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SIZING AND GEOMETRY OPTIMIZATION OF CABLE-STAYED BRIDGES

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Abstract—This paper describes a method which sets steel cable-stayed bridge design in a multi-objective optimization context with goals of minimum cost and stress. The cable anchor positions on the main girder and pylon and the cross-sectional sizes of the structural members are dealt with as design variables. A Pareto solution is found by means of an entropy-based optimization algorithm. Illustrative examples for both the fixed and variable geometry problems show the significance of dealing with cable anchor positions as design variables. The influence of the stresses arising during the erection procedure on the optimum solutions is also emphasized.

1. INTRODUCTION

Long span contemporary cable-stayed bridges are very appealing aesthetically and provide an economical solution for medium to long span bridges (150–700 m). The use of stay cables to support the main deck directly from towers distinguishes cable-stayed bridges from suspension bridges. The increasing popularity of this type of structure can be attributed to: (a) cost, (b) efficient utilization of structural materials, (c) increased stiffness over suspension bridges, (d) efficient and fast mode of construction, (e) relatively small size of the substructure, (f) low maintenance costs.

One of the great challenges to the structural engineers for the 1990s is the application of this kind of bridge to longer span lengths. There are plans in Japan for the construction of Tatara bridge, which is to span about 1500 m. The increase in span length, together with the trend to more shallow or slender stiffening girders has raised concern about their behaviour under both service and environmental dynamic loading such as traffic, wind and earthquake loading. In cable-stayed systems the tension in the steel cables produces a compression in the deck and tower. The deck behaves essentially as a beam on an elastic foundation. The elastic support for the girder is provided by the cables at their attachment points. These supports become more and more effective as the stiffness of the cables increases. Although the stays are always made with high-strength steel, there are several arrangements for the deck and pylons. In the present study steel was considered for all the structural elements.

Cable-stayed bridges are statically indeterminate and their structural behaviour is greatly affected by the cable arrangement and stiffness distribution in the cables, deck and pylons. Some authors have done parametric studies in which the structural element stiffness, anchorage positions, side-to-central span ratio, etc., were considered [1–3]. However, few attempts have been made to use optimization techniques. Some papers published in Japan by the end of the 1970s address the sizing problem either by optimality criteria or mathematical programming (sequence of linear programs). In a recent work Fleury's mixed variable method has been used to optimize the cross-section of the cables, the equivalent thicknesses of the upper and lower flanges of the pylon and deck box girders and the cable anchor positions on the main girder and pylon [4].

In this paper the cable-stayed design problem is posed in a multi-objective optimization format with goals of minimum cost and stress and a Pareto solution is sought. This problem is equivalent to a minimax formulation which is discontinuous and non-differentiable, both of which attributes make its numerical solution by direct means difficult. An entropy-based technique is used to determine the minimax solution via the minimization of a convex scalar function. Although the scalar function is unconstrained and differentiable, the stress goal functions do not have an explicit algebraic form. Explicit approximations were made by taking Taylor series expansions truncated after the linear term. The evaluation of the design sensitivities was done by the semi-analytical method.

The proposed optimum design method has been applied to a three-span steel cable-stayed bridge under various design conditions. The significance of dealing with cable anchor positions on the main girder and pylon as design variables is demonstrated through numerical results showing that considerable savings can be made. The influence of the erection sequence in the optimum solution is also emphasized.

2. PROBLEM FORMULATION

2.1. *Analysis*

The various longitudinal structural types of arrangements are: (a) radiating; (b) harp; (c) star; (d) fan. Although the first type is the most efficient from a structural point of view other arrangements are sometimes preferred for aesthetic reasons. A radiating model was considered for the present study, though the methodology described might be used with any other. A finite element package was chosen to analyse the structure.

Cable-stayed bridges in the rigorous sense behave non-linearly when loaded. Non-linearity is introduced because of: (a) the non-linear axial force elongation relationship for the inclined cable stays due to the sag caused by their own weight; (b) the non-linear axial force and bending moment deformation relationships for the towers and longitudinal girder elements under combined bending and axial forces; (c) the geometry change caused by large displacements in this type of structure under normal as well as environmental design loads; (d) non-linear constitutive stress–strain relationships for the materials of the structural elements.

Consider a cable-stayed bridge in service conditions and a set of loading cases. If the number of degrees of freedom is denoted by n, the nodal displacements and nodal loads can be represented by the n vectors $\mathbf{u}_j$ and $\mathbf{P}_j$. The problem of analysing the structure reduces to the solution of a system of non-linear equations that, when assembled for the whole structure can be represented in matrix form as

$$K\mathbf{u}_j = -\mathbf{P}_j + \mathbf{R}_j, \qquad (1)$$

where K is the stiffness matrix containing coefficients of the unknowns $\mathbf{u}_j$ and $\mathbf{R}_j$ is a column vector corresponding to the difference between this non-linear analysis and the results which would be obtained by a strictly linear analysis. The solution of these equations by a suitable iterative procedure will give values for $\mathbf{u}_j$.

Comparative analysis between the linear and non-linear behaviour for this type of structure in service conditions shows that the differences between these models can be neglected (less than 1%). Parametric studies performed by the authors and in [5] lead to identical conclusions. It is assumed that the Young's modulus considered for the stays in the analysis is the secant Ernst value corresponding to the expected stress level and stress variation range. Since the stress level in the stays for service load conditions is considerably high, this value differs very little from the instantaneous Young's modulus and therefore non-linear effects due to cause (a) may be expected to be very small. Also, reason (d) was discarded from our research, once only linear elastic material behaviour was assumed for structural (and, particularly, for stays) steel.

Although some authors focused their investigations on the two-dimensional static non-linear analysis of cable-stayed bridges, very few attempts have been made to solve the tridimensional problem. Bearing in mind the complex design conditions that must be considered, namely those corresponding to transversally asymmetric load conditions, as well as the perspectives

