

# COMPARISON BETWEEN MODAL AND STEP-BY-STEP APPROACHES IN THE OPTIMIZATION OF CABLE-STAYED BRIDGES SUBJECTED TO SEISMIC LOADS

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## ABSTRACT

Earthquake-resistant provisions are essential features of the design of cable-stayed bridges built on seismic regions and must therefore be accounted for in any structural optimization approach. The response to seismic loads depends on geometric and mechanic structural characteristics and, once these change continuously throughout the optimization process, adequate sensitivity analysis algorithms must be used. Time-history and modal/spectral approaches were both considered for such purpose, their relative merits and limitations being discussed. The optimization is formulated as a multiple goals problem seeking to improve objectives such as cost, stresses and displacements. An integrated analysis-optimization finite element program including these features was developed from MODULEF (INRIA, 1992) open code. An illustrative example is used to highlight the main issues of both methods and for a comparative study.

## KEYWORDS

-- Cable-stayed bridges; optimization; sensitivity analysis; modal/spectral analysis; time-history method;

## INTRODUCTION

The use of an analytical analysis method for seismic design requires that some simplified model is used which can provide suitable appraisal of the support excitation induced by the ground vibration waves.

The most widely used approaches for seismic structural analysis are the modal superposition method combined with response spectra analysis and the time-history (step-by-step) method.

In the former, one obtains a set of pseudo-static forces which represent an envelope of the instantaneous critical state of the structure throughout the whole seismic excitation. This is obtained by combining the maximum modal forces assigned to the modes with lower eigenfrequencies. For many years the Square Root of Sum of Squares method (SRSS) was used for this purpose. However, this method is inadequate when the structures possess closely spaced frequencies and in three-dimensional structures with strong mode coupling, as is the case with cable-stayed bridges. This fact led to the development of the Complete Quadratic Combination (CQC) method, originally formulated by Kiureghian (1981) and followed in this study. The method uses the concept of correlation coefficients, becoming the SRSS solution when all the frequencies are well separated. The inertial forces induced by support excitation are computed by using spectral accelerations,

displacements or velocities for each representative mode, displacements or velocities, available from a standard response spectra.

In the time-history approach, the integration of the dynamic equilibrium equations either by a direct or a modal procedure is done. A complete set of instantaneous solutions for the successive time steps is evaluated, the excitation being provided through an accelerogram. Although this data should have been recorded on site or in a near region from an actual seismic event, this is seldom the case because there is not much available records of seisms of adequate intensity. The alternative is the use of an artificial accelerogram generated from spectral density power functions, and this approach was adopted here.

Although there is an extensive literature and a number of actual design applications concerning both methods, very little attention has been paid, however, to its behaviour in the context of structural optimization. Differently from other kinds of loads, seismic action is highly dependant on the stiffness and mass characteristics of the structure, which permanently vary with the design variables values through the optimization process, no explicit relation existing to describe this variation. In the structural optimization context, all the load conditions that the final optimized structure will be required to resist must be available. Therefore, specific algorithms for the sensitivity analysis of the structural responses with respect to changes in the design variables need to be derived and computed.

## PROGRAMMING IMPLEMENTATION

The relative advantages and inconvenients of both modal/spectral and time-history procedures in the context of an ordinary seismic analysis are well known. Modal superposition is unsuitable for strongly non-linear problems, while time-history approach may effectively be used assuming a non-linear equation solver is available. However, short-to-medium span cable-stayed bridges show mild non-linear behaviour, resulting essentially from the sag effect of the cables. The use of the equivalent Ernst modulus concept (Walther *et al.*, 1985) may thus be used to model the stress-dependant stiffness of the cables, allowing for a linear methodology to be used elsewhere.

