impact. However, it is difficult to treat the imperfections since there is not sufficient information about their distribution and additional difficulty is coming from the fact that they are not considered explicitly in the design formulation of EN 1993-1-1 [2], being incorporated in the imperfection factor α.

Additionally, the influence of modelling with nominal properties of the basic input variables is also studied. This matter is of particular interest, since the number of simulations can be significantly reduced, if it can be proven that it is acceptable to adopt the approach.

The column length L and the imperfection factor α are considered constant. Therefore, the basic input variables are – yield strength f_y, Young's modulus E, cross-section width b, cross-section height h, flange thickness – t_f; and web thickness t_w. Estimates for their mean values and standard deviations are given in section 6.1.2.

6.1.1 CASES

The cases which are modelled are presented. As it was already explained, for each case the number of normalized slenderness ratios is six and for each slenderness100 specimens are considered.

Table 6.1 Cases

Basic variables considered	Variables	$N_{slenderness}$	Subtotal
Nominal	1	46	46
Variability f_y	100	6	600
Variability **CS**	100	6	600
Variability **E**	100	6	600
Variability f_y**+CS**	100	6	600
Variability f_y**+E**	100	6	600
Variability f_y**+RS(**f_y**)**	100	6	600
Variability f_y**+CS+E**	100	6	600
*Variability (***GI+RS***)*	100	6	600
Variability **GI, RS=const**	100	6	600
Variability **GI=const, RS**	100	6	600
Variability f_y**+CS+E+GI+RS**	100	6	600
		Total:	**6646**

- Firstly, 46 cases with nominal parameters are run, for different member lengths. They are used as a reference case and also when the concept of modelling with nominal parameters is discussed;
- Secondly, each variable is modelled separately and the remaining parameters are left nominal in order to study the influence of each parameter;
- Subsequently, combinations of variables are considered, in order to obtain the influence of parameters when they are grouped;
- Finally, all variables are combined together;

6.1.2 SAMPLES

In order to simulate experiments, samples of the random variables were generated. For that, assumptions about the parameters of the distribution were made. Additionally, these samples should correspond to more or less plausibly to reality, because the results are very much influenced of the input.

Another difficulty is arising from the fact that there is not sufficient data for some of the basic variables. For instance, the distribution of the yield stress is very well known, due to the fact that it is standardized and therefore it needs to be checked if it corresponds to the reality. It is also known that it can be described by a lognormal distribution. An interesting fact about the distribution of the yield stress is that the nominal value is a guaranteed value; hence the scatter is always higher than the nominal. This is very advantageous feature which is further analysed.

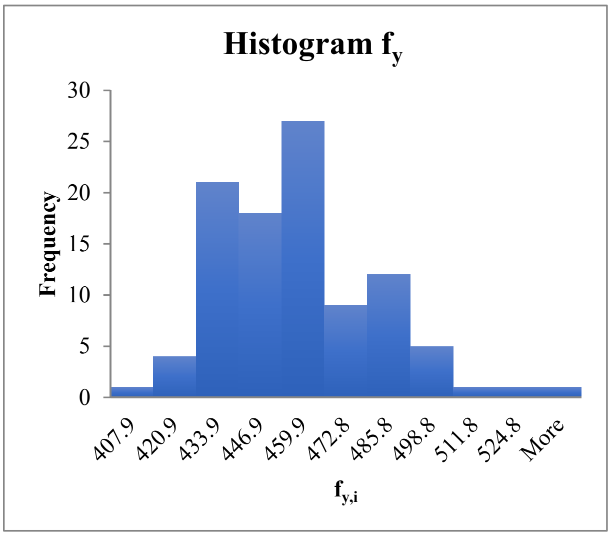

Figure 6.1Histogram fy

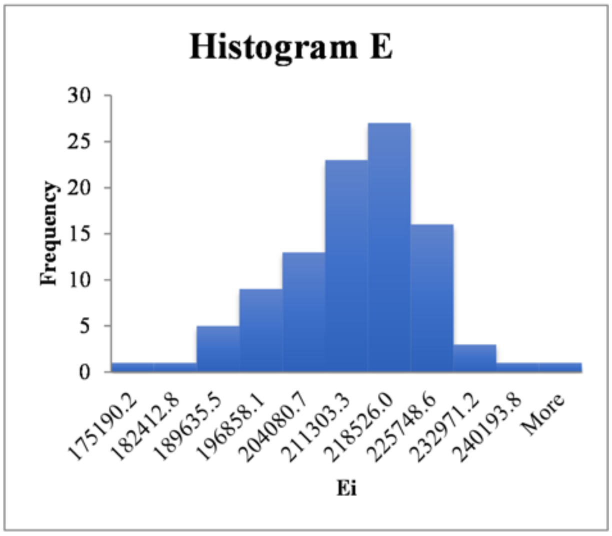

Figure 6.2 Histogram E

n=100, [MPa]		
$f_{y,nom}$	$f_{y,m}$	σ_{fy}
355	455	24.6

n=100, [MPa]		
E_{nom}	E_m	σ_E
210000	210000	10500

The population of the geometrical properties is known to follow the Gaussian distribution. However, here the nominal properties are not guaranteed values and it is observed that the mean value is close to the nominal value, which in turn means that there is part of the distribution which exhibits unfavourable quantities from engineering stand point.

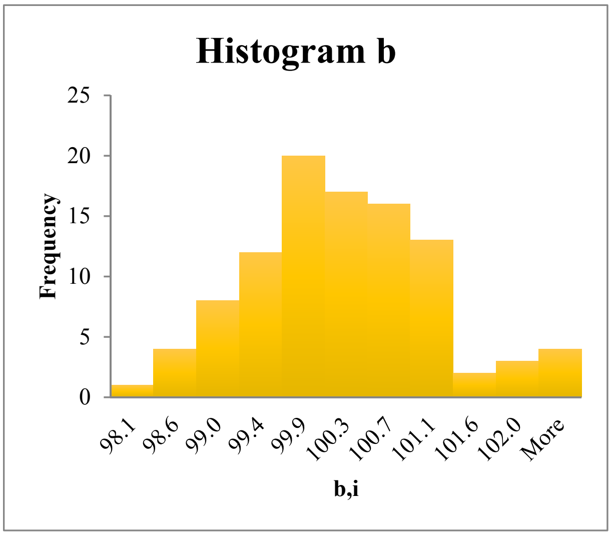

Figure 6.3 Histogram b

n=100, [mm]		
b_{nom}	b_m	σ_b
100	100	0.9

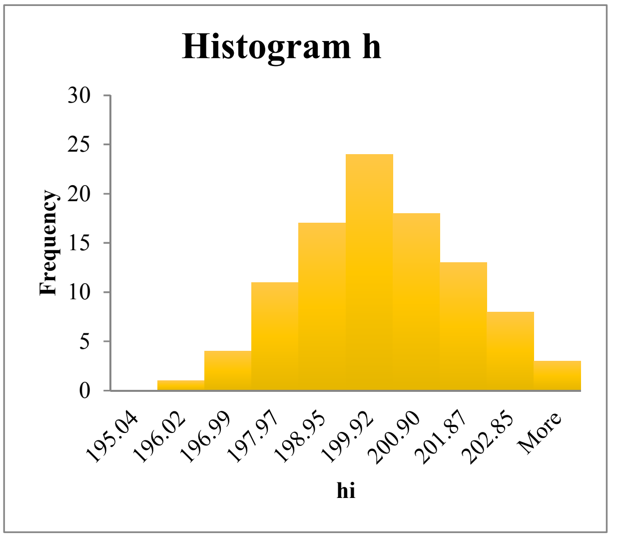

Figure 6.4 Histogram h

n=100, [mm]		
h_{nom}	h_m	σ_h
200	200	1.8

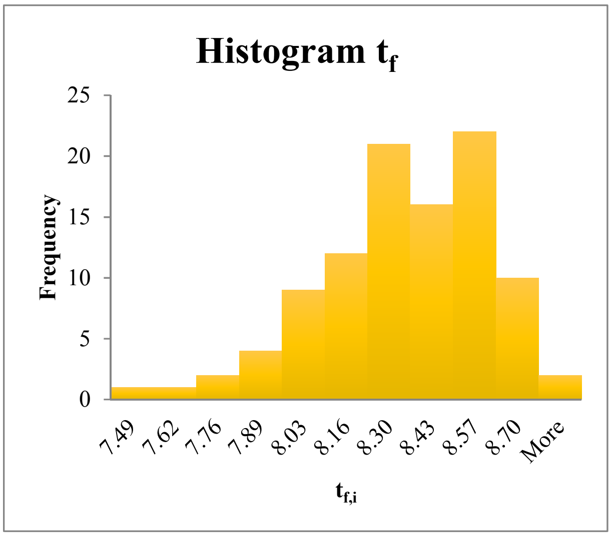

Figure 6.5 Histogram t_f

n=100, [mm]		
$t_{f,nom}$	$t_{f,m}$	σ_{tf}
8.5	8.29	0.2486

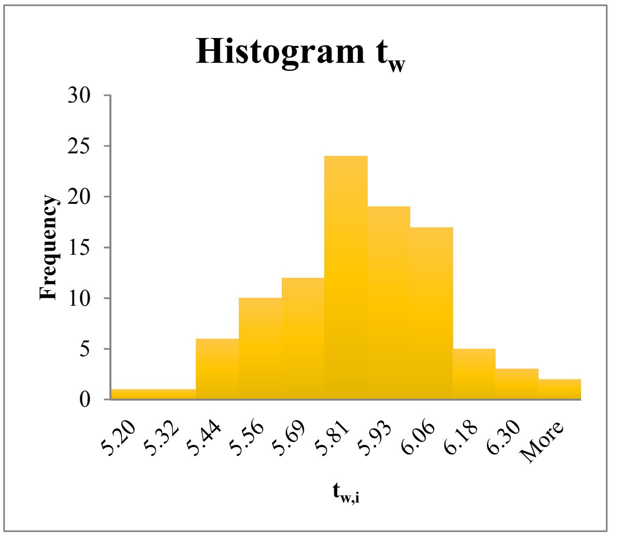

Figure 6.6 Histogram t_w

n=100, [mm]		
$t_{w,nom}$	$t_{w,m}$	σ_{tw}
5.6	5.74	0.2296

The population of the modulus of elasticity is also known to follow the Gaussian distribution. However, there is not much data on measurements of the modulus of elasticity, due to the fact that it is very difficult to measure the modulus; moreover, even if the same measurement is considered, two different people may obtain with very different results. In this example recommendations from [15] are adopted.