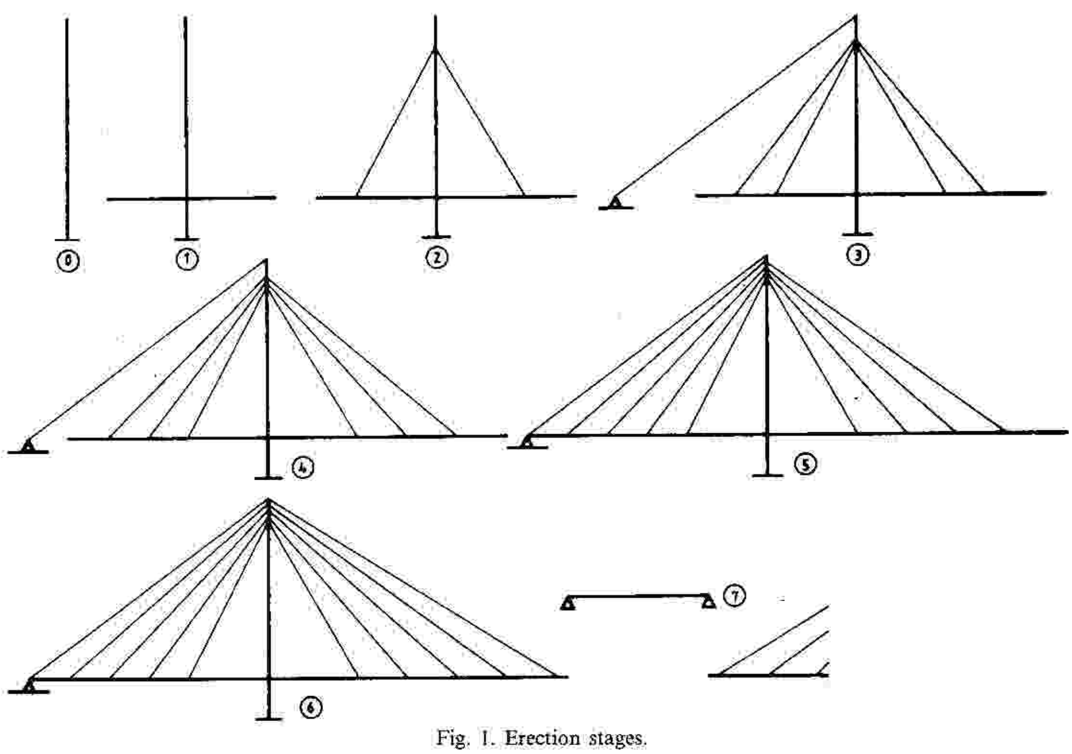

Fig. 1. Erection stages.

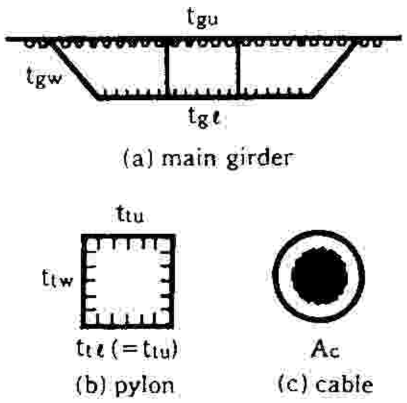

Fig. 2.

of increasing spans and complex geometries, a nonlinear tridimensional analysis is a fundamental step for a reliable design. In spite of that, the two-dimensional analysis remains a valuable tool for a simple and efficient estimation on the feasibility of an assumed model. In fact, the comparative study between both models (with static traffic loads applied on the transverse section symmetry axis) has confirmed this aspect, leading to similar results.

2.2. *Erection procedure*

The cantilever method was considered for the erection sequence. This procedure progresses by the assembly of modular units consisting of girder and deck segments with their associated cables on alternate sides of a supporting tower or pylon. Therefore it is possible to avoid large displacements and stresses in the pylons, while allowing for the clearance requirements to be met. During this process a sequence of partial structures arises until the last segment is joined to adjacent sections or anchor piers and the bridge reaches its final form. These intermediate substructures are generally more flexible than the completed structure and are subject to erection loads. Temporary supports, counterweights and transient adjustments in the cable tensions may be required to prevent over-stressing of the partial structures. Furthermore, the spatial position of the control points of the partial substructures should be checked to ensure that construction tolerances do not accumulate to the point where the bridge fails to achieve its correct geometry. In order to establish an acceptable procedure for the bridge erection it is necessary for the designer to analyse all of the partial substructures and verify their stresses and configurations. The optimization algorithm proposed in this work considers the erection stages depicted in Fig. 1, but can also be used if a different erection scheme is chosen.

3. OPTIMIZATION

3.1. *Decision variables*

The shapes of the cross-sections of the main girder and pylon are assumed to be the box types represented in Fig. 2. The span lengths, number of cables, height and width of the elements of the main girder and pylon, material types to be used for each structural element, are the preassigned constant design parameters. For the sizing optimization, the decision variables selected are:

The equivalent plate thickness of each pylon element ($t_{tu} = t_{tl}$ in Fig. 2);

The equivalent plate thickness of the upper and bottom flanges of each main girder element ($t_{gu} = t_{gl}$ in Fig. 2);

Cross-sectional area of each cable (A_c in Fig. 2).

The thickness of the flange plates are dealt with as the converted thickness which includes the contributions of the longitudinal stiffeners. To simplify notation the vector of these sizing design variables will be referred to as x in the following sections.

For the study with varying geometry the distance from the left pier to each cable anchor position in the deck y_k (Fig. 3) and the height of the lowest stay in

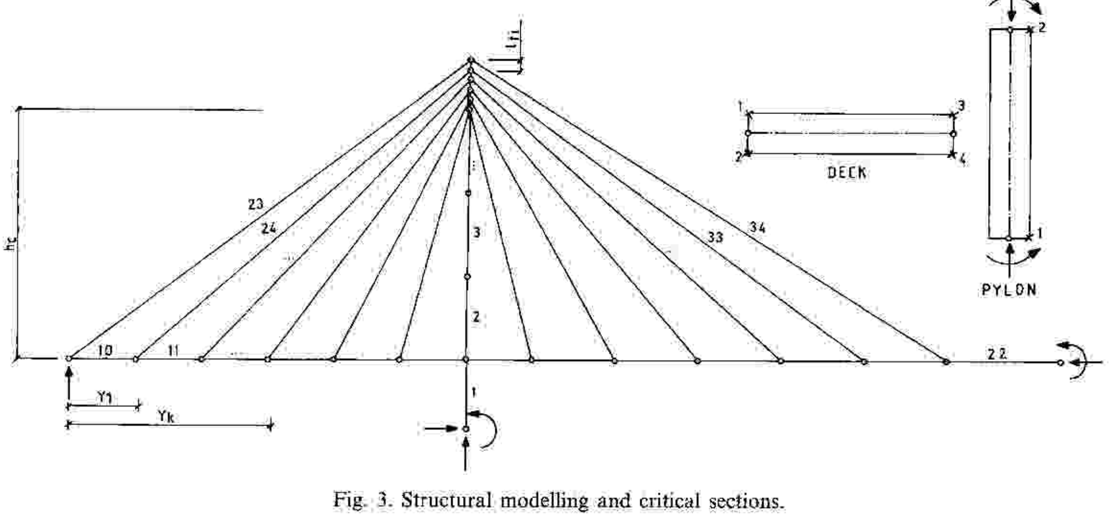

Fig. 3. Structural modelling and critical sections.

the pylon from the axis of the main girder h_c are dealt with as geometric design variables defining vector **y**. The distances of each cable in the pylon are assumed as the preassigned constant values (l_{ti}). Variable linking is used to meet symmetry requirements and reduce the number of design variables.

The assumption of a symmetrical structural model is broken by the consideration of a horizontal deck-to-pylon bond only at the left pylons. Comparative analysis between the overall structure and a half-bridge with a vertical slide at the middle span section showed, however, very little difference in the force distributions and displacements for both cases when longitudinally symmetrical load conditions are considered. This latter model was therefore chosen, leading to great computing time savings.

