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CFD based evaluation of the serviceability conditions of a cable stayed bridge under wind load

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ABSTRACT: This paper presents a new methodology for the analysis of serviceability limit states of vibrations caused by wind action on slender structures, based on the appropriate conjugation of an algorithm of Computational Fluid Dynamics (Control Volume Method) with an algorithm of linear or geometrically non-linear analysis of structures. The computer code developed on the basis of this methodology was applied to the evaluation of the serviceability conditions of a cable stayed bridge with a Π deck cross-section, under wind load. Some of the most interesting results associated with the evaluation of the corresponding response acceleration are presented and compared with available human body acceptance criteria for vibrations (comfort evaluation) present in the technical literature.

1 INTRODUCTION

Long span bridges are very flexible structures that can be affected by aeroelastic phenomena, like vortex-shedding / lock-in or flutter.

The analysis of the dynamic behaviour of such bridges under wind excitation is usually performed on the basis of experimental tests on physical models in wind tunnels (Simiu and Scanlan 1986).

As an alternative to such procedure, some numerical methodologies have been developed, namely in terms of flutter analysis (Scanlan 1971, Jones and Scanlan 1998), though they are still based on some coefficients (flutter derivatives or Scanlan coefficients) whose evaluation still involves usually the use of experimental tests (forced or free vibration tests).

An attempt to overcome such limitations consists in using algorithms of Computational Fluid Dynamics, that permit the numerical simulation of the air flow around the deck cross-sections. This type of applications has been considered for the evaluation of force or Scanlan coefficients, but not for integral aeroelastic analyses of slender bridge decks, taking into account the temporal evolution of the dynamic flow-structure interaction.

After recent progress in computer technology, the authors could develop and implement a new numerical methodology for the aeroelastic analysis of slender structures (Lopes 2001, Lopes *et al.* 2002, Lopes *et al.* 2004a). This computational algorithm is a time incremental approach based on two numerical algorithms working together: one of them determines the fluid flow action and the other one evaluates the structural response. The Finite Element Method (*FEM*) is used to model the structural dynamic behaviour, which can be idealised as geometrically non-linear. The numerical procedure used to calculate the fluid flow and its action on structures is based on the Finite Volume Method (*FVM*). It is considered a viscous incompressible unsteady turbulent bidimensional air flow solved on a structured control volume mesh. The mentioned algorithm uses an iterative sub-process to achieve the correspondence between aeroelastic forces and structural movements at every time step.

However, most of the applications performed deal with the evaluation of the critical velocity, also known as critical flutter velocity. This procedure can be understood as a verification of the structural safety in terms of ultimate limit state. But, as mentioned by the Eurocodes, it may be also needed to verify the serviceability limit states of vibrations caused by wind action. In the particular case of very flexible bridges under wind action, this verification can be done in terms of undesirable effects for users (discomfort), comparing the evaluated acceleration (or velocity) r.m.s. (or peak) values with human body acceptance criteria for vibrations.

In this context, this paper presents the application of the above mentioned computer algorithm to the evaluation of the serviceability conditions of a cable stayed bridge with a Π cross-section, under wind load. Some of the most interesting results associated with the evaluation of the corresponding response acceleration values are presented and compared with available human body acceptance criteria for vibrations (comfort evaluation) listed in Bachmann (1987), ISO (1989), CEB (1991) and NRCC (1990).

2 NONLINEAR COUPLED FLUID-STRUCTURE AEROELASTIC ANALYSIS

The computational algorithm developed to simulate aeroelastic phenomena in slender structures is a time incremental approach based on two numerical algorithms working together: one of them determines the fluid flow action and the other one evaluates the structural response. The numerical procedure used to calculate the fluid flow and its action on structures is based on the *FVM*. The *FEM* is used to model the structural dynamic behaviour, which can be idealised as geometrically non-linear.