Another property which is difficult to find are the residual stresses. Apparently, their measurement is very expensive and therefore, there are not many results. Distribution is assumed from [16]. Moreover, this distribution has a mean value which is much lower than the nominal, which is also favourable.

Finally, for the population of the member imperfection, it is also very difficult to find information, even though it is being standardized. For this example, the statistical parameters given in [16] are adopted.

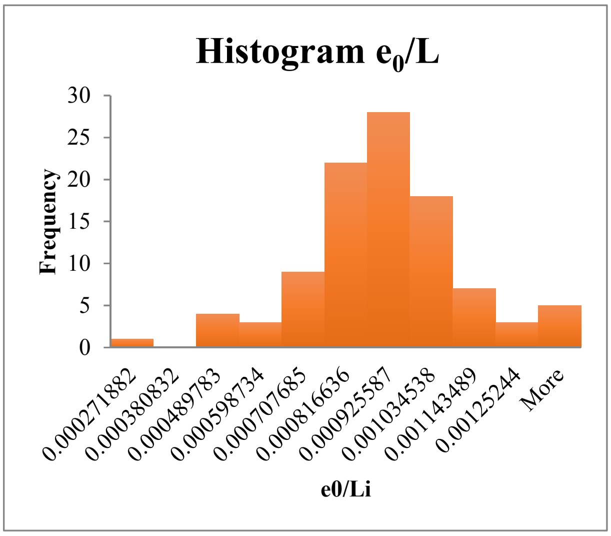

Figure 6.7 Histogram $e_{0/L}$

n=100, [-]		
$e0/L_{nom}$	$e0/L_m$	$\sigma_{e0/L}$
0.001	0.00085	0.0002

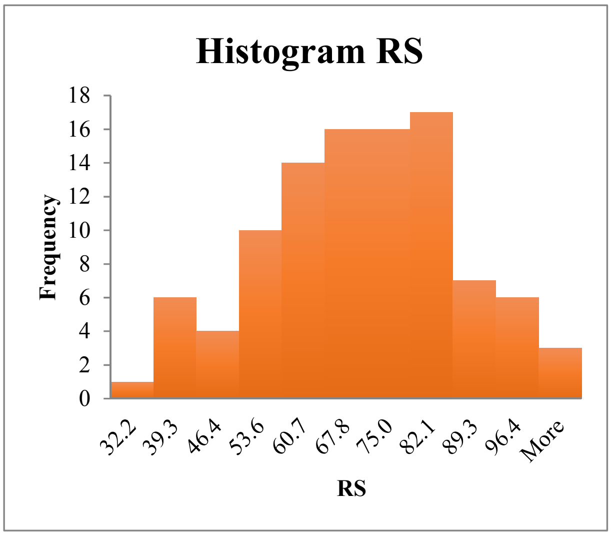

Figure 6.8 Histogram residual stresses

n=100, [MPa]		
RS_{nom}	RS_m	σ_{RS}
106.5	71	17.75

6.2 NUMERICAL MODELLING [17]

In order to obtain "experimental" results [17], large number of finite element simulations was carried out. Using numerical models for experimental results is reported to be useful way of obtaining results [5], because the output can be monitored, and in case of real experiments sometimes it is not possible to capture the real behaviour.

In this experimental programme, finite element software product Abaqus 6.12[18] was used. Each column is modelled with four-node linear shell elements (S4) with six degrees of freedom.

Advanced analyses were performed using geometrical and material nonlinearities with imperfections, also known as GMNIA. This type of analysis allows to capture the second order effects which are essential for the stability problem.

Material nonlinearity is incorporated in the model by using elastic-plastic constitutive law based on Von-Misses yield criterion.

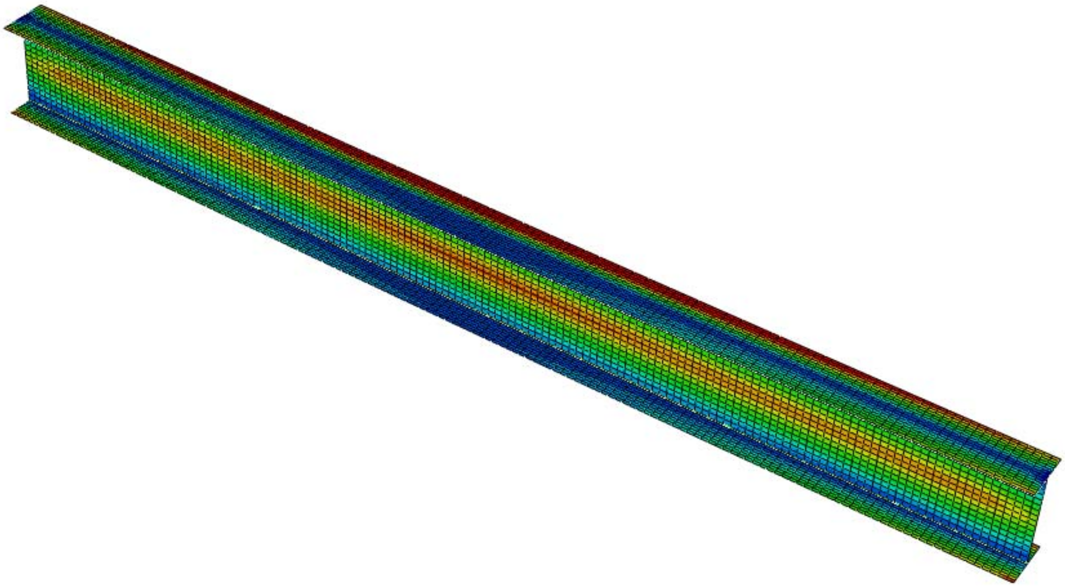

Figure 6.9 Finite element model

Geometrical imperfection were modelled using initial sinusoidal imperfection of amplitude e_0=L/1000. The imperfection is introduced in the weak axis of the cross-section.

The load is applied using load stepping routine, in which the increment size is chosen in order to meet the accuracy and convergence criteria. The equilibrium equations are solved for each increment using Newton-Raphson iteration technique.

The adopted mesh is 16 sub-divisions in the web and flanges and 100 divisions along the members axis.

The boundary conditions are implemented as end fork conditions in the shell model. The following restraints are used - vertical (δ_y) and transverse (δ_z) displacements and rotation about xx axis (ϕ_x) are prevented at supports. In addition, longitudinal displacement (δ_x) is prevented in one end. End cross-sections are modelled to remain straight.

6.3 RESULTS AND DISCUSSION

6.3.1 INTRODUCTION

In this section, the results are presented and further discussed based on comparisons between different approaches and considering different variables. The assessment is performed based on the partial safety factor γ_M. Different types of comparisons are used:

- Firstly, the influence of each basic variable: all basic variables are modelled separately, and the other variables are kept with nominal characteristics, in order to compare the impact of each basic variable on the resistance function;
- Secondly, the combinations of random variables are considered; in reality they are always acting together and their combined impact is being analysed;
- Subsequently, the influence of establishing a model using only nominal properties for basic variables is considered. Their variability is further accounted using the procedure of Annex D;
- Finally, the simplified procedures are applied in order to illustrate previously defined trends in section 5.

The column buckling formulation of EN 1993-1-1 [2] is adopted for resistance function. Based on that resistance function, partial safety factors are calculated using the procedure proposed in Annex D of EN 1990. The procedure is applied as shown in *Figure 3.5.* The theoretical quantities of the resistance function $r_{t,i}$ are calculated and further analysed with the experimental ones $r_{e,i} = N_b$. The errors terms δ_i are then obtained, leading to the coefficient of variation V_δ having three different quantities for each slenderness respectively. Furthermore, the sensitivity of the resistance function to the basic input variables is studied. Since the resistance function is a complex function, then the computation of the coefficient of variation V_{rt} involves partial derivatives of the resistance function at each variable. The partial derivatives are computed numerically using expression 6.1 and very small values of "ΔX_i" for each test specimen separately, leading to a scatter of $\gamma_{M,i}$ for each specimen.

$$\frac{\partial r_t}{\partial X_i} \approx \frac{r_t(X_1,...,X_i+\Delta X_i,....X_j)-r_t(X_1,...,X_i,....X_j)}{\Delta X_i} \tag{6.1}$$

Finally, the mean value of γ_M for each slenderness range is obtained and further compared in the cases listed above.

