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TIME-DEPENDENT ANALYSIS AND CABLE STRETCHING FORCE OPTIMIZATION OF CONCRETE CABLE-STAYED BRIDGES

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Summary. *This paper presents a procedure to determine the cable forces on cable-stayed bridges using an entropy-based optimization algorithm. It includes an analytic sensitivity analysis module, which provides the structural behaviour responses to changes in the design variables. A finite-element approach is used for structural analysis which includes the time-dependent effects of creep and shrinkage of concrete, and the construction stages of the bridge erected by the balanced cantilever method. The main objective is to find the cable stay forces to correct errors during the erection stages to control the bridge dead load geometry condition by requiring the stresses in the structural elements to remain within allowable limits.*

1 INTRODUCTION

Cable-stayed bridges are an elegant and efficient structural solution, and their use has been steadily increasing. While steel was the dominating structural material, prestressed and composite decks have increasingly been used.

The design of cable-stayed bridges can be made iteratively. The engineer seeks, through the change of some parameters, the best solution that satisfies a set of criteria. The complexity of the task rises with the model dimension and the specific problems that need to be solved. Therefore, optimization naturally arises as a tool to aid in the design of cable-stayed bridges, in view of obtaining economic and structural efficient solutions.

Most of the work on cable-stayed bridge optimization dealt with steel bridges, concerning the prestressing forces on the cable stays[1-5]. Simões and Negrão presented an algorithm for the geometric and cross-sectional optimization of the structural members, using 2D and 3D modelling. They studied also the optimization considering a box-girder deck and the seismic action effect[6-9].

When the bridge is executed in prestressed concrete (PSC) it is necessary to include the evolutionary nature of the structure. This is due to the time-dependent effects, namely, the creep and shrinkage of concrete and the relaxation of the prestressing steel, and changes in the stresses and displacements during the construction stages.

Khalil et al[10] using step-by-step time integration studied the influence of the time-dependent effects in the 2D structural analysis of cable-stayed bridges with PSC decks. Cluley and Shepherd[11] analyzed the time-dependent effects and computed the equivalent nodal loads to implement in a finite-element computer program using 3D modelling. Cruz et al[12] developed a nonlinear step-by-step analysis model of planar frame concrete structures. The model simulates segmental construction processes and accounts for the nonlinear time-dependent material properties, the structural effect of the delayed deformations and the second-order effects. Somja and de Ville de Goyet[13] presented a method based on the fictious loading age method with improvements to take into account recovery. Therefore time-dependent effects are important to control the geometry and internal forces during the construction and service life of the structure.

This paper outlines a method using an entropy-based optimization algorithm to find the prestressing forces in the stay cables of a PSC cable-stayed bridge so that the stresses in the stays and the structure remain in the allowable range during the construction process and a desired final condition is achieved at the time of completion of the structure. In the optimization process an analytical sensitivity analysis is used to evaluate the structural responses to changes in the design variables. The structural analysis accounts for the time-dependent effects and stages of the construction where the balanced cantilever method is employed.

2 GEOMETRY CONTROL AND CABLE FORCES

2.1 Erection procedures

The erection method used in the construction of a cable-stayed bridge clearly depends on the size of the structure, the structural system and the conditions found at the intended location. In general, there are four different construction methods possible:

- Construction on temporary supports;
- Construction by rotation;
- Construction by incremental launching;
- Construction by the balanced cantilever method.

The balanced cantilever method is the most used method for the erection of cable-stayed bridges and for that is used in this work. In the cantilever construction method a form traveler is attached to the previously casted segment and carries the formwork for the new segment that is to be cast. In each erection stage new segments are installed and then supported by new cables.

In this method no falsework or centering is required, leaving traffic under the spans unobstructed during construction. If the bridge spans are too high above ground and if the terrain under the spans is inaccessible fast construction can be achieved with cantilevering. To obtain the forces on the stay cables, a detailed analysis of the construction stages by the cantilever method is required since the structural scheme changes in each construction stage affecting the stresses and displacements.

2.2 Cable forces adjustment

Cable tensioning is used to adjust and control the stress distribution and the geometry of cable-stayed bridges. Three different approaches to adjust the cable stay prestressing force distribution have been proposed in the literature: the “optimization method”[1-9], the “force equilibrium method”[14] and the “zero displacement method”[15]. Several of these solutions are based on the final configuration of the structure and do not take into account the actual construction process and/or the time-dependent effects, both influencing considerably the geometry and distribution of internal forces in the completed structure.