2.1 *Fluid flow simulation*

The implemented program, based on the Finite Volume Method (Patankar 1980, Ferziger 1996, Versteeg 1995), is suitable to simulate incompressible and isothermal bidimensional unsteady fluid flows around obstacles. It is assumed that the flow's domain may be discretised in a Cartesian and structured control volume mesh, whose faces have vertical and horizontal directions.

The equations taken from the integration of general transport equations in differential forms are discretised by using the hybrid differentiation scheme. In order to reduce false diffusion, a refined mesh around boards of the obstacle is considered and the *QUICK* (Quadratic Upstream Interpolation for Convective Kinematics) differentiation scheme is also used in deferred correction context (Ferziger 1996). The stability is preserved by the use of base scheme (hybrid) to set up all coefficients of every equation, and by taking into consideration all the differences to the adequate scheme in source term. Due to their complexity and extensity, hybrid and *QUICK* coefficients, as source and deferred correction related equations, are not indicated here, but can be found in Lopes (2001).

Alternate value field resulting from first derivatives of pressure or velocity is avoided on the basis of a staggered grid approach, which is the basis of the *SIMPLE* (Semi-Implicit Method for Pressure-Linked Equations) procedure also used to ensure correct linkage between pressure and velocity field values. All these methods are iterative algorithms and, when other scalars (like turbulent quantities) are coupled to the momentum equations, the calculation has to be done sequentially. In order to ensure stability of the iteration process of this strongly non-linear problem, all these methods require under-relaxation what will be mentioned.

In this algorithm, the high Reynolds number $k - \varepsilon$ turbulence diffusion model is applied to simulate the flow turbulence (Rodi 1980, Tennekes 1980, Hossain 1982).

It should be mentioned that the probability of instability grows as the flow velocity increases (high convective). In this case, it is possible to use the same under-relaxation factors in order to get both stability and a solution procedure for transient calculations. Considering this procedure, the corresponding time interval for convective flows can be set up from the following inequality

$$\Delta t \le \frac{\mathbf{min}(\delta_n)}{2U} \tag{1}$$

where δ_n represents the distance between the central points of two adjacent control volumes and U represents the velocity of the free flow.

The model to simulate fluid flows is completed by defining boundary conditions, which can be separated into two parts: one of them for obstacle walls and the other for all limits of the considered external flow's domain (inlet and outlet).

All relevant equations and values used for modulation of turbulence and for defining boundary conditions at both obstacle walls and remaining boundary conditions in inlet and outlet regions can also be consulted in Lopes (2001).

The convergence criterions for pressure-correction equations and for the remaining equations are respectively

$$\frac{1}{n}\sum_n \frac{\left\| {}^i b \right\|}{\rho U} \le 10^{-4} \quad \text{and} \quad \frac{1}{n}\sum_n \frac{\left\| {}^i\phi - {}^{i-1}\phi \right\|}{\phi_{inlet}} \le 10^{-4} \tag{2}$$

where n is the number of control volumes, ρ is the density of the fluid, ${}^i b$ is the source term at the i^{th} iteration, ${}^i\phi$ is the field of the generic property value calculated at the i^{th} iteration and ϕ_{inlet} is the correspondent field value in the inlet domain.

2.2 *Structural analysis*

The Finite Element Method is used to model the structural behaviour (Bathe 1982, Clough 1993, Zienkiewick 1989). The simulation of the dynamic behaviour is based on the incremental Newmark Method and the corresponding integration parameters are set up according to Newmark's initial proposal (constant-average-acceleration-method). Structural damping is introduced by assuming a Rayleigh damping matrix, where the mass and stiffness matrix coefficients are evaluated by adopting two particular modal damping factors. The numerical procedures, based on an Updated Lagrangian formulation, allow the consideration of global large displacements (geometrical non-linear behaviour). However, small element deformations were assumed to evaluate the structural response.

