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OPTIMIZATION OF CABLE-STAYED BRIDGES SUBJECTED TO EARTHQUAKES WITH NON-LINEAR BEHAVIOUR

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Earthquake-resistant provisions are essential features of the design of cable-stayed bridges built on seismic prone areas. Optimization can be employed to reduce cost and enhance geometrical and/or mechanical properties of the structure. The structural analysis programme developed here allows for a three-dimensional representation of cable-stayed bridges. Sensitivity analysis is carried out by analytic means both for the combined modal analysis/response spectra and the time-history methods. This enables the prediction of the variation of the structural response to earthquakes with respect to changes in the design variables. The optimization consists of a problem of multiple goals seeking to improve objectives such as cost, stresses, code of practice and erection requirements. This optimization problem turns out to be equivalent to the minimization of an unconstrained convex scalar function, which can be done by conventional quasi-Newton methods. Illustrative examples are given describing the features and results of both procedures.

Keywords: Cable-stayed bridges; multicriteria optimization; earthquakes

INTRODUCTION

In countries of high seismicity, the structural safety of large structures such as cable-stayed bridges against earthquake vibrations is a major concern of the design. If the application of some optimization technique at a preliminary design stage is to be considered, the

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usefulness and reliability of their results will depend upon the consideration of this action.

The application of optimization techniques to improve the design of cable-stayed bridges undergoing earthquake vibrations requires appropriate structural and sensitivity analysis programmes. The vibrational characteristics of this type of structure are highly dependent on the stiffness and mass distribution and there is not an explicit form for this relation. Special procedures must therefore be developed to account for the changes in such quantities as the structural configuration evolves. Three-dimensional modelling and the capability of handling geometric non-linearities are required for such study [8].

The usual methods for seismic analysis are either modal/spectral analysis or the time-history (step-by-step) process. Both methods have been extensively described in the literature and checked by a number of real designs. Their relative advantages and inconvenience are well known and will be underlined in the next section, both in the context of ordinary dynamic analysis and structural optimization.

COMPARATIVE ISSUES

Modal superposition is an intrinsically linear process and it is therefore likely to be found adequate only in linear or mildly nonlinear problems. Short-to-medium span cable-stayed bridges typically display the latter kind of behaviour, essentially resulting from the sag effect of the cables. The use of the Ernst modulus concept [11] may thus be used to model the stress-dependant stiffness of the cables, allowing for the use of a linear methodology.

The response spectra procedure used to be unsuitable for the dynamic analysis of structures with close frequencies and strong mode coupling which are typical of cable-stayed bridges. However, with the advent of the Complete Quadratic Combination (CQC) method [12] for the modal effects combination, this limitation was overcome. The method introduces the concept of correlation coefficients for mode coupling and converges to the traditional SRSS solution when all the frequencies are well separated.

The modal/spectral approach provides a set of pseudo-static forces leading to an envelope of the critical structural responses throughout the vibration process. Therefore the processing time needed to solve the equations is slightly increased, because the stiffness matrix is factorised just once for all load cases. As to the pseudo-static loads generation, the solution of the eigenvalue problem followed by the mode combination and CQC rule must be sequentially accessed, and here the computational cost depends upon the number of vibration modes whose contributions are accounted for. Theoretically, an exact description of the dynamic structural behaviour would be provided by considering the whole mode set. However, this is computationally expensive and inaccurate results for higher frequency modes are to be expected. A strategy considering a limited number of modes is recommended. Only the lowest modes with significant participation factors are selected to contribute to the seismic forces. This approach greatly improves the effectiveness of the method without compromising its accuracy.

Once the pseudo-static forces set is obtained from the modal/spectral analysis, the remaining analysis and optimization sequence can proceed as in an ordinary static problem. This means that this step can be recursively called as a black-box where the main programme inputs data concerning the structural description and from which it gets the forces set (and their sensitivities needed for the optimization). Once these are stored in disk, all the remaining information may be cleaned up. Whenever the available memory is a critical issue, the live memory may be set free for use in the main analysis core. This allows the method to be coded in a separate set of subroutines, called at the start of each analysis-optimization cycle, resulting in a clear programme structure, as can be seen in the block diagram of Figure 1.

Step-by-step methodology is also, in principle, relatively easy to implement. However, most finite element packages in which this tool is available proceed either with static or dynamic solutions, but not both at a time. Considering that structural optimization requires information from (possibly) many load cases, of both static or psudo-static (dead and live load, wind, *etc.*) and dynamic (seismic action) nature, the possibility of sequential solution of both types of load cases, prior to the optimization step, must be available. Although no reference to this problem was found in the literature, it could obviously be handled

