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MULTICRITERIA OPTIMIZATION OF PRESTRESSED CONCRETE BRIDGE WITH DISCRETE DESIGN VARIABLES

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ABSTRACT

This paper describes an application of a branch and bound method for optimum design of reinforced and prestressed concrete structures with discrete design variables. The approach integrates multicriteria optimization strategy with a commercial finite element program. The design problem is posed directly as a multicriteria optimization with goals of minimum cost, stresses and displacements. The multicriteria problem is folded into a single envelop function based on maximum entropy formulation, where pareto optimum achieve a comprise among the objectives and constraints and represent more meaningful and rational results in practice. The approach provides the practical engineer with a direct and systematical way to design the structure. The validity of the proposed design method is examined by means of numerical examples.

INTRODUCTION

In really practical structural design, in the last three decades, much work has been done in the structural optimization, in addition to considerable development in optimization theory[1]. However, structural optimization has not been used extensively for civil engineering structures [3](as opposed to mechanical or aerospace structures). It is said that two reasons to cause the situation are: firstly, the difficulty in choosing the meaningful objective that includes all relevant criteria; and secondly, different design representations between the communications of structural optimization and engineering practice.

In engineering practice of structural design, most final designs are based on prefabricated products composed in different ways. Therefore, design variables consist to an extent, from a set of prefabricated elements specified in specialized catalogues. In most of the available literature of structural optimization, the very necessary practical requirement that the optimum design should employ only the standard discrete component is omitted. The omission of this very practical requirement has been one of several contributory factors to the lack of use of structural optimization in engineering practice.

Unlike mechanical or aerospace structures, it is not easy nor obvious to find one dominating criterion in the civil engineering structures but several (possibly conflicting) criteria such as: minimum cost, maximum safety, minimum stresses, minimum deflections and so forth. The advantage of multicriteria is that it simultaneously considers all competing objectives and convergence to a Pareto solution for which it is not possible to improve one merit function without seriously impairing others. It is very natural and suitable for the practical design of civil engineering structures.

The optimization of prestressed beams, which possesses a large number of constraints and occasionally conflicting objectives, is dealt within this work. The problem is posed as a multicriteria optimization with goals of minimum cost, stresses and displacements. The problem is solved through the minimization of a convex function involving one control parameter obtained by folding all goals and constraints into a single envelope. The single objective optimization problem is approximated by first order Taylor's series expansion of structural response and then solved by the public domain Non-Linear Programming NLP program TN/GAMS[6] based on the truncated-Newton algorithm. All structural responses are gained by using the commercial finite element program (ADINA). The branch and bound method is used to satisfy the practical requirement of discrete design variables.

The numerical examples illustrated the reliability of the application of such a direct design method of a prestressed beam with different parameters. The comparison between numerical results and real engineering show its considerable improvement of the practical design. The design variables consist of the areas of prestressing and mild steel, overall depth and depth and web thickness. The actual cost construction is composed of prestressing, mild steel, concrete and formwork costs. Other possible objectives such as initial camber, minimum depth of girder were introduced also.

MULTICRITERIA OPTIMIZATION PROBLEM AND PARETO OPTIMUM

The multicriteria (multicriterion, multiobjective, vector) optimization problem may be formulated as follows: to determine a vector of design variables that satisfy the constraints and minimize a vector of objective function. Mathematically, this can be stated as follow:

$$\text{Min} \quad \mathbf{Z} = \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})] \qquad \mathbf{x} \in \Omega \tag{1}$$

where $\mathbf{f}$ = vector of objective functions; f_i = component objective function ($i=1,2,\ldots,m$); Ω = feasible set to which $\mathbf{x}$ belongs and is a subset of R^n

$$\Omega = \{\mathbf{x} \in R^n : \mathbf{g}(\mathbf{x}) \le 0,\ \mathbf{h}(\mathbf{x}) = 0\} \tag{2}$$

Usually there exists no unique point which would give an optimum for all m criteria simultaneously. The concept of Pareto Optimum is introduced as a solution to multicriteria optimization.