Once the pseudo-static forces set is obtained from the modal/spectral analysis, the remaining analysis and optimization sequence is that of an ordinary static problem. This means that this step can be recursively called as a black-box where the main program inputs data concerning structural description and from which it gets the force set. After this, all the live memory may be cleaned up to be used by the main analysis core. This allows for the method to be coded in a separate set of subroutines, called at the start of each analysis-optimization cycle, resulting in a clear program structure.

Step-by-step methodology is also, in principle, relatively easy to implement. However, most finite element packages in which this tool is available proceed either with static or dynamic solutions, but not both at the same time. Considering that structural optimization requires information from (possibly) many load cases, of both static or pseudo-static (dead and live load, wind, etc.) and dynamic (seismic action) nature, the possibility of sequential solution of both types of load cases, prior to the optimization step, must be available. This could obviously be made by hand, preparing a set of load cases at a time, but at the cost of considerable pre-processing user effort, because it had to be repeated in every analysis-optimization cycle. For that reason, an automatic procedure was implemented.

Both methods are considerably more expensive in computing time than the static case, but while most of the computational effort for modal/spectral analysis corresponds to the solution of the eigenvalue problem, in the case of time-history approach it results from the step-by-step procedure solution spanning the excitation period.

Therefore modal/spectral approach seems to have advantages, because it is well known that the most significant contributions for the pseudo-static forces set result from the modes with lower frequencies. This allows for the eigenvalue search domain to be shortened. This advantage in ordinary seismic analysis may be lost in structural optimization. Eigenvector derivatives, required for the sensitivity analysis, are computed from

a convenient expansion of eigenvectors but, for the purpose of accuracy, requires a considerable number of terms to be employed. As a consequence, a relatively wide search domain must again be adopted for that case.

Time-history approach requires the solution of a large number of instantaneous results throughout the time domain. In structural analysis this is done for both the real and the pseudo-load vectors, for each time-step. The total number of right-hand sides equals the real number of load cases plus one times the number of design variables. Even though the stiffness matrix factorisation is done once, time-history analysis is time-consuming, specially when a significant number of design variables is being used.

Although the unconditionally stable algorithms, used for time-history solution, allow for the use of a large time-step, accuracy requirements restrict that possibility. Furthermore, given the nature of the problem, the time-step must be small enough to capture, from one side, the pattern of the exciting accelerogram and, on the other hand, the effects of the most representative vibration modes of the structure. If one grossly assumes that these modes are contained within the frequency range 0.1-3Hz (which is usually the case with cable-stayed bridges), and that a minimum of five time-steps (Kiureghian, 1993) is required to a coarse modelling of the behaviour within a single period, magnitudes of about 0.1s or less will result for the recommended time-step.

Besides, and since Rayleigh damping is usually considered, the method needs an eigenvalue solver. Moreover an eigenvalue sensitivity analysis must be carried out, because the damping matrix depends on the two first frequencies, which are structural dependent. This is a major disadvantage from the programming point of view, because these tasks use most of the code developed in the modal/spectral approach.

## MODAL/SPECTRAL METHOD

A brief description on the sensitivity analysis for the modal/spectral approach will be made next. The maximum modal force for any mode under consideration is obtained from the expression

$$f_{i,\max} = \underline{\mathbf{M}} \phi_i \frac{\vartheta_i}{M_i} S_{ai}(\xi_i, f_i) \quad (1)$$

where  $\underline{\mathbf{M}}$  stands for mass matrix,  $\phi_i$  the  $i$ th eigenvector,  $\vartheta_i$  and  $M_i$  the participation factor and generalized mass for the mode and  $S_{ai}$  the spectral acceleration, depending on the damping ratio and the frequency of the mode.