In this section, if it is not otherwise specified, the method which is used to obtain the partial safety factor is the procedure of Annex D using the partial derivatives.

The simplified procedures are applied as proposed in section 4.

6.3.2 INFLUENCE OF EACH BASIC VARIABLE

The influence of each basic variable is considered as each of them is modelled separately as a random variable and the remaining properties are kept with nominal characteristics. Furthermore, the procedure of Annex D is applied using the partial derivatives only at the respective variable and partial safety factors $\gamma_{M,i}$ are calculated for each column. Finally, the mean value of each slenderness case is obtained and further plotted in *Figure 6.10*.

Observing *Figure 6.10,* the favourable effect of the yield stress distribution is clear. Through all slenderness ranges, the partial safety factors are always lower than unity and more or less homogeneous.

On the contrary, if the modulus of elasticity is considered as only variable, it is almost in every case presenting with partial safety factors which are higher than one. This issue can be addressed to the fact that the mean value of the distribution is considered equal to the nominal one, and therefore there is range in which the modulus has lower quantity than the nominal. Hence the partial safety factor should "compensate" for those lower values. It is can be noticed that within low-to-intermediate slenderness range, the partial safety factors are lower since in that range it is has minor influence on the stability problem (see $\bar{\lambda}$ =0.3 and 0.6). However, in the medium-to-high slenderness range partial safety factors of 1.1 are observed.

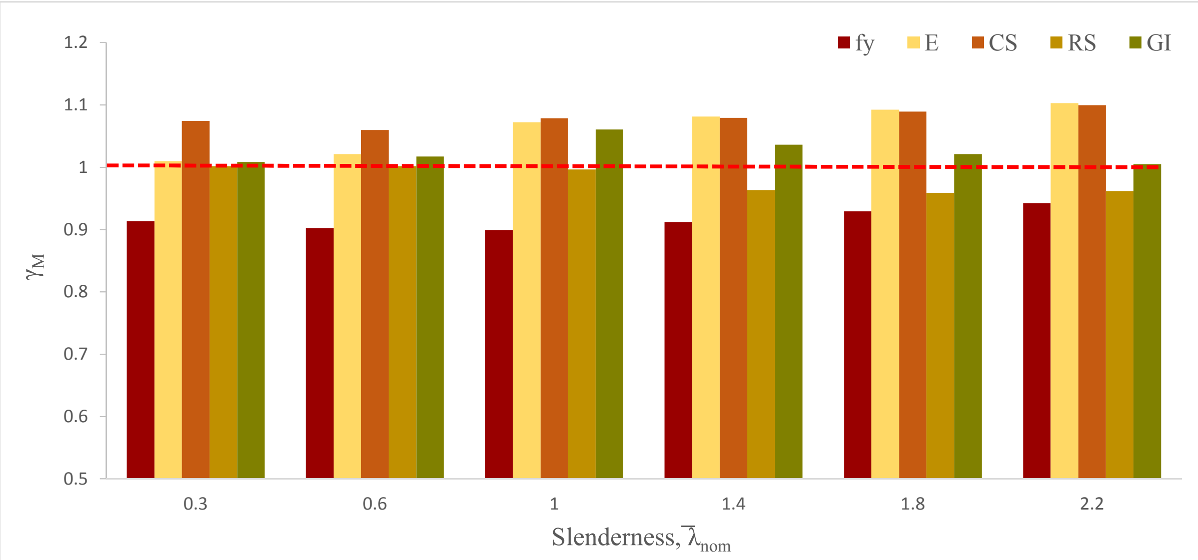

Figure 6.10 Mean value of partial safety factor for each variable

Similarly to the modulus of elasticity, the geometrical properties have mean value around the nominal value and therefore the partial safety factors are accounting for that by resulting in higher values. Unlike the modulus of elasticity, the geometrical properties exhibit more or less the same partial safety factor through all slenderness cases, due to fact that they are entering the resistance function both through the area and the moment of inertia. As the area is more important in the low-to-intermediate slenderness range, the inertia has more influence in the high slenderness range.

When considering the imperfection, their influence is only accounted through the difference of experimental value when compared to the nominal value, because they are incorporated in the design model of EN 1993-1-1 and they are not explicitly considered in the resistance function and therefore, it is not possible to obtain coefficient of variation V_{rt} when they are considered as a random variables.

As the residual stresses have a mean value lower than the nominal, the favourable effect, indeed, is seen by partial safety factors lower than unity. The geometrical imperfection however is presenting partial safety factor higher than unity in the intermediate slenderness range, where it actually cannot be neglected.

Further comparison of the imperfections is presented in *Figure 6.11,* where two additional cases are considered, namely: i) imperfections action together; ii) residual stresses changing according the change of the yield stress. Influence of modelling with residual stresses as a function of the nominal or the actual yield stress is found negligible. On the other hand, when both imperfection types are combined, for intermediate slenderness values, differences between both residual stresses and geometrical imperfection modelled separately, are found.

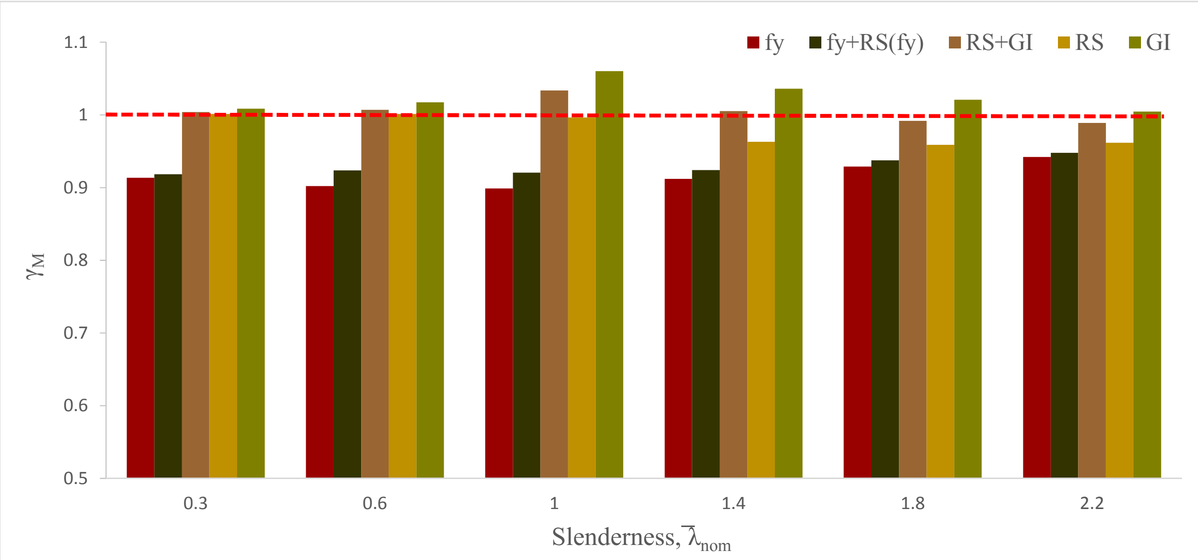

Figure 6.11 Mean value of partial safety factor for each variable

In reality, it is not possible to distinguish between basic variables. Since their nature is random, the basic variables act together and it is not possible to isolate them. However, it was useful for better understanding to try to quantify the influence of each variable.

6.3.3 COMBINATIONS OF BASIC VARIABLES

The combinations of variables are chosen as each of them is modelled separately with the random variables considered and the remaining properties are kept with nominal characteristics, similarly to the previous section. Furthermore, the procedure of Annex D is applied using the partial derivatives only at the respective variables and partial safety factors $\gamma_{M,i}$ are calculated for each column. Finally, the mean value of each slenderness case is obtained and further plotted in *Figure 6.12*.

The following combinations of variables are considered:

- Yield stress and cross-section properties **(fy+CS)** – this combination it is aimed at evaluating the influence of the variability of the cross-section relatively to the yield stress. The partial derivatives of the geometrical properties are performed with respect to the area and moment of inertia;
- Yield stress and modulus of elasticity **(fy+E)** – similarly to the previous combination, here the impact of the Young's modulus relatively to the yield stress is evaluated;
- Yield stress, cross-section properties and modulus of elasticity **(fy+CS+E)** – finally all the basic variables which are considered explicitly in the formulation of the EN 1993-1-1 are combined in order to obtain a realistic value of the partial safety factor;
- Yield stress, cross-section properties and modulus of elasticity; geometrical imperfection and residual stresses **(fy+CS+E+GI+RS)** – this combination is compared with the previous in order to see the influence;

Figure 6.12 summarizes the partial safety factors for the various slenderness ranges considered. In all cases the partial safety factors obtained considering the yield stress as the only variable is added.