In this incremental algorithm, the main purpose at every incremental time step Δt consists in reducing the non-balanced structural forces as much as possible, with the intention of obtaining the updated structural shape ${}^{t+\Delta t}a$. With this intend, this process involves an iterative sub-process with the purpose of evaluating the global increment of displacements ${}^{t+\Delta t}\Delta a$ at the corresponding Δt, which will be added to the displacements at the previous time instant ${}^{t}a$.

In any incremental time interval, the convergence criterion for non-balanced forces at the i^{th} iteration is

$$\frac{1}{n}\sum_{n}\frac{\left\| {}^{i}a - {}^{i-1}a \right\|}{L_{ref}} \leq 10^{-8} \tag{3}$$

where n is the number of degrees of freedom and L_{ref} is a reference length (for instance, maximum structural dimension).

2.3 *Aeroelastic algorithm*

A structural system is submitted to several forces when immersed in a fluid flow. If the structure is flexible, these forces are called aeroelastic forces, and they depend, not only on the flow characteristics around the structural system, but also on the structural flexibility (Naudascher 1994).

The present algorithm uses an iterative sub-process to achieve the convergence between aeroelastic forces and structural movements at the end of every time step.

The iterative sub-process begins based on the prediction about the movements at the end of each time step, by using the following linear extrapolation

$${}^{t+\Delta t}\ddot{a}_k = 2\,{}^{t}\ddot{a}_k - {}^{t-\Delta t}\ddot{a}_k \tag{4}$$

Then, the algorithm solves the flow equations and calculates the aeroelastic forces. Now, it is possible to determine the corresponding structural movements. If those movements are not in good agreement with the predictions, these predictions must be corrected and this sub-process should be reinitiated until convergence is achieved. The convergence criterion is similar to equation (3).

Due to the characteristics of bidimensional fluid flow simulation, this algorithm considers several transversal cross sections along the slender part of the structure where the aeroelastic forces are calculated. This simplified procedure assumes that the flow is normal to the longitudinal axis of the slender structure. Moreover, the flow around one section is simulated by itself and is considered independent from the other sections.

As it is mentioned above, this aeroelastic algorithm does not consider the three-dimensional flow effects, which constitutes the weakest feature of the presented fluid-structure model. However, it is expected that the three-dimensional effects, associated to the variation of the flow and structural geometry along a third spatial dimension, are not very significant for long cable-stayed or suspension bridges. This means that there are not considerable effects coming from the flow parallel to longitudinal deck axis, and the geometry variations are only localised in a few sections, which is probably insufficient to deeply change the characteristics of the global dynamic aeroelastic forces acting on the bridge deck.

The structural movements in fluid flow are modelled indirectly by changing the velocity components (v_1 and v_2) of the fluid flow at external inlet boundary domain as described in Lopes (2001).

3 ANALYSIS OF THE SERVICEABILITY CONDITIONS

The effects of the aeroelastic action have oscillatory features, usually characterized by the amplitude and the frequency.

If the free fluid flow velocity is bigger than a certain value, called critical velocity, the structural system has divergent oscillations, commonly known as flutter phenomenon. After enough time, the structure may have developed enlarged straining, plasticity included, or even may have fallen down. From the designer point of view, one finds out to have the highest possible flutter velocity.

On the other hand, when the free fluid flow velocity is less than the critical velocity, the structural system also shows some continuous vibrations, whose characterisation depends on vortex-shedding of the cross section. Generally, this vibration does not develop enlarge straining, but can lead to unacceptable vibration exposure, from human comfort point of view, or even materials fatigue.

These vibrations become important (large amplitudes) when the aeroelastic forces are synchronised with a structural frequency (a condition called resonance). This phenomenon, called "lock-in", is well known in aeroelastic field. Eurocode 1 (1991) suggests avoiding this synchronisation, but this phenomenon has been observed (Larsen 1999).