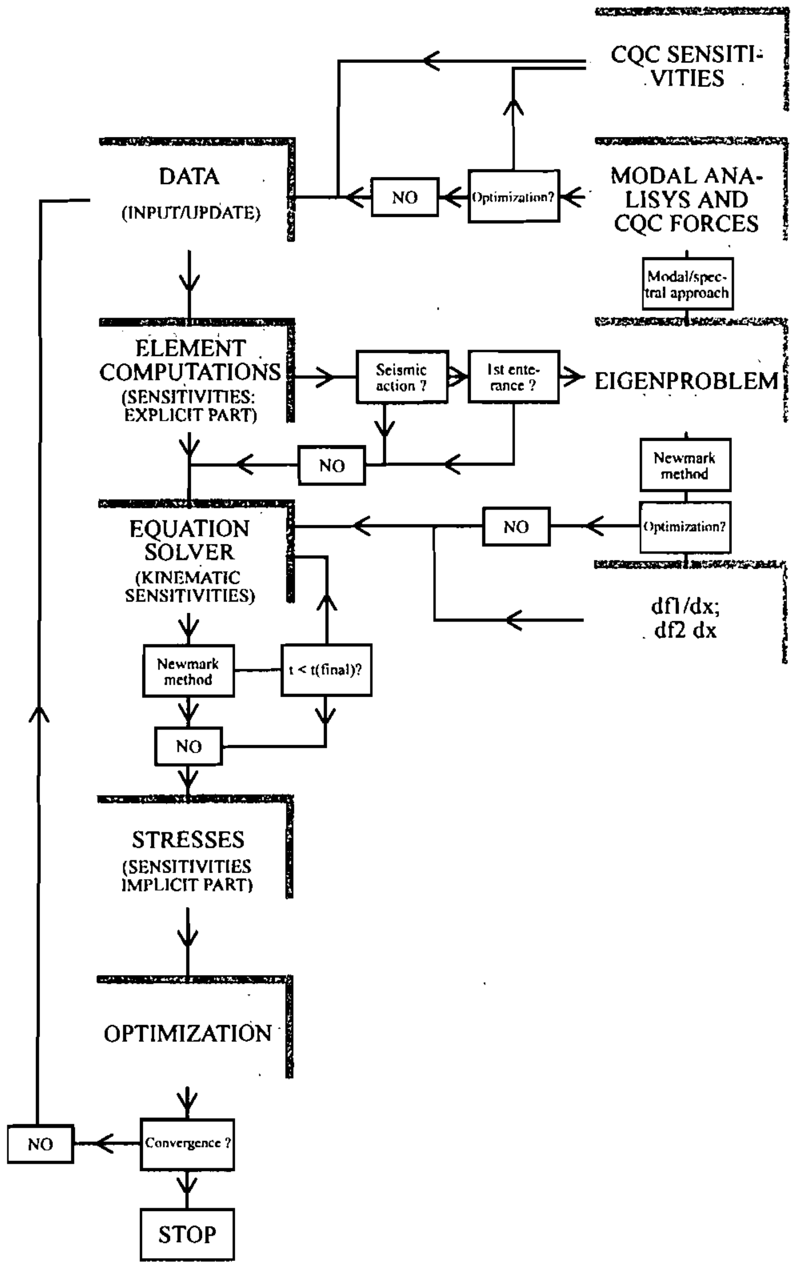

FIGURE 1 Programme block diagram.

manually, by firstly processing the static load cases set and proceeding afterwards with the dynamic analysis. This requires, however, a considerable pre- and post-processing user effort, because the process must be repeated in every analysis-optimization cycle. For that reason, an automatic procedure was implemented to govern the process. This resulted in a somewhat dense nesting of the step-by-step procedure within the main stream of the analysis core, as can be seen in Figure 1, which represents a disadvantage from the point of view of code implementation. No loss of generality results from the procedure, however, and purely static, dynamic or mixed problems may be handled.

Both methods are considerably more expensive in computing time than the static case, but they show remarkable differences in the way they share time between the various tasks. Most of the computational effort in the modal/spectral approach is spent in the solution of the eigenproblem, while the step-by-step procedure solution spanning the excitation period consumes most of the time required for time-history approach. However, while one can limit the computational effort of the modal/spectral approach by tightening the frequency-search domain, not much can be done with regard to time saving with the step-by-step method, and this is one of its comparative weaknesses. Although unconditionally stable algorithms such as Newmark or Wilson-θ methods allow for the use of large time-steps, accuracy poses much more severe requirements. Bathe [2] states that time-step must be shorter than 1/5 of the shortest significant period of the structure, so that its time-varying effects may be captured. In cable-stayed bridges, main global mode frequencies are contained within the range 0.2 – 5.0 Hz, which means that time-steps of at most 0.1s must be considered. With Portuguese RSA [7] and other Codes of Practice prescribing excitation periods of as much as 30 seconds, this results in 300 instantaneous solutions.

The huge amount of information resulting from this process constitutes the other main disadvantage of the step-by-step approach, because it requires tens or hundreds of times the storage space of a static or a modal/spectral procedure. As it is out of the question to keep it in live memory, an intensive use of slow disk storage must be undertaken, which slows up the process. However, in this case one can make use of the concept of a *time-grid*, a strategy by which broader time-steps for selective instantaneous solutions retaining are adopted, the remaining ones being skipped. These time-steps may be prescribed in a number of ways, either considering a constant integer multiple of the equations' time-step or by pre-defining alternate narrow and large steps so that peak response regions may be accounted for in more detail. Nevertheless, the solution still must be evaluated at all minor time-steps, because the recurrent integration scheme needs that information to evolve in time.

A further disadvantage of the step-by-step approach results from consideration of the Rayleigh damping model, in which the damping matrix is assumed to be a linear combination of stiffness and mass matrices, the coefficients of this combination depending on the two first circular frequencies. The eigenproblem solver must therefore be implemented for this single purpose and, although the eigenvalue search domain may be substantially reduced, a considerable computational effort is still required.

Although there is an extensive literature and a number of actual design applications concerning both methods, very little attention has been paid to its behaviour in the context of structural optimization. In the case of the modal/spectral approach, particularly, no reference was found by the authors regarding such a problem.

Most of the arguments previously pointed out in the context of ordinary dynamic analysis apply to the case of structural optimization. However, greater concern must be dedicated to the computational cost of the solution process, because it is well known that structural optimization is much more expensive than ordinary analysis and dynamic analysis is an expensive process on its own.

Sensitivity analysis represent most of the computational effort in structural optimization problems. Either direct or adjoint methods may be used in providing analytical solutions for derivatives. The former was implemented in the programme used in this study, because the number of constraints is typically much larger than that of the design variables, for this type of structure. Besides, both stresses and displacements derivatives are required when deflection control is taken into account, as is usually required in the design practice of cable-stayed bridges. If N is the number of design variables under consideration, the direct method leads to the solution of $N + 1$ right-hand sides for each prescribed load case, corresponding to the true load vector and one pseudo-load vector for each design variable. The amount of data involved in the solution process is therefore nearly proportional to the number of design variables and, considering current values for N to be within the range 20 – 80, gives an idea of the cost of the optimization problem in relation to the ordinary analysis. This aspect may be critical in the step-by-step method, where this volume of data is generated in every time-step, which makes indispensable the use of a time-grid procedure.