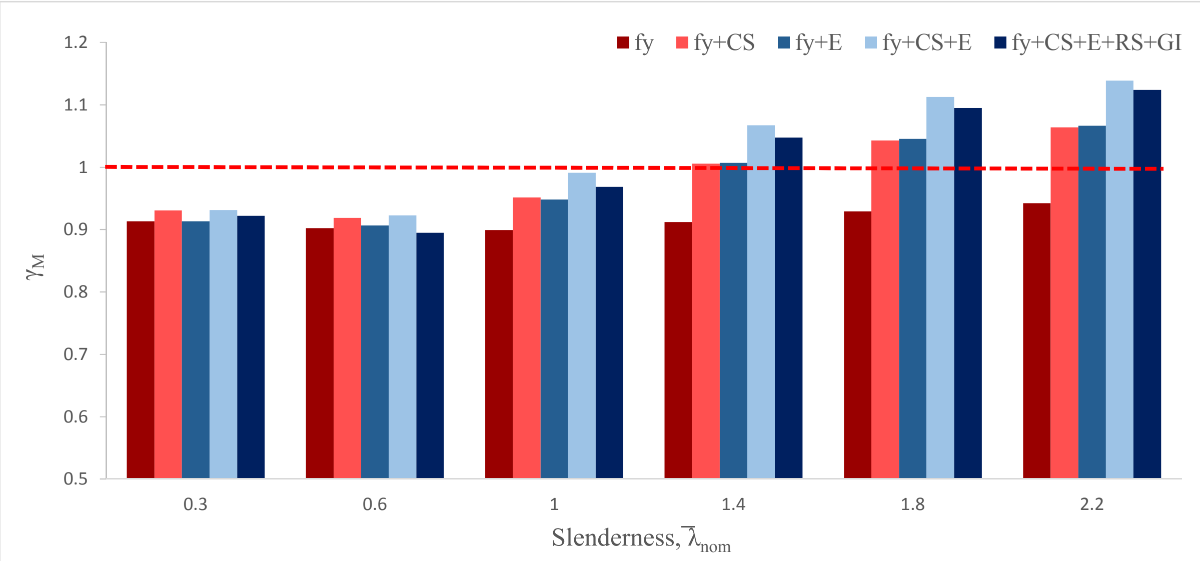

Figure 6.12 Combination of variables

In the previous sections, it was already mentioned that the distribution of the yield stress possesses useful features. Here, it can be observed that it is not valid only when the yield stress is considered as only variable. When comparing the results for *fy+E* and *fy+CS* it is seen that in the low and intermediate slenderness range *fy* manages to cover for the variability of the area and modulus of elasticity. However, in the high slenderness this is not true anymore since the modulus of elasticity and the inertia of the cross-section become more influential due to the second order effects. The same trend is observed when all variables are combined.

It can be also noticed that for $\overline{\lambda}$ =1.4, 1.8 and 2.2, the partial safety factor should be higher than unity, which is not the case in the present version of EN 1993-1-1[2]. This result is explained by already mentioned importance of the modulus of elasticity and inertia in the high-slenderness range. In addition, in the calibration of the buckling curves [16], the scatter of the modulus of elasticity and moment of inertia was not considered which in turn leads to the observed differences.

6.3.4 INFLUENCE OF MODELLING WITH NOMINAL PARAMETERS

A useful feature is the possibility to model the element with nominal properties and further apply the procedure of Annex D with the variability of the basic variables, it would be very useful , since it would reduce significantly the number of finite element simulations. In such case, the partial derivatives of the resistance function are performed at the nominal value of each basic variable.

Modelling with nominal properties would mean that the coefficient of variation V_δ, which is obtained from the differences between the numerical and theoretical results is the same regardless of the input variables. Moreover, it means that epistemic uncertainty is constant for the various slenderness cases, which in turn is not the reality. In *Figure 6.13* it is noticed that the differences between the experimental and theoretical estimations for the reduction factor χ are not constant thorough the buckling curve, therefore the model uncertainty differs for every slenderness.

In order to reduce the error between different slenderness ranges, for the vicinity of each slenderness case in this example, additional simulations are performed in order to obtain the coefficient of variation of the model V_δ, because if all are considered together the scatter would be higher and it would lead to a single γ_M factor.

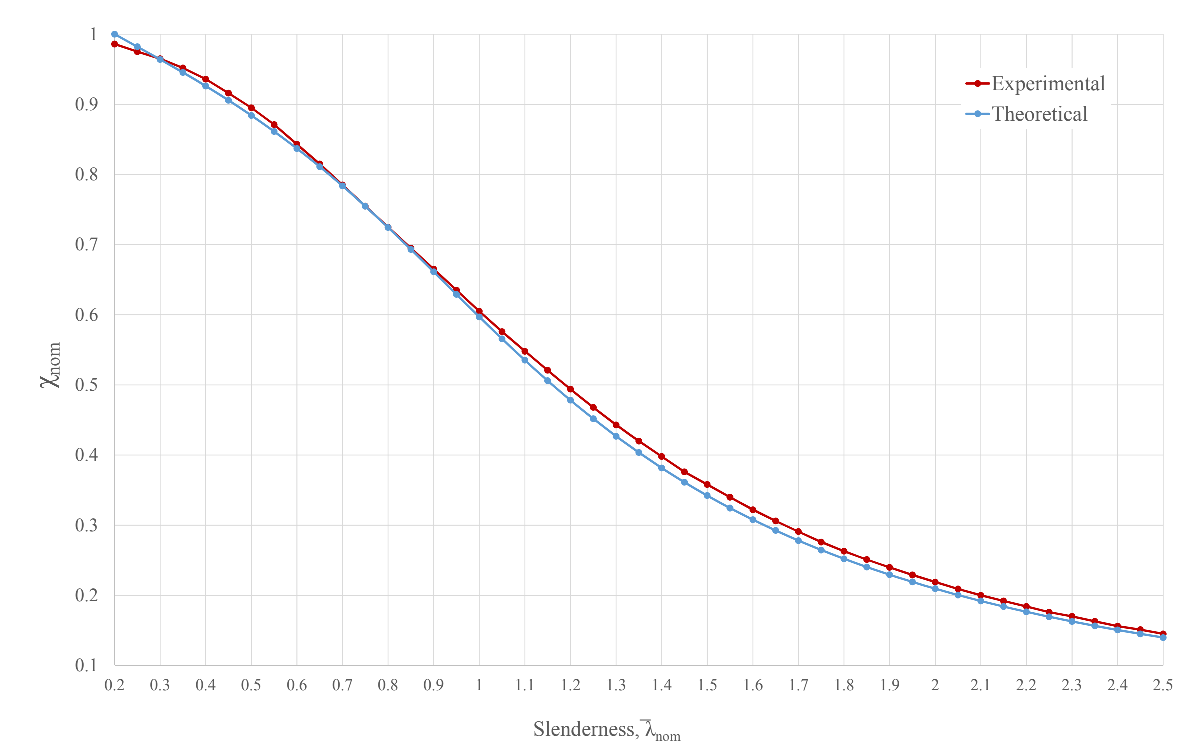

Figure 6.13 Nominal experimental and nominal theoretical

Several combinations are considered as it can be seen from *Figure 6.14* to *Figure 6.17,* the application of the assumption that the models are built with nominal characteristics and the influence of the variability is further considered in the application on the procedure of Annex D, leads to safe results. However, the differences are not constant throughout the different slenderness case as it was expected.

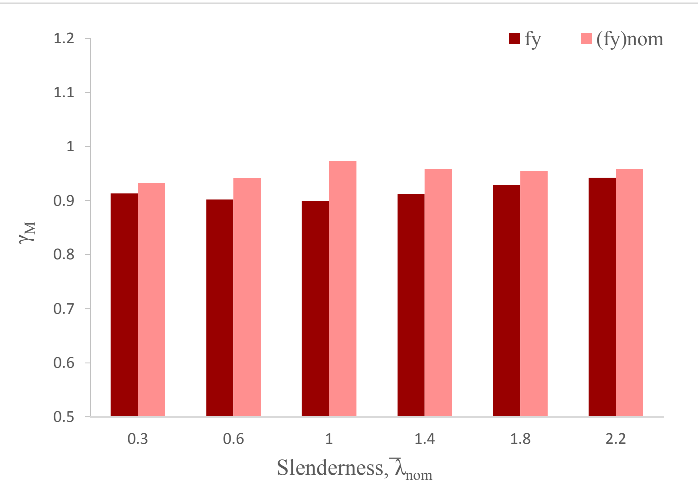

Figure 6.14 Fy vs (Fy)nom

Table 6.2 fy vs (fy)nom

	fy	(fy)nom	Error
λ=0.3	0.9135	0.9326	2.09%
λ=0.6	0.9024	0.9422	4.41%
λ=1.0	0.8992	0.9738	8.30%
λ=1.4	0.9123	0.9590	5.12%
λ=1.8	0.9292	0.9549	2.77%
λ=2.2	0.9423	0.9582	1.68%

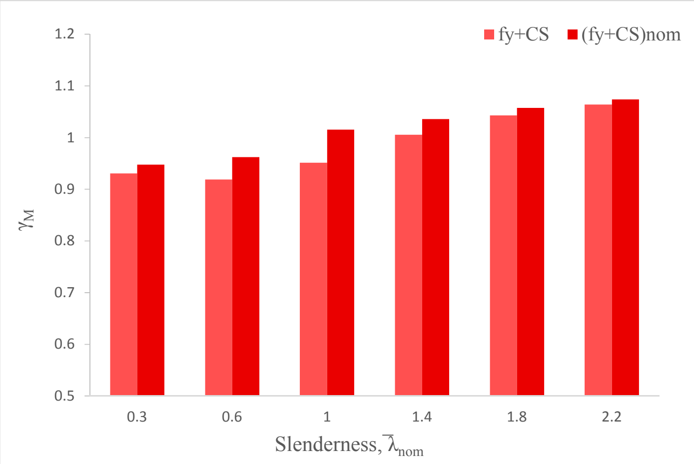

Figure 6.15 Fy+CS vs (Fy+CS)nom

Table 6.3 fy+CS vs (fy+CS)nom

	fy+CS	(fy+CS)nom	Error
λ=0.3	0.9309	0.9477	1.80%
λ=0.6	0.9187	0.9621	4.73%
λ=1.0	0.9517	1.0155	6.70%
λ=1.4	1.0058	1.0360	3.01%
λ=1.8	1.0427	1.0572	1.39%
λ=2.2	1.0638	1.0741	0.97%