So, considering the possibility of occurrence of these effects, it seems interesting to develop a methodology for the analysis of the serviceability conditions. In this case, the mentioned analysis will be done by comparing the evaluated acceleration values of movements with available human body acceptance criteria for vibrations (comfort evaluation) listed in Bachmann (1987), ISO (1989), CEB (1991) and NRCC (1990).

The acceleration peak values of structural movements are evaluated by using the described program. With regard to the establishment of human body acceptance criteria for vibrations, two points must be referred: (i) there are not specific values set up for bridge cases; (ii) human body acceptance criteria for vibrations depend on body position (standing, sitting, or lying), on activity (working, walking, resting, etc), on main oscillations' direction (vertical or horizontal), on magnitude and frequency of vibrations, and also on the duration of the exposure. The last parameter is certainly one the most important, though it is difficult to find adequate references for it. In the present case it is assumed that the users of the bridge are standing, resting, and the vibrations are vertical. Table 1 resumes the information of some human body acceptance criteria for the found vibrations, where *a* means acceleration and *g* is the gravity.

Table 1. Human body acceptance criteria for vibrations.

Refer.	Human body acceptance criteria		Obs.
Bachmann (1987)	Perceptible	a≤5%g	Pedestrian structures
	Uncomfortable	5%g<a≤10%g	
ISO 2631 (1989)	Perceptible	0,1%g<a≤2,2%g	Public Transport
	A little uncomfortable	2,2%g<a≤4,5%g	
	Fairly uncomfortable	3,5%g<a≤7,1%g	
	Uncomfortable	5,7%g<a≤11%g	
CEB (1991)	Perceptible	0,5%g<a≤1,5%g	Building structures
	Disturbing	1,5%g<a≤5%g	
	Very disturbing	5%g<a≤15%g	
NBCC (1990)	Perceptible	a≤4%g	Rhythmic activities
	Uncomfortable	4%g<a≤7%g	

4 APPLICATION

This methodology is applied to the analysis of the serviceability conditions of a cable stayed bridge, with a Π-shaped deck cross-section (Fig. 1-2). The pylons were modelled considering *72* beam elements, and the deck with *96* plate-membrane elements. The deck and pylons, made of reinforced concrete, were connected by strong bars at section 5 (Fig. 1). The cables, made of steel, were modelled considering *56* bar elements, whose elasticity modules were updated based on Ernst's module, and the corresponding initial stresses were set up with the intention of having very small vertical deck displacements when the bridge is at rest and loaded by the quasi-permanent action ("*dead load condition*"). In this case, the mid span deflection is less than *0.02m*. Additionally, in order to avoid the lateral instability of the deck's vertical plates, the deck was strengthened, each *20m*, with *50* transversal plates of *0.4m* thickness. To be solved numerically by the structural algorithm, there are *214* nodes (*1284* degrees of freedom), which constitutes a very hard task for every PC. Table 2 presents the general mechanical characteristics considered. Table 3 shows the first natural frequencies and respective mode types. Structural damping is idealised on the basis of a Rayleigh damping matrix, whose composition is determined by assuming modal damping factors of *0.5%* for the first vertical bending and torsional modes. The evaluation of the aeroelastic forces is made by simulating the fluid flow around seven cross-sections: *3, 7, 10, 13* and the symmetric ones (Fig. 1). The fluid (air at standard conditions) flow mesh is built by using *97x56* control volumes (with a minimum dimension of *7E-2m* and a maximum of *91E-2m*). The distances from the faces of the deck's cross section to the boundary domain are fixed so as to obtain forces not dependent upon those distances. Globally, in this example, the aeroelastic algorithm running in a Pentium IV based PC takes about 30 minutes to simulate only one real second.

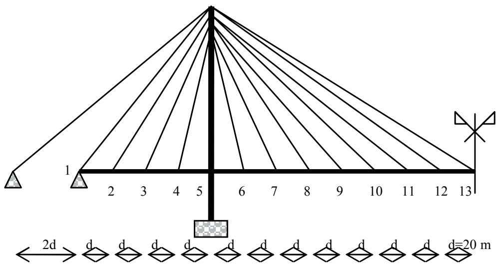

Figure 1. Geometry of the cable stayed bridge.