In the “optimization method” the cable forces are determined based on certain functions related to structural efficiency or economy. Objective functions to be minimized are often the volume/cost of the starting trial design, the total strain energy, the work produced by the cable forces, the bending stresses and the tolerance deflection for the geometry control.

In the majority of the research works the concept of influence matrix of cable forces is used. Analyzing the structure response of a unit prestress applied at each cable, the influence value of all the targets can be obtained. In other works the gradients of the objective function and the design constraints are computed using finite differences which require a great amount of computational time.

In the present research work the objective is to apply to concrete cable-stayed bridges the strategy developed by Simões and Negrão[6-9] to steel bridges, taking into account the construction stages and the time-dependent material behaviour of concrete.

The optimization method used, besides the determination of the cable stay forces, allows the minimization of the structure cost and ensures that the stresses remain within allowable limits and the final desired geometry is achieved. The structural response to changes in the design variables is computed by a discrete direct sensitivity analysis which requires some programming work but is a computationally efficient procedure.

2.3 Construction stage analysis

The internal forces as well as the geometry of a complete cable-stayed bridge highly depend on the erection sequence of the structure. The cable forces at the time of installation clearly differ form those of the final dead load condition. Moreover, the geometric profile of the girder during the construction greatly changes and it is important to ensure that the cantilever ends finally meet at the bridge closure. Another reason for considering the erection process is that sometimes the structural behavior during construction might be much more critical than that of the final stage. In order to complete the design of the bridge, the stresses in the cables, the girder and the pylon need to be checked not only in the final but also in intermediate erection stages. Therefore, the internal forces in members of the bridge structure at each erection stage have to be known. The construction stage analysis can de performed by backward analysis or forward analysis. The forward analysis reflects the real construction sequence, whereas the backward analysis is performed by regarding the state of the final structure.

To follow the time-dependent effects in the construction stages, a forward analysis is used here.

2.4 Construction control and monitoring

During the erection of cable-stayed bridges by the cantilevering method the initial cable forces may not lead to the desired final condition. Unexpected discrepancies between the predicted and the real structure in a given construction stage may occur caused by inaccuracies of model parameters. In order to avoid the accumulation of these discrepancies and to ensure a safe erection process, it is necessary to carry out a detailed simulation analysis and a continuous monitoring throughout the erection stages. This way the discrepancies can be detected and the erection can be controlled by certain cable stays adjustments.

3 STRUCTURAL ANALYSIS

3.1 Static linear analysis

The structural analysis was done by means of a finite element computer program developed specifically for that purpose, because code availability was a fundamental requirement in order to the necessary further developments, namely, sensitivity analysis and structural optimization. The bridge was modelled as a two-dimensional structure using bar and beam (Euler-Bernoulli formulation) elements. A computer program developed in MATLAB was used to solve the structural analysis problem. By now, to simplify the problem formulation, cable-sag and other geometrical non-linear effects were neglected. Therefore, the algorithm will be improved in the future.

3.2 Time-dependent effects

In concrete cable-stayed bridges, the change of the stresses and deformations during the erection process is significantly influenced by time-dependent processes such as creep and shrinkage.

The total strain at time t of a concrete specimen uniaxially loaded at time t_0 can be written as the sum of the stress dependent, $\varepsilon_{c\sigma}(t,t_0)$, and stress independent, $\varepsilon_{cn}(t)$, strains:

$$\varepsilon_c(t) = \varepsilon_{c\sigma}(t,t_0) + \varepsilon_{cn}(t) = [\varepsilon_{ci}(t_0) + \varepsilon_{cc}(t,t_0)] + [\varepsilon_{cs}(t) + \varepsilon_{cT}(t)] \tag{1}$$

where $\varepsilon_{ci}(t_0)$ is the instantaneous strain, $\varepsilon_{cc}(t,t_0)$ the creep strain, $\varepsilon_{cs}(t)$ the shrinkage strain and $\varepsilon_{cT}(t)$ the thermal strain. If the stresses are less than $0{,}45 f_{ck}(t_0)$ the principle of superposition is valid and the creep strain varies linearly with the applied stress:

$$\varepsilon_{c\sigma}(t,t_0) = \frac{\sigma_c(t_0)}{E_c(t_0)} + \varphi(t,t_0)\frac{\sigma_c(t_0)}{E_{c28}} = \sigma_c(t_0)\left[\frac{1}{E_c(t_0)} + \frac{\varphi(t,t_0)}{E_{c28}}\right] = J(t,t_0)\cdot\sigma_c(t_0) \tag{2}$$

where $\varphi(t,t_0)$ is the creep coefficient, E_{c28} is the concrete modulus of elasticity at 28 days and $J(t,t_0)$ is the creep function. So that (1) can be rewritten as:

$$\varepsilon_c(t) = J(t,t_0)\cdot\sigma_c(t_0) + \varepsilon_{cn}(t) \tag{3}$$

In a cable-stayed bridge the stresses continually change during both the construction phase and the service life of the structure. Under variable stresses and using the superposition (3) can be rewritten as:

$$\varepsilon_c(t) = J(t,t_0)\cdot\sigma_c(t_0) + \int_{t_0}^{t} J(t,\tau)\frac{\partial\sigma_c(\tau)}{\partial\tau}d\tau + \varepsilon_{cn}(t) \tag{4}$$

Several approaches have been proposed to solve this equation, simplified methods, step-by-step numerical integration and approximation of the creep function. In this work the creep function is approximated by a Dirichlet series[16] leading to:

$$J(t,t_0) \cong \frac{1}{E_c(t_0)} + \frac{1}{E_{c28}}\sum_{i=1}^{n} a_j(t_0)\left(1 - e^{-\alpha_j(t-t_0)}\right) \tag{5}$$

which corresponds to admit a rheological model composed by one Hooke model an *n* Kelvin models (Fig. 1)

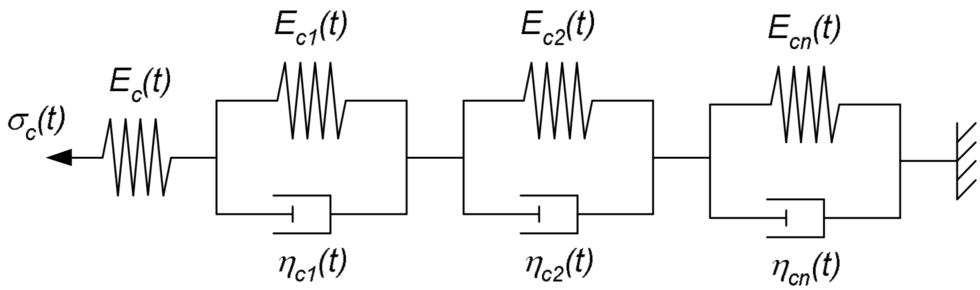

Figure 1: Rheological model of the creep function approximation by a Dirichlet series

where the viscosity coefficients, $\eta_{cj}(t)$, and modulus of elasticity, $E_{cj}(t)$, vary with time

$$\eta_{cj}(t) = \frac{E_{cm}}{\alpha_j a_j(t)} \tag{6}$$

$$E_{cj}(t) = E_{cm}\left[\frac{1}{a_j(t)} + \frac{1}{\alpha_j a_j^2(t)}\frac{da_j(t)}{dt}\right] \tag{7}$$

To solve the integral in (4) the time is divided into several intervals $\Delta t_k = t_k - t_{k-1}$ and assuming in each one the following simplifications

$$\frac{d\sigma_c(t)}{dt} = \frac{\Delta\sigma_c^k}{\Delta t^k} = const \tag{8}$$

$$a_j^k = const \tag{9}$$

$$E_c^k = const \tag{10}$$

and after several mathematical manipulations it is possible to write the incremental creep strain

$$\Delta\varepsilon_{cc}^{k*} = \sum_{i=1}^{n}\left(1 - e^{-\alpha_j(t-t_0)}\right)\gamma(t_{k-1}) \tag{11}$$

where

$$\gamma(t_k) = e^{-\alpha_j \Delta t_k}\gamma(t_{k-1}) + \lambda_j^k \frac{a_j^k}{E_{c28}}\Delta\sigma_c^k \tag{12}$$

$$\lambda_j^k = \frac{1 - e^{-\alpha_j \Delta t_k}}{\alpha_j \Delta t_k} \tag{13}$$

and n is the number of terms of the Dirichlet series and the coefficients a_j are obtained from a curve fitting using the least squares method. The coefficients $1/\alpha_j$ are called retardation times and are chosen to cover the range of time values for the creep coefficients calculation.

