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Reliability of portal frames with discrete design variables*

L.M. da Cruz Simões

Dep. Engenharia Civil, Universidade de Coimbra, P-3049 Coimbra, Portugal

Abstract Virtually all structural optimization based on system reliability was conducted considering continuous design variables. The solution of the reliability-based design problem is obtained by solving alternatively a reliability assessment problem and an optimal sizing program until the best reliability-based design occurs. The reliability assessment problem is formulated as a linearly constrained concave quadratic program. By introducing the concept of segmental members, the discrete optimum design is achieved based upon linear programming. Examples are solved by employing the proposed computational technique.

1 Introduction

Reliability-based structural optimization has attracted intensive attention of researchers in the past two decades and considerable results have been produced. It has been more widely accepted that structural reliability should be taken into account in such decision-making processes as structural optimization. While component reliability is still considered in the design process, system reliability has been emphasized more and included in structural systems optimization. This includes trusses of uniaxial components and frames of bending components. These studies have overwhelmingly dealt with structural optimization involving continuous design variables. It is however of practical importance to consider discrete design variables. The major purpose of this paper is to describe a first-order second-moment reliability-based approach to the discrete optimum design of ductile frames. This design has a preassigned reliability level against plastic collapse and simultaneously minimizes the prescribed objective function. Most of the work on plastic reliability analysis is based upon the upper bound theorem of plasticity. According to this an upper bound on the system's reliability can be evaluated on the basis of a set of collapse mechanisms. The solution of the reliability-based design problem is obtained by solving alternatively a reliability assessment problem and an optimal sizing program until the best reliability-based design occurs. The procedure for identifying stochastically most relevant failure mechanisms consists of the minimization of a quadratic concave function over a linear domain. The discrete optimal sizing problem is combinatorial in nature and significantly more difficult than the continuous problem. This paper describes a method for discrete optimal design based upon the concept of segmental members. The segmental optimum design is found by linear programming and its volume is a close lower bound to the discrete solution. A simple method for achieving a discrete design from the segmental solution is described. Frame examples are given highlighting significant differences between discrete and continuous optimum solutions both for the cases where yield is governed by a single bending moment and when the interaction of bending moment and axial force prevails.

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2 Reliability-based design

2.1 *Problem formulation*

The process of selection of the optimum solution is highly complex involving both qualitative and quantitative factors that must be considered simultaneously. There are diverging opinions on many basic issues from the very definition of reliability-based optimization including the definition of the objective function and the constraints to its application in structural design practice. The reliability-based optimization problem here consists of member size selection for given detailing arrangements and specified probabilities of failure against collapse. If the plastic capacities are proportional to the volume of material required,

$$\min V = \mathbf{f}(\mathbf{x}), \tag{1a}$$

subject to,

$$\mathbf{g}_1(\mathbf{x}, \mathbf{x}^*, \mathbf{l}, \delta, \delta_*, \theta_*, \mathbf{u}*) = 0, \tag{1b}$$

$$\mathbf{g}_2(\mathbf{x}, \mathbf{x}^*, \mathbf{l}, \delta, \delta_*, \theta_*, \mathbf{u}*) \leq 0, \tag{1c}$$

$$\mathbf{g}_3(\mathbf{x}) \leq 0, \tag{1d}$$

$$P_r[Z_k \leq 0] \leq P_{fs} \quad k = 1, \ldots, m, \tag{1e}$$

$$P_r[\cup_{k=1,m} Z_k \leq 0] \leq P_{fo}, \tag{1f}$$

where $\mathbf{x}, \mathbf{x}_*, \mathbf{l}, \mathbf{u} * \delta, \theta_*, \delta_*$ are the vectors of discrete design variables, random plastic capacities, random loads, stress resultant rates, nodal displacements, total critical section rotations and total nodal displacements, respectively. The objective function and the constraints (1b)-(1d) are linear. The above formulation is different from plastic limit synthesis problems, owing to single mode failure probability constraints (1e) and the system failure probability constraint (1f). An alternative approach minimizes the probability of collapse or unserviceability for fixed volume of material, which give Pareto solutions to the general multi-objective reliability-based optimization.

2.2 *Collapse of ductile structures*

The probability of failure via the k-th individual collapse mode p_k can be obtained from the probability that a certain performance function Z_k

