Computer Aided Optimum Design of Structures: Application
Editors: C.A. Brebbia and S. Hernandez
Computational Mechanics Publications 1989 / Springer Verlag
ISBN 1-85312-035-9

Reliability-Based Plastic Synthesis of Reinforced Concrete Slabs

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ABSTRACT

A mathematical programming technique is described which minimizes the total average volume of steel reinforcement of a reinforced concrete slab for a specified failure probability. The structure is discretized into triangular finite elements and the structural material is assumed to exhibit a perfectly-plastic behaviour so that plastic collapse is the only possible failure mode. It consists of solving alternatively a quadratic convex maximization and a convex minimization until the best reliability-based design against collapse occurs. The proposed computational technique is illustrated on a reinforced concrete slab.

INTRODUCTION

Methods of optimization based on deterministic safety concepts, while minimizing the weight of an element may be changing its level of safety. This is a major limitation of deterministic optimization formulations, in which the inherent random nature of both structural loading and strength is not included and, consequently, the safety criteria are not specified in terms of a risk value. Clearly, the proper tool for the assessment and analysis of such uncertainties requires methods and concepts of reliability. While this basic design philosophy has been accepted for some time, reliability-based optimization programs have not enjoyed the same popularity as their deterministic counterparts. At least three reasons for this may be suggested. First, there are several different procedures for handling the uncertainty in similar structural design situations (Ellingwood and Galambos[1], Shinozuka[2]). This affords the possibility of nonuniform reliability levels in similar structural design situations. Second, in present design practice it is usually assumed that structural systems are as safe as their components. However, in recent years consideration of both ductile and brittle behaviour including load and strength correlation has permitted considerable improvement of solutions to system reliability problems (Ditlevsen[3], Ang and Ma[4], Chou, McIntosh and Corotis[5]). Finally, the NP-hard nature of the mathematical programming problems involving the reliability assessment of structures with nonlinear behaviour lead individuals (Nafday, Corotis and Cohon[6], Rashedi and Moses[7]) to adopt procedures based on plastic limit analysis that may overlook stochastic important modes. This paper describes a first order second moment reliability-based approach to the optimum design of reinforced concrete slabs. The loads and strengths are treated as random variables and failure is defined as the event that the load effect exceeds the plastic resistance of the structure.

The general structural configuration including the dimensions of the concrete section, reinforcement lever arm and length of the steel reinforcement are specified in a deterministic manner. This mathematical program has a preassigned reliability level against plastic collapse and simultaneously minimizes the prescribed objective function. The formulation consists of minimizing a linear function such that the mechanisms with the higher reliability indices are associated with probabilities of failure that are lower than prespecified values. This min-min problem becomes a modified parametric optimal design and a cutting plane method is employed to solve it. By fixing the design variables, stochastic important collapse mechanisms are automatically generated and this information is used in the global problem to find a solution closer to the optimum design.

PROBLEM STATEMENT

Assumptions

The basic assumptions adopted herein for the structural reliability optimization of ductile reinforced concrete slabs are: (a) the yield lines are straight; (b) shear deformation is neglected; (c) the load and yield line positions are deterministic; (d) the loads and plastic moments are assumed to be random variables; (e) the statistical dependence among loads and among plastic moments is accounted for through the coefficients of correlation between loads and between plastic moments; (f) the necessary statistical information is assumed to be available; (g) a reinforced concrete slab fails by collapse when enough yield lines make possible the occurrence of a plastic mechanism; (h) local fracture, instability and other possible causes of failure are avoided.

Formulation

The reliability based design optimization problem can be formulated in general form as follows:

$$\text{Min } z = f(d) \tag{1a}$$

subject to,

$$g_1(d,m,\lambda,\Theta,u,\Theta_*,u_*) = 0 \tag{1b}$$

$$g_2(d,m,\lambda,\Theta,u,\Theta_*,u_*) \leq 0 \tag{1c}$$

$$g_3(d) \leq 0 \tag{1d}$$

$$\Pr\,[Z_k \leq 0] \leq p_s \qquad k = 1,\dots,m \tag{1e}$$

$$\Pr\,[\cup_{k=1,m} Z_k \leq 0] \leq p_o \tag{1f}$$

where $d,m,\lambda,\Theta,u,\Theta_*,u_*$ are the vectors of design variables, random bending moment resistances, random loads, yield line rotations, nodal displacements, total yield line rotations and total nodal displacements, respectively. The objective function and the constraints (1b)-(1d) are linear. The above formulation is different from plastic limit synthesis problems, owing to single mode failure probability constraints (1e) and the system failure probability constraint (1f).