A vector $\mathbf{x}^* \in \Omega$ is Pareto optimum for problem (1) if, and only if, there exists no $\mathbf{x} \in \Omega$ such as $f_i(\mathbf{x}) \le f_i(\mathbf{x}^*)$, for $i = 1,2,\ldots,m$ with $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one j. In other words, $\mathbf{x}^*$ is a Pareto optimum if there is no feasible solution $\mathbf{x}$, which decreases some objective without simultaneously causing a serious increase of at least another objective function.

SOLUTION OF MULTICRITERIA OPTIMIZATION PROBLEM: MAXIMUM ENTROPY FORMULATION APPROACH

The maximum entropy formulation, first published by Jaynes in 1957[5], is recognized as a fundamental concept in information theory. It determines a less biased possibility for a problem in a random process via maximum entropy direction. In recent years, it has emerged as an important and powerful formulation in a wide variety of different fields, and it has found applications in many disciplines throughout science and technology as well as in the structural optimization [8]. In the present paper, the maximum entropy formulation is used to achieve the Pareto solution of a multicriteria optimization problem.

Several approaches, already applied in operation research and control theory, have been proposed in the literature for the solution of multicriteria optimization problems [2]. In the study, the approach based on the Maximum Entropy Formulation are accepted, because of its concrete foundation.

Entropy is most commonly known in physics in connection with the second law of thermodynamics - the entropy law - which states that entropy (or amount of disorder), in any closed conservative thermodynamics system tends to maximum. A fundamental step in using entropy in the new context unrelated to thermodynamics was provided by Shannon [7] who realized that entropy could be used to measure other types of disorder by using the following algebraic form to measure the amount of uncertainty in any discrete probability distribution.

$$S = -k \sum p_i \ln p_i \tag{3}$$

where p_i is the probability of event i , k is a constant

The Multicriteria optimization process could be considered as a deductive process. Given a set of objective function f and some constraint functions, the process commences without any numerical information. An initial point is then chosen and information is calculated about the gradient of objective function and constraint function. The numerical information is then used to determine the current situation to infer where the next trial point should be placed via the maximum entropy direction. $\mathbf{x}_k = D(\mathbf{x}_{k-1})$

The Kreisselmeier-Steinhauser function [4] defined in (4) could be considered as the measure of entropy in the multicriteria optimization process of (1). The most rational direction of the process may be the maximum KS norm direction, so that the multicriteria optimization problem (1) was replaced by maximizing a single KS norm function (4).

$$KS(f^*,g) = \frac{1}{\rho} \ln\left(\sum e^{\rho f_i^*} + \sum e^{\rho g_i}\right) \tag{4}$$

$$f^* = \frac{f - f_{k-1}}{f_{k-1}} \tag{5}$$

where f^* is the set of gradient of objective functions.

It has been proved [4] that for any positive value of ρ, the KS norm is always more positive than the most positive constraint.

$$Min(f^*,g) \leq KS(f^*) \leq Min(f^*,g) - \frac{\ln(m+n)}{\rho} \tag{6}$$

where m is the number of objective functions, and n is the number of constraints

Because of the behaviour of the KS norm, it is obvious that when ρ becomes sufficiently large, the result of

$$\text{Minimum } [KS(f^*, g)] \tag{7}$$

is the Pareto-optimal solution of multicriteria optimization problem (1). Therefore the multicriteria unconstraint optimization problem (1) can be solved by minimizing a single continuous and differentiable function (7).

Until now, the multicriteria optimization problem has been converted into a single objective function optimization problem.