Using CQC method, the forces so computed for the  $P$  considered modes are combined to obtain the resulting force through the expression

$$f_k = \sqrt{\sum_{i=1}^P \sum_{j=1}^P f_{ki} \rho_{ij} f_{kj}} \quad (2)$$

Correlation coefficients  $\rho_{ij}$  depend on damping ratios  $\xi_i$  and frequencies ratio  $r = \min(\omega_i, \omega_j) / \max(\omega_i, \omega_j)$ :

$$\rho_{ij} = \frac{8\xi_i^2(1+r)r^{3/2}}{(1-r^2)^2 + 4\xi_i^2 r(1+r)^2} \quad (3)$$

Sensitivity analysis of the seismic pseudo-loads is based on the differentiation of equation (2) with respect to the design variable set. This requires, successively, the derivatives of modal forces and correlation parameters and, for these, the eigenvalues and eigenvectors derivatives for the  $P$  modes. The pseudo-static force derivative has the form

$$\frac{df_k}{dx} = \frac{\sum_{i=1}^P \sum_{j=1}^P \left[ \frac{df_{ki}}{dx} \rho_{ij} f_{kj} + f_{ki} \frac{d\rho_{ij}}{dx} f_{kj} + f_{ki} \rho_{ij} \frac{df_{kj}}{dx} \right]}{2 \sqrt{\sum_{i=1}^P \sum_{j=1}^P f_{ki} \rho_{ij} f_{kj}}} \quad (4)$$

and, for its computation, the  $i$ th modal maximum force derivative will be found to be

$$\frac{d}{dx} f_{i,max} = \mathbf{M} \left[ \frac{d\phi_i}{dx} \frac{\vartheta_i}{M_i} \mathbf{S}_{a,i} + \phi_i \left( \left( \frac{1}{M_i} \frac{d\vartheta_i}{dx} - \frac{\vartheta_i}{M_i^2} \frac{dM_i}{dx} \right) \mathbf{S}_{a,i} + \frac{\vartheta_i}{M_i} \frac{d\mathbf{S}_{a,i}}{dx} \right) \right] + \frac{d\mathbf{M}}{dx} \phi_i \frac{\vartheta_i}{M_i} \mathbf{S}_{a,i} \quad (5)$$

## TIME-HISTORY METHOD

Newmark's method was adopted for step-by-step integration of the dynamic equilibrium equations, in view of its proven effectiveness and formal simplicity. Rayleigh damping is assumed and therefore a previous eigenvalue solution is made to supply the two lower frequencies and their sensitivities. The access to this module occurs whenever an updated structure is generated.

Sensitivity analysis algorithm is directly derived from the differentiation of the discrete instantaneous dynamic equilibrium equations and the procedure evolves in time as the ordinary problem. It relies on chain computation of a pseudo-load vector for next time-step, from which the acceleration and its gradient with respect to the design variables can be obtained. The instantaneous relative displacements and velocities can then be evaluated by using equations (8) and the gradient of the accelerations previously found.

$$\mathbf{K}_{ef} \mathbf{u}_{t+\Delta t} = \mathbf{P}_{ef,t+\Delta t} \quad (6)$$

$$\mathbf{P}_{ef,t+\Delta t} = \mathbf{P}_{t+\Delta t} + \mathbf{M} \left( (a_0 + \alpha a_1) \ddot{\mathbf{u}}_t + (a_2 + \alpha a_4) \dot{\mathbf{u}}_t + (a_3 + \alpha a_5) \mathbf{u}_t \right) + \mathbf{K} \left( \beta a_1 \mathbf{u}_t + \beta a_4 \dot{\mathbf{u}}_t + \beta a_5 \ddot{\mathbf{u}}_t \right) \quad (7)$$

$$\dot{\mathbf{u}}_{t+\Delta t} = \dot{\mathbf{u}}_t + \left[ (1-\delta) \ddot{\mathbf{u}}_t + \delta \ddot{\mathbf{u}}_{t+\Delta t} \right] \Delta t \quad \mathbf{u}_{t+\Delta t} = \mathbf{u}_t + \dot{\mathbf{u}}_t \Delta t + \left[ \left( \frac{1}{2} - \eta \right) \ddot{\mathbf{u}}_t + \eta \ddot{\mathbf{u}}_{t+\Delta t} \right] \Delta t^2 \quad (8)$$

The effective stiffness matrix  $\mathbf{K}_{ef}$  in (6) is a linear combination of the stiffness and mass matrices. Dot means time differentiation. The coefficients  $a_i$  depend on the time-step and the Newmark constant parameters  $\delta$  and  $\eta$ .  $\alpha$  and  $\beta$  are Rayleigh's parameters for damping matrix and are implicit functions of the design variables through the basic frequencies  $\omega_1$  and  $\omega_2$ .