The incremental constitutive equation for the time interval Δt_k is given by

$$\Delta\sigma_c^k = E_c^{k*}\left(\Delta\varepsilon_c^k - \Delta\varepsilon_{cc}^k - \Delta\varepsilon_{cn}^k\right) \tag{14}$$

where E_c^{k*} is the equivalent modulus of elasticity to consider in the time interval Δt_k. The main advantage of the creep function approximation by a Dirichlet series with respect to the step-by-step time integration is that the storage of the entire stress history is not needed which saves computational time.

In this work the Eurocode 2[17] creep model was used, where the creep function is calculated as

$$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0) \tag{15}$$

where φ_0 is the notional creep coefficient and $\beta_c(t,t_0)$ is the time function describing the development of creep with time. The notional creep coefficient is estimated as

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \tag{16}$$

with

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0.1 \cdot \sqrt[3]{h_0}}, \text{ for } f_{cm} \leq 35 \text{ MPa}$$
$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1 \right] \cdot \alpha_2, \text{ for } f_{cm} > 35 \text{ MPa} \tag{17}$$

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} \tag{18}$$

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{\,0.20})} \tag{19}$$

The time-development function is described by

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{(\beta_H + t - t_0)} \right]^{0,3} \tag{20}$$

with

$$\beta_H = 1{,}5\left[1 + (0.012RH)^{18}\right]h_0 + 250 \leq 1500, \text{ for } f_{cm} \leq 35 \text{ MPa}$$
$$\beta_H = 1{,}5\left[1 + (0.012RH)^{18}\right]h_0 + 250\alpha_3 \leq 1500\alpha_3, \text{ for } f_{cm} > 35 \text{ MPa} \tag{21}$$

$$\alpha_1 = \left[\frac{35}{f_{cm}} \right]^{0,7} \qquad \alpha_2 = \left[\frac{35}{f_{cm}} \right]^{0,2} \qquad \alpha_3 = \left[\frac{35}{f_{cm}} \right]^{0,5} \tag{22}$$

According to Eurocode 2[17] the total shrinkage strain, $\varepsilon_{cs}(t)$, which is also time-dependent is the sum of the autogenous and the drying shrinkage. The drying shrinkage can be computed by

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \tag{23}$$

with

$$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^{\,3}}} \tag{24}$$

$$\varepsilon_{cd,0} = 0.85\left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}} \right) \right] \times 10^{-6} \times \beta_{RH} \tag{25}$$

$$\beta_{RH} = 1.55\left[1 - \left(\frac{RH}{RH_0} \right)^3 \right] \tag{26}$$

The autogenous shrinkage develops due to chemical reactions during hardening in the early age of concrete and it can be expressed as

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) \tag{27}$$

where

$$\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10) \times 10^{-6} \tag{28}$$

$$\beta_{as}(t) = 1 - \exp\left(-0.2t^{0,5}\right) \tag{29}$$

The aging of concrete leads to an increase in the strength and modulus of elasticity, and according to Eurocode 2[17] this can be expressed by:

$$f_{cm}(t) = \exp\left\{ s\left[1 - \left(\frac{28}{t}\right)^{1/2}\right]\right\} f_{cm} \tag{30}$$

$$E_{cm}(t) = \left(\frac{f_{cm}(t)}{f_{cm}}\right)^{0.3} E_{cm} \tag{31}$$

Since the stresses in the stay cables in service conditions are limited to $0.45 f_{pk}$, the problem formulation was simplified neglecting the relaxation phenomenon because it occurs for stresses higher than $0.50 f_{pk}$.