Sensitivity analysis for modal/spectral approach requires the computation of eigenvalues and eigenvectors derivatives. Specific problems related to these tasks may degrade or even compromise the effectiveness of this method.

The existence of coalescent modes may be a serious source of trouble. Multiple frequencies may arise as temporary occurrences as the structure configuration changes, or may be intrinsically related to the structure, as is the case with symmetric and antisymmetric transverse bending of pylons. Special procedures [3, 4, 6] address the solution for the first kind of coalescence, but it was shown by Haug and Rousselet [5] that only directional derivatives can be obtained in such situations. As to the latter case, no solution has been suggested up to the present.

Nevertheless, no serious problems were detected when using the modal/spectral approach in the examples of this paper and many other previously tested. Two sorts of reasons provide an explanation for that fact: i) when the base excitation is such that small participation factors result for the coalescent modes, these are discarded from the mode superposition process and their derivatives are no longer required; ii) in the other case, the resulting inaccuracies are limited by moderate move limits and correct seismic pseudo-loads are re-evaluated at every new cycle of design-optimization.

The eigenvectors derivatives are computed according to the Lancaster procedure [1], by mean of a linear expansion involving the eigenvector set. This approach converges to the exact solution when terms corresponding to all eigenvectors are included, but the computational cost of such a process is prohibitive. Therefore, a limited set of vibration modes is considered, but its number must be large enough to provide accurate results. Thus a broad eigenvalue search domain must be initially set, so that a large number of eigenvectors may be captured, which greatly affects the effectiveness of the modal/spectral approach.

Response spectra are based on collections of empirical and discrete values of displacements, velocities or accelerations. Therefore, no standard analytical expression exists to describe them. This information is used to compute modal forces in modal/spectral seismic analysis. In optimization, modal forces sensitivities are required and, by using the chain derivation rule, this leads to the need to obtain derivatives of the spectral values. For this purpose, polynomial interpolation was used to analytically describe the various response spectra as functions of the frequency.

As stated before, the step-by-step procedure requires previous access to the eigenvalue solver, so that the Rayleigh damping matrix may be built using the two lowest circular frequencies. In the context of structural optimization, its sensitivities must also be provided, which requires the use of some routines used by the modal/spectral approach. Most of this code must therefore be implemented whatever is the method used to perform optimization with seismic analysis.

The excitation source is provided through the input of recorded or artificial accelerograms, the latter approach being used in this study. The same accelerogram must be used throughout the optimization process. Otherwise, predicted and computed values of stresses and displacements would not match from one analysis-optimization cycle to the next. The programme was provided with an accelerogram generator and, to fill this requirement, its access is inhibited after the first cycle. Of course, no guarantee exists that an optimized solution obtained by using an accelerogram will be feasible under a different one. Therefore, some checking must be made *a posteriori*, which constitutes a serious inconvenience, because the process must be restarted if infeasibility is found. An optimized solution computed through the modal/spectral approach, on the other hand, will always be feasible, because the response spectra are prescribed in Codes of Practice as unchangeable quantities.

MODAL/SPECTRAL METHOD

A brief description of the sensitivity analysis for the modal/spectral approach will be made next. The maximum modal force for any mode under consideration is obtained from the expression

$$\underline{\mathbf{f}}_{i,\max} = \underline{\mathbf{M}}\underline{\boldsymbol{\phi}}_i \frac{\boldsymbol{\vartheta}_i}{M_i} \underline{\mathbf{S}}_{ai}(\xi_i, f_i) \tag{1}$$

where $\underline{\mathbf{M}}$ stands for mass matrix, $\underline{\boldsymbol{\phi}}_i$ the ith eigenvector, ϑ_i and M_i the participation factor and generalised mass for the mode and $\underline{\mathbf{S}}_{ai}$ the spectral acceleration, depending on the damping ratio and the frequency of the mode.

Using the CQC method, the forces so computed for the P considered modes are combined to obtain the resulting force through the expression

$$f_k = \left(\sum_{i=1}^{P} \sum_{j=1}^{P} f_{ki}\, \rho_{ij}\, f_{kj} \right)^{1/2} \tag{2}$$

Correlation coefficients ρ_{ij} depend on damping ratios ξ_i and frequencies ratio $r = \min(\omega_i, \omega_j)/\max(\omega_i, \omega_j)$:

$$\rho_{ij} = \frac{8\xi^2(1+r)r^{3/2}}{(1-r^2)^2 + 4\xi^2 r(1+r)^2} \tag{3}$$

Sensitivity analysis of the seismic pseudo-forces is based on the differentiation of Eq. (2) with respect to the design variables. This requires the derivatives of modal forces and correlation parameters and, for these, the eigenvalues and eigenvectors derivatives for the P modes. The pseudo-static force derivative has the form

$$\frac{df_k}{dx} = \sum_{i=1}^{P} \sum_{j=1}^{P} \left[\frac{df_{ki}}{dx} \rho_{ij} f_{kj} + f_{ki} \frac{d\rho_{ij}}{dx} f_{kj} + f_{ki}\, \rho_{ij} \frac{df_{kj}}{dx} \right] \Bigg/ \left(4 \sum_{i=1}^{P} \sum_{j=1}^{P} f_{ki} \rho_{ij} f_{kj} \right)^{1/2} \tag{4}$$

and, for its computation, the ith modal maximum force derivative will be found to be

$$\frac{d}{dx}\underline{\mathbf{f}}_{i,\max} = \underline{\mathbf{M}} \left[\frac{d\underline{\boldsymbol{\phi}}_i}{dx} \frac{\boldsymbol{\vartheta}_i}{M_i} \underline{\mathbf{S}}_{a,i} + \underline{\boldsymbol{\phi}}_i \left(\left(\frac{1}{M_i} \frac{d\boldsymbol{\vartheta}_i}{dx} - \frac{\boldsymbol{\vartheta}_i}{M_i^2} \frac{dM_i}{dx} \right) \underline{\mathbf{S}}_{a,i} + \frac{\boldsymbol{\vartheta}_i}{M_i} \frac{d\underline{\mathbf{S}}_{a,i}}{dx} \right) \right]$$
$$+ \frac{d\underline{\mathbf{M}}}{dx} \underline{\boldsymbol{\phi}}_i \frac{\boldsymbol{\vartheta}_i}{M_i} \underline{\mathbf{S}}_{a,i} \tag{5}$$

TIME-HISTORY METHOD

The direct integration procedure of Newmark's method was considered in the step-by-step procedure, due to its effectiveness and formal simplicity. Rayleigh damping is assumed and therefore a previous eigenvalue solution is made to supply the two lower frequencies and thier sensitivities. The access to this module occurs whenever an updated structure is generated.