The step-by-step sensitivity analysis main expressions are as follows

$$\frac{d\mathbf{K}_{ef}}{dx} = a_1 \frac{d\beta}{dx} \mathbf{K} + (1 + a_1 \beta) \frac{d\mathbf{K}}{dx} + a_1 \frac{d\alpha}{dx} \mathbf{M} + (a_0 + a_1 \alpha) \frac{d\mathbf{M}}{dx} \quad (9)$$

$$\frac{d\mathbf{P}_{ef,t+\Delta t}}{dx} = \frac{d\mathbf{P}_{t+\Delta t}}{dx} + \frac{d\mathbf{M}}{dx} \left( \mathbf{A}_{1,t} + \alpha \mathbf{A}_{2,t} \right) + \mathbf{M} \left( \frac{d\mathbf{A}_{1,t}}{dx} + \frac{d\alpha}{dx} \mathbf{A}_{2,t} + \alpha \frac{d\mathbf{A}_{2,t}}{dx} \right) + \frac{d\beta}{dx} \mathbf{K} \mathbf{A}_{2,t} + \beta \frac{d\mathbf{K}}{dx} \mathbf{A}_{2,t} + \beta \mathbf{K} \frac{d\mathbf{A}_{2,t}}{dx} \quad (10)$$

with

$$\mathbf{A}_{1,t} = a_0 \mathbf{u}_t + a_2 \dot{\mathbf{u}}_t + a_3 \ddot{\mathbf{u}}_t \quad \mathbf{A}_{2,t} = a_1 \mathbf{u}_t + a_4 \dot{\mathbf{u}}_t + a_5 \ddot{\mathbf{u}}_t \quad (11)$$

and Rayleigh coefficients derivatives are

$$\frac{d\beta}{dx} = \frac{2\xi}{(\omega_1 + \omega_2)^2} \left( \frac{d\omega_1}{dx} + \frac{d\omega_2}{dx} \right) \quad \frac{d\alpha}{dx} = \left( \frac{d\omega_1}{dx} \omega_2 + \omega_1 \frac{d\omega_2}{dx} \right) \beta + \omega_1 \omega_2 \frac{d\beta}{dx} \quad (12)$$

## APPLICATION EXAMPLE

Both methods are used for the optimization of the 250m symmetric cable-stayed bridge shown in Fig. 1. The deck is 20m wide and is supported by two longitudinal asymmetric I-shaped beams. Symmetric I beams spaced



every 5m support the slab and the stiffening effect in the transverse direction is modelled by using diagonal bar elements between consecutive beams. Rectangular hollow cross sections are adopted for the pylons. Vertical connection between the deck and the pylons is the only considered. Live load all over the deck and seismic vibration under dead load condition are used as load cases. For the modal/spectral approach, the Portuguese code of practice RSA response spectras are used as excitation source. For time-history approach, artificial accelerograms are derived from the standard spectral density power function available in that same code. Earthquakes of both types I (moderate magnitude and a near focus) and II (intense magnitude and a far focus) were studied, corresponding to time spans of 10s and 30s, respectively, for the step-by-step approach. Volume unit cost was assumed to be the same for all kinds of structural elements. Allowable stresses of 200MPa and 500MPa were assigned to beam and cable elements, respectively.