3.2. *Objectives*

The Pareto design of a cable-stayed bridge consists of minimizing a whole set of goals cast in a normalized form by finding an optimal set of sizing x and geometric y design variables. One of the goals is that some measure representing the cost of the structure should be as small as possible. Although the overall cost of a bridge depends on several technical, economical and social aspects some of which are difficult to quantify, the conventionally adopted value is the cost of materials used, eventually including erection costs. The measure chosen in this work is the sum of the sizing variables each weighted by the costs per unit thickness (or cross-sectional area), i.e.

$$C(x, y) = \Sigma_{i=1,n} w(y)_i x_i, \tag{2}$$

where $w(y)$ is a function of the geometric variables only for the cables. Reducing the value of the cost would be desirable. If some reference cost $\underline{C}$ is specified, this goal can be written in the form:

$$g_1(x, y) = C(X, Y)/\underline{C} - 1 \leqslant 0. \tag{3a}$$

A second set of goals arises from the imposition of lower limits on the sizing variables, namely minimum cable cross-sections and flange equivalent thicknesses to prevent topology changes

$$g_2(x) = -\frac{x_i}{x_L} + 1 \leqslant 0, \tag{3b}$$

where x_i is the ith sizing variable and x_L its lower bound. Similar bounds must be considered for the geometric design variables:

$$g_3(y) = \frac{y_k}{y_U} - 1 \leqslant 0 \tag{3c}$$

$$g_4(y) = -\frac{y_k}{y_L} + 1 \leqslant 0, \tag{3d}$$

where y_k is the kth geometric variable and y_L and y_U are its upper and lower bounds, respectively. The following set of goals is imposed to avoid the intersection of the cables:

$$g_5(y) = -\frac{y_k}{y_{k-1}} + 1 \leqslant 0. \tag{3e}$$

Further goals arise from the requirement that the stresses at each cable and elements of the main girder and pylon should be as small as possible. The working stresses result from two different sets of loading conditions: dead load in the cantilever system during erection and traffic and asphalt pavement loads acting on the deck at the service stage. The critical stresses at the upper and lower flange plates in each finite element of the deck and tower are inspected at the points shown in Fig. 3.

The following objectives represent the stresses in stays and the critical stresses in the deck and pylons, respectively:

$$g_6(x, y) = \frac{\sigma_k}{\sigma_{ku}} - 1 \leqslant 0, \tag{3f}$$

where σ_k and σ_{ku} are the tensile stresses due to the design loads and the allowable stress, respectively. The tensile and compression stresses at the main girder element are:

$$g_7(x, y) = \frac{\sigma_a}{\sigma_{au}} - 1 \leqslant 0 \tag{3g}$$

$$g_8(x, y) = -\frac{\sigma_a}{\sigma_{al}} + 1 \leqslant 0, \tag{3h}$$

where σ_a, σ_{au} and σ_{al} are the working stress and the allowable stresses in tension and compression, respectively. The goals which represent the stresses at the critical points of the pylon are:

$$g_9(x, y) = \frac{\sigma_c + \sigma_{bc}}{T} - 1 \leqslant 0, \tag{3i}$$

where T represents the allowable compression stress and σ_c, σ_{bc} are compression stresses due to the axial forces and bending moments, respectively. In general the upper and lower bound design variables and stresses are assumed to be constant. For compressed members, besides the allowable stress of the material, safety against local buckling should be included by means of an allowable stress that depends on the material elastic properties, the aspect ratio of the plate and its boundary conditions. The expression adopted here is [7]:

$$f = 7.0 \frac{\pi^2 E}{12(1 - \nu^2)(b/t^2)}, \tag{4}$$

which depends both on the sizing and geometric variables. For compressed sections, the smallest (in modulus) of the allowable compression stress σ_{al} of the material and f was chosen as the limiting value T.

3.3. *Minimax formulation*

The objective of this work is to minimize all the goals over decision variables x and y. This is achieved by the minimax optimization problem:

$$\text{Min}_{x,y}\text{Max}_j\langle g_1, g_2, \ldots, g_9\rangle. \quad (5)$$

Minimax problems are discontinuous and non-differentiable both of which attributes make its numerical solution by direct means difficult. Entropy is a natural measure of the amount of disorder (or information) in a system. A high entropy value corresponds to chaos, while a low value identifies an ordered system. In the case of optimization, entropy can be interpreted as a measure of the degree of optimality. Reference [8] explores the relationship between minimax and scalar function optimization. Specifically it is shown that a minimax problem can be solved indirectly by minimizing a continuously differentiable scalar function. The Shannon/Jaynes maximum entropy principle plays a key role in these class of problems, hence the characterization of these methods is entropy based. The proof of the validity of these assumptions is very simple requiring only the use of Cauchy's arithmetic–geometric mean inequality. In consequence, all the results can be derived entirely deterministically and without recourse to probabilistic interpretations. The minimax optimization problem for cable-stayed bridge design with goals defined by eqns. (3a)–(3i) is solved in this work by the scalar minimization:

$$\text{Min}_{x,y}(1/\rho)\ln\left\{\sum_{j=1,J}\exp[\rho g_j(x,y)]\right\}, \quad (6)$$

with a sequence of increasing positive ρ values.

3.4. *Scalar function optimization*

Problem (6) is unconstrained and differentiable which in theory gives a wide choice of possible numerical solution methods. However since the stress goal functions do not have an explicit algebraic form and are only obtained numerically from the analysis results of a particular design, the strategy adopted was to solve (6) by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking series expansions of all the goal functions truncated after the linear term. This gives:

$$\text{Min}(1/\rho)\ln\left\{\sum_{j=1,J}\exp\rho\,[g_j(\mathbf{x}_o,\mathbf{y}_o) + \sum_{i=1,n}(\partial g_j/\partial x_i)_o(x_i - x_{io}) + \sum_{k=1,m}(\partial g_j/\partial y_k)_o(y_k - y_{ko})]\right\}, \quad (7)$$

where n, m and J are the number of sizing, geometric design variables and goal functions, respectively. $(\mathbf{x}_o, \mathbf{y}_o)$ is the current vector of those design variables at which the Taylor series expansion is made. Problem (7) is an approximation of problem (6) if the values of all the $g_j(\mathbf{x}_o, \mathbf{y}_o)$ and $(\partial g_j/\partial x_i)_o$, $(\partial g_j/\partial y_k)_o$ are known numerically. Given such values, problem (7) can be solved directly by any standard unconstrained optimization method.

Solving (7) for particular numerical values of $g_j(\mathbf{x}_o, \mathbf{y}_o)$ forms only one iteration of the complete solution of problem (6). The solution vector $(\mathbf{x}_1, \mathbf{y}_1)$ of such an iteration represents a new design which must be analysed and gives new values for $g_j(\mathbf{x}_1, \mathbf{y}_1)$ and $(\partial g_j/\partial x_i)_1$, $(\partial g_j/\partial y_k)_1$ to replace those corresponding to $(\mathbf{x}_o, \mathbf{y}_o)$ in (7). Iterations continue until changes in the design variables become small. During these iterations the control parameter ρ must be increased to ensure that a minimax solution is found. In this work, an initial value between 10 and 30 has been considered, increasing to 100 in subsequent iterations.