$$Z_k = U_k - E_k = \mathbf{x}^t \theta_* - \mathbf{l}^t \delta_* \tag{2}$$

is negative. In (2) U_k and E_k are the internal and external random works associated with the k-th collapse mode. The strength of the structure with respect to the k-th mode, U_k, is represented by a linear combination of random plastic capacities $\mathbf{x}$, and the load effect with respect to this mode, E_k, is represented by a linear combination of random loads $\mathbf{l}$. The probability of occurrence of the k-th mode $P(F_k) = P(Z_k \leq 0)$ can be evaluated exactly if the distribution functions of the plastic capacities are available, and if the statistical correlation structure of the plastic moment capacities $\mathbf{C_x}$ and applied loads $\mathbf{C_l}$ is known. In practice the distribution functions of $\mathbf{x}$ and $\mathbf{l}$ are not known, and usually only the first two statistical moments (means $\mu_{\mathbf{x}}$, $\mu_{\mathbf{l}}$ and a measure of uncertainty: coefficients of variation $\Omega_{\mathbf{x}}, \Omega_{\mathbf{l}}$ or variances $\sigma^2_{\mathbf{x}}, \sigma^2_{\mathbf{l}}$) are available. The correlation coefficients between pairs of plastic capacities $\rho(x_i, x_j)$ and between pairs of loads $\rho(\ell_\ell, \ell_m)$ can usually only be estimated by engineering judgment. Consistent with a first-order second-moment reliability analysis, the failure probability may be measured entirely with a function of the first and second moments of random parameters. It is assumed that safety with regard to plastic collapse via the failure mode k depends only on reliability index β_k, that is defined as the shortest distance from the origin to a failure surface in the reduced random variables coordinate system,

$$\beta_k = \mu_{Z_k}/\sigma_{Z_k}\,. \tag{3}$$

It is important to note that the standard deviation of the safety margin of an individual collapse mode, σ_{Z_k}, and the probability of occurrence of this mode, $P(F_k)$, increases as the statistical positive dependence between plastic moments and/or between loads that are active in producing the mechanism increases.

2.3 Overall probability of failure

In general, the admissible failure probability for structural design is very low. Hence, approximation by Cornell's first-order upper bound is a conservative estimate of the overall probability of failure,

$$\max_{\text{all}\,k}[P(F_k)] \leq P_{fo} \leq 1 - \Pi_{k=1,m}[1 - P(F_k)]\,. \tag{4}$$

The lower bound, which represents the probability of occurrence of the most critical mode (dominant mode) is obtained by assuming the mode failure events F_k to be perfectly dependent, and the upper bound is derived by assuming independence between mode failure events. Narrower bounds can be obtained by taking into account the probabilities of joint failure events such as $P(F_i \cap F_j)$ which means the probability that both events F_i and F_j will simultaneously occur. The resulting closed-form solutions for the lower and upper bounds are as follows:

$$P_f \geq P(F_1) + \sum_{i=2}^{m} \max\left\{\left[P(F_i) - \sum_{j=1}^{i-1} P(F_i \cap F_j)\right]; 0\right\}, \tag{5}$$

$$P_f \leq \sum_{i=1}^{m} P(F_i) - \sum_{i=2}^{m} \max_{j<i} P(F_i \cap F_j)\,. \tag{6}$$

The above bounds can be further approximated using Ditlevsen's method of conditional bounding (Ditlevsen 1979). This is accomplished by using a Gaussian distribution space in which it is always possible to determine three numbers β_i, β_j and ρ_{ij} for each pair of collapse modes F_i and F_j such that if $\rho_{ij} > 0$ (i.e. if F_i and F_j are positively dependent)

$$P(F_i \cap F_j) \geq \max\left\{\Phi(-\beta_j)\Phi\left(-\frac{\beta_i - \beta_j\rho_{ij}}{\sqrt{1-\rho_{ij}^2}}\right), \Phi(-\beta_i)\Phi\left(-\frac{\beta_j - \beta_i\rho_{ij}}{\sqrt{1-\rho_{ij}^2}}\right)\right\}, \tag{7}$$

$$P(F_i \cap F_j) \leq \Phi(-\beta_j).\Phi\left(-\frac{\beta_i - \beta_j\rho_{ij}}{\sqrt{1-\rho_{ij}^2}}\right) + \Phi(-\beta_i).\Phi\left(-\frac{\beta_j - \beta_i\rho_{ij}}{\sqrt{1-\rho_{ij}^2}}\right), \tag{8}$$

in which β_i and β_j are the safety indices of the i-th and the j-th failure mode, ρ_{ij} is the correlation coefficient between the i-th and the j-th failure mode, and $\Phi()$ is the standardized normal probability distribution function.

The probabilities of the joint events $P(F_i \cap F_j)$ in (5) and (6) are then approximated with the appropriate sides of (7) and (8). For example, if F_i and F_j are positively dependent, for the lower bounds (5) it is necessary to use the approximation given by the upper bound (7).

By rewriting (5) and (6), Moses and Kinser (1976) expressed the overall probability of collapse of a system in the following way:

$$P_f = P(F_1) + \sum_{i=2}^{m} a_i P(F_i)\,, \tag{9}$$

where

$$a_i = P(S_1 \cap S_2 \cap \ldots \cap S_{i-1} \mid F_i) \tag{10}$$

is the conditional probability that the first $i-1$ modes survive given that mode i occurs. Note that the failure modes are arranged so that $P(F_1) \geq P(F_2) \geq \ldots \geq P(F_i) \geq \ldots \geq P(F_m)$ because the value of the conditional probability (9) depends on the ordering of failure modes.