28 design variables were considered, all respecting to cross sectional parameters. Variables 1,2 and 3 are upper and lower flanges width and web height for the main girders, constant values being assumed throughout the span for aesthetic reasons. Variables 4 through 12 are flanges and web thicknesses for respectively the outer part of side spans, the neighbourhood of the pylons and the inner part of central span. Variables 13 to 20 correspond to pylons cross-sectional dimensions and thicknesses for both the lengths below and above the deck. Finally, variables 21 through 28 are the cross-sectional areas of each couple of stays, starting from the outer to the inner cables. Values for the starting and final (optimized) design are listed in Table 1 for both modal/spectral and time-history approaches.

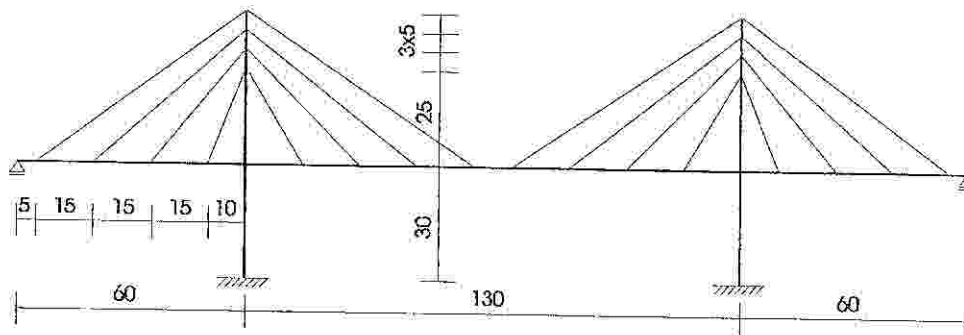


Fig. 1. Cable-stayed bridge geometry

Table 1. Initial and optimized values of designed variables

var	start.	mod/sp.	time-h.	var	start.	mod/sp.	time-h.	var	start.	mod/sp.	time-h.
x1	1.000	0.741	0.843	x11	.0200	.0367	.0226	x21	.0060	.0129	.0087
x2	1.000	0.532	0.539	x12	.0200	.0100	.0100	x22	.0060	.0035	.0036
x3	2.000	1.198	1.376	x13	3.000	4.304	3.617	x23	.0050	.0047	.0048
x4	.0200	.0164	.0124	x14	3.000	4.782	3.046	x24	.0050	.0061	.0050
x5	.0200	.0326	.0209	x15	.0250	.0108	.0108	x25	.0050	.0041	.0051
x6	.0200	.0100	.0010	x16	.0250	.0127	.0274	x26	.0050	.0045	.0054
x7	.0300	.0397	.0322	x17	2.000	4.177	4.141	x27	.0050	.0054	.0057
x8	.0300	.0129	.0129	x18	2.000	2.266	2.055	x28	.0050	.0038	.0035
x9	.0300	.0129	.0129	x19	.0200	.0100	.0100	Vol	165.00	128.71	130.68
x10	.0200	.0103	.0100	x20	.0200	.0100	.0100	%	-	22.00	20.80

Move limits of 10% were allowed for design variables through an optimization cycle. Very similar optimal designs were found from both methods, as can be seen by values in Table 1. A global cost reduction of about 21-22% was achieved.

Figure 2 shows the relative weight of each computational task within each method. Time-integration procedures takes most of the time in the step-by-step approach. For modal/spectral approach, three modes with the highest participation factors were considered in computing the pseudo-static loads. The eigenvalue problem search domain was set in the range 0-300rad<sup>2</sup>/s<sup>2</sup>, grossly corresponding to the frequency range 0-3Hz. About 20 modes, depending on the current solution, were detected within these bounds. No coalescence situations were detected throughout the process.