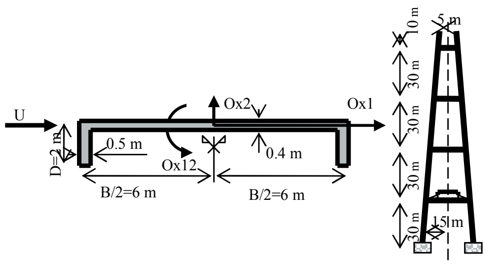

Figure 2. Π-shaped cross-section and pylon of the cable stayed bridge.

Table 2. Mechanical characteristics of the cable stayed bridge.

Pylons	Deck	Cables
γ=25kN/m^3	γ=25kN/m^3	γ=77kN/m^3
EA=3.84E8kN	E=3.2E7kN/m^2	A=7.85E-3m^2
EI_1=2.88E8kNm^2	ν=0.2	E(initial)=2E8kPa
EI_3=5.12E8kNm^2		

Table 3. Natural frequencies of the cable stayed bridge.

	1st mode		2nd mode		3rd mode	
Direction	No.	Freq.(Hz)	No.	Freq.(Hz)	No.	Freq.(Hz)
Vert./Bending	1	0.398	3	0.454	8	0.623
Horiz./Drag	2	0.440	5	0.571	7	0.609
Torsion	4	0.529	6	0.584	10	0.996

With the purpose of simulating the "*lock-in*" phenomenon, it is considered only the first (1st vertical), third (2nd vertical) and fourth (1st torsional) frequencies. Usually, the horizontal vibration is not significant due to positive horizontal aeroelastic damping (Lopes 2004b). So the corresponding three velocities of the free fluid flow, which is defined indirectly by the dimensionless Reynolds number (Re=ρUD/μ, where ρ and μ represent the density and the dynamic viscosity of the fluid), are: Re=8.9E5 (U=6.7m/s), Re=9.8E5 (U=7.4m/s) and Re=1.15E6 (U=8.65m/s). The used incremental time step is 5E-3s for dynamic structural analysis and, for each of these used intervals, five incremental time steps were adopted for fluid flow simulations. The simulation considers two phases: the structure is fixed in a first instance and is free to deform in a second one. Before releasing the structure, the velocity of the free flow is elevated to the pre-defined value and the simulation is led to a stable condition according to oscillatory aeroelastic forces. After that, the time account starts and the structure is liberated.

It is presented in Table 4 some static values corresponding to the mean value, the corresponding root-mean-square (*RMS*) and the predominant frequencies of the aerodynamic force coefficients, when the flow around a fixed cross-section is considered.

Table 5 and Figures 3-4 present some more significant results concerning displacements and aeroelastic forces at the mid-span section (section number 13 in Fig. 1), at U=6.7m/s free flow velocity and for certain time intervals.

Table 4. Relevant values got when the cross-section is at rest.

U (m/s)	Variable	Mean value	RMS	Freq. (Hz)
6.7	Moment coeff.	-0.313	0.204	0.425
	Lift coefficient	-0.452	0.194	0.425
	Drag coefficient	1.41	0.326	0.425
7.4	Moment coeff.	-0.320	0.194	0.473
	Lift coefficient	-0.416	0.172	0.471
	Drag coefficient	1.42	0.306	0.473
8.65	Moment coeff.	-0.318	0.196	0.546
	Lift coefficient	-0.435	0.179	0.544
	Drag coefficient	1.42	0.312	0.546

Table 5. Results at 6.7m/s flow velocity.

