OPTIMISATION OF CABLE-STAYED BRIDGES SUBJECTED TO DYNAMIC LOADING

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ABSTRACT

A procedure for the application of sensitivity analysis and optimisation to cable-stayed bridges undergoing seismic pseudo-static loads is discussed in this paper. The seismic behaviour of a structure depends on its stiffness and mass properties and these change throughout the optimisation process because the design variables are cross-section dimensions and geometry-related parameters. The seismic loads are computed according to the Portuguese Safety and Actions Code of Practice (RSA) through a modal analysis and response spectra approach. The contribution of the modes is computed via the Complete Quadratic Combination Method. The sensitivity analysis is conducted by analytical means, the seismic forces set and their derivatives being evaluated for all updated intermediate structures. An integrated analysis-optimisation computer program is briefly described and used to solve a symmetric three span cable stayed bridge.

KEYWORDS

Cable-stayed bridges; optimisation; seismic loading; modal analysis; CQC method.

INTRODUCTION

Large structures such as cable-stayed bridges are a natural domain for the application of optimisation techniques.

The second author is developing an integrated analysis-optimisation program where most of repetitive and time-consuming tasks are automated so as to demand a minimum effort from the user. Particular attention was paid to specific and conditioning aspects of cable-stayed bridges design, such as consideration of erection stages, non-linear behaviour of cables and automatic evaluation of seismic action. The subject of this paper concerns mostly this last feature.

PC and DOS were the selected platform and environment and PharLap DOS-Extender was used to overcome memory restrictions. A downsized version of public domain program MODULEF (INRIA, 1991) was used for the analysis.

Routines were developed for automatic remeshing, mechanical properties and loading updating.

Structure-independent (live loads) and structure-dependent (seismic loads) loading require different procedures. The first type of loading remains constant through the whole optimisation process. Seismic loads, however, depend on the mass and stiffness structure characteristics and since these change continuously according to the values of the design variables, they need to be recomputed at each iteration, as well as their sensitivities. Self-weight, although a structure-dependent loading, is dealt with in the same way as live loads, because it is an explicit function of the mechanic and geometric properties of the structure, which are always known.

Several types of design variables were programmed at element level, the relation between these and the element mesh being established in each problem through a *dependency matrix* automatically generated at pre-processing time.

SENSITIVITY ANALYSIS

In view of accurate results and reduction in processing time the analytical procedure was carried out for sensitivity analysis. Differentiation of the equilibrium equations leads to

$$\frac{\partial \underline{\mathbf{K}}}{\partial \mathbf{x}}\underline{\mathbf{u}}+\underline{\mathbf{K}}\frac{\partial \underline{\mathbf{u}}}{\partial \mathbf{x}}=-\frac{\partial \underline{\mathbf{P}}}{\partial \mathbf{x}} \qquad \frac{\partial \underline{\mathbf{u}}}{\partial \mathbf{x}}=\underline{\mathbf{K}}^{-1}\,\underline{\mathbf{Q}}_p \qquad \text{with} \qquad \underline{\mathbf{Q}}_p=-\frac{\partial \underline{\mathbf{P}}}{\partial \mathbf{x}}-\frac{\partial \underline{\mathbf{K}}}{\partial \mathbf{x}}\underline{\mathbf{u}} \tag{1a-c}$$

The second term of the virtual pseudo-load expression is not explicit and must be computed after solving the system equation for the real load cases. The first term may be either explicit or not, depending on the kind of loading. So, when dead and live loads only are considered, it may be directly evaluated at element level, simultaneously with mass, stiffness and real right-hand sides. If seismic action is to be considered, one has

$$\underline{\mathbf{P}}=\underline{\mathbf{P}}(\underline{\mathbf{X}},\underline{\mathbf{W}}) \qquad \text{and the first term in (1c) becomes} \qquad \frac{\partial \mathbf{P}}{\partial x}+\sum_{L}\frac{\partial \mathbf{P}}{\partial \omega}\frac{\partial \omega}{\partial x} \tag{2}$$

in which $\underline{\mathbf{X}}=\{x_1, x_2, \ldots, x_{NV-1}, x_{NV}\}^T$ and $\underline{\mathbf{W}}=\{\omega_1, \omega_2, \ldots, \omega_{L-1}, \omega_L\}^T$ are the design variable set and the vector of L most important eigenfrequencies. For the seismic loading it is compulsory to find the solution of the eigenproblem and its sensitivities.