Single Mode Failure Probability Constraint

The probability of failure via the k-th individual collapse mode p_k can be obtained from the probability that a certain performance function Z_k :

$$Z_k = U_k - E_k = m^{+t}\,\Theta_*^{+} + m^{-t}\,\Theta_*^{-} - F^t\,u_* \tag{2}$$

is negative. In eq.(2) U_k and E_k are the internal and external random works associated with the k-th collapse mode. Consistent with a first-order second-moment reliability analysis, the failure probability may be measured entirely with a function of the first and second moments of random parameters. It is assumed that safety with regard to plastic collapse via the failure mode k depends only on reliability index β_k, that is defined as the shortest distance from the origin to a failure surface in the reduced random variables coordinate system.

$$\beta_k = \mu_{Z_k} / \sigma_{Z_k} \tag{3}$$

and the single mode failure probability constraint can be rewritten as,

$$1 - \Phi(\beta_k) \leq p_s \quad \Rightarrow \quad \beta_k \geq \Phi^{-1}(1-p_s) \tag{4}$$

where $\Phi(\,)$ stands for the cumulative standard normal distribution function. It is important to note that the standard deviation of the safety margin of an individual collapse mode and the probability of occurrence of this mode increases as the statistical positive dependence between plastic moments and/or between loads that are active in producing the mechanism increases and vice-versa. To check whether the probability constraint condition is satisfied, an optimization problem called inner problem must be solved.

Multi-Mode Failure Probability Constraint

Evaluation of the failure probability appearing in constraints of eq. (1f) is one of the major concerns in the solution of reliability-based optimization problems. Since multiple integration with respect to random parameters must be executed and correlations of each failure mode must be known a priori, in a multi-mode failure probability, an exact calculation of probability Pr[ ] is practically impossible, without resorting to an approximation method. In general, the admissible failure probability for structural design is very low. Hence, approximation by Cornell's first-order upper bound is suitable to calculate multi-mode failure probability and it can be used for the designer's benefit, since it is conservative,

$$\mathrm{Max}_{\mathrm{all}\,k}\,[\Pr(Z_k)] \leq p_f \leq 1 - \Pi_{k=1,m}\,[1 - \Pr(Z_k)] \tag{5}$$

The lower bound, which represents the probability of occurrence of the most critical mode (dominant mode) is obtained by assuming the mode failure events Z_k to be perfectly dependent, and the upper bound is derived by assuming independence between mode failure events. Since the multi-mode failure probability is a summation of single mode failure probabilities, the constraint (1e) can be stated as,

$$\Sigma_{k=1,m} \Pr\,[Z_k \leq 0] = \Sigma_{k=1,m}\, p_k \leq p_o \tag{6}$$

For optimization purposes and since the failure probabilities are very small comparing with other variables, the multi-mode probability constraint can be replaced by the convex approximation,

$$\ln \Sigma_{k=1,m}\, e^{-a\,\beta_k^{\,b}} \leq \ln p_o \tag{7}$$

where p_o denotes an admissible system failure probability, a and b are constants. For practical purposes and uncorrelated modes one may use a=1.48 and b=1.36, but the quality of the approximation (7) needs to be calibrated "a posteriori". Improved bounds can also be obtained by using Vanmarcke's[8] concept of failure mode decomposition or Ditlevsen's[3] method of conditional bounding, which take into account the probabilities of joint failure events.