The choice of a large control parameter ρ for a very infeasible design point may cause overflow problems. To prevent this from happening, (6) can be replaced by

$$\text{Min}_{x,y}\left\{g_M(x,y) + (1/\rho) \times \ln\left(\sum_{j=1,J}\exp\{\rho[g_j(x,y) - g_M(x,y)]\}\right)\right\}, \quad (8)$$

where $g_M(x,y)$ is the largest of the goals $g_j(x,y)$ and ρ is a positive constant (say 10^3). The scalar minimization (8) is parameter independent.

4. SENSITIVITY ANALYSIS

To formulate and solve the explicit approximation problems (7) numerical functions are required for all the goal functions and their derivatives with respect to the design variables. Assuming that the costs per unit volume are constant, the material cost of the bridge is known explicitly in terms of the sizing design variables and need not be considered further. However it is necessary to find the derivatives of the cable length with respect to the geometric variables. The derivatives of the stresses with respect to the design variables require an approximation of the member stresses σ which are implicit functions of x and y. Given some design variable values a full analysis of the cable-stayed bridge will give numerical values for all the nodal displacement under all loading cases. The first derivative of each element of u with respect to each design variable is also required to find stress derivatives and their calculation is a considerable task. One way of evaluating $\partial u_j/\partial x_i$ is to calculate them from analytical expressions as follows. The cable-stayed bridge equilibrium equation may be differentiated with respect to a particular design variable as follows:

$$\partial K/\partial x_i u_j + K\partial u_j/\partial x_i = -\partial P_j/\partial x_i + \partial R_j/\partial x_i. \quad (9a)$$

Hence

$$\partial u_j/\partial x_i = K^{-1}\{-\partial P_j/\partial x_i + \partial R_j/\partial x_i - \partial K/\partial x_i u_j\} = -K^{-1}Q_p. \quad (9b)$$

The $\partial P_j/\partial x_i$ terms that refer to the transverse applied loading are all 0. Analytical expressions for the $\partial K/\partial x_i$ and $\partial R_j/\partial x_i$ terms come from analytical

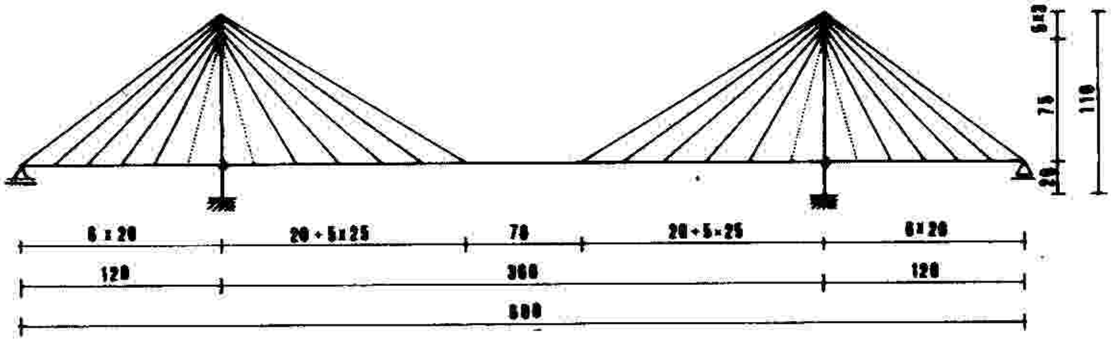

Fig. 4. Bridge geometry.

differentiation of the left hand side of the residuals and equilibrium equations, respectively. For the bridge in service conditions the $\partial R_j/\partial x_i$ terms can be neglected on the basis of the assumptions stated in Sec. 2.1. Care must be taken, particularly with the stiffness matrix derivatives, that the appropriate equations are differentiated and combined together according to the structure of the stiffness matrix itself. The evaluation of the design sensitivities by the above analytical approach is computationally efficient, but requires great care. An alternative means of calculating sensitivities which is simpler to implement is to use the semi-analytic method. It consists of the following steps:

1. Given a proper step length vector $\Delta \mathbf{x}_i = \{0; 0; \ldots; 0; \Delta \mathbf{x}_i; 0; \ldots; 0\}$, the difference approximation of pseudo-load vector Q_p in (9b) is

$$Q_p = \Sigma_e E[-K_e(x + \Delta x_i)u + K_e(x)u + P_e(x + \Delta x_i) - P_e(x)]/\Delta x_i \quad (10a)$$

where subscript e denotes the eth element and E is the set of finite elements related to the design variable x_i.

2. Compute $\partial u/\partial x_i$ from

$$\partial u/\partial x_i = K - 1Q_p. \quad (10b)$$

3. Estimate the first order approximation of the displacement vector at the design point

$$u(x + \Delta x_i) \cong u(x) + \partial u/\partial x_i \cdot \Delta x_i \quad (10c)$$

4. Obtain the sensitivities of the responses $R(x, u)$ by local differences:

$$\partial R/\partial x_i \cong [R(x + \Delta x_i, u + \Delta u) - R(x, u)]/\Delta x_i. \quad (10d)$$

The procedure to compute the sensitivities of the responses with respect to the y decision variables is similar; however it should be remembered that the elements of the stress matrix are functions of the geometric design variables.

5. APPLICATIONS

Several steel cable-bridge optimization problems have been solved by the proposed method. In this section the numerical results of the three-span bridge shown in Figs 4–6 are presented and discussed. The design constants relating to the loading conditions and mechanical properties of the structural steels and described in Tables 1 and 2 are similar to those used in [4]. Lower limits (0.1 cm^2) were set for the cable cross-sectional area.

Three different design problems were studied, two with fixed geometry (with and without erection stresses) and one with variable geometry.

For the examples where sizing are the only design variables, the symmetrical design model used implies that the number of independent variables and goals are 47 and 83, respectively. The x design variables are: 2 × 13 upper and lower plates thicknesses for

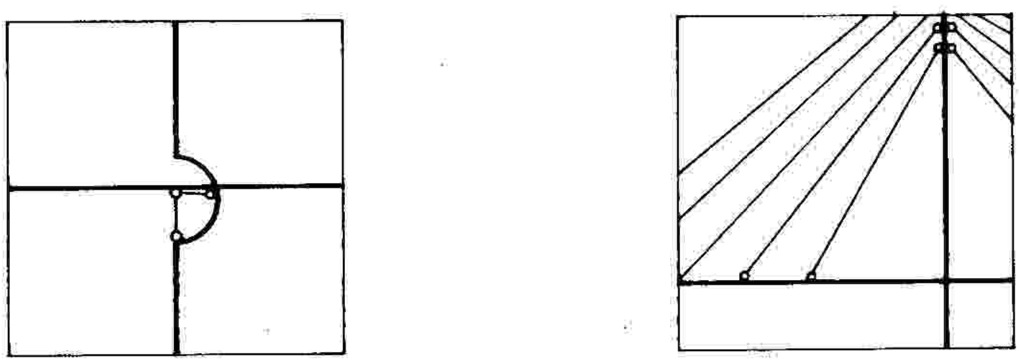

Fig. 5. Connections.