The sensitivity analysis algorithm is directly derived from the differentiation of the discrete instantaneous dynamic equilibrium equations and the procedure evolves in time as the ordinary problem. It relies on chain computation of a pseudo-load vector for the next time-step, from which the acceleration and its gradient with respect to the design variables can be obtained. The instantaneous relative displacements and velocities can then be evaluated by using Eq. (8) and the gradient of the accelerations previously found. Some authors [10, 13] have derived these expressions for the case of generic dynamic loading. In this particular study, the time-dependant loading reduces to the inertial forces due to the base shaking in accordance with the accelerogram pattern, their sensitivities with respect to the design variables being null. The main expressions of the Newmark method in the ordinary analysis form are

$$\underline{\mathbf{K}}_{ef}\underline{\mathbf{u}}_{t+\Delta t} = \underline{\mathbf{P}}_{ef,t+\Delta t} \tag{6}$$

$$\begin{aligned}\underline{\mathbf{P}}_{ef,t+\Delta t} = \underline{\mathbf{P}}_{t+\Delta t} &+ \underline{\mathbf{M}}((a_0+\alpha a_1)\underline{\mathbf{u}}_t + (a_2+\alpha a_4)\underline{\dot{\mathbf{u}}}_t + (a_3+\alpha a_5)\underline{\ddot{\mathbf{u}}}_t) \\ &+ \underline{\mathbf{K}}(\beta a_1\underline{\mathbf{u}}_t + \beta a_4\underline{\dot{\mathbf{u}}}_t + \beta a_5\underline{\ddot{\mathbf{u}}}_t)\end{aligned} \tag{7}$$

$$\begin{aligned}\underline{\dot{\mathbf{u}}}_{t+\Delta t} &= \underline{\dot{\mathbf{u}}}_t + [(1-\delta)\underline{\ddot{\mathbf{u}}}_t + \delta\underline{\ddot{\mathbf{u}}}_{t+\Delta t}]\Delta t \\ \underline{\mathbf{u}}_{t+\Delta t} &= \underline{\mathbf{u}}_t + \underline{\dot{\mathbf{u}}}_t\Delta t + \left[\left(\frac{1}{2}-\eta\right)\underline{\ddot{\mathbf{u}}}_t + \eta\underline{\ddot{\mathbf{u}}}_{t+\Delta t}\right]\Delta t^2\end{aligned} \tag{8}$$

The effective stiffness matrix $\underline{\mathbf{K}}_{ef}$ in (6) is a linear combination of the stiffness and mass matrices. Dot means time differentiation. The coefficients a_i depend on the time-step and the Newmark constant parameters δ and η. α and β are Rayleigh's parameters for the damping matrix and are implicit functions of the design variables through the basic frequencies ω_1 and ω_2.

The step-by-step sensitivity analysis main expressions are as follows

$$\frac{d\underline{\mathbf{K}}_{ef}}{dx} = a_1\frac{d\beta}{dx}\underline{\mathbf{K}} + (1 + a_1\beta)\frac{d\underline{\mathbf{K}}}{dx} + a_1\frac{d\alpha}{dx}\underline{\mathbf{M}} + (a_0 + a_1\alpha)\frac{d\underline{\mathbf{M}}}{dx} \tag{9}$$

$$\begin{aligned} \frac{d\underline{\mathbf{P}}_{ef,t+\Delta t}}{dx} &= \frac{d\underline{\mathbf{P}}_{t+\Delta t}}{dx} + \frac{d\underline{\mathbf{M}}}{dx}(\underline{\mathbf{A}}_{1,t} + \alpha\underline{\mathbf{A}}_{2,t}) \\ &+ \underline{\mathbf{M}}\left(\frac{d\underline{\mathbf{A}}_{1,t}}{dx} + \frac{d\alpha}{dx}\underline{\mathbf{A}}_{2,t} + \alpha\frac{d\underline{\mathbf{A}}_{2,t}}{dx}\right) \\ &+ \frac{d\beta}{dx}\underline{\mathbf{K}}\underline{\mathbf{A}}_{2,t} + \beta\frac{d\underline{\mathbf{K}}}{dx}\underline{\mathbf{A}}_{2,t} + \beta\underline{\mathbf{K}}\frac{d\underline{\mathbf{A}}_{2,t}}{dx} \end{aligned} \tag{10}$$

with

$$\underline{\mathbf{A}}_{1,t} = a_0\underline{\mathbf{u}}_t + a_2\dot{\underline{\mathbf{u}}}_t + a_3\ddot{\underline{\mathbf{u}}}_t \qquad \underline{\mathbf{A}}_{2,t} = a_1\underline{\mathbf{u}}_t + a_4\dot{\underline{\mathbf{u}}}_t + a_5\ddot{\underline{\mathbf{u}}}_t \tag{11}$$

and Rayleigh coefficients derivatives are

$$\frac{d\beta}{dx} = -\frac{2\xi}{(\omega_1+\omega_2)^2}\left(\frac{d\omega_1}{dx}+\frac{d\omega_2}{dx}\right) \quad \frac{d\alpha}{dx} = \left(\frac{d\omega_1}{dx}\omega_2+\omega_1\frac{d\omega_2}{dx}\right)\beta+\omega_1\omega_2\frac{d\beta}{dx} \tag{12}$$

OPTIMIZATION FORMULATION

The optimization objectives to simultaneously minimize stresses, material cost/volume and displacements, in order to achieve some global cost reduction. This is a multi-objective problem leading to a Pareto solution. The problem is equivalent to a minimax formulation that is discontinuous and non-differentiable, making its numerical solution by direct means difficult.