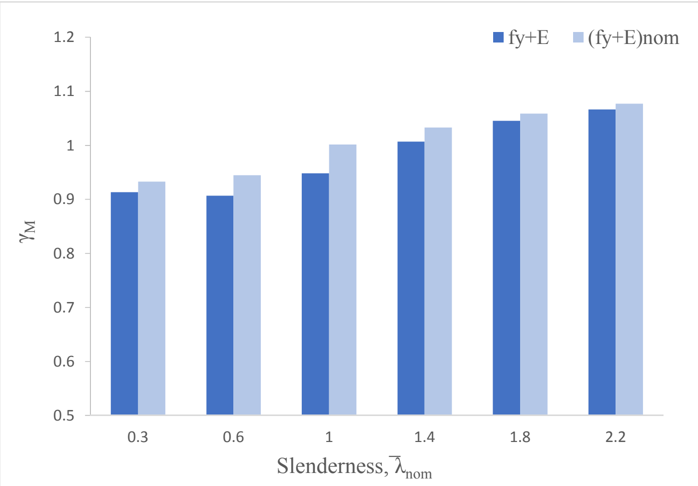

Figure 6.16 Fy+E vs (Fy+E)nom

Table 6.4 fy+E vs (fy+E)nom

	fy+E	(fy+E)nom	Error
λ=0.3	0.9136	0.9328	2.10%
λ=0.6	0.9069	0.9448	4.19%
λ=1.0	0.9483	1.0014	5.60%
λ=1.4	1.0068	1.0329	2.59%
λ=1.8	1.0455	1.0587	1.27%
λ=2.2	1.0667	1.0772	0.99%

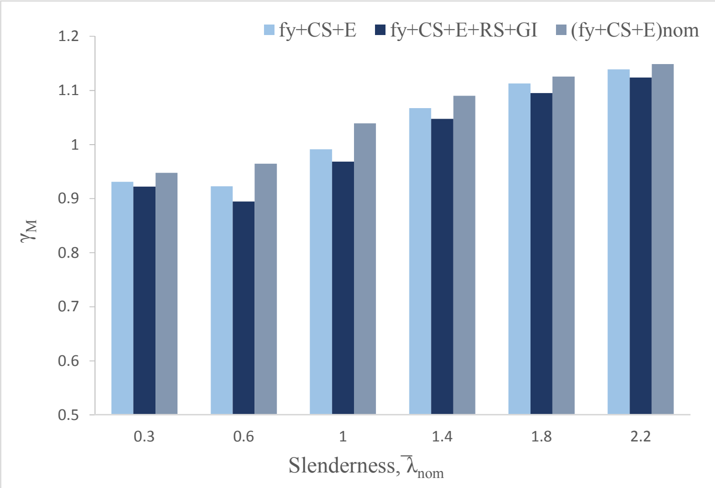

Figure 6.17 Fy +CS+E vs (Fy+CS+E)nom

Table 6.5 fy +CS+E vs (fy+CS+E)nom

	fy+CS+E	(fy+CS+E)nom	Error
λ=0.3	0.9312	0.9479	1.79%
λ=0.6	0.9231	0.9647	4.51%
λ=1.0	0.9914	1.0394	4.84%
λ=1.4	1.0672	1.0901	2.15%
λ=1.8	1.1131	1.1255	1.12%
λ=2.2	1.1387	1.1488	0.89%

6.3.5 INFLUENCE OF GEOMETRICAL PARAMETERS

In previous sections the partial derivatives for the cross-section properties were performed with respect to area and moment of inertia. In this section, the influence of performing the derivatives at each cross-section parameters or as more global – area and moment of inertia is considered. Figure 6.18 summarizes the partial safety factors obtained both alternatives, moreover, it presents the same comparison based on models with nominal properties. Very small differences are observed and therefore, the assumption is considered as valid.

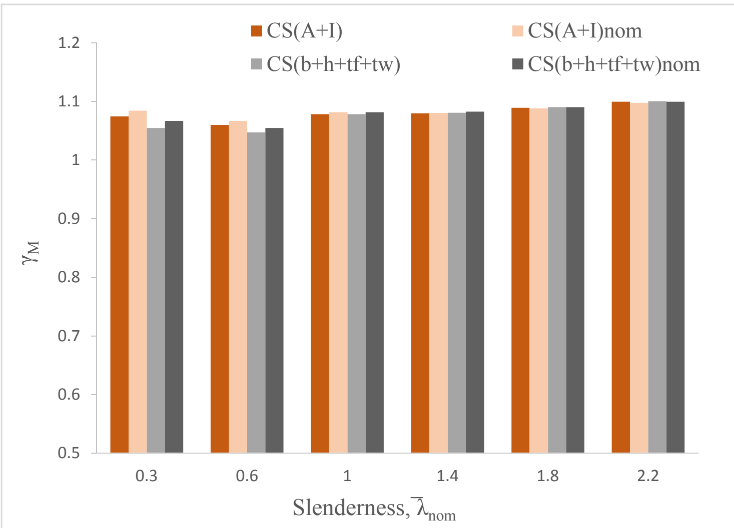

Figure 6.18A+I vs. $b+h+t_f+t_w$

Additionally, the influence of area and inertia separately is studied. The results comply with the discussion previously carried out – that the area has more influence in the low-to-intermediate slenderness, as the inertia in the intermediate-to-high. It is also observed that neglecting the inertia can lead to high differences. Moreover, the difference remains more or less constant no matter if the variable is combined with the yield stress or not.

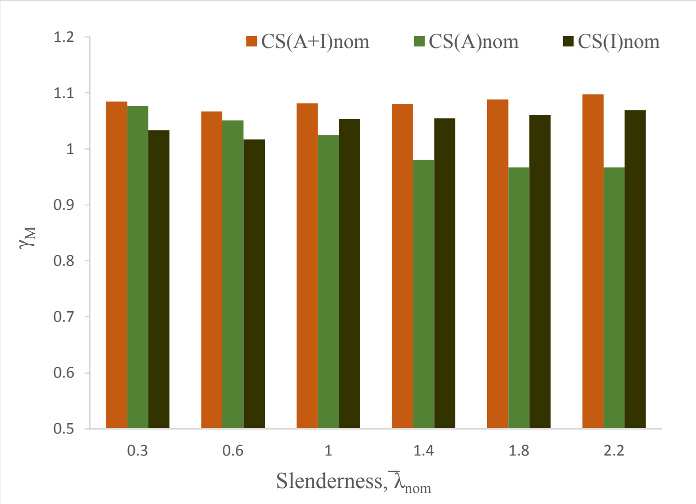

Figure 6.19Influence of cross-section area and inertia

Table 6.6 Influence of area

	CS(A+I)nom	CS(A)nom	Error
λ=0.3	1.0844	1.0769	-0.696%
λ=0.6	1.0668	1.0508	-1.50%
λ=1.0	1.0814	1.0250	-5.21%
λ=1.4	1.0802	0.9808	-9.20%
λ=1.8	1.0880	0.9670	-11.12%
λ=2.2	1.0974	0.9670	-11.88%

Table 6.7 Influence of inertia

	CS(A+I)nom	CS(I)nom	Error
λ=0.3	1.0844	1.0333	-4.712%
λ=0.6	1.0668	1.0169	-4.68%
λ=1.0	1.0814	1.0536	-2.57%
λ=1.4	1.0802	1.0547	-2.36%
λ=1.8	1.0880	1.0611	-2.48%
λ=2.2	1.0974	1.0694	-2.55%

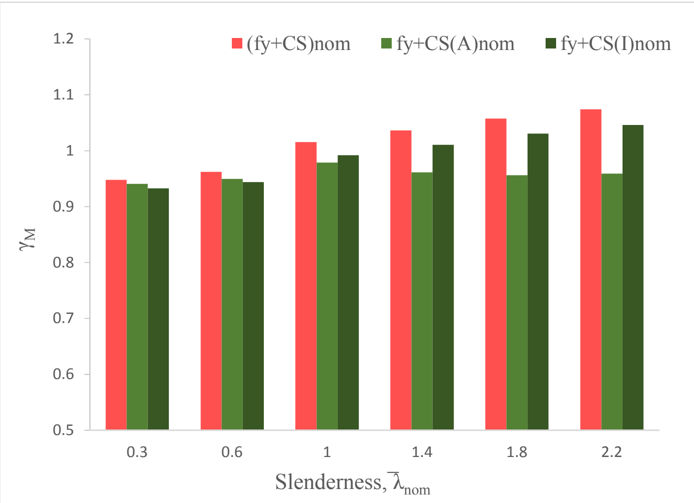

Figure 6.20Influence of cross-section area and inertia

Table 6.8 Influence of area

	(fy+CS)nom	fy+CS(A)nom	Error
λ=0.3	0.9477	0.9405	-0.763%
λ=0.6	0.9621	0.9494	-1.33%
λ=1.0	1.0155	0.9785	-3.64%
λ=1.4	1.0360	0.9614	-7.21%
λ=1.8	1.0572	0.9563	-9.55%
λ=2.2	1.0741	0.9591	-10.70%

Table 6.9Influence of inertia

	(fy+CS)nom	fy+CS(I)nom	Error
λ=0.3	0.9477	0.9327	-1.585%
λ=0.6	0.9621	0.9438	-1.90%
λ=1.0	1.0155	0.9918	-2.33%
λ=1.4	1.0360	1.0106	-2.46%
λ=1.8	1.0572	1.0304	-2.54%
λ=2.2	1.0741	1.0463	-2.59%

6.3.6 SIMPLIFIED PROCEDURES

The simplified procedures are applied in the context presented in previous sections.