SEISMIC ANALYSIS BY RESPONSE SPECTRA AND MODE SUPERPOSITION

Both step-by-step methods or modal analysis can be used to solve problems involving seismic loading conditions. Spectral analysis associated with modal analysis is accepted in most codes of practice and has some comparative advantages with respect to the step-by-step method: It generates a smaller amount of data for the analysis-optimisation module, corresponding to a load case for each seismic load case (and each design variable); It may be used recursively as a black box where input data and results are exchanged with the main analysis and optimisation core, with no need of significant changes of this one. Therefore, all the main memory may be set free at the end of a task to be used by the other. For these reasons, it was the selected method for this work, together with Complete Quadratic Combination Method (CQC) for seismic forces evaluation. However, its use is limited to linear elastic behaviour.

PSEUDO-STATIC SEISMIC FORCES EVALUATION

The evaluation of seismic loads follows the usual procedure of mode superposition. The subspace method (Bathe, 1976) has been used to find the most important eigenvalues and the associated eigenvectors. Using the response spectra approach, and denoting by $\underline{\mathbf{M}}$ and $\underline{\mathbf{f}}_n$ the mass matrix and the eigenvector related to mode n, L_n the modal participation factor, M_n the generalised mass and $S_{pa,n}$ the spectral acceleration (extracted from the response spectra) for the same mode, the maximum seismic force contribution of mode n is known to be

$$\underline{\mathbf{f}}_{n(max)} = \underline{\mathbf{M}}\,\underline{\phi}_n \frac{L_n}{M_n} \mathbf{S}_{pa,n} \tag{3}$$

CQC (Wilson, 1981) method is then used to obtain the maximum design forces

$$f_k = \sqrt{\sum_i \sum_j f_{ki}\,\rho_{ij}\,f_{kj}} \qquad \text{with} \qquad \rho_{ij} = \frac{8\xi^2(1+r)r^{3/2}}{(1-r^2)^2 + 4\xi^2 r(1+r)^2} \tag{4a-b}$$

k denoting component k of the maximum forces vector and ξ the damping coefficient, assumed to be constant for all modes. r is the frequencies ratio $r=\min(\omega_i,\omega_j)/\max(\omega_i,\omega_j)$.

PSEUDO-STATIC FORCES SENSITIVITY ANALYSIS

Sensitivities for the seismic maximum design forces (5) are obtained by chain derivation of that equation, in which all the parameters involved are explicit or implicit functions of $\underline{\mathbf{X}}$.

$$\frac{\partial f_k}{\partial x} = \frac{1}{2f_k} \sum_i \sum_j \left(\frac{\partial f_{ki}}{\partial x}\rho_{ij} f_{kj} + f_{ki}\frac{\partial \rho_{ij}}{\partial x} f_{kj} + f_{ki}\,\rho_{ij}\frac{\partial f_{kj}}{\partial x}\right) \tag{5}$$

where a non zero value of f_i will be ensured either because of the processor idiosyncrasies or by imposing some twilight minimum. For maximum modal forces derivatives one has

$$\frac{\partial \underline{\mathbf{f}}_n}{\partial x} = \underline{\mathbf{M}}\left[\frac{\partial \underline{\phi}_n}{\partial x}\frac{L_n}{M_n}S_{pa,n} + \underline{\phi}_n\left(\left(\frac{1}{M_n}\frac{\partial L_n}{\partial x} - \frac{L_n}{M_n^2}\frac{\partial M_n}{\partial x}\right)S_{pa,n} + \frac{L_n}{M_n}\frac{\partial S_{pa,n}}{\partial x}\right)\right] + \frac{\partial \underline{\mathbf{M}}}{\partial x}\underline{\phi}_n\frac{L_n}{M_n}S_{pa,n} \tag{6}$$

This expression requires finding modal participation factors, generalised modal mass and spectral acceleration derivatives. The former two are obtained by using

$$\frac{\partial L_n}{\partial x} = \left[\frac{\partial \underline{\phi}_n^T}{\partial x}\underline{\mathbf{M}} + \underline{\phi}_n^T\frac{\partial \underline{\mathbf{M}}}{\partial x}\right]\underline{\mathbf{r}} \qquad \frac{\partial M_n}{\partial x} = \left[\frac{\partial \underline{\phi}_n^T}{\partial x}\underline{\mathbf{M}}\,\underline{\phi}_n + \underline{\phi}_n^T\frac{\partial \underline{\mathbf{M}}}{\partial x}\underline{\phi}_n + \underline{\phi}_n^T\underline{\mathbf{M}}\frac{\partial \underline{\phi}_n}{\partial x}\right] \tag{7a-b}$$