Using the concept of *information entropy* introduced by Shannon, Templeman [9] showed that the solution of the minimax problem could be indirectly obtained through the minimization of an unconstrained convex scalar function

$$\underset{\underline{\mathbf{x}}\in\underline{\mathbf{X}}}{\text{Minimize}}\,\frac{1}{\rho}\ln\sum_{i=1}^{M}\exp(\rho g_i(\underline{\mathbf{x}})) \tag{13}$$

where ρ is a parameter which must be progressively increased as the optimization proceeds and $g_i(\underline{\mathbf{x}})$ are the goals to be simultaneously reduced.

For numerical effect, this form may be replaced by an explicit approximation, formulated by taking series expansions of all goals truncated after the linear term:

$$\underset{\underline{\mathbf{x}}\in\underline{\mathbf{X}}}{\text{Minimize}}\,\frac{1}{\rho}\ln\sum_{i=1}^{M}\exp\left(\rho\left[g_i(\underline{\mathbf{x}}_o)+\sum_{j=1}^{N}\frac{dg_i(\underline{\mathbf{x}}_o)}{dx_j}\Delta x_j\right]\right) \tag{14}$$

NUMERICAL EXAMPLE

A three-span symmetric cable-stayed bridge with the geometry represented in Figure 2 will be subjected to optimization with both methods. The bridge is 300 m long, with side spans of 70 m. The deck stands 30 m above the foundation level and the total pylon height is 75 m. The deck consists of two side stiffening girders supporting transverse beams 5 m spaced. These support the deck slab, which is assumed to have local load bearing and transverse stiffening functions, the latter being modelled by diagonal bars with proper stiffness, defining a truss in the deck plane.

A two-plane semi-harp cable arrangement is considered, spacing between anchor positions being of 10 m on deck and 5 m on pylons. The pylons are clamped in foundation, no top bracing having been considered. Only a vertical link was considered for the deck-to-pylon connection, which leads to large horizontal sway of the deck during seismic excitation but results in some isolation effect with respect to it.

Three static loadings are accounted for, together with the seismic action: live load all over the span, on the side spans, or on the central span only. A traffic live load of 4KN/m^2 was used, according to the RSA prescriptions. Allowable stresses of 500 and 200 Mpa were

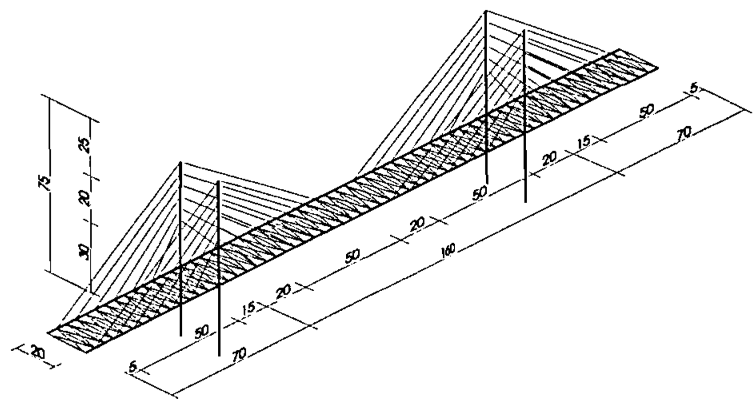

FIGURE 2 Model geometry.

assumed for the cables and the remaining structural elements, respectively. Although buckling provisions could also have been prescribed, they were not considered in this study. All the structural elements were assigned the cost factor of 1.000.

Asymmetric I-shaped cross sections were assigned to the stiffening girders, because the coupled effects of axial force and bending moments recommend the use of different widths and thicknesses of flanges. Axial force in transverse beams is neglectable and so symmetric I-type cross sections were used in modelling them. For the pylons, hollow rectangular cross sections were considered, which maximize both bending and torsional characteristics.

Although the programme developed by the authors for integrated analysis-optimization study of cable-stayed bridges allows for the use of sizing, shape and cable stretching design variables, only the first type has been considered in this example.

Table I lists the adopted design variables, their starting and optimized values for both methods and imposed bounds. Headers $\underline{\mathbf{X}}(m\text{-}s)$ and $\underline{\mathbf{X}}(t\text{-}h)$ stand for optimal values for modal-spectral and time-history approaches, respectively. Three regions of constant sectional geometry were specified for the stiffening girders. Region 1 covers side spans up to the inner cable anchor position. Region 2 spans the lengths on both sides of pylons defined by the inner cables anchor positions. Region 3 is the remaining central span.