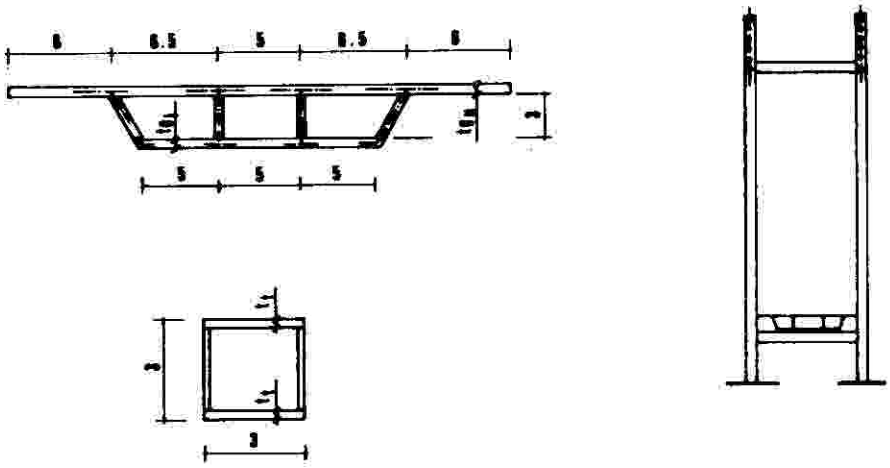

Fig. 6. Cross-sectional geometry.

13 deck elements; 9 plate thicknesses for 9 pylon elements; 12 cross-sectional areas for 12 pairs of stays. The objectives are the cost function and the stresses at the critical points: the compression stress at each end of the pylon elements (18 stresses); the stresses at both flanges of each deck element (52 stresses); the stresses at the stays (12 stresses).

An initial structural analysis shows that the lowest cables would be subjected to a compression stress. Therefore symbolic cross-section areas (0.1 cm^2) were assigned to these stays to simulate their very small contribution. Although this procedure allows for their eventual re-introduction in further iterations, this situation never happened. Hence, the number of significant stresses in the cables is 10.

For the variable geometry example and neglecting the lowest cables in the side and centre span, 10 geometric decision variables (y) were considered on top of the 47 sizing variables referred to above. They correspond to cable anchor locations in the main girder and the height of the lowest cable in the pylon. The distances between successive cables in the pylon were set as 3.0 m.

The optimization algorithm works both for feasible and infeasible designs. Its convergence rate improves when the initial starting point is close to the boundary, satisfying and eventually violating the goals by small amounts. The initial prescribed values were:

$$t_{gu}, t_{gl} = 30 \text{ mm, (deck)}$$

$$t_{tu}, t_{tl} = 30 \text{ mm, (pylons)}$$

$$A_c = 600 \text{ cm}^2\text{, (pair of cables).}$$

To investigate the significance of dealing with sizing design variables without erection stresses, erection stresses and sizing and geometric design variables, the optimum solutions for these three cases are summarized in Tables 3–6. Figure 7 shows the cable arrangement for the variable geometry case. The maximum and minimum bending moments and axial force distributions, and upper and lower flange

Table 1. Loadings

Erection stage	Uniformly distributed dead load	Deck†	80 kN/m
		Pylons‡	20 kN/m
		Structural steel	77 kN/m^3
	Concentrated load	Deck	100 kN
Service stage	Uniformly distributed loads on deck	Dead load†,§	68 kN/m
		Live load†	45 kN/m

†Total width of the deck.
‡Acting vertically.
§Cumulative with the erection time loads.

Table 2. Structural steels and cost factors

Region	Steel class	E (GPa)	σ (MPa)	Cost factor
Deck	SS41	210	137	500
Pylons (below cable anch. pos.)	SM50	210	181	700
Pylons (above cable anch. pos.)	SM58	210	255	700
Cables	Prestress, steel	200	510	900

Table 3. Cross-sectional areas of the cables

Fixed geometry		Variable geometry	
Stay	A_c (cm^2)	Stay	A_c (cm^2)
1	367.8	1	333.8
2	93.6	2	118.6
3	40.3	3	80.0
4	177.1	4	177.0
5	60.5	5	177.0
6	0.1	6	0.1
7	0.1	7	0.1
8	90.6	8	151.0
9	173.4	9	141.3
10	96.0	10	135.2
11	117.4	11	277.2
12	353.8	12	260.6

Table 4. Upper and lower flange thicknesses of the main girder elements at the optimum

F. Elem.	T_{gu}–T_{gl} (mm) (fixed)	T_{gu}–T_{gl} (mm) (variab.)
1	18.0–15.0	18.0–15.0
2	18.0–15.0	18.0–15.0
3	18.0–15.0	18.0–15.0
4	18.0–15.0	18.0–15.0
5	18.0–15.0	18.0–15.0
6	18.0–46.9	18.0–44.6
7	18.0–47.4	18.0–44.8
8	18.0–15.0	18.0–15.0
9	18.0–15.0	18.0–15.0
10	18.0–15.0	18.0–15.0
11	18.0–15.0	18.0–15.0
12	18.0–15.0	18.0–15.0
13	18.0–21.8	18.0–25.4

thickness distributions at the optimum solutions are depicted in Figs 8 and 9.

Optimum solutions are obtained after 9 and 5 iterations for the fixed and variable geometry cases, respectively (Fig. 10). The optimum height of the lowest cable in the pylon from the axis of the main girder is 64.4 m.

The example with variable geometry gives a solution 4% more economical than when sizing are the only design variables to be considered. The first and third examples (sizing variables only without erection stresses, variable geometry) allows a direct comparison with the results presented in [4]. The algorithm developed in this work leads to optimum solutions 15% and 10% cheaper, respectively.

The two cables in this case are nearly parallel and anchored at the end support at the optimum. The sectional area of the top cable is 2–6 times larger than those of the middle cables. The iterative process shows that stays 6 and 7 are unnecessary, as their cross-sectional areas can be reduced to the lower limit.

Table 5. Flange thicknesses of the pylon positions (variable geometry)

F. Elem.	T_t (mm) (fixed)	T_t (mm) (variable)
1	21.6	21.4
2	15.4	12.6
3	12.0	12.6
4	12.2	12.6
5	26.2	15.0
6	23.2	12.6
7	15.2	21.6
8	12.0	12.6
9	12.0	12.6

Table 6. Flange thicknesses of the cable anchor positions (variable geometry)

Cable	Y initial (m)	Y optimum (m)
1	0.00	0.00
2	20.00	4.30
3	40.00	22.50
4	60.00	56.20
5	80.00	73.60
6	100.00†	100.00†
7	140.00†	140.00†
8	165.00	165.90
9	190.00	179.40
10	215.00	211.10
11	240.00	229.20
12	265.00	281.50

†Unnecessary cables.