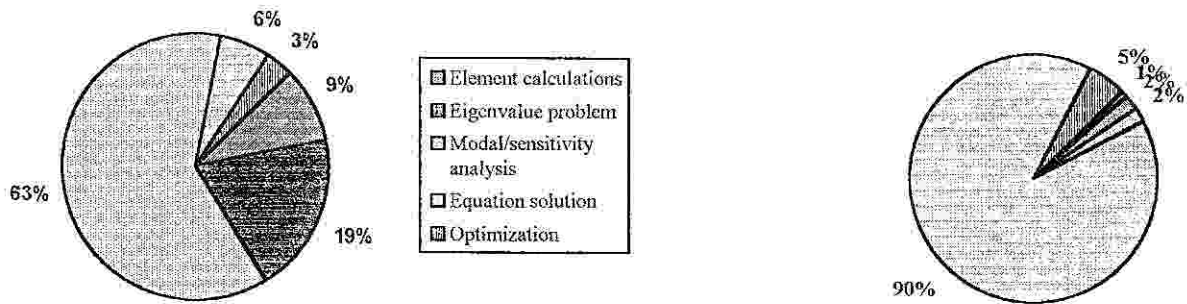


Fig. 2. Computation time of each task for modal/spectral and time-history

Figure 3 illustrates the absolute processing time spent until convergence (8 iterations). A cost decrease of 1% was considered as stopping criterion. The higher cost of optimization stage in the time-history approach results from the larger number of goals defining the multi-objective function.

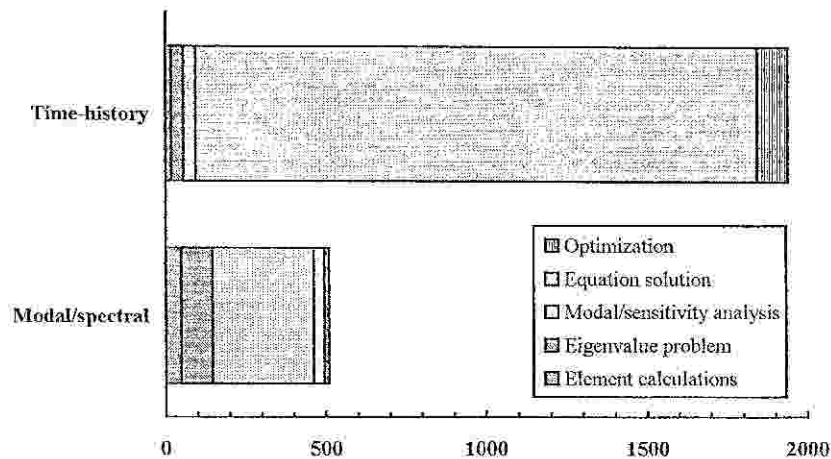


Fig. 3. Absolute computation time until convergence (8 iterations)

## DISCUSSION

In the specific context of structural optimization, modal/spectral approach requires careful control to avoid numeric overflow. The method of Rogers (1970) is used to compute eigenvectors sensitivities. The weak point of that method arises when eigenvalue coincide, in which case the algorithm fails. This might happen during the optimization process in two ways. First, the same value corresponds to two or more frequencies with intrinsically different vibration modes, at some intermediate design. This situation is temporary because the frequencies separate, when the design variable set evolves to the next solution. The other situation is related to intrinsically equal frequencies, such as those corresponding to symmetric and anti-symmetric transversal modes of unbraced pylons, in which case both frequencies will change identically throughout the process. This case can be excluded from the very beginning of the optimization process by taking measures to prevent the occurrence of those modes (such as adopting top bracing of pylons) or throwing them outside the considered frequency range (by adequate geometry or cross-section parameter selection).

In a strict sense, coalescence will barely occur, because the computational idiosyncrasies will cause slight numerical differences to occur between the formally equal eigenfrequencies. However, the numeric accuracy will probably result affected in such situations and, therefore, a criterion based in some specified tolerance must be used as a control procedure. Special procedures (Haftka *et al*, 1992) for handling coalescent eigenvalues were also implemented.