TABLE I Design variables: meanings, values and bounds

Variable	*Description*	$\underline{X}(init)$	$\underline{X}(low)$	$\underline{X}(upp.)$	$\underline{X}(m\text{-}s)$	$\underline{X}(t\text{-}h)$
1	Width of upper flange of stiffening girders/region 1	1.000	0.500	1.500	0.767	0.739
2	Width of bot. flange of stiffening girders/region 1	1.000	0.500	1.500	1.202	1.500
3	Thick. upper flange of stiffening girders/region 1	30	15	40	16	20
4	Thick. bottom flange of stiffening girders/region 1	30	15	40	19	32
5	Height of stiffening girders/region 1	3.000	1.000	4.000	4.000	4.000
6	Thickness of web of stiffening girders/region 1	30	15	40	15	15
7	Width of upper flange of stiffening girders/region 2	1.000	0.500	1.500	0.500	0.542
8	Width of bot. flange of stiffening girders/region 2	1.000	0.500	1.500	1.500	1.500
9	Thick. upper flange of stiffening girders/region 2	30	15	40	15	15
10	Thick. bottom flange of stiffening girders/region 2	30	15	40	33	36
11	Height of stiffening girders/region 2	3.000	1.000	4.000	2.539	4.000
12	Thickness of web of stiffening girders/region 2	30	15	40	15	15
13	Width of upper flange of stiffening girders/region 3	1.000	0.500	1.500	1.022	0.926
14	Width of bot. flange of stiffening girders/region 3	1.000	0.500	1.500	0.973	1.500
15	Thick. upper flange of stiffening girders/region 3	20	15	40	15	15
16	Thick. bottom flange of stiffening girders/region 3	20	15	40	16	15
17	Height of stiffening girders/region 3	2.500	1.000	4.000	1.000	1.000
18	Thickness of web of stiffening girders/region 3	20	15	40	15	15
19	Width of pylons cross section below deck	3.000	2.000	5.000	3.656	2.000
20	Height of pylons cross section below deck	3.000	2.000	5.000	5.000	3.856
21	Thickness of transverse walls of pylons below deck	25	15	40	15	15
22	Thickness of longitud. walls of pylons below deck	25	15	40	17	15
23	Width of pylons cross section above deck	2.500	2.000	5.000	2.779	2.000
24	Height of pylons cross section above deck	2.500	2.000	5.000	2.000	2.000
25	Thickness of transverse walls of pylons above deck	20	15	40	15	15
26	Thickness of longitud. walls of pylons above deck	20	15	40	15	15
27	Cross-sectional area of cables 1 and 24	90	0	1000	112	98
28	Cross-sectional area of cables 2 and 23	50	0	1000	27	30
29	Cross-sectional area of cables 3 and 22	50	0	1000	25	34

30	Cross-sectional area of cables 4 and 21	50	0	1000	43	65
31	Cross-sectional area of cables 5 and 20	50	0	1000	49	45
32	Cross-sectional area of cables 6 and 19	50	0	1000	81	70
33	Cross-sectional area of cables 7 and 18	50	0	1000	43	29
34	Cross-sectional area of cables 8 and 17	50	0	1000	40	49
35	Cross-sectional area of cables 9 and 16	50	0	1000	33	37
36	Cross-sectional area of cables 10 and 15	50	0	1000	53	47
37	Cross-sectional area of cables 11 and 14	50	0	1000	27	31
38	Cross-sectional area of cables 12 and 13	70	0	1000	77	76
39	Width of transverse beams flanges	0.500	0.400	1.000	0.400	0.400
40	Height of transverse beams	0.800	0.500	2.000	1.000	1.000
41	Thickness of web and flanges of trnasverse beams	20	15	40	15	15

The Portuguese Code RSA requires checking of seismic effects under two kinds of seismic events: a strong earthquake with a large focus distance, and a moderate intensity earthquake with a near focus. The latter leads to higher spectral accelerations for the typical range of dominant frequencies of cable-stayed bridges and was adopted. Although multiple directions of excitation may be simultaneously accounted for, only longitudinal ground vibration was considered in this example.

Both optimized designs show overall similar trends in the values of design variables, confirming that the modal-spectral approach leads to extreme stresses in good agreement with maximum values obtained from a complete time discription of the dynamic response. Some localised differences may however be noticed. Deck longitudinal girders are stiffer in regions 2 and 3 in the time-history rather than in the modal-spectral approach, while the reverse trend is noticeable with respect to pylons. Back-staying effects of outer cables leads to large cross-sectional areas in both methods, the large slope producing the same effect in central stays.

Figures 3 through 5 represent initial and final stress distributions in stiffening girders, for both the static and dynamic load cases. It may be seen from them that non-seismic load cases produce higher stresses than seismic vibration, which confirms the effectiveness of floating or vertical-link of deck-to-pylon connections in deck seismic isolation. Comparison between stress diagrams in the starting design for both modal-spectral and the set of time-history instantaneous stress distributions again reveals the good agreement of both methods.

Cost reduction of about 25 % was achieved with the modal-spectral approach, while 27 % resulted from the use of time-history method. Figures 6–7 show the cost reduction/iteration curves.

In the modal-spectral approach, the frequency search domain was set in the range 0–3 Hz, 20 modes having been detected within this interval in the starting design and a few more in the optimized design, due to the structure stiffness reduction. 4 modes with the highest participation factors were accounted for in computing the pseudo-static seismic loads. Coalescence procedures needed not to be invoked, because pylons transverse symmetric and antisymmetric modes, although within the search domain, revealed neglectable participation factors for the considered direction.

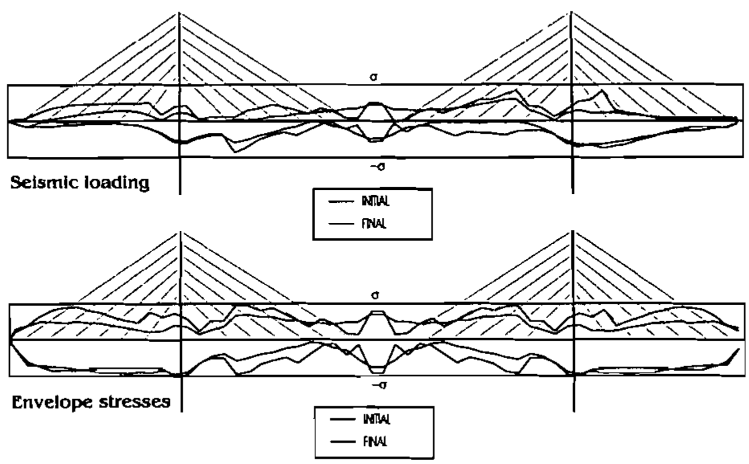

FIGURE 3 Stresses in stiffening girders - modal/spectral approach.