3.3 Simulation of the time-dependent effects

The time-dependent effects can be simulated in the finite element (FE) analysis of the structure by equivalent nodal forces corresponding to the non-mechanical deformations that induce a displacements field from which is calculated the actual deformation state. The stresses are computed using only the elastic constitutive relationship between stresses and mechanical origin deformations. Knowing the strains due to creep and shrinkage, the equivalent nodal forces can be computed as initial deformations using the finite element formulation for each time interval Δt_k. The equivalent nodal forces due to creep are given by

$$\Delta f_{cc}(t_k) = \int_V B^T \cdot D^k \cdot \Delta\varepsilon_{cc}^k dV \tag{32}$$

and due do shrinkage by

$$\Delta f_{cs}(t_k) = \int_V B^T \cdot D^k \cdot \Delta\varepsilon_{cs}^k dV \tag{33}$$

where B is the deformation matrix and D^k is the elasticity matrix for the time interval, updated according to the value of the concrete modulus of elasticity.

The temperature variation effect can also be included in the analysis

$$\Delta f_T(t_k) = \int_V B^T \cdot D^k \cdot \Delta\varepsilon_T^k dV = \int_V B^T \cdot D^k \cdot \alpha_T \cdot \Delta T^k dV \tag{34}$$

where α_T is the linear thermal expansion coefficient and ΔT^k is the temperature variation in the time interval Δt_k.

Knowing the incremental nodal forces due to the time-dependent effects is possible to write the incremental equilibrium equations for a given Δt_k

$$K^k \cdot \Delta u^k = \Delta F^k + \Delta f_c^{\ k} + \Delta f_s^{\ k} + \Delta f_T^{\ k} \tag{35}$$

where ΔF^k is the updated force vector of the structure due to changes in the external applied loading.

3.4 Construction stages

The modeling and analysis of the construction stages was performed using a forward analysis procedure. The forward analysis is performed following the sequence of erection stages in construction. In the first construction stage, only the pylon and the start of the cantilevers are activated and then in the following stages the other deck segments and stay cables are erected and the corresponding loads are applied. The analysis is carried out stage by stage until the bridge girder is completely erected and the results are continuously accumulated. Each new segment is installed tangentially to the existing one.

The forward analysis allows the knowledge of the stresses and displacements of the structure throughout the construction which allows the direct consideration of the time-dependent effects.

4 SENSITIVITY ANALYSIS AND OPTIMIZATION

4.1 Sensitivity analysis

Iterative optimization algorithms need to know the way a change in each design variable will affect the requirements expressed as goals. This is the task of the sensitivity analysis and represents most of the computational effort required for structural optimization. The evolution of the problem depends on a critical way on the accuracy with which these values are computed. Given the availability of the source code, the discrete nature of cable-stayed bridge structures and the large number of constraints (stresses and displacements) under control, the analytical discrete direct method was used for the sake of sensitivity analysis. The expressions for this method are obtained by differentiating the equilibrium equations

$$K \cdot u = F \tag{36}$$

the following expression is obtained:

$$\frac{dK}{dx_i} u + K \frac{du}{dx_i} = \frac{dF}{dx_i} \tag{37}$$

which can be rewritten in the form

$$K \frac{du}{dx_i} = \frac{dF}{dx_i} - \frac{dK}{dx_i} u = Q_{vi} \tag{38}$$

where Q_{vi} is the virtual pseudo-load vector of the system with respect to the ith design variable. The displacement sensitivities can be expressed as:

$$\frac{du}{dx_i} = K^{-1} \cdot Q_{vi} \tag{39}$$

which requires pre-programming and storing the stiffness matrix and right-hand side derivatives so the displacement derivatives may be computed by the solution of N pseudo-load right hand sides.

The stress derivatives are accurately determined from the chain derivation of the finite element stress matrix

$$\sigma = D \cdot B^e \cdot u^e \tag{40}$$

$$\frac{d\sigma}{dx_i} = \frac{d\left(D \cdot B^e\right)}{dx_i} \cdot u^e + D \cdot B^e \cdot \frac{du^e}{dx_i} \tag{41}$$

The first term of right-hand side may be directly computed during the computation of element contribution for the global system, on the condition that derivative expressions are pre-programmed and called on that stage. Since the displacement derivatives are known the second term on the right-hand side is easily computed. The explicit form of matrix derivatives depends on the type of element. For two-dimensional bar and beam elements their calculation is a straightforward task.