The method suggested by Vanmarcke (1973) reduces the number of survival events to one, such that

$$a_i \leq \min_{j=1}^{i-1} P(S_j \mid F_i) = a_i^*\,. \tag{11}$$

Therefore, $a_1^* = a_1 = 1, a_2^* = a_2$ and

$$P_f = P(F_1) + \sum_{i=2}^{m} a_i P(F_i) \tag{12}$$

is an upper bound to the overall probability of collapse. Using a first-order approach, Vanmarcke introduced a useful approximation of the conditional probability $P(S_j \mid F_i)$ in terms of the safety indices β_i and β_j and of the coefficient of correlation ρ_{ij} between the failure modes F_i and F_j as follows:

$$P(S_j \mid F_i) \cong 1 - \frac{\Phi[-\max(\beta_j/\mid\rho_{ij}\mid, \beta_i)]}{\Phi[-\beta_i]}\,, \tag{13}$$

in which it is assumed that the probability of occurrence of the i-th mode $P(F_i) = \Phi(-\beta_i)$ depends on β_i only.

2.4 Solution method

The system's reliability involves the enumeration of the stochastically most important mechanisms for a given design. Therefore the reliability constraints are not known explicitly and the reliability-based design problem is a bi-level program. The solution method for this class of problems is divided in two alternation sub-procedures which are repeated until the vector of design variables converges: (a) an optimization procedure for assessing the structural reliability (inner problem), that finds the stochastic most important mechanism and enumerates other relevant collapse modes for a given value of the design variables (plastic capacities); (b) an optimization of the discrete optimal design problem (outer problem), that finds the vector of average plastic capacities giving the least volume solution that satisfies failure requirements.

3 Inner problem: reliability assessment

3.1 Assumptions

The following assumptions are considered. (1) The general structural configuration including the lengths of all prismatic and straight members is specified in a fixed (deterministic) manner. (2) Plastic collapse is the only possible failure mode. (3) The effects of shear and torsion are not considered. (4) The ultimate plastic capacities for both beam and column critical sections, which form the vector **x** are random but their position is deterministic. (5) Reliability analysis with random variable loading has meaning only for one load. When more than one load exists, a load combination problem is invoked to produce an equivalent single load effect. (6) The magnitudes of static loads which form the load vector **l** are random but their locations are deterministic.

3.2 Computation of the reliability index

For Gaussian random variables, the identification of the stochastic most relevant mechanism consists of minimizing the reliability index β given by (Shinozuka 1983)

$$\beta = \frac{[\mu_x^t \theta * - \mu_l^t \delta *]}{\sqrt{\theta *^t \sigma_x^t C_x \sigma_x \theta * + \delta *^t \sigma_l^t C_l \sigma_l \delta *}}, \tag{14}$$

subject to the compatibility relations

$$B^t N u* = 0, \tag{15}$$

where B represents the mesh matrix, $u*$ the strain resultant rates and N is a matrix containing the normals of the yield polytope. The linear incidence equations are

$$\theta * = J_\theta u*; \quad \delta * = J_\delta \delta, \tag{16}$$

where the incidence matrices J_θ and J_δ are obtained by associating the strain-resultant rates $u*$ represented by the same random variables $\theta*$ and the displacements of the point loads δ linked by the same random variables $\delta*$. Sign constraints on the variables need also to be considered

$$u_* \geq 0, \quad \delta * \geq 0, \tag{17}$$

and the displacement rates δ that correspond to the loads L are evaluated in terms of $u*$

$$\delta = B_o^t N u *. \tag{18}$$

If the probability distribution functions of the random variables are not Gaussian, the Rosenblatt transformation may be used.

This mathematical program belongs to the class of fractional programming problems. The minimization of β shares its solutions with the quadratic concave minimization (Simões 1990)

$$\max -1/\beta^2 = -\theta *^t \sigma_x^t C_x \sigma_x \theta * - \delta *^t \sigma_l^t C_l \sigma_l \delta *, \tag{19a}$$

subject to

$$\mu_x^t \theta * - \mu_l^t \delta * = 1, \tag{19b}$$

$$\theta * = J_\theta u*; \quad \delta * = J_\delta \delta, \tag{19c}$$

$$B^t N u* = 0; \quad \delta = B_o^t N u*, \tag{19d}$$

$$u_* \geq 0, \quad \theta * \geq 0. \tag{19e}$$

This problem cannot be solved by convex programming techniques because of the possibility of nonglobal local minima. The global optimum of these programs gives the plastic deformations for the stochastic most important mechanism and the reduced random variables can be evaluated using

$$x' = -\sigma_x \theta * \beta^2, \tag{20a}$$

$$l' = \sigma_l \delta * \beta^2. \tag{20b}$$

3.3 Solution of a concave quadratic programming

Although theoretically large-scale 0-1 mixed integer programming are NP-hard problems, they can be solved in a reasonable time (Marsten 1987), providing most of the variables are continuous.