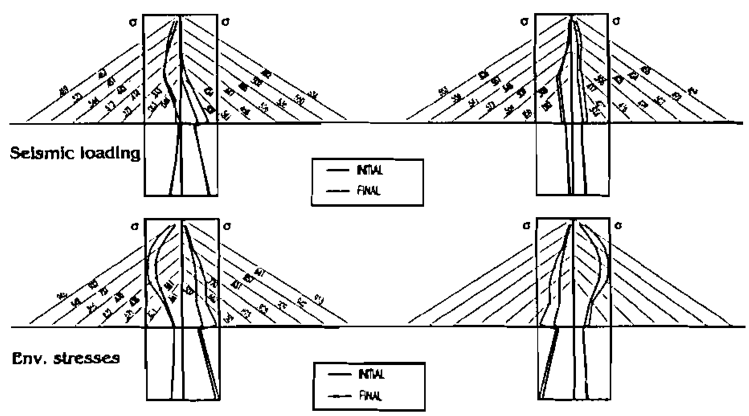

FIGURE 4 Stresses in pylons and stays - modal/spectral approach.

For the step-by-step scheme, a time-step size of 0.1 s was adopted. An artificial accelerogram, computed from the spectral density power values of RSA and the Amin and Yang [2] modulation function, was used, spanning a time gap of 14 s. Instantaneous solutions spaced every 2 seconds were retained for goal selection. These 8 complete structural descriptions involve about 10000 stresses all over the bridge, a secondary filtering process having been used for selection of the highest 5000 values.

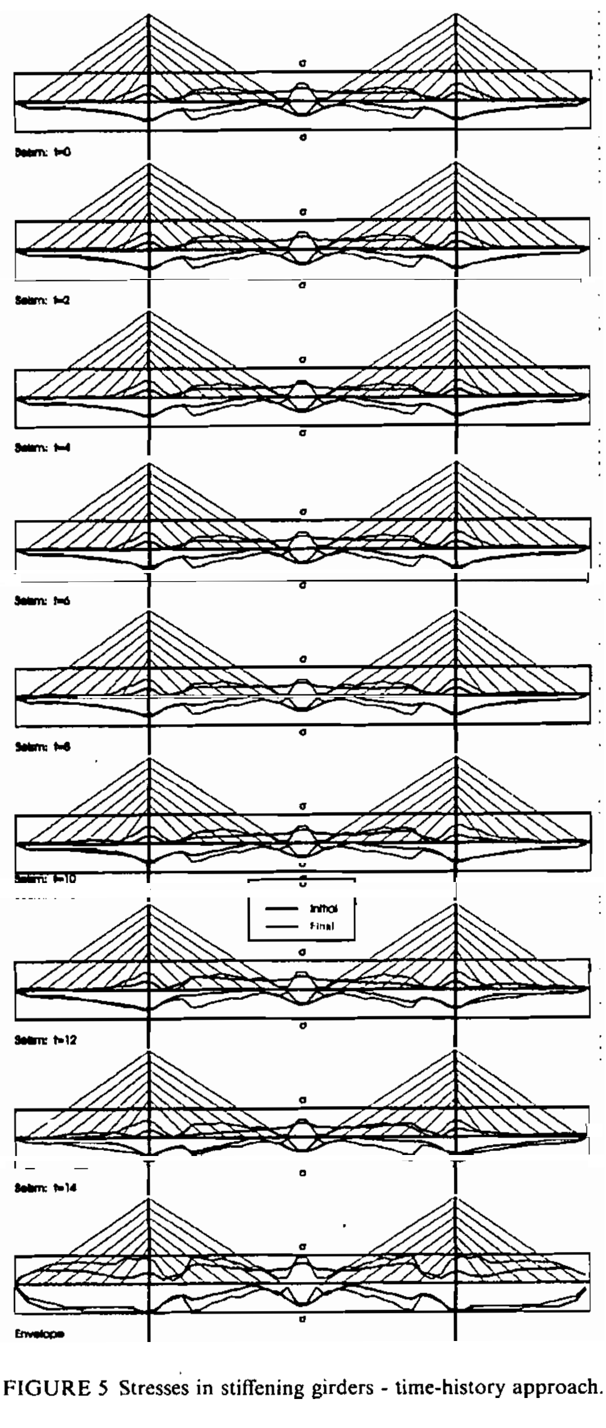

FIGURE 5 Stresses in stiffening girders - time-history approach.

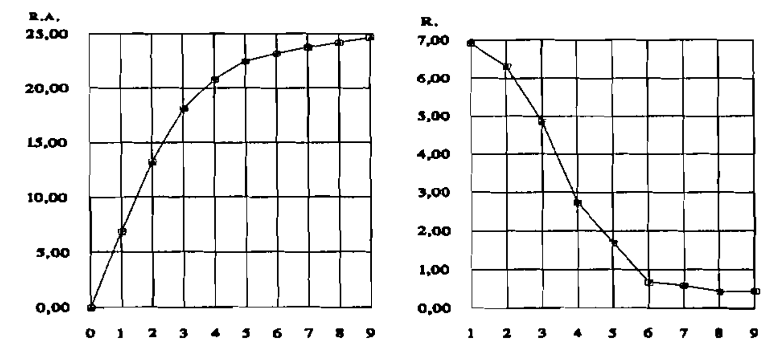

FIGURE 6 Accumulated and per iteration cost reduction - modal/spectral approach.

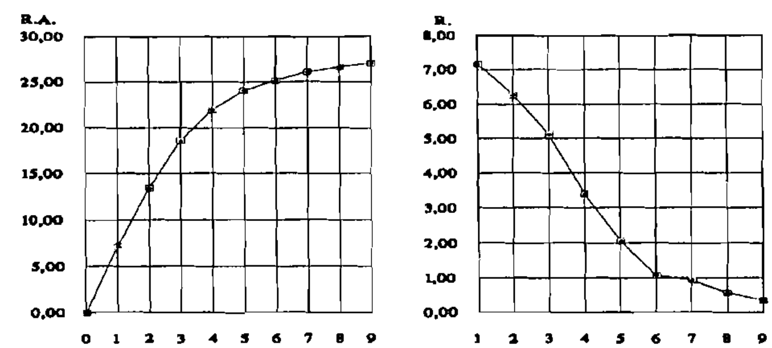

FIGURE 7 Accumulated and per iteration cost reduction - time-history approach.