The large amount of results and, consequently the corresponding requirements of live and mass memory necessary for handling and storage is probably the main handicap of time-history procedure. The amount of data supplied by modal/spectral method to the main analysis-optimization program corresponds to that of a static load case and the respective sensitivities for each considered seismic load case. Memory requirements are therefore only slightly increased with respect to the static problem.

Time-history approach uses and generates that same volume of data and results for every time-step of the solution process. This means that, unless some filtering method is adopted, the system will run out of memory even for small- or mean-sized problems. Memory is not a critical issue in big computational systems, but in personal computers or workstations, which this study is targeted to, downsizing becomes necessary and a judicious control of memory is fundamental.

Filtering procedure consists of retaining results from just a selected number of time-steps, the intermediate information being lost or stored in disk, for the case in which post-processing of time-dependant quantities is to be done. The distribution of the time points has to be close enough to preclude the possibility of significant peak responses between two points. On the other hand, it must be large enough to significantly reduce the amount of data to deal with. An initial time mesh of one eighth of the first natural frequency is used being reduced to one eighth of the third lowest frequency near the optimum solution

The total number of objectives concerning stresses and displacements for the static problem amounts typically to a few hundred. This is a reasonable number to handle when constructing the envelope function replacing the original multiobjective formulation. However, if a large number of time-steps needs to be considered after primary filtering, this number may raise to a few thousands, only a few of which are nearly critical. As a consequence, the objective function may loose sensitivity with respect to those critical objectives. A second filtering operation, when time-history method of dynamic analysis is employed, in which only a limited number with the most critical objectives is used, is adopted as a standard procedure.

This procedure may obviously be used whatever the method considered for dynamic analysis. However, given that the total number of objectives resulting from a modal/spectral approach is usually moderate, its use is not as indispensable as with time-history method.

The linearization of the sensitivities within the move limits may introduce inaccuracies in the time-history approach. The rate of change of stresses and displacements depends on the current design point. Updated prediction of these values is made by using a truncated Taylor's series in which terms concerning second derivatives and higher are neglected. This has the effect of introducing discrepancies between predicted and actual structural responses which depend on the smoothness of the first derivatives functions and the variation of each design variable. For move limits of 10%, such errors are usually within the range 0-20%. As the optimization sequence evolves, tighter limits must be chosen to reduce the error and avoiding unfeasible solutions.

In modal/spectral analysis, in which all load cases are independent, the first order linearization used to compute sensitivities has no effects on the effectiveness of the process and convergence is similar to that of an ordinary static problem. However, in time-history approach each instantaneous solution in time is set by using data from the previous one and so does sensitivity analysis. This may cause the error to propagate through the time domain and predictions of one later time-step will probably lack accuracy, if the number of steps is large.

Either a tighter move limit must be chosen so as that the final accumulated error remains reasonably bounded, or second derivatives must be used in the sensitivity analysis, to prevent this problem. The latter results in a considerable increase in computing effort and data produced and was not adopted.

It is worth to make a remark concerning the reliability of optimized designs obtained by using specific accelerograms. Both true and artificial accelerograms are data sets expressing a single occurrence of a random process. Therefore, it is recommended that a design should be checked for three different accelerograms to improve its effectiveness to withstand earthquakes. A standard response spectra as defined in the codes of

practice can be used for the optimization with modal/spectral dynamic analysis, the seismic excitation remaining unchanged for the same structural conditions.

## CONCLUSION

Both modal/spectral and time-history approaches lead to suitable solutions of structural optimization of cable-stayed bridges undergoing seismic vibrations. The former, however, is more effective from a computational point of view, considering both the processing time and memory requirements. It is known that results from modal analysis used in conjunction with CQC method are in good agreement with those of time-history approach in linear analysis. The results seem to indicate that the same applies to sensitivity analysis, although a more exhaustive research would be recommendable to check such conclusion. In spite of its higher computational cost, time-history approach still must be used in large span cable-stayed bridges, in which strongly non-linear effects invalidate the use of modal/spectral approach.

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