Concerning spectral acceleration derivative and since the response spectra is built from empirical and experimental data, no explicit analytic expression is available. To overcome this difficulty, an analytical expression was fitted to the discrete available data which is composed by a non-linear followed by a constant value curve. The approximation made to the response spectra is such that no discontinuity occurs both in spectral acceleration and its derivative. As to CQC expression derivatives, one has

$$\frac{\partial}{\partial x}\rho_{ij} = \frac{\partial}{\partial x}\frac{8\xi^2(1+r)r^{3/2}}{(1-r^2)^2 + 4\xi^2 r(1+r)^2} = \frac{\partial}{\partial x}\frac{A}{B} = \frac{\frac{\partial A}{\partial x}B - A\frac{\partial B}{\partial x}}{B^2} \tag{8}$$

$$\frac{\partial A}{\partial x} = 8\xi^2\left(r^{3/2} + \tfrac{3}{2}(1+r)r^{1/2}\right)\frac{\partial r}{\partial x} \qquad \frac{\partial B}{\partial x} = 4(1+r)\left[r(1+3\xi^2) + \xi^2 - 1\right]\frac{\partial r}{\partial x} \tag{9a-b}$$

$$\frac{\partial r}{\partial x} = \frac{\frac{\partial \omega_j}{\partial x}\omega_i - \frac{\partial \omega_i}{\partial x}\omega_j}{\omega_i^2} \qquad \omega_j =< \omega_i \tag{10}$$

The frequencies and eigenvalues relates through $\lambda=\omega^2$, the latter being numerically evaluated, and so

$$\frac{\partial \omega}{\partial x} = \frac{1}{2\sqrt{\lambda}}\frac{\partial \lambda}{\partial x} \tag{11}$$

For this purpose the method described by (Haftka, 1992) is used:

$$\frac{d\lambda_n}{dx} = \frac{\underline{\phi}_n^T\left[\frac{d\underline{\underline{\mathbf{K}}}}{dx} - \lambda\frac{d\underline{\underline{\mathbf{M}}}}{dx}\right]\underline{\phi}_n}{\underline{\phi}_n^T\underline{\underline{\mathbf{M}}}\,\underline{\phi}_n} \tag{12}$$

Eigenvector derivatives are also required, and among the available methods the one based on eigenvector linear combination was adopted, because a limited number of terms provide enough accuracy for the desired purpose:

$$\frac{\partial \underline{\phi}_n}{\partial x} = \sum_{j=1}^{P} c_{nj}\,\underline{\phi}_j \quad (P =< L) \qquad c_{nj} = \frac{\underline{\phi}_j^T\left(\frac{d\underline{\underline{\mathbf{K}}}}{dx} - \lambda_n\frac{d\underline{\underline{\mathbf{M}}}}{dx}\right)}{\left(\lambda_n - \lambda_j\right)\underline{\phi}_j^T\underline{\underline{\mathbf{M}}}\,\underline{\phi}_j} \quad j \neq n \qquad c_{nn} = -\frac{1}{2}\underline{\phi}_n^T\frac{d\underline{\underline{\mathbf{M}}}}{dx}\underline{\phi}_n \tag{13a-c}$$

Note that an higher number of modes is needed for evaluating (13) than the combined CQC forces, because there is a loss of accuracy of numeric evaluation of derivatives and its values strongly condition the whole process.

Expressions (12)-(13) assume that only separate eigenvalues are being dealt with and so, if coalescence occurs at the initial structural solution or as a consequence of changes in the mechanical properties through the optimisation process, numeric overflow may cause the process to crash. However (Haftka, 1992) states that slight differences between eigenvalues usually avoid that occurrence and the effectiveness of this statement was confirmed in the tested examples, in which the eigenvalues were computed in double precision up to the 5th significant digit. Some authors (Leung, 1990) (Chen *et al*, 1993) address this problem and suggest alternative approaches.

OPTIMISATION METHOD

The cable-stayed design problem is posed in a multi-objective optimisation format with goals of minimum cost and stress and a Pareto solution is sought. This problem is equivalent to a minimax formulation which is discontinuous and non-differentiable, both of which attributes make its numerical solution by direct means difficult. An entropy-based technique (Templeman, 1989) is used to determine the minimax solution via the minimisation of a convex scalar function. Although the scalar function is unconstrained and differentiable, the stress goal functions do not have an explicit algebraic form. Explicit approximations were made by taking Taylor series expansions truncated after the linear term. Assigning the normalised volume (or cost) to objective g_1 and normalised stresses and/or displacements all over the structures to objectives $g_2, g_3, \ldots, g_J$, one has for the scalar function and its approximation

$$F(\underline{\mathbf{X}}) = \frac{1}{\rho}\ln\left(\sum_{j=1,J} e^{\rho g_j}\right) \qquad F(\underline{\mathbf{X}}) = \frac{1}{\rho}\ln\left\{\sum_{j=1,J} e^{\rho\left[g_j(\underline{X}_o) + \sum_{i=1,NV}\frac{\partial g_j}{\partial x_i}(x_i - x_{oi})\right]}\right\} \tag{14a-b}$$

ρ is a positive control parameter initially set by the user at values usually in the range 10-100. During the iterative process it must be increased so as to ensure that a minimax solution is found for function (14). Index 0 in the approximation function denotes starting values, computed by the analysis. The solution of this equation constitutes the starting design for the next iteration and convergence is attained when the objective function $F(\underline{\mathbf{X}})$ improvement from one iteration to the following is smaller then some pre-defined value.