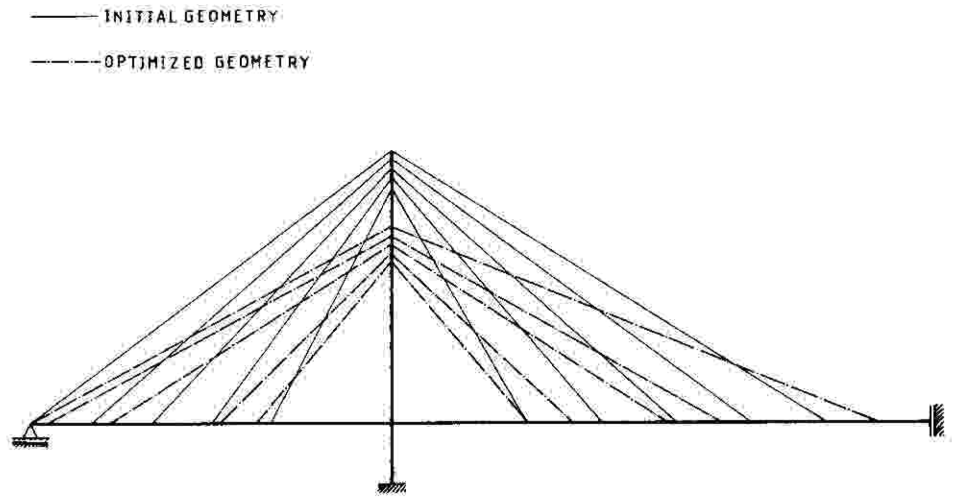

Fig. 7. Initial and final geometries.

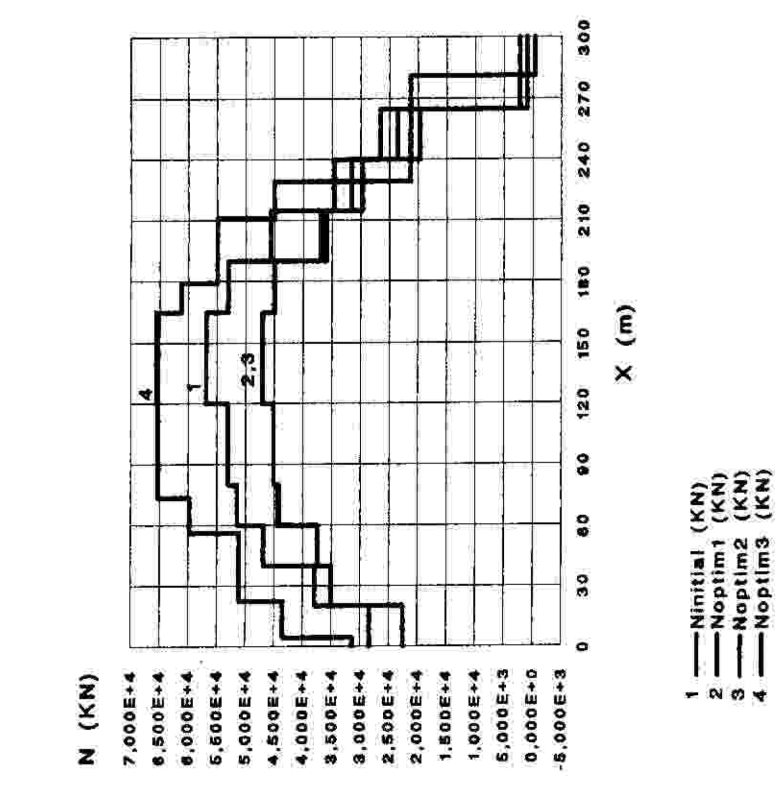
N (KN)
7,000E+4
6,500E+4
6,000E+4
5,500E+4
5,000E+4
4,500E+4
4,000E+4
3,500E+4
3,000E+4
2,500E+4
2,000E+4
1,500E+4
1,000E+4
5,000E+3
0,000E+0
-5,000E+3
0
30
60
90
120
150
180
210
240
270
300
X (m)
4
1
2,3
1 Ninitial (KN)
2 Noptim1 (KN)
3 Noptim2 (KN)
4 Noptim3 (KN)

Fig. 8(a).

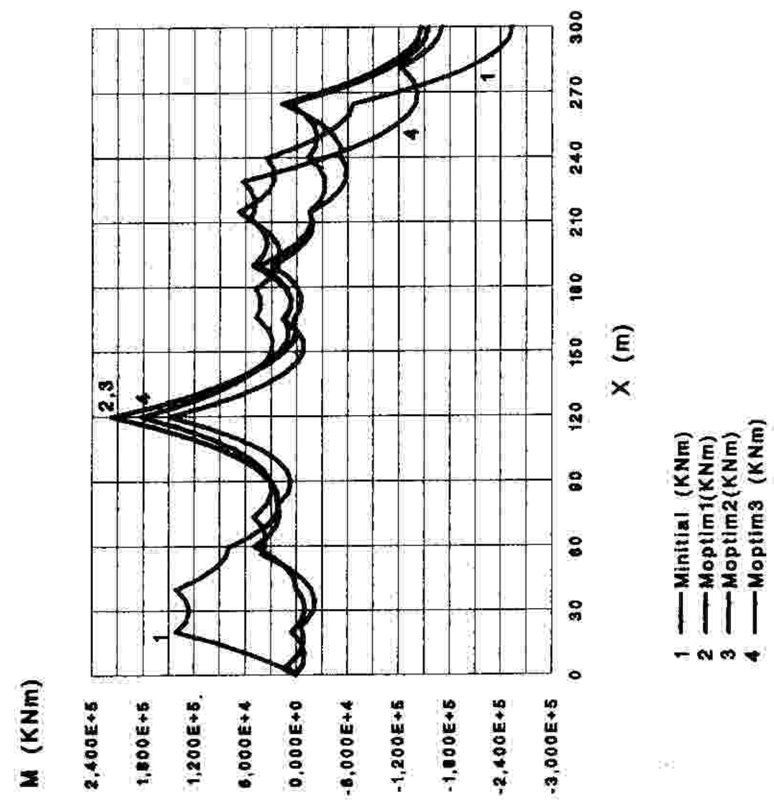
M (KNm)
2,400E+5
1,800E+5
1,200E+5
6,000E+4
0,000E+0
-6,000E+4
-1,200E+5
-1,800E+5
-2,400E+5
-3,000E+5
0
30
60
90
120
150
180
210
240
270
300
X (m)
1
2,3
4
4
1
1 Minitial (KNm)
2 Moptim1(KNm)
3 Moptim2(KNm)
4 Moptim3 (KNm)

Fig. 8(b).