The motivation for considering concave quadratic minimization is similar to that for problems of 0-1 integer linear programming. The computational method presented by Rosen (1983) for finding the global minimum of a quadratic concave function over a polyhedral set takes advantage of the ellipsoid-like level surfaces of the objective function to find a good initial vertex and to eliminate a rectangular domain (enclosed in a level surface) from further consideration. The basic step used is to initially determine a rectangular domain which contains the projection of the domain on the space of the nonlinear variables. This can be done by a multiple-cost-row LP with n objective functions. Then a linear underestimating function is computed and a linear underestimating problem is solved to give lower and upper bounds for the global optimum. This solution also gives a bound on the relative error in the function value of this incumbent vertex. If the incumbent is not a satisfactory approximation to the global optimum, a guaranteed $\mathcal{E}$-approximate solution is obtained by solving a single 0-1 mixed integer programming problem. This integer problem is formulated by a piecewise linear underestimation of the separable problem (Simões 1991).

3.4 Enumeration of other stochastic important mechanisms

An appraisal of the current procedures for generating the stochastic most representative failure modes indicate that they are variously dependent on simulation, trial-and-error, perturbation, human judgement, complex heuristic strategies, or approximations, either for choosing the appropriate starting points or for continuing the method at different stages. Some of the methods generate the modes in random order and thus many of the important modes may be missed without ever knowing about them (Nafday *et al.*

1987). There remains the need for a reliable algorithmic approach. The techniques used here to find the stochastic most representative mechanism were employed to enumerate other stochastic important modes by assigning the incumbent solution at a desired level of significance, larger than the global solution. Since the domain is partitioned with respect to the nonlinear variables only (random loads and resisting moments), it is possible that within the same range of bounds other mechanisms exist (and are not identified). Moreover, mechanisms with plastic hinges at different locations but associated with the same values of the random variables might be overlooked. For this reason, after finding the stochastic dominant mode over each of the subregions, either a branch and bound procedure or a vertex enumeration and ranking must be employed to enumerate the remaining stochastic relevant mechanisms.

4 Outer problem: discrete optimum design

4.1 Assumptions

Consistent with a first-order second-moment reliability approach, the minimum statistical information required for the evaluation of the optimum solution is: (a) the mean values of the loads which make up the vector μ_1, the coefficients of variation of the loads which make up the vector Ω_1, and the coefficients of correlation between paris of loads which form a square symmetrical correlation matrix denoted C_1; (b) the coefficients of variation of the plastic capacities, which make up the vector Ω_X and the coefficients of correlation between pairs of plastic capacities which form a square symmetrical matrix denoted C_X.

4.2 Reliability against collapse

By fixing the design variables, the reliability assessment problem (19) gives the activation parameters $\theta*$ and displacement rate $\delta*$ associated with the stochastic most important mechanism and other relevant modes. Clearly, the reliability analysis for another set of design variables (but the same mechanism) would give proportional activation parameters and displacement rates. For a pre-specified reliability index $\beta*$ and m mechanisms, single mode probability constraints will be satisfied if,

$$\mu_x^t \theta *_k - \mu_1^t \delta *_k \geq$$

$$\geq \beta * \left[\sqrt{\theta *^t \sigma_X^t C_X \sigma_X \theta * - \delta *^t \sigma_1^t C_1 \sigma_1 \delta *} \right], \qquad (21)$$

where $k = 1, \ldots, m$. It can be shown that these constraints are convex with respect to the design variables. Under mild requirements, the multi-mode constraints arising from (4), (5), (6) and (11) can be assumed convex and they can be approximated by the affine terms of the Taylor series expansion.

4.3 Formulation

If the plastic capacities are proportional to the volume of material required and their choice is limited to a discrete set, this problem can be expressed as

$$\min V(\mathrm{x}) = \mathrm{c}^t \mathrm{x}, \qquad (22a)$$

subject to

$$P_f = P_f(\mathrm{x}) \leq P_f*, \qquad (22b)$$

$$\mu_X \in S = \{s_d : d = 1, \ldots, D\}, \qquad (22c)$$

where c is the vector of member lengths. The objective function is a linear function of the characteristic values of plastic capacities x which can be directly related to the mean values μ_X. The reliability constraints are nonlinear. The direction solution of problem (22) is difficult because of its nonlinearity and the discreteness requirement stated by (22c).

The rigorous discrete optimum design is a NP-hard-problem, significantly more difficult than the continuous problem. The continuous optimum design forms a lower bound to the discrete optimum and it is usually assumed that the continuous plastic capacities should somehow be rounded up or down to discrete sizes. This rounded process turns out also to be a combinatorial problem. The method described next introduces the artificial concepts of segmental members and segmental optimum design (Templeman 1983), and provides a close lower bound to the discrete reliability-based design.