4.2 Structural optimization

An entropy-based optimization algorithm was used to found the cables prestressing forces. In minimization problems, a solution vector is said to be Pareto optimal if no other feasible vector exists that could decrease one objective function without increasing at least another one. The optimum vector usually exists in practical problems and is not unique. The design variables are the cables prestressing forces and are represented by x_i, respectively, and the global design variable vector is

$$x = \{x_1, x_2, x_3, \ldots, x_N\}^T \tag{42}$$

Using an entropy-based multicriteria approach, the problem is formulated as the minimization of an unconstrained convex scalar function which may be solved by conventional quasi-Newton methods, with which an optimal solution (in the Pareto sense) is achieved for each starting trial design. Representing the stress constraints by

$$\begin{aligned} g_j(x) &= \frac{\sigma}{\sigma_t} - 1 \le 0 \\ g_j(x) &= \frac{\sigma}{\sigma_c} - 1 \le 0 \end{aligned} \tag{43}$$

where σ, σ_t and σ_c are the acting stress and the allowable stresses in tension and compression, respectively, and the kinematic constraints by

$$g_j(x) = \frac{|\delta|}{\delta_0} - 1 \le 0 \tag{44}$$

where δ_0 are limit values for the deflections or displacements in certain points of the structure, namely the cable anchor points on the deck and the horizontal displacement in the top of the towers, the optimization problem may be posed as

$$F(x) = \frac{1}{\rho} \ln\left[\sum_{j=1}^{M} e^{\rho(g(x))}\right] \tag{45}$$

This function depends only on one control parameter, ρ, which must be steadily increased through the optimization process.

The goal functions $g_j(x)$ do not have an explicit algebraic form in most cases and the strategy adopted was to solve Eq. (45) by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking Taylor series expansions of all the goal functions $g_j(x)$ truncated after the linear term. This gives:

$$\min F(x) = \frac{1}{\rho}\ln\left[\sum_{j=1}^{M} e^{\rho\left(g_{0j}(x)+\sum_{i=1}^{N}\frac{dg_{0j}(x)}{dx_i}\Delta x_i\right)}\right] \tag{46}$$

where N and M are, respectively, the number of design variables and the number of behavior constraints. $g_{0j}(x)$ and $dg_{0j}(x)/dx_i$ are the goals and their derivatives evaluated for the current design variable vector (x_0), at which the Taylor series expansion is made. Solving Eq. (46) for particular numerical values of $g_{0j}(x)$ forms only one iteration of the complete solution of problem, Eq. (46). The solution vector (x_1) of such iteration represents a new design that must be analyzed and gives new values for $g_{1j}(x)$, $dg_{1j}(x)/dx_i$ i and (x_1), to replace those corresponding to (x_0) in Eq. (46). Iterations continue until changes in the design variables become small. Move limits are imposed to ensure the accuracy of the explicit approximation. During these iterations the control parameter ρ must not be decreased to ensure that a multi-objective solution is found.

The sensitivity analysis and optimization processes were carried out using an optimization module implemented in MATLAB.

5 NUMERICAL EXAMPLE

To illustrate the features of the proposed method a numerical example of a cable-stayed bridge was developed. This example is composed by an asymmetrical concrete cable-stayed bridge with a total length of 84 m, with a main span of 56 m and a lateral span of 28 m. Pylon total height is of 30 m with the deck placed 10 m above the foundation. Fig. 2 shows the geometry of the bridge example.