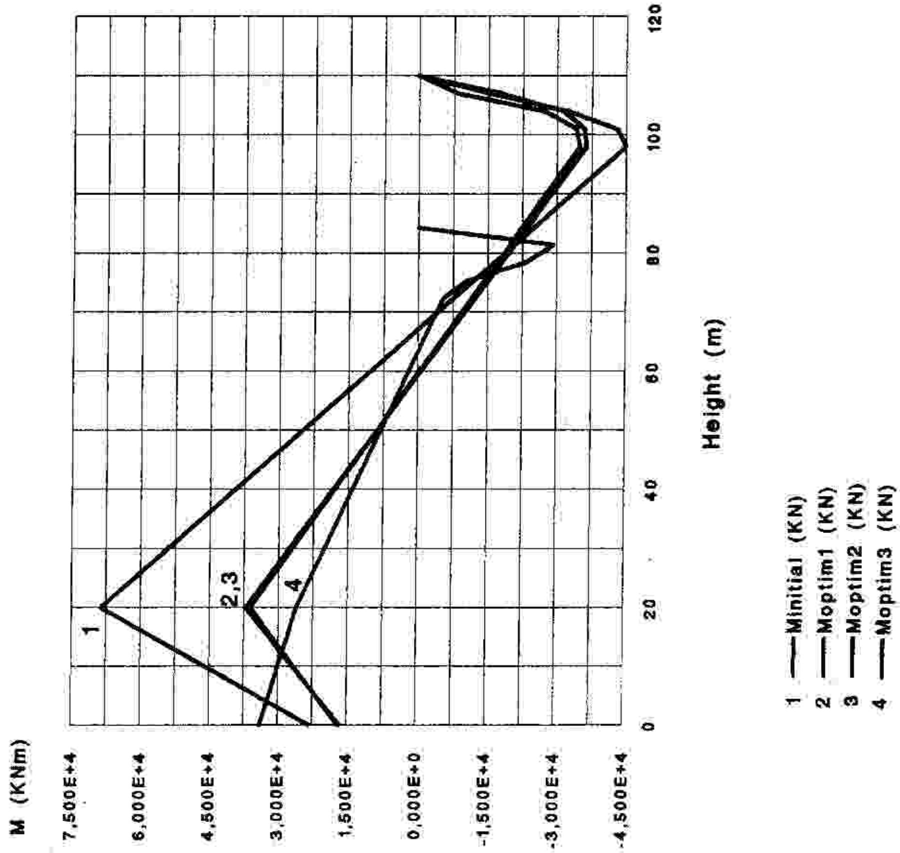

Fig. 8(d).

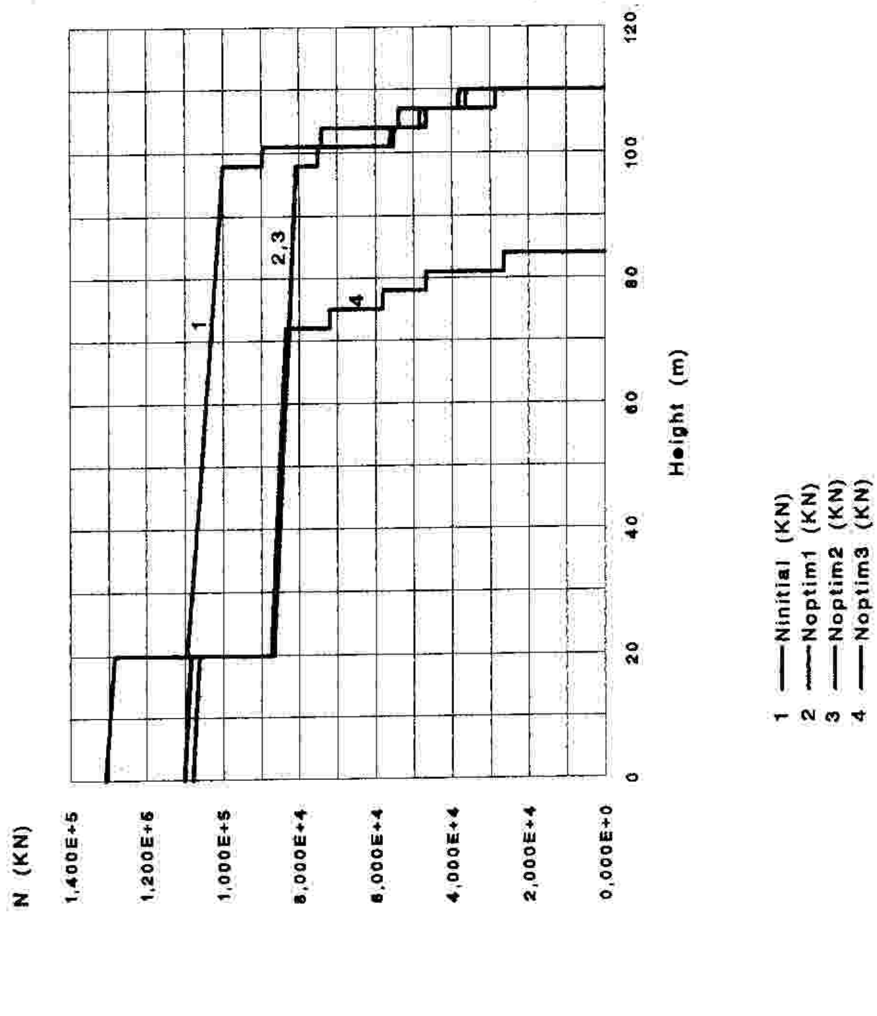

Fig. 8(c).