4.4 Segmental optimum design

Problem (21) assumes that each member is of known length and has an unknown average plastic capacity. Assume instead that each member of the frame is composed of a total of D segments, each with an average plastic capacity equal to one of the discrete sizes such that all sizes are represented among segments.

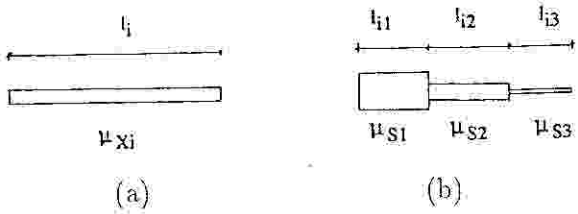

Fig. 1. (a) Conventional member, (b) segmental member

The plastic capacities of all segments represented in Fig. 1 are known but the segment lengths are unknown. If problem (21) is reformulatd with this segmental assumption in place of uniform members the following problem is obtained:

$$\min V(c) = \sum_{i=1,n} \sum_{d=1,D} \mu_{Sd} c_{id}, \qquad (23a)$$

subject to,

$$P_f(X) + \sum_{i=1,n} \sum_{d=1,D} (\partial P_f / \partial \mu_{Sd})_i / c_i c_{id} \leq P_f*, \qquad (23b)$$

$$\sum_{d=1,D} c_{id} = c_i, \quad i = 1, \ldots, n, \qquad (23c)$$

$$c_{id} \geq 0, \qquad (23d)$$

where $P_f(X)$ is the structural reliability corresponding to the vector of plastic capacities x and $(\partial P_f / \partial \mu_{Sd})_i$ is the change in P_f when the plastic capacity of member i is replaced by $(\mu_{Sd})_i$. Therefore $(\partial P_f / \partial \mu_{Sd})_i$ are implicit functions of the plastic capacities and will change as the design variables are changed.

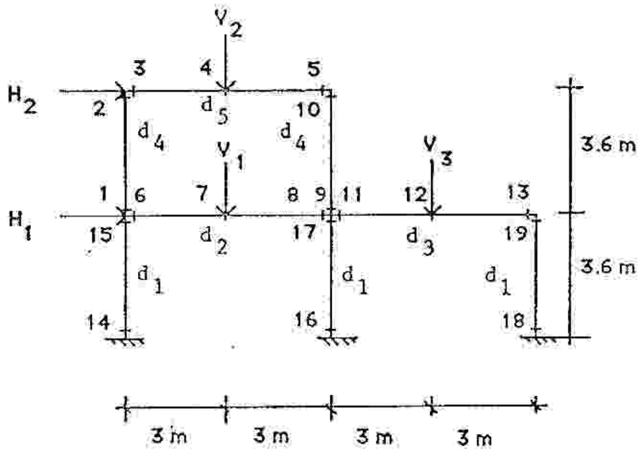

Fig. 2. Frame example

The variables of problem (23) are the lengths c_{id} of all segments of all members. The additional constraints in problem (23), which do not appear in problem (22), ensure length equivalence of conventional and segmental members and the non-negativity requirement for all segment lengths which are clearly necessary. This problem may be solved by an LP algorithm yielding what can be termed segmental optimum design. Several features of this problem can be deduced. Firstly, its average volume will be globally minimum for the specified values of $(\partial P_f/\partial \mu_{Sd})_i$. Secondly, the average volume of the segmental optimum design is a lower bound to the average volume of the discrete optimum design, because the discrete requirements are relaxed. Thirdly, in the segmental optimum design there can be at most one (m in the case of Cornell's lower bound) multi-segment member and at least $n-1(n-m)$ members composed of a single segment. It is clear from this that the segmental optimum design forms a very useful lower bound to the discrete reliability-based design.

4.5 Achieving a discrete optimum design

The discrete optimum design must have only one segment of discrete size per member. In the segmental optimum design most members will satisfy this requirement but there will be one multi-segment member which does not (a few, when Cornell's lower bound is considered). An obvious rounding scheme is to increase the plastic capacity of all the multi-segment members until they have the same average discrete plastic capacity within the member. The result will be a feasible discrete design which may be the required solution or an upper bound to the discrete optimum. Because only one or at most a few members are concerned in this operation, the design obtained should have an average volume fractionally larger than the optimum solution. A further refinement of the rounded-up design may be possible. It consists of determining whether any complete members in the discrete design can be replaced by complete members of smaller volume without violating any constraints. The slack variable(s) of the reliability constraint(s) will be in the basic set and must remain there with positive or zero values. The other basic variables are segment lengths with a value equal to the physical length of the member. The objective function coefficients will indicate several candidate segment length variables in the non-basic set which, if they entered the basis would reduce the average volume of the structure. Each candidate can be examined and pivoted into the basis provided it pivots a complete segment variable out of the basis and does not violate the reliability constraint(s). This procedure may tighten the already close bounds on the reliability-based design, although it is not possible to guarantee convergence to the global optimum.