SOLUTION OF SINGLE OBJECTIVE OPTIMIZATION PROBLEM WITH DISCRETE DESIGN VARIABLES

After the multicriteria optimization is transferred into an equivalent single-objective problem based on the maximum entropy formulation, the latter may be solved using standard non-linear programming(NLP) with discrete design variables constrained by the equilibrium equations of finite element methods. The problem could be formulated as follows.

$$\begin{aligned}
&\text{Minimize} && KS(\mathbf{x}) && \text{(8a)}\\
&\text{Such that:} && \mathbf{h}(\mathbf{x}) = 0 && \text{(8b)}\\
& && \mathbf{g}(\mathbf{x}) \le 0 && \text{(8c)}\\
& && \mathbf{x}^{lower} \le \mathbf{x} \le \mathbf{x}^{upper} && \text{(8d)}\\
& && x_i \in S_i = (s_m : m = 1,\ldots,M_i) &&
\end{aligned}$$

where $\mathbf{h}$ = vector of equal constraints, $\mathbf{x}$ = vectors of design variables in which each design variables x_i must be selected from the finite set S_i which contains M_i discrete sizes.

It is obvious that equations(8a-c) are very complex non-linear programming problem. As the computational expensive results of (8b), it isn't suitable to use the standard NLP directly. The iterative method was employed in the study. At the start of each iteration, the nonlinear KS norm (8a) was linearized at the current point $\mathbf{x}_k$ using first-order Taylor's series expansion, i.e.

$$\text{Minimize} \qquad KS(x) = KS(\mathbf{x}_k) + KS'(\mathbf{x}_k)\,(\,\mathbf{x} - \mathbf{x}_k\,) \tag{9a}$$

$$KS'(x) = \frac{\partial KS}{\partial g}\cdot\frac{\Delta g}{\Delta x} + \frac{\partial KS}{\partial f}\cdot\frac{\partial f}{\partial x} \tag{9b}$$

The overall finite difference approach was implemented to calculate the sensitivity information.

The solution to the above discrete programming problem proceeds as follows. First the requirement of discrete is relaxed, and a solution of continuous problems is obtained. If the solution is such that the requirement of discrete is satisfied, an optimal design is obtained. If any one of design variable x_i of the results in the first step violate the original requirement, ie. the result locates between two discrete values of ordered discrete set ($d_1, d_2, \ldots, d_p$)

$$d_j < x_j < d_{j+1} \tag{10}$$

two new subproblems are generated; one with a new lower bound $x_j \ge d_{j+1}$, and another with a new upper bound $x_j \le d_j$. The procedure is carried out until all design variables satisfy the discrete requirement.

NUMERICAL EXAMPLE

The fully and partially prestressed concrete girder optimal design problem will be formulated next as a mathematical program to access the efficiency of two procedures deduced above. The requirements, based on the EUROCODE, include serviceability limit state (SLS) constraints, ultimate flexural limit state (ULS) constraints, as well as a guarantee of balanced failure.

DESIGN VARIABLES

The prestressed concrete girders with the cross section, tendon layout and loading in FIG.1 is to be designed for two objective functions:(1) Minimize the total cost;(2) minimize the initial camber. The nine design variables are the girder depth b, the areas of prestressed steel and reinforcement A_{ps} and A_s, the tendon eccentricity at support section e_e and the thicknesses of top and bottom slabs and web t_t, t_b and t_w, ie.

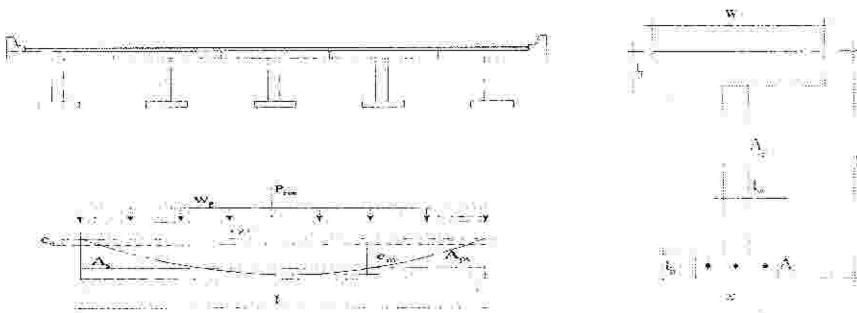


FIG.1 Structure Geometry

x_1:	b	x_2:	$e_e + e_m$	x_3:	half of straight tendon
x_4:	t_b	x_5:	W_0	x_6:	$2t_w$
x_7:	t_t	x_8:	A_{ps}	x_9:	A_s