Fig. 8(a–d). Bending moment and axial force diagrams

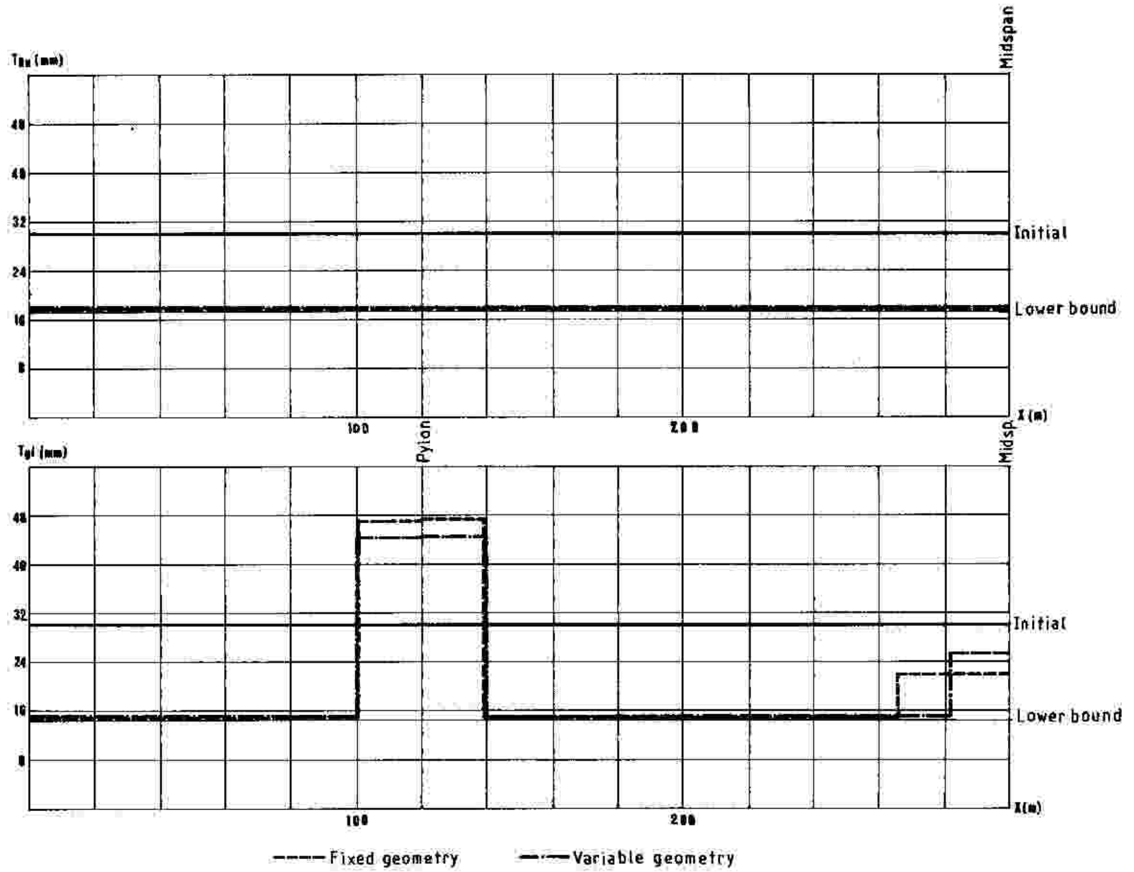

Fig. 9. Flange thickness distributions.

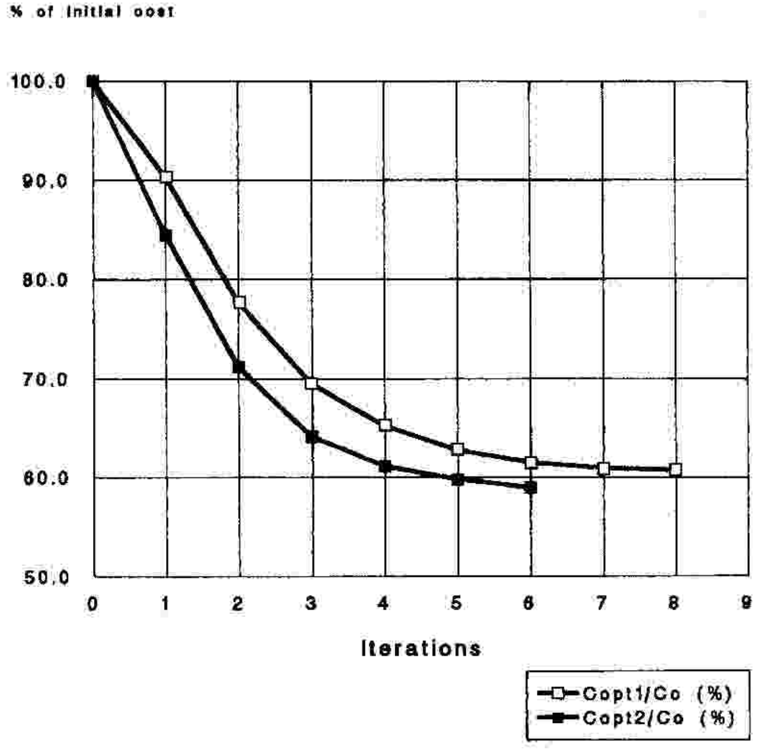

Fig. 10. Iteration history.

Table 7. Absolute and relative cost gains at the optimum

		Fixed geometry		Variable geometry	
Region	Initial volume	Opt. volume	Δ%	Opt. volume	Δ%
Deck	901.5	592.9	−34.2	591.9	−34.3
Pylons	229.4	113.7	−50.4	86.1	−62.5
Cables	147.6	84.0	−43.1	87.0	−41.1
Global	1278.5	790.6	−38.2	765.0	−40.2
Cost	744,150	451,600	−39.3	434,500	−41.6

All the optimized solutions show similar bending moment distributions on the deck. The maximum positive and negative bending moments are of the same order of magnitude. Critical compression stresses occur near the pylon as a result of the combined action of maximum negative bending moments and axial forces. However, at the mid-span section where the maximum positive bending moment occurs and the axial force is close to zero, bending is the most critical stress. Hence the optimum thicknesses are much greater at the deck elements near the pylon than at the mid-span. The large maximum bending moments acting at the centre point of the deck can be sustained even with the upper and lower flange thicknesses at their lower limits. On most of the other deck elements lower bounds prevailed for the flange thicknesses.

In the variable geometry example the optimum cable arrangement determined tends to reduce the critical load peaks of the maximum and minimum bending moments at the main girder and pylon. It is seen from Fig. 8 local peaks of the minimum bending moments at the middle support of the girder are smaller than in the fixed geometry case. In the optimum solutions the horizontal components of the tension caused by the dead load in the left and right cables of each set in the pylon are well balanced and the magnitude of the maximum and minimum bending moments are reduced substantially when compared with the initial design.

The percental variation in the volume was slightly less than that of the cost, which highlights preferential incidence of the optimization algorithm on those components with higher unit costs (cables and pylons). Table 7 shows these values.

6. CONCLUSIONS

This paper has shown that the optimization of cable-stayed bridges can be done efficiently by the proposed entropy-based algorithm. The optimization process has three major elements: analysis, sensitivity analysis and optimization. Because the non-linear analysis can only be done numerically, it is not possible to obtain closed form algebraic expressions for most of the functions needed in the optimization model. Consequently the optimization process must be based upon approximate models which employ numerical functions and first derivative values calculated for the particular designs. The non-linear and iterative nature of the analysis procedure mean that the calculation of the necessary sensitivity information must inevitably be expensive in programming and effort and computer time by whatever means it is done.

The minimax formulation adopted here allows the simultaneous optimization and control of many different engineering goals. Pareto solutions can be obtained efficiently for the multi-objective problem through the minimization of a non-linear convex function.

The examples solved in the course of this work provided considerable insight into the behaviour of cable-stayed bridges. The total cost of the steel cable bridge is affected by the cable anchor positions on the main girder and pylon. Stresses due to the erection sequence change the optimum solution and must also be considered. Hence the treatment of the cable anchor positions and the height of the pylon as design variables and a consideration of the stresses arising during construction are extremely significant.

Finally this work has only touched the surface of optimizing the design of cable-stayed bridges, nevertheless it has shown there are potential savings to be made through the use of optimization. This algorithm can also be used with more complex structural modelling, namely the formulation of more complex constitutive laws (concrete stress–strain relationship, creep, etc.) as well different types of loading (dynamic and/or thermal effects, etc.).

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