This solution can be found by using an alternative branch and bound strategy. A large number of solutions with round-up and round-down members generated by pivotal operations can *a priori* be excluded by enforcing the bounds provided by the segmental method.

5 Numerical example and discussion

5.1 Two-bay, two-storey frame

The minimum volume of the rigid portal frame with fixed geometry, represented in Fig. 2, satisfying given reliability requirements is to be found. Five discrete design variables corresponding to average plastic capacities of the columns and beams are considered for the discrete optimum design. The mean values of the loads and the coefficients of variation of loads and plastic capacities are read as input data as follows:

μ_{V1}=169 kN; Ω_{V1}=0.15; μ_{V2}=89 kN; Ω_{V2}=0.25;
μ_{V3}=116 kN; Ω_{V3}=0.25;
μ_{H1}=62 kN; Ω_{H1}=0.25; μ_{H2}=31 kN; Ω_{H2}=0.25;
$\Omega_{X_1}=\Omega_{X_2}=\Omega_{X_3}=\Omega_{X_4}=\Omega_{X_5}$=0.15.

For the purpose of discrete design it was assumed that eight different bar sizes were available for each member. The member sections were chosen from a commonly available prefabricated size (Portuguese INP). For normally distributed load and plastic capacities, statistic independence between random loads was assumed, except the horizontal loads which were perfectly correlated. For normally distributed load and plastic capacities, perfect correlation within members and column-column correlation was considered.

The sizing given by deterministic plastic limit synthesis can be very inadequate when, for a fixed volume of material the structural reliability is the major concern. It was shown (Simões 1991) that the probability of failure against collapse can be reduced from 50 to 150 times by just redistributing the plastic capacities by the structural members according to the probability-based design.

Figure 3 shows the least volume material required to satisfy reliability requirements evaluated according to Cornell's upper bound. The segmental solution (SC) is marginally heavier than the continuous solution (CS) (1.3-2%) and is represented by the same line. As expected, because there is only one probability constraint which is active, there is only one multi-segment member in this segmental optimum design. Rounding up this segmental optimum design to a fully discrete design is a trivial operation (SLS) and leads to a volume increase of (1.6-3.9%) increase with respect to the continuous solution. By allocating the discrete size associated with the greatest length in the segmental members (SLI) results in a more unsafe structure: the probability of failure

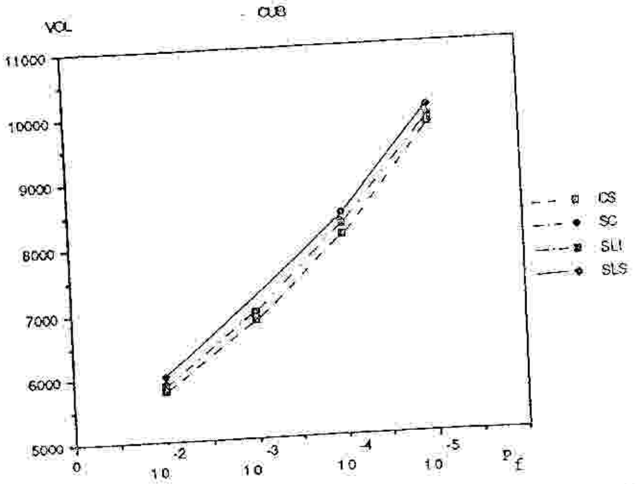

Fig. 3. Effect of prescribed risk level on optimum solution. Cornell's upper bound used to compute plastic collapse probabilities

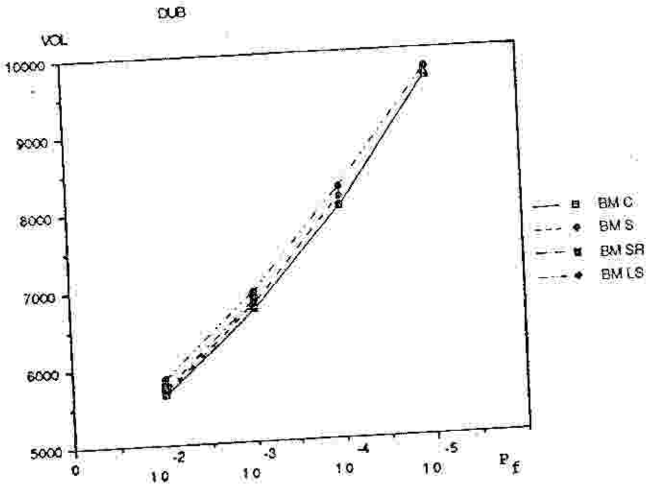

Fig. 4. Effect of prescribed risk level on optimum solution. Ditlevsen's upper bound used to compute plastic collapse probabilities

may be up to 20% higher with respect to the probability of failure of the segmental solution.