CONSTRAINTS:

① Ultimate limit state constraints

The *Strength limit-states* used in flexural design are evaluated according to EUROCODE. The constraints could be expressed

$$M_{ul} \geq 1.3M_d + 1.5M_l \qquad (11)$$

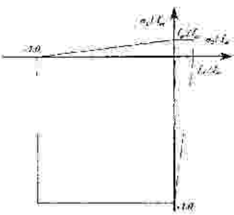
where = the factored moment strength of the cross section; = the elastic moment due to dead loads and worst arrangement of live loads, respectively.

For the full prestressed concrete design method, the prestressing steel is the only contribution to the constraint. The partially prestressed concrete design concept was adopted, where the reinforcement makes contribution to the constraint.

② *Serviceability limit-state* constraints

Two working load cases are considered, ie., the initial prestressing without live load and the live load with prestressed force with 15% prestress loses. The commercial finite element program ADINA was used to simulate structural responses under the two loads cases. The concrete is modeled using 9-node plane strain elements and the prestressed tendon and reinforcement steel by 3-node truss elements (Fig.2). It was assumed that, the girder carries 9.343 kN/m.lane uniform design load, and concentrated design force 80 kN/lane in the midspan. Instead of practical design code, a general two dimensional failure envelope under principle stresses failure criteria of concrete material in tension and compression in two principle stresses directions as described in Fig.2 are employed as constraints. For each selected point in the concrete elements, the following constraints were set:

$$\sigma_1/f_{cc} + 1.0 > 0.0$$
$$\sigma_2/f_{cc} + 1.0 > 0.0$$
$$1.0 - \sigma_1/f_{ct} > 0.0$$
$$1.0 - \sigma_2/f_{ct} > 0.0$$
$$\sigma/f \quad \sigma/f + 1.0 > 0.0$$
$$- \sigma_2/f_{ct} + \sigma_1/f_{cc} + 1.0 > 0.0$$



Similarly, for each prestressed tendon and mild steel elements:

$$1.0 - \sigma_s/f_{yk} > 0.0$$
$$1.0 - \sigma_{ps}/f_y > 0.0$$

FIG.1 SLS constraints

The concrete (C50/60) has f_{cc} =0.3333×10^8 N/m^2 , f_{ct} = -0.2800×10^7 N/m^2 , ρ_c = 0.2450×10^5 N/m^3, E_c = 0.37×10^{11} N/m^2, E_{c} = 0.34×10^{11} N/m^2 , γ = 0.2. Prestressing Steel has f_{ps} = .1785×10^{10} N/m^2, $f_{yk} = f_{ps} / \gamma_{ps}$ = .1552×10^{10} N/m^2 , E = 2.0×10^{11} N/m. The mild steel has f_s = .4500×10^9 N/m^2 , $f_y = f_s / \gamma_s$ = .3910×10^9 N/m^2 . Prestress losses of 15% between transfer and service are assumed.

③ Guarantee of balanced failure

The absolute lower and upper bounds were set according to geometrical constraints, and some practical experience. For example, the reinforcement area for a reinforced concrete beam should be

$$0.15\% < 100A_s/A_c < 4.0\% \qquad (12)$$

The minimum reinforcement is provided mainly to control thermal and shrinkage cracking, while the maximum steel is determined largely from the practical need to achieve adequate compaction of the concrete around reinforcement.