Variable	Time interval	Amplitude	Frequency (Hz)
Deflection	50-100 s	1.65e-2m	0.406
	100-150 s	1.90e-2m	0.404
Lift coefficient	50-100 s	0.301	0.404
	100-150 s	0.314	0.406
Vertical acceleration	50-100 s	0.109m/s2	0.406
	100-150 s	0.124m/s2	0.404

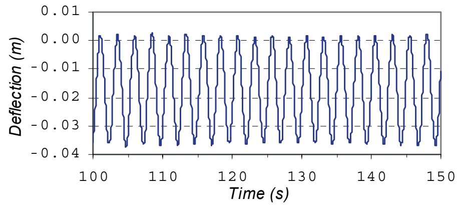

Figure 3: 100-150s deflection at 6.7m/s.

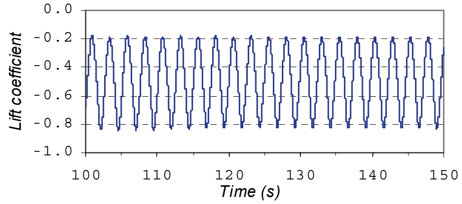

Figure 4: 100-150s lift coefficient at 6.7m/s.

Table 6 presents some results concerning displacements and aeroelastic forces at section number 10 (Fig. 1), at U=7.4m/s free flow velocity and for 100-150 seconds time interval.

Table 7 presents some results concerning displacements and aeroelastic forces at the mid-span section, at U=8.65m/s free flow velocity and for 100-150 seconds time interval.

Table 6. Results at 7.4m/s flow velocity.

Variable	Amplitude	Frequency (Hz)
Deflection	2.07e-3m	0.406
Lift coefficient	0.262	0.464
Vertical acceleration	0.014m/s2	0.406

Table 7. Results at 8.65m/s flow velocity.

Variable	Amplitude	Frequency (Hz)
Rotation	7.57e-4rad	0.544
Moment coefficient	0.294	0.544
Angular acceleration	0.0125rad/s2	0.544

Based on the above results, it is possible to draw the following conclusions:

- For the considered velocities of the free fluid flow, no mechanism of instability (growing amplitudes) was observed.
- For the presented structure under considered wind actions, the oscillations' stability was quickly reached (it takes less than 2 minutes of simulation).
- The evaluated maximum amplitude of accelerations, calculated for the first frequency case, is $0.124 m/s^2 \approx 1.2\%g$, which can be classified as a perceptible case.
- For the third frequency (2nd case), the evaluated maximum amplitude of accelerations is almost not

perceptible (low value). In this case, it was not possible to synchronise the second vertical frequency, since the main vertical movement's frequency evaluated coincides with the first one, not the third one. One thinks that, the horizontal frequency, which is closer to the third frequency, has an important damping effect on it.

o The evaluated maximum amplitude for the fourth frequency is $0.0125 rad/s^2$, which can produce a vertical acceleration of $0.075 m/s^2$ over the deck.

o Considering the wind action, the structure with this particular Π-shaped deck cross section, which is very sensitive to flutter (Lopes 2004c), demonstrates to have a reasonable behaviour at serviceability limit states, as the evaluated maximum amplitude of accelerations is $0.124 m/s^2$.

5 CONCLUSIONS

The results presented here illustrate an application of a new numerical methodology for the analysis of serviceability condition of bridge structures under wind action. Generally, this kind of aeroelastic phenomenon does not lead to any mechanism of instability, but can generate human body discomfort when the amplitude of acceleration reaches certain limit values. Typically, the amplitude of acceleration is high if both forces and displacements have a particular frequency of oscillation ("lock-in").

Although the conclusions drawn from the specific case of a cable stayed bridge, with a Π-shaped deck cross section, can not be directly extrapolated to other situations, the methodology presented in this paper can be applied to other cases with different shapes of the deck cross section.

It is worth mentioning that, it will be important to have specific rules in terms of characteristics of incoming fluid flow, serviceability limit states definition and maximum time period of analysis, in order to evaluate this kind of "lock-in" phenomenon.

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