1) **P1 (f_y)** – as it was presented in section 4.3, it is based on expression (3.9) and only the variability of the yield stress is taken into account. As a result, from the fact that the mean and standard deviation of the yield strength are represented by single values, global partial safety coefficients γ_M are obtained for each slenderness range.*Figure 6.21* and *Table 6.10* summarize the results obtained with this procedure.
   In addition, *Figure 6.21* and *Table 6.10* present results for the procedure in ***f)*** using the partial derivatives *only for f_y, it is "acting as if"* the yield strength is the only variable. Nonetheless, the percent difference is computed with regard to (global) ***f_y+CS+E.***
   Observing *Table 6.10,* it can be pointed out that P1 provides results which are *unsafe* when compared to ***f_y+CS+E.***
   These results confirm the trends observed in section 5.3. Nevertheless, it should be noticed that the percentual difference might be different for the same V_δ since the distributions of the basic variables are different from the ones used in section 5.

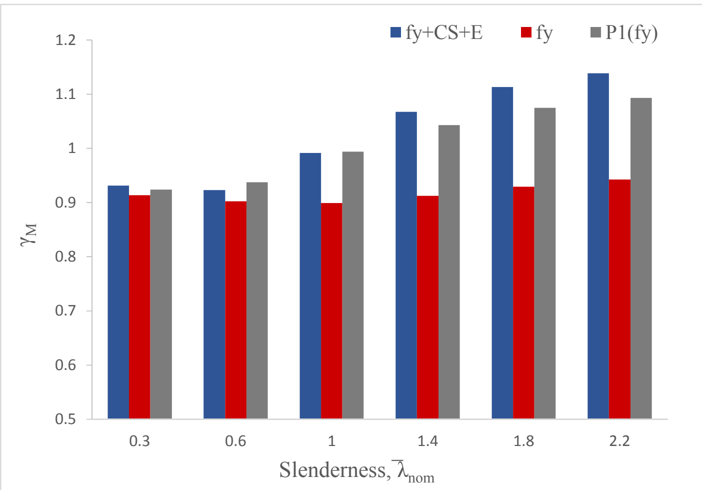

Figure 6.21 Comparison of partial safety factors

Table 6.10 Difference P1(fy)

	fy+CS+E	fy	P1 (fy)	Error
λ=0.3	0.9312	0.9135	0.9237	-0.81%
λ=0.6	0.9231	0.9024	0.9376	1.57%
λ=1.0	0.9914	0.8992	0.9939	0.26%
λ=1.4	1.0672	0.9123	1.0430	-2.26%
λ=1.8	1.1131	0.9292	1.0745	-3.46%
λ=2.2	1.1387	0.9423	1.0932	-4.00%

2) **P2 (*f_y+A)*** – as an extension of the assumptions above, here the geometrical properties are included are basic variables. P2 is assumed as presented in section 4.4. In this case, the random variables are the yield strength and the cross-section area. The mean value and standard deviation of the area are calculated using Eq. (3.23) and Eq. (3.24). Similarly to P1, single results for the partial safety coefficient γ_M are obtained. *Figure 6.22* and *Table 6.11*summarize the results obtained with this alternative;

   It is noticed for $\bar{\lambda}$ =2.2, the procedure starts to become unsafe, unlike the observations in section 5, where the procedure was showing only safe-sided results. However, this "misfit" is explained by the fact that the distribution in this example has much lower coefficient of variation for the area ~2.3%, instead of 4% which were used in section 5.

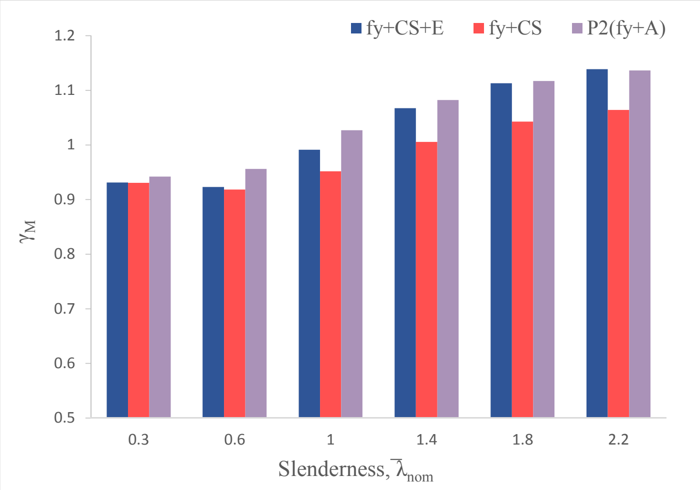

Figure 6.22 Comparison of partial safety factors

Table 6.11 Difference P2(fy+A)

	fy+CS+E	fy+CS	P2 (fy+A)	Error
λ=0.3	0.9312	0.9309	0.9423	1.19%
λ=0.6	0.9231	0.9187	0.9562	3.58%
λ=1.0	0.9914	0.9517	1.0270	3.59%
λ=1.4	1.0672	1.0058	1.0826	1.44%
λ=1.8	1.1131	1.0427	1.1170	0.36%
λ=2.2	1.1387	1.0638	1.1364	-0.20%

3) **P2 (*f_y+A+I*)** – same as 2, but with the additional consideration of the moment of inertia as a random variable. Correspondingly to **P2 (*f_y+A*)**, the mean value and standard deviation of the moment of inertia are calculated using Eq. (3.23) and Eq. (3.24). Here also single results for the partial safety coefficient γ_M are obtained. *Figure 6.23* and *Table 6.12* summarize the results obtained with this alternative;
Moreover, *Figure 6.23* and *Table 6.12* present results for the procedure in ***f_y+CS*** using the partial derivatives in case that the yield strength, column cross-section dimensions are the basic variables and the variability of the Young's modulus is neglected. However, the percent difference is evaluated with regard to ***f_y+CS+E.***

Comparing *Table 6.11* and *Table 6.12,* it can be observed, that **P2 (*f_y+A)*** and **P2 (*f_y+A+I)*** provide results which are on the safe side when associated to ***f_y+CS+E.*** However, this difference increases with increased number of variables.

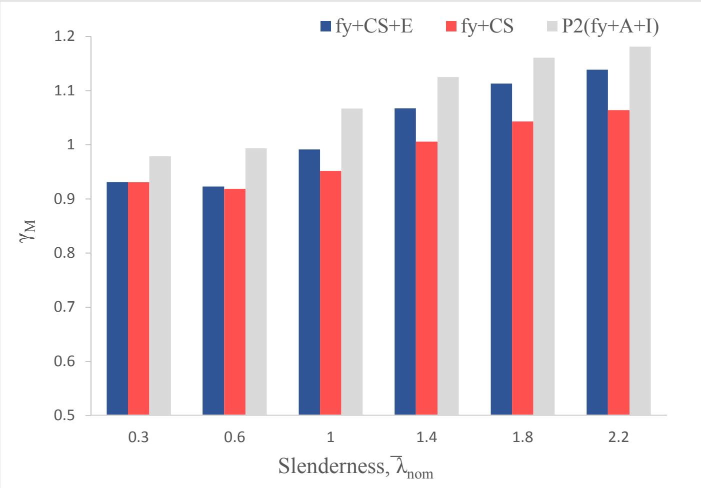

Figure 6.23 Comparison of partial safety factors

Table 6.12Difference P2(fy+A+I)

	fy+CS+E	fy+CS	P2 (fy+A+I)	Error
λ=0.3	0.9312	0.9309	0.9791	5.15%
λ=0.6	0.9231	0.9187	0.9934	7.61%
λ=1.0	0.9914	0.9517	1.0666	7.59%
λ=1.4	1.0672	1.0058	1.1248	5.40%
λ=1.8	1.1131	1.0427	1.1607	4.28%
λ=2.2	1.1387	1.0638	1.1808	3.70%

4) **P2 (*f_y+A+I+E)*** – finally variability of yield stress, Young's modulus and cross-sectional properties are grouped together using P2. Results are presented in *Figure 6.24*and *Table 6.13*.