The absolute computational cost of both solutions is displayed in Figure 8, where the processing time for the static problem with the same number of design variables is also represented, for purposes of comparison.

Figures 9 and 10 and show how modal-spectral and time-history approaches share the processing time among the most relevant computational tasks. It can be seen that modal-spectral method spends as much as 82% of the total time in solving the eigenvalue and sensitivity/modal analysis stages. As to the step-by-step approach, 90% of the whole elapsed time is dedicated to the solution of the dynamic equilibrium equations and their sensitivities.

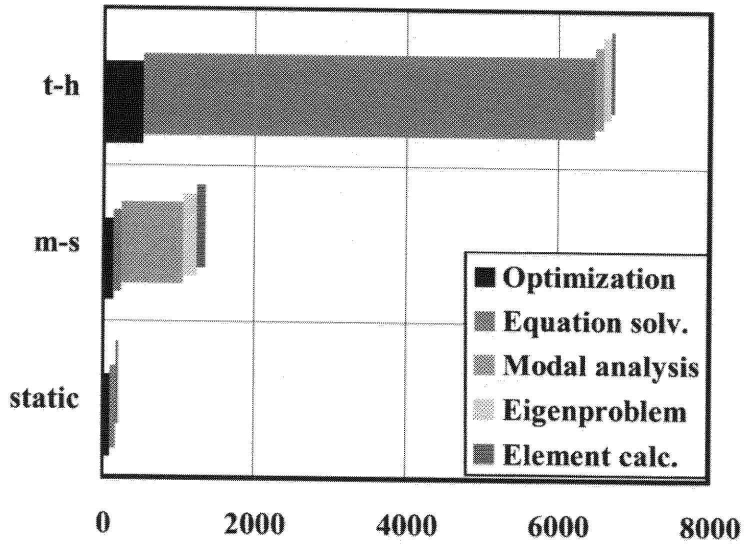

FIGURE 8 Absolute processing time (minutes).

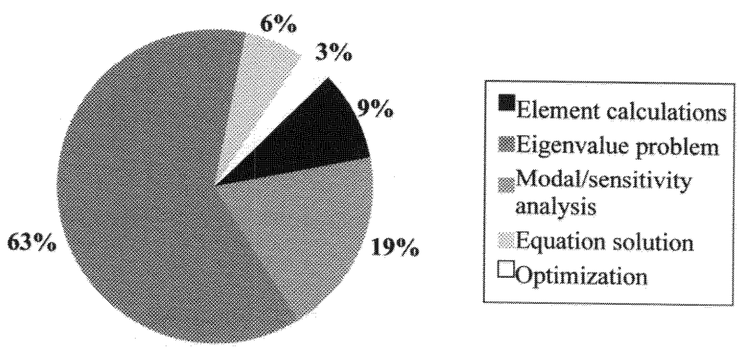

FIGURE 9 Share of time among tasks - modal/spectral approach.

CONCLUSIONS

Both modal/spectral and time-history approaches lead to suitable solutions for structural optimization of cable-stayed bridges undergoing seismic vibrations. However, each method poses specific problems related with either the code implementation or runtime

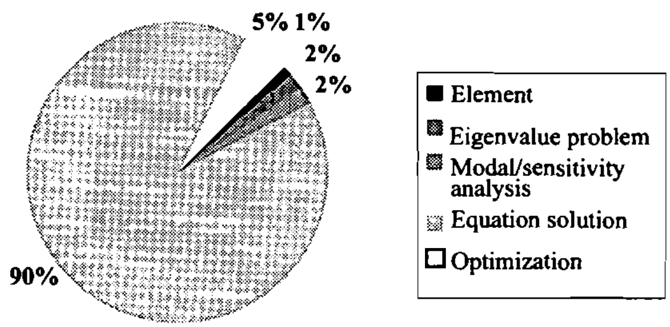

FIGURE 10 Share of time among tasks - time-history approach.

stages and their relative merit depends strongly on the problem under study. As a final balance, the following issues may be pointed out:

The modal/spectral approach may be implemented according to a black-box philosophy, allowing for a clear programme structure and an effective memory strategy. Also, the code may easily exported to other applications. A moderate volume of data is produced and transferred to the analysis/optimization core, corresponding to that of a single load case for each ground shaking direction. Although computationally expensive in relation to a similar static problem, the method is much cheaper than the time-history procedure. However, attention must be paid to accuracy problems or even to runtime crash resulting from coalescence situations. Furthermore, the basic assumptions of the method make it inadequate for strongly non-linear problems, which may be the case with large span cable-stayed bridges.

As to the time-history approach, the main disadvantage appears to be the high computational cost of solution. The issue, however, may not be critical in some instances. The solution process is stable and straightforward, no numerical disturbances being likely to occur, and that may be a factor for preference with respect to the modal/spectral approach. However, no guarantee of solution feasibility exists and the optimum design candidate must be checked with at least three different accelerograms and one may have to restart the whole process. An enormous volume of information is generated by the solution procedure of the dynamic equilibrium equations, and the use of primary and eventually secondary filtering of stresses and sensitivities is mandatory for large finite element discretizations. The method still requires the implementation of most of the code used by the modal/spectral approach, and specific code segments are deeply nested in the main analysis core, making the programme structure somewhat confusing. The adequacy of the time-history procedure for the solution of strongly non-linear problems may be the decisive argument in the case of large cable-stayed bridges. However, non-linear formulation in ordinary analysis is comparatively much more expensive than a linear approach. Inclusion of optimization, in turn, greatly increases the computational cost of the solution process, and this is particularly of concern in the case of the step-by-step method, as seen from the previous example. Therefore, the feasibility of a truly non-linear dynamic approach in the context of structural optimization must be seriously questioned.

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