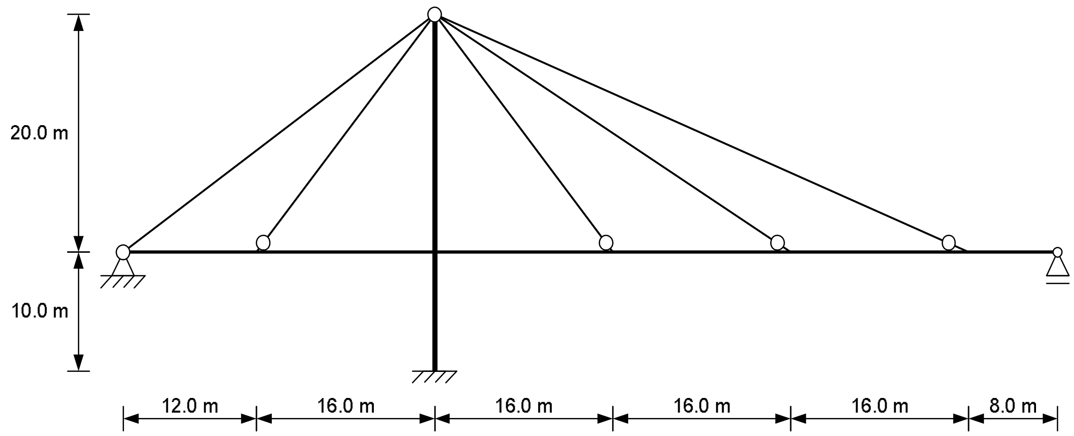

Figure 2: Bridge geometry

A C30/37 concrete (f_{ck} = 30 MPa) was considered for the deck and the pylon. For the cables prestressing steel an allowable stress corresponding to 40% of the ultimate tensile strength, f_{pk} = 1860 MPa was considered. Self-weigh values of 25 kN/m^3 and 77 kN/m^3 were considered respectively for the concrete and the prestressing steel. A value of 6 kN/m^2 was considered for the non structural dead load of the deck.

Since the objective of this work is to find the cable prestressing forces, the overall geometry of the bridge and also the cross-section geometry of deck and pylon were considered fixed. A slab-type cross-section was adopted for the bridge deck and a rectangular hollow section was considered for the pylon.

The considered construction stages are represented in Fig. 3. The deck-to-pylon connection is only vertical, except for the early erection stages in which structure stability requires additional links.

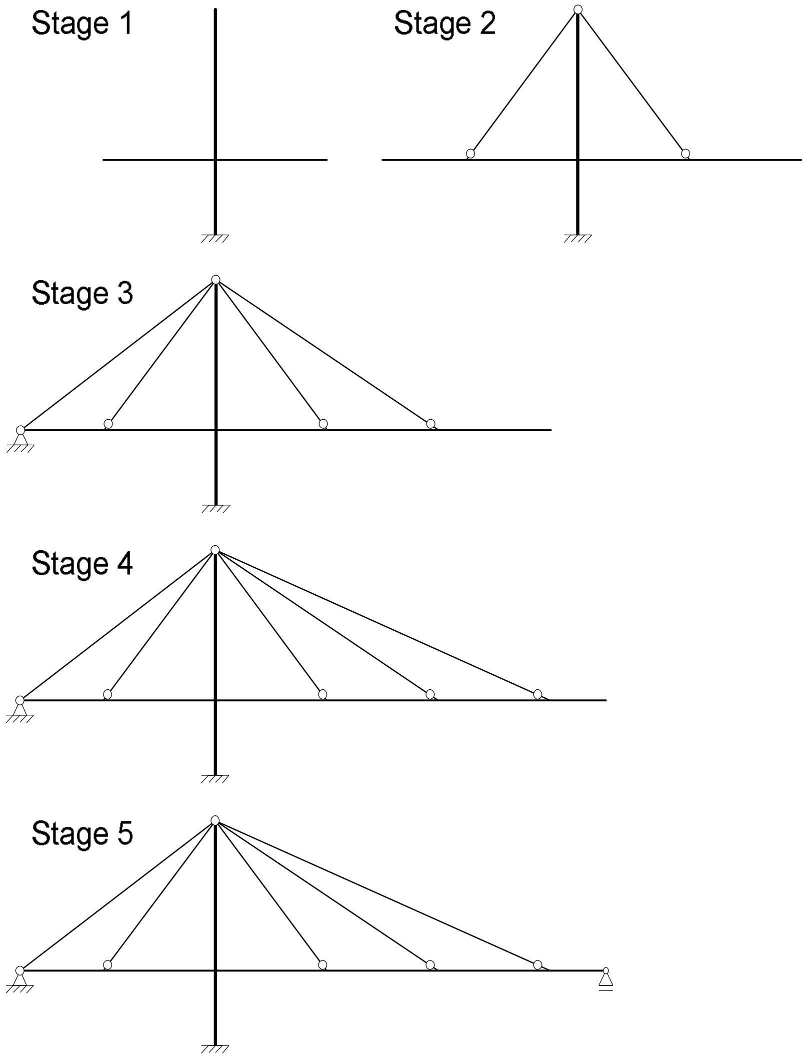

Figure 3: Construction stages

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