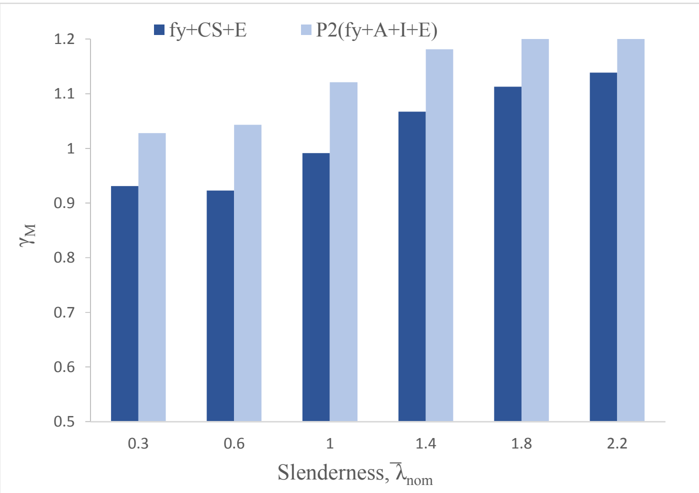

Figure 6.24 Comparison of partial safety factors

Table 6.13 Difference P2(fy+A+I+E)

	fy+CS+E	P2(fy+A+I+E)	Error
λ=0.3	0.9312	1.0281	10.41%
λ=0.6	0.9231	1.0431	13.00%
λ=1.0	0.9914	1.1213	13.11%
λ=1.4	1.0672	1.1814	10.70%
λ=1.8	1.1131	1.2190	9.52%
λ=2.2	1.1387	1.2405	8.94%

The example confirms the trends shown in section 5.3. However, it should be noted that it is not possible to directly compare results since the distributions used in sections 5.3 and 6 have different parameters – ratio between mean and nominal values, standard deviation. Despite that fact, the trends drawn in section 5 for the simplified procedures are confirmed with the example in chapter 6.

7 BRIEF ANALYSIS OF THE STATISTICAL DEPENDENCY BETWEEN BASIC VARIABLES

Throughout the previous chapters, there was always the same assumption which in fact is the assumption adopted in EN 1990, i.e., the basic variables are treated as statistically independent. However, it is not always the case. For example, the yield stress is dependent on the plate thickness; or the cross-section dimensions are dependent to each other due to the fact that the element mass is strictly monitored, the modulus of elasticity and the elastic strain, etc.

It was already explained that it is very hard to obtain information about the distributions of the basic variables. Therefore, it is even more difficult to obtain the magnitude of the correlation between basic variables. However, it is worth to try to quantify the impact of the correlation, i.e. if it is safe-sided (within certain reasonable limits) to neglect it.

In this chapter, the statistical dependence of the yield stress and thickness of the plate is treated. It is a negative correlation, i.e. when one parameter is increasing, the other is decreasing.

7.1 CORRELATION BASED ON EXPERIMENTS

In [19], a large number (689) of tests was performed on yield stress and geometrical properties. Based on that data, an attempt to estimate the correlation coefficient is performed. As the yield stress is dependent on the plate thickness, firstly the scatter of flange thickness and yield stress is considered.

689 tests from [19] on S235 steel are considered for this analysis. The geometrical properties were measured on both H and I sections, namely IPE80, IPE120, IPE 140, IPE 140, IPE 160, IPE 180, IPE 240, IPE 270, HEB 100, HEB 140, HEB 180. Although all flange thicknesses are smaller than 16mm, a certain trend can be seen when observing the scatter on *Figure 7.1*.

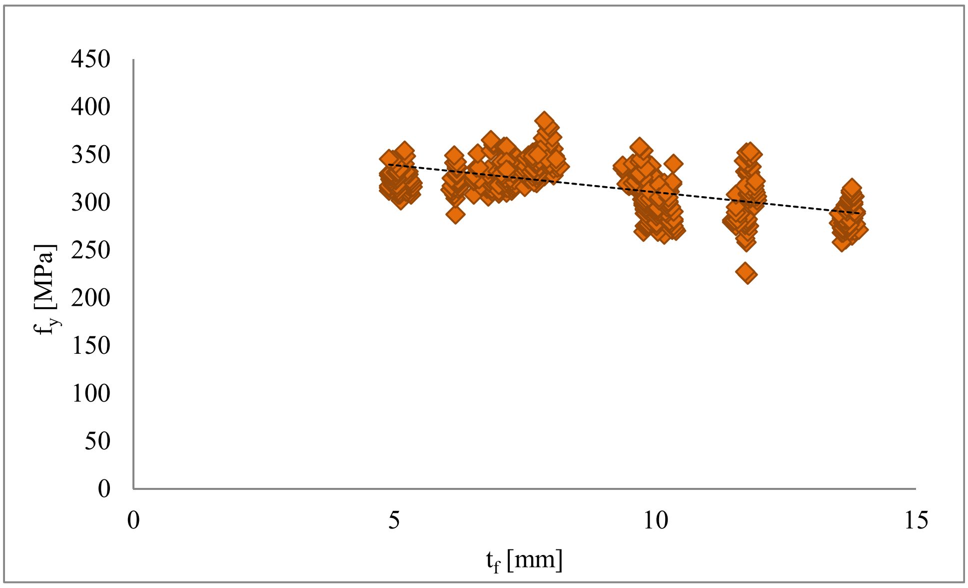

Figure 7.1 Correlation between yield stress and flange thickness

The correlation coefficient is obtained based on expression (2.25), and the value of $\rho = -0.611$ is found. *Figure 7.1* clearly shows the negative correlation between the variables, yet it is confirmed by the calculation.

Additionally, the correlation between the cross-section area and yield stress is studied and correlation coefficient is found equal to *-0.681,* similarly to the one above. The scatter is shown in *Figure 7.2.*

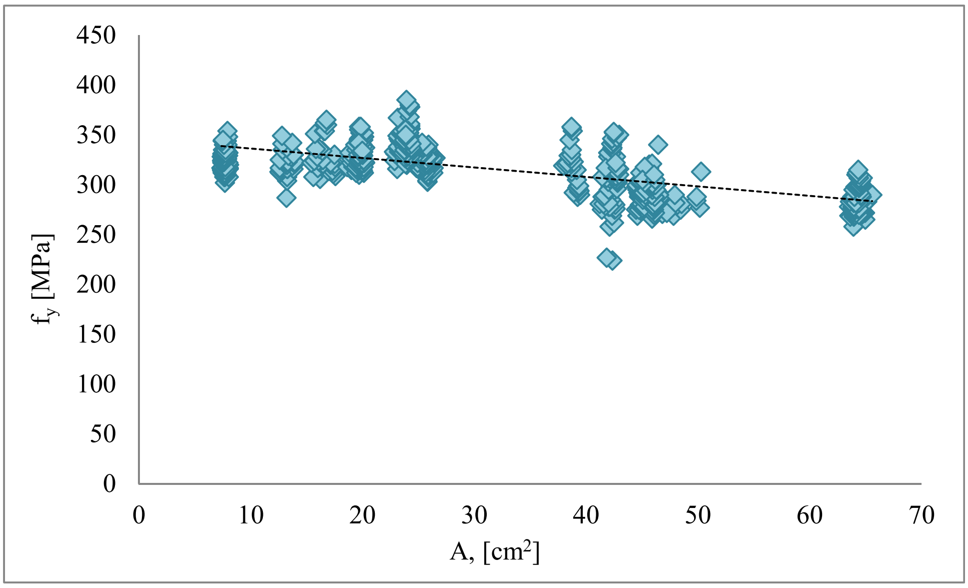

Figure 7.2 Correlation between yield stress and cross-section area

7.2 Implementation of the correlation

The correlation between basic input variables in a function was discussed in sections 2.6 and 2.7. There it can be seen that the statistical dependence of the variables has no impact on the mean value of the function, however additional cross terms appear in the variance of the function, i.e. in the error propagation. On the other hand, the procedure adopted in Annex D of EN 1990 that was presented in section 3.3, considers statistical independence of the basic input variables and therefore, the coefficient of variation V_{rt} is calculated using expression (2.29), obtained from expression (2.28). Here, in order to account for the correlation between basic variables, expression (2.28) and the coefficient of variation V_{rt} becomes:

$$V_{rt}^{\ 2} = \frac{1}{g(\underline{X_m})^2}\left[\sum_{j=1}^{k}\left(\frac{\partial g(X_j)}{\partial X_j}\sigma_j\right)^2 + \sum_{i,j=1}^{n}\sum_{i\neq j}^{n}\rho_{i,j}\sigma_{X_i}\sigma_{X_j}\frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j}\right] \qquad (7.1)$$

The remaining part of the procedure remains unchanged.

In order to consider the correlation in terms of difference between partial safety factors, the samples in section 5.2 are used and the comparison is performed using the same assumptions as in section 5.3.1 as here a correlation between area and yield stress is assumed. Moreover, the correlation coefficient is assumed equal to *-0.681* as calculated for the experimental results in the previous section. Additionally, assumption for the coefficient of variation V_δ was also made.

Figure 7.3 summarizes the results. It was found that higher difference between considering the correlation or not is found for $V_\delta=0$. The maximum observed difference is 3.6%, meaning that including the covariance would result in 3.6% lower partial safety factors. Therefore, neglecting the correlation is actually safe-sided and the simplification does not lead to high differences.