APPLICATION EXAMPLE

With the purpose of testing the behaviour of this method, an example with simplified conditions was modelled. It consists of a 250m long steel cable-stayed bridge, the central span being 130m. The initial mechanical characteristics were set up by full stress design, considering two asymmetric I-typed cross-sections for main girders supporting transverse symmetric I beams which in turn support the deck. Pylons cross section are of hollow rectangular type. A uniformly distributed load on deck and seismic action acting both in longitudinal, transversal and vertical directions were the considered load cases, for the latter, according to the RSA prescriptions, the response spectra ordinates being reduced in 1/3.

The shape modes 1 to 5 for the initial design are represented in Figure 1. It may be seen that the design variables which affect most strongly the longitudinal stiffness (and therefore mode 1) were greatly reduced, resulting in a significant reduction in frequency and in the associated static forces.

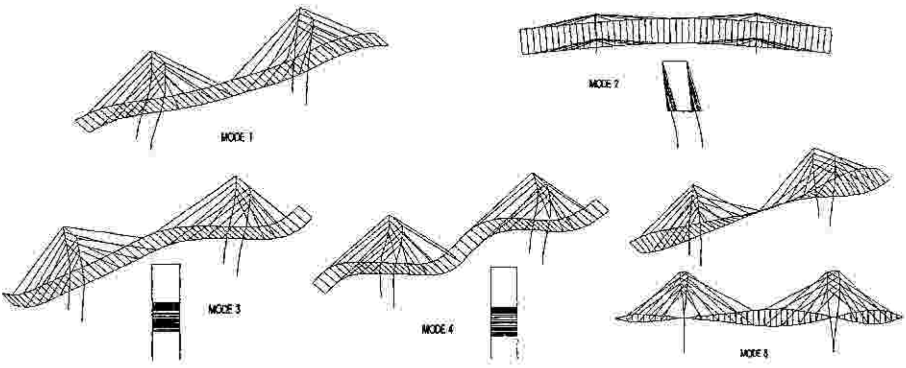

Fig. 1. Mode shapes 1 to 5.

The main 5 modes remain the same, but modes 2 and 3 change their relative position (optimised structure is relatively stiffer in the transversal direction). Initial and final values of first 5 eigenfrequencies are listed in Table 1. Base shear is reduced from the initial value of 2805KN to 1720KN after optimisation. A global cost reduction of 22.5% with respect to the starting value is achieved. It may be seen that the average reduction in seismic forces is greater then that of gravity loads, which indicate a more favourable distribution of mass and stiffness. Initial and final maximum stress distribution in girders and pylons are shown in Figure 2. For the evaluation of seismic forces the contributions from 5 first modes were considered. However, when the corresponding eigenvectors sensitivities were computed according to expression (13), 59 eigenvectors in the specified range ($\lambda = < 500$ or $f = < 3.55$Hz) were considered in the expansion.

Table 1 - Frequencies (Hz) for first five modes

	f_1	f_2	f_3	f_4	f_5
Initial	0.370	0.552	0.620	0.968	1.103
Optimised	0.214	0.553	0.470	0.664	0.785

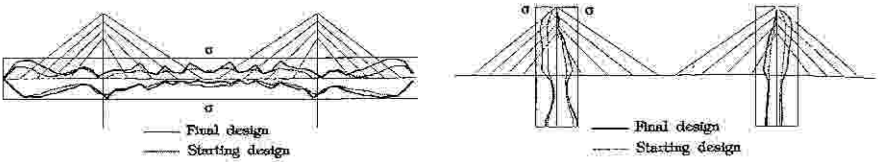

Fig. 2. Maximum stresses on pylons and deck

CONCLUDING REMARKS

Although the design of a cable-stayed bridge with a satisfying seismic behaviour is a time-consuming task, the use of an integrated analysis-optimisation program can greatly enhance the efficiency of the process, allowing for improved solutions to be obtained from a starting coarse design. Pareto solution supplied by the optimisation method chosen is not necessarily unique and several design alternatives might be found. Some features that were not discussed in the present example, such as consideration of erection stages, spatial variability of seismic excitation or simultaneous consideration of various seismic actions are already available or currently being developed.

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