Similar conclusions can be drawn when Ditlevsen's upper bound is used to compute the structural reliability of the frame (Fig. 4). BM C meaning the continuum optimum solution when the bending moment is the only stress resultant. BM S and BN LS represent the segmental solutions and the designs obtained by rounding up all segmental members, respectively. BM SR is the solution given by allocating the discrete size associated with the greatest length in the segmental members.

Figure 5 shows the solution obtained when the statistical correlation of mechanisms (Cornell's lower bound) is assumed. The segmental solution (BM S) almost coincides with the continuous solution (BM C) and the number of multi-segment members equals the number of design variables. The discrete solution obtained by rounding-up these members (BM LS) leads to a 6-9% increase of the volume required. Moreover,

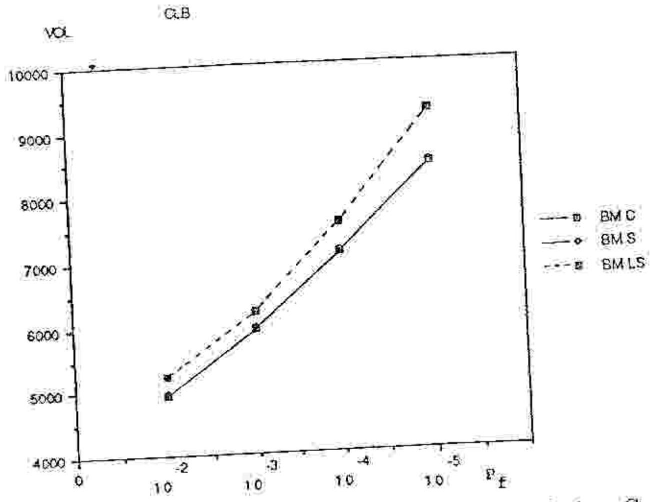

Fig. 5. Effect of prescribed risk level on optimum solution. Cornell's lower bound used to compute plastic collapse probabilities

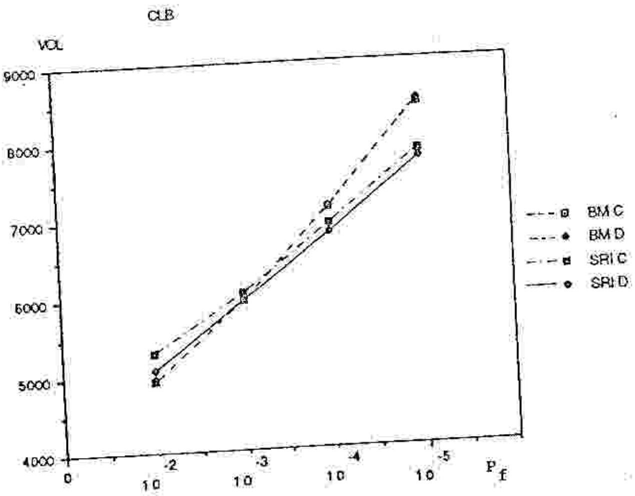

Fig. 6. Effect of prescribed risk level against plastic collapse with stress-resultant interaction on optimum solution. Cornell's lower bound

it may not be possible to refine further this discrete design because the collapse mechanism probabilities are strongly dependent on the individual design variable values.

Figures 6-10 illustrate the results of the example frame with (SRI) or without (BM) stress-resultant interaction when the structural reliability is evaluated according to Cornell's lower bound, Ditlevsen's lower bound, Ditlevsen's upper bound, Vanmarcke's upper bound and Cornell's upper bound, respectively. Even for this small scale example, the computation required up to 24 and 40 mechanisms for the BM and SRI cases, respectively. The volume of material needed to satisfy the reliability requirements with increasing value of P_f is larger when the only stress-resultant is the bending moment. This is caused by the larger number of plastic hinges required to form mechanisms when the bending moments interact with axial forces. The optimum solutions also show an increase of the first storey column plastic capacity and a reduction of the design variable requirements for the remaining

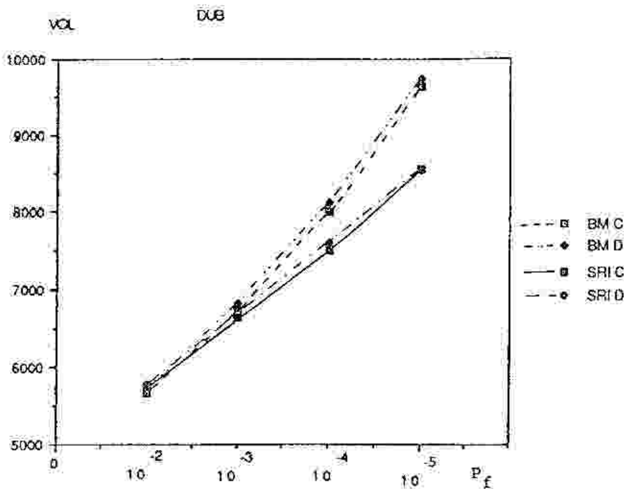

Fig. 7. Effect of prescribed risk level against plastic collapse with stress-resultant interaction on optimum solution. Ditlevsen's lower bound