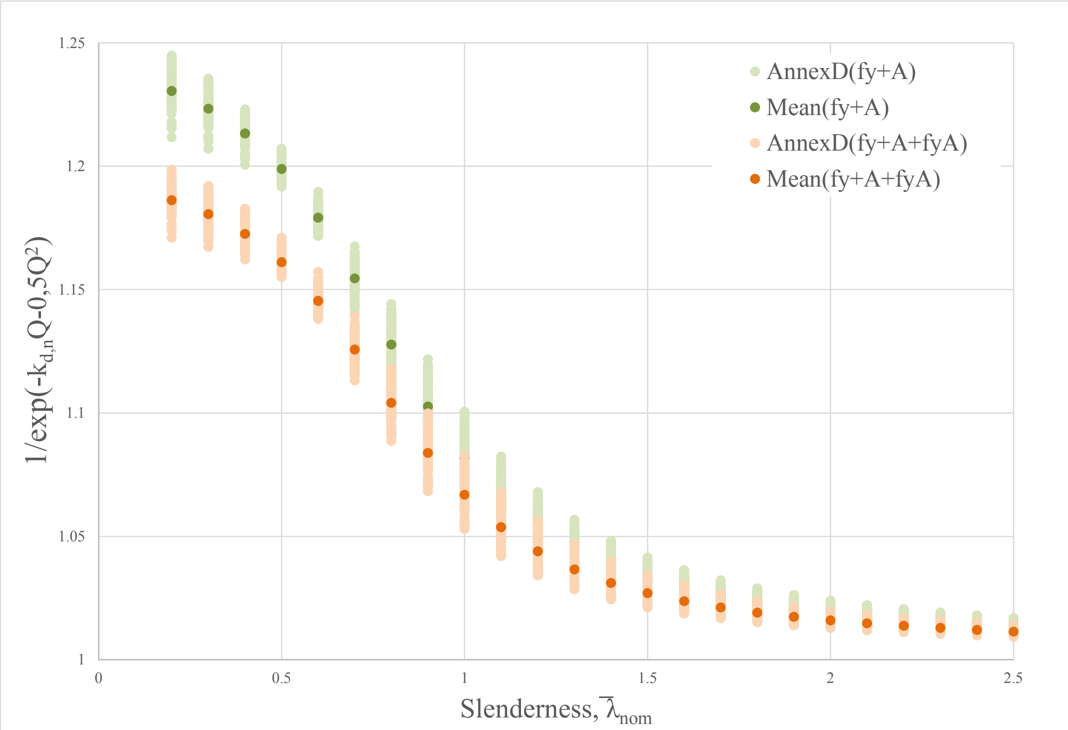

Figure 7.3 Correlation coefficient -0.681

Additionally, an extreme case of -1.0 is also considered. Here the same assumption for $V_\delta=0$, leads to 5.6% difference, which is also safe-sided.

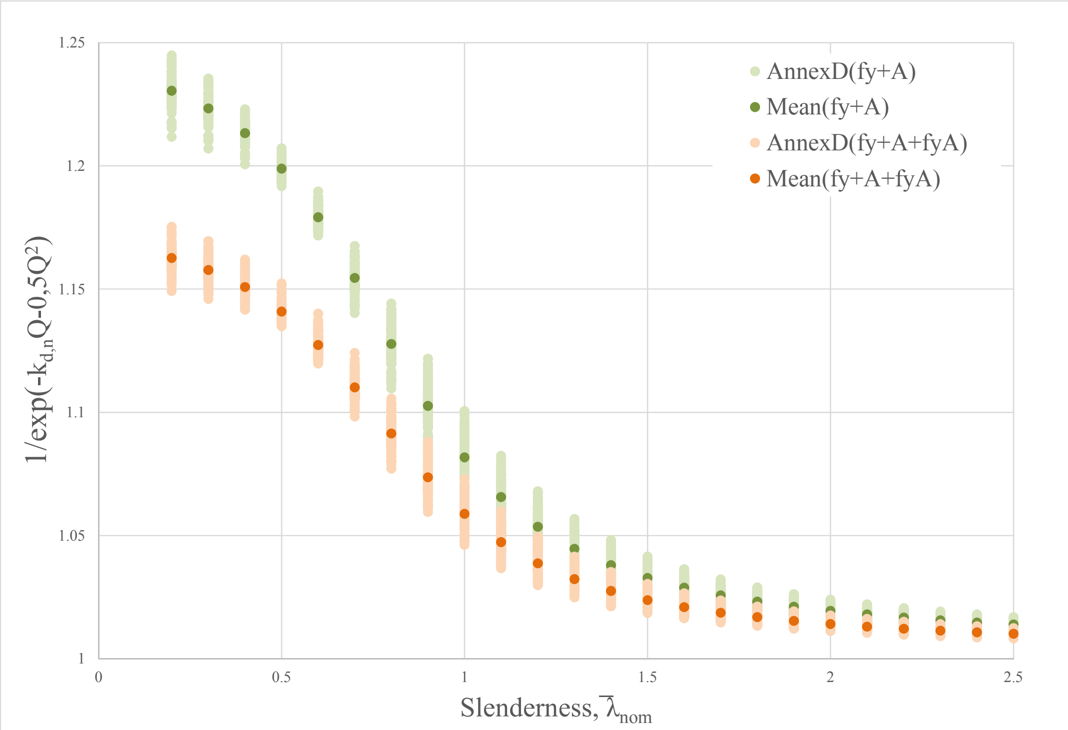

Figure 7.4 Correlation coefficient -1.0

In this section, an attempt to take into account statistical dependence of the basic input variables was presented. The assumption of correlation between yield stress and geometrical properties was adopted. That correlation is negative and it leads to safe-sided result, if it is neglected. However, these observations might not be valid for other parameters which exhibit positive correlation or other statistical data. On the contrary, the observed differences were not very high (5.6%) for an extreme case.

8 CONCLUSIONS AND FUTURE RESEARCH

8.1 CONCLUSIONS

This study summarized different possibilities for the safety assessment of design rules focusing on the buckling resistance of steel members. The various simplifications were explained and assessed based on assumed distributions for the basic variables. Different basic variables were considered, including: i) material properties; ii) geometrical properties. Moreover, the influence of each basic variable was studied in the numerical example in section 6.

It was shown that simplified procedures which include the variability of the yield strength as the only basic variable may be unsafe for certain slenderness ranges when compared to the Annex D procedure considering all relevant basic input variables.

When geometrical properties were included, P2 always showed results on the safe side. However, a clear trend of increasing the error between P2 and the "full" Annex D with increased number of variables considered was noted in both the numerical validation of chapter 5 and the example presented in chapter 6. This results from the simplified procedure simply adding the variability of the input variables, while the Annex D procedure using the partial derivatives for V_{rt}, accounts for the function and each variable gives its contribution according to its contribution in the resistance function, at the respective slenderness value. In other words, the simplified procedures inevitably calculate a "too large" value of V_{rt}.

Additionally, it is noticed that the difference between the "full Annex D" and the simplified procedure for each slenderness range become more homogeneous with an increased number of basic random variables. This tendency can be seen as very useful in case of adopting adjustment function or factors for the simplified procedures, as proposed in [14].

Moreover, the influence of each variable was discussed based on the numerical example in section 6. It was shown that the distribution of the yield stress has favourable effect on the partial safety factor γ_M i.e. it reduces its value. On the contrary, the distributions of the geometrical properties and Young's modulus lead to higher partial safety factors. It was observed that when the basic variables are combined, the yield stress may compensate for the distributions of geometry and modulus of elasticity. However, in the high slenderness range, the favourable effect was not enough due to the higher importance of the stiffness parameters.

A try to assess the partial safety factors based on models with nominal properties was also presented. It was shown that the differences are small, nevertheless not constant. Keeping in mind those observations, it is not possible to conclude that models with nominal properties can be used for safety assessment.

Additionally, the scatter of member imperfection and residual stresses was also included in the numerical models. Imperfections, being basic variables incorporated in the design procedure, are difficult to assess explicitly, therefore their impact was only accounted in the coefficient of variation of the model V_{δ}.

Although all the numerical comparisons were performed only on the basis of the flexural buckling of columns, the conclusions should be equally valid to other stability phenomena (LTB of beams, TB and LTB of columns and the buckling resistance of beam-columns). These other stability problems will be addressed in the near future to confirm this.

In EN 1990 and in the simplified procedures, the same assumption was adopted – the basic variables are statistically independent. In chapter 7, an attempt to consider correlation between basic variables was proposed. The correlation between yield stress and plate thickness was only accounted for. In order to evaluate reasonable correlation coefficient, statistical data from real experiments was used. It was shown that if the statistical dependence is neglected, the differences are not very high and the result is safe sided, for the case considered. However, different results may be observed when other basic variables are used. Therefore, the topic should be further explored.

8.2 FUTURE RESEARCH

The scope of this study has certain limitations and therefore it should be further extended in the following directions:

- Here, the safety assessment was performed based only on the flexural buckling of columns, as already mentioned it is expected that the conclusions made would be equally valid for other stability modes, however it is worth to extend the study for LTB of beams, TB and LTB of columns and the buckling resistance of beam-columns in order to confirm the same trend. Furthermore, the concept should be tested for other failure modes such as ductile and brittle failure modes.
- In chapter 6, it was discussed that many parameters lack sufficient statistical characterization. Consequently, it would be very useful, if such data can be collected. In order to achieve that, a European database is developed under SAFEBRICTILE project. It can provide the needed statistical characterization and allow to give guidance to the designers and researchers on which basic variables should be considered;
- The safety assessment procedure should be developed, in order to provide clear guidance to designers and researchers on how to assess new design rules. In addition, a guidance on which basic variables are relevant for the different failure modes is also required;
- Finally, and attempt to include statistical dependence was presented, however it was only focused on correlation between plate thickness and yield stress. As already mentioned, it would be useful to extend the study to correlation between other basic variables and check if it is safe-sided to assume statistical independence of the relevant basic variables.

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