④ Absolute upper and lower boundaries

In addition to the relative bounds, the following absolute bounds are set:

x_1:	(*span*/24, *span*/8)	x_2:	(x_4+thickness, x_2-x_7-*thickness*)
x_3:	(*span*/9, 2**span*/9)	x_4:	(0.5*thickness*, 1.5*thickness*)
x_5:	(3*width*/8, 5*width*/8)	x_6:	(4.0*thickness*, 6.0*thickness*)
x_7:	(1.5*thickness*, 3.0*thickness*)	x_8:	(0.000015A_c, 0.0004A_c)
x_9:	(0.000015A_c, 0.0004A_c)		

OBJECTIVE FUNCTIONS

The primary-objective function was the total cost per-unit length, which includes the costs of concrete, mild steel, prestressing steel and formwork and it was stated as:

$$f_1 = LC_f^o + V_cC_c + A_sC_s + A_{ps}C_{ps} \qquad (13)$$

where C_f^o, C_c, C_s, and C_{ps} = unit cost of framework, concrete, mild steel and prestressed steel per length, volume, and areas respectively. In the paper, we assumed C_f^o = *76.0 ESC/m*, C_c = *350.0 ESC/m*^{2}, C_s = *5,000 ESC/m*^{2}, C_p = *19,500 ESC/m*^{2} .

The other objective function was designed as the minimization of the initial camber due to prestressing and own weight. The objective and all constraints described after are governed by the finite element program, ie.

$$f_2 = u = K^{-1}P \text{ or } exp[k(u_1 - \delta)] \tag{14}$$

NUMERICAL RESULT

The parameter ρ in (4) is a very important parameter in the algorithms[9]. In the following example ρ = 20.0 is assumed.

Two sets of objective functions (ie. single objective function of total cost of unit length, and the multi-objective functions of cost and initial camber) with different parameters and objective functions were implemented. The overall geometrical scales are set *0.05 m.*; the internal one x_2 is considered as continuous. The areas of prestressed steel and reinforcement are treated with scale of *5 cm²*. The results are shown in Table 1-2.

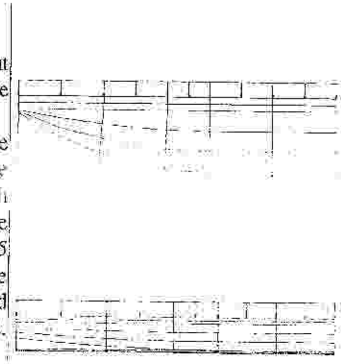


FIG.2 Initial and Finial Design

It was observed that:

1. All pareto solutions in the table were acceptable and feasible;
2. More than one local optimum point could be achieved. All of them improved the real practical design considerable (decreasing 25% total cost.);
3. All results would be considered as real practical design.

Table 1. Single Objective Solutions

	$obj1\times10^{4}$	x1	x2	x3	x4	x5	x6	x7	$x8\times10^{-2}$	$x9\times10^{-2}$	Iteration
	S$/m	*m*	*m*	*m*	*m*	*m*	*m*	*m*	*m²*	*m²*	
Initial	6179	3.75	1.90	6.00	.30	5.00	1.20	.60	2.36	2.36	
U10	2986	1.30	.6874	5.80	.10	2.95	0.95	.30	.150	.190	24
U20	3051	1.40	.8270	5.90	.10	3.05	0.85	.30	.155	.200	11
U30	3028	1.40	.7014	5.15	.10	2.95	0.85	.30	.175	.045	10
Initial	4373	2.50	1.250	4.00	.20	4.00	1.00	.40	1.224	1.224	
M10	2964	1.25	.6265	6.00	.10	2.95	0.95	.30	.165	.165	19
M20	3145	1.55	.8465	4.65	.10	2.95	1.00	.30	.390	.030	15
M30	3070	1.50	.8302	4.40	.10	2.95	0.90	.30	.135	.095	8