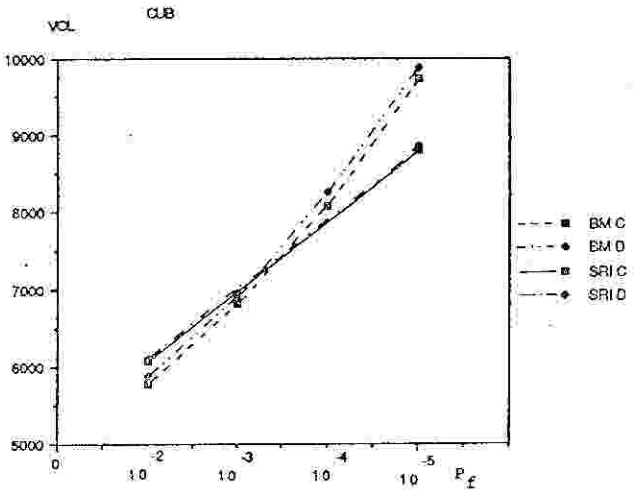

Fig. 8. Effect of prescribed risk level against plastic collapse with stress-resultant interaction on optimum solution. Ditlevsen's upper bound

members when the SRI is considered.

6 Conclusions

The use of probabilistic concepts in structural design need not be restricted because of the ideal design problem's complexity. The solution method for the optimum design of portal frames with interacting stress-resultants consists of two alternating sub-procedures: (a) an optimization of the stochastic most important modes; (b) an optimization of the convex outer problem that includes the cost function. The present investigation has discussed the influence of discrete design variables on the minimization of the total expected volume of material for a specified failure probability. The proposed technique illustrates that the stochastic dominant modes differ and the resulting probabilities of failure may change considerably, whenever the interaction of bending and

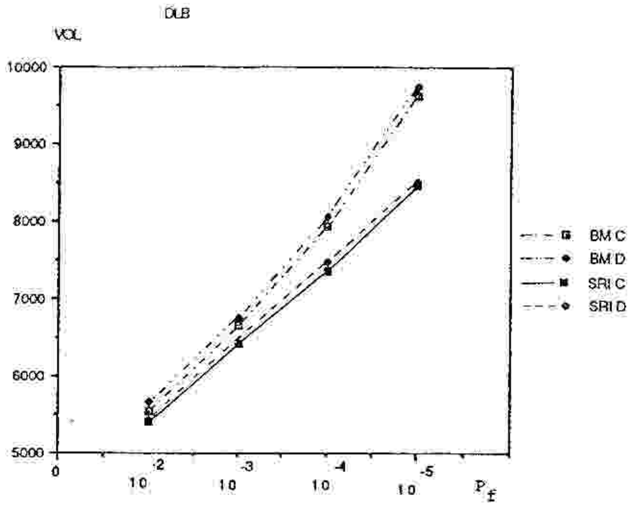

Fig. 9. Effect of prescribed risk level against plastic collapse with stress-resultant interaction on optimum solution. Vanmarke's upper bound

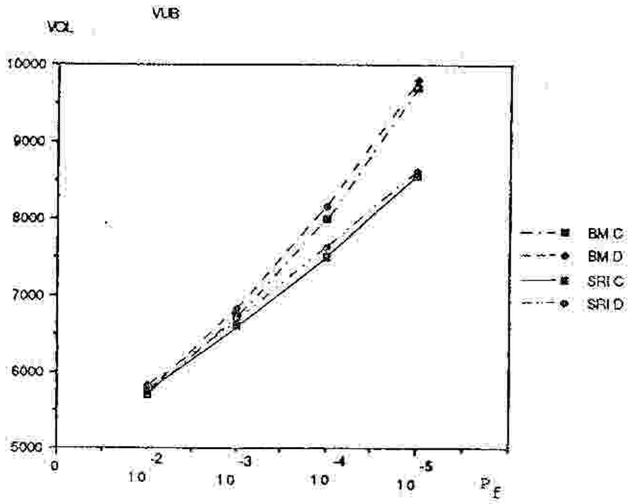

Fig. 10. Effect of prescribed risk level against plastic collapse with stress-resultant interaction on optimum solution. Cornell's upper bound

axial forces is considered instead of a predominant bending action. Preliminary results on larger scale examples seem to indicate that the allocation of more plastic capacities to the lower storeys is more notorious when stress resultant interaction is considered.

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