Table 2. Multi- Objectives Pareto Solutions

	$obj1\times10^{4}$	obj2	x1	x2	x3	x4	x5	x6	x7	x8	$x9\times10^{-2}$	Iteration
	$\$/m$	cm	m	m	m	m	m	m	m	m^2	m^2	
Initial	6179	1.360	3.75	1.90	6.0	.30	5.00	1.20	.60	2.36	2.36	
U30	3094	.7853	1.35	.7858	2.55	.20	3.05	1.25	.30	.120	.145	8
U20	3052	.7887	1.35	.7919	2.20	.15	3.00	1.25	.30	.125	.180	14
U10	3186	.8092	1.35	.8521	2.15	.35	2.95	1.25	.30	1.85	.190	24
Initial	4373	1.428	2.50	1.25	4.00	.20	4.00	1.00	.40	1.224	1.224	
M10	3152	.7654	1.30	.7649	2.25	.35	2.95	1.25	.30	.145	.190	20
M20	3164	.7884	1.30	.7558	2.35	.30	3.10	1.25	.30	.165	.245	10
M30	3084	.7957	1.35	.7290	2.10	.20	3.00	1.15	.30	.135	.105	8

U/M: initial point on the Upper boundaries and in the Middle of boundaries

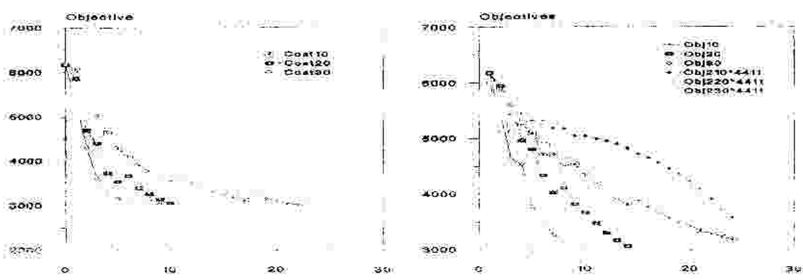


FIG.4 Typical Multicriteria and Single Objective Optimization Processes

CONCLUSION

An automated comprehensive procedure in the study can be used to achieve a practical solution for prestressed concrete beam, without the need to guess the amount of initial points. The multicriteria optimization based on the maximum entropy formulation approach, presented in this paper, demonstrates the potential of the advanced optimization strategy coupled with finite element techniques, to solve a variety of prestressed and reinforcement concrete design problems in a quite efficient way. The major merits of the approach are: (1) discrete design variables are introduced to satisfy the practical requirements; (2) a very concrete theoretical foundation to determine the direction for a multicriteria problem to the non-unique Pareto Solution which includes possible (even conflicting) objective functions for a given structural design problem; (3) satisfaction of all constraints (very easy to convergence into feasible designs); and (4) because a general FEM was employed, there is the suitability of a variety of structural designs in the civil engineering, which had been widely used in the mechanical and aero-space engineering.

More comprehensive structural design representation should be investigated which should consider separately structural details, such as anchorage, prestressed concrete block and bridge bearing.

NOTATION

$\gamma_{c,s,ps}$	:	Partial safety factor for concrete, reinforcement and prestressing steel
f_c'	:	Compressive strength of concrete
$f_{cc,ct}$	:	Flexural compressive and tensile strength of concrete
$f'_{cc,ct}$	:	Initial flexural compressive and tensile strength of concrete
ρ_c	:	the density of concrete
τ_{Rd}	:	basic design shear strength of concrete
f_{ps}	:	Tensile strength of prestressing steel
f_{yk}	:	Characteristic tensile strength of prestressing steel
f_s	:	design strength of reinforcing steel
f_y	:	yield stress of reinforcing steel
w_g	:	own weight
P_{con}	:	Concentrated loan load
E, E_c, E_{ci}		Steel and Concrete Young's modulus
γ	:	Poisson's ratio of Concrete Material
φ	:	Safety factor
x	:	Design variables
f	:	objective functions
h, **g**	:	vectors of constraints
K	:	stiffness matrix
P	:	load vector

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