SOLUTION METHOD

The solution method can be divided in two alternating subprocedures:
a) an optimization procedure for the nonconvex inner problem, that finds the stochastic most important mechanism and enumerates other relevant collapse modes for a given value of the design variables (plastic moments of resistance).
b) an optimization of the convex outer problem on the design variables, that is the solution with least cost satisfying serviceability and technological requirements and the reliability constraints (1c)-(1f). In order to satisfy these, the number of reliability constraints is expanded to include several stochastic dominant modes.

The procedure is repeated until the vector of design variables converges. Since these two procedures are themselves mathematical programmes, any suitable techniques can be applied. The single mode failure probability constraints are checked by using the results of the inner problem. This form of reliability-based optimization is similar to the parametric optimization problem treated by Kwak and Haug[9] except that the domains of subproblems are functions not only of random parameters but also of design variables. This problem was overcome by Lee and Kwak[10] for elastic trusses, but is was assumed that the solution of each inner problem is unique. This is not true for structures with plastic behaviour, because each collapse mechanism is a local solution of the inner problem.

INNER PROBLEM

Structural relations

The optimal solution to the discretized problem may provide a bound to the solution of the continuous problem, but the existence and nature of such a bound depends on the form of the finite element modelling. A particularly simple kinematic formulation corresponds to the automation of the yield-line method. For any selected F.E. pattern, the plastic flexural deformations are confined to the element boundaries whilst the interiors of the elements remain undeformed plastically. Such deformations correspond to the collapse mode. As the deformations are to be described through the displacements of the nodes of triangular finite elements, it follows that such deformations are necessarily compatible. The rotations Θ_i^e of the outward normals to the three edges of the single element of Fig.1, may then be expressed in terms of the corner vertical displacements u_j in the following way (Munro and Da Fonseca[11]):

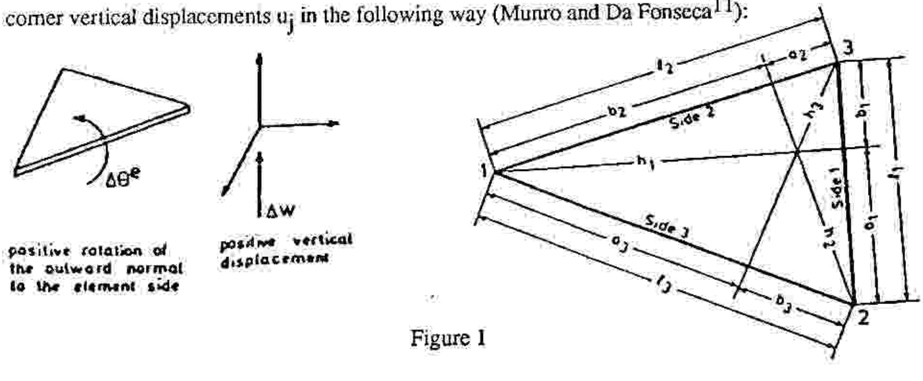

Figure 1

probability may be measured entirely with a function of the first and second moments of random parameters. It is assumed that safety with regard to plastic collapse via the failure mode k depends only on reliability index β_k, that is defined as the shortest distance from the origin to a failure surface in the reduced random variables coordinate system.

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$$\begin{vmatrix} \Theta_1^e \\ \Theta_2^e \\ \Theta_3^e \end{vmatrix} = \begin{vmatrix} -1/h_1 & b_1/(l_1 h_1) & a_1/(l_1 h_1) \\ a_2/(l_2 h_2) & -1/h_2 & b_2/(l_2 h_2) \\ b_3/(l_3 h_3) & a_3/(l_3 h_3) & -1/h_3 \end{vmatrix} \begin{vmatrix} u_1 \\ u_2 \\ u_3 \end{vmatrix} \quad (8)$$

For an interelement side the total angular discontinuity Θ_i is clearly the algebraic sum of the rotations Θ_i^e of the two outward normals with respect to the two finite elements sharing such a side. If the boundary conditions are taken into account, then the number of nodal vertical displacements to be considered is merely the number (NC) of corner nodes free to undergo that form of displacement, and the vector Θ will contain one component for each finite element side which can sustain the corresponding bending moment. If there are NS such angular discontinuities, the assemblage of relations (8) for all finite elements is thus readily performed and may be written in the following compact form:

$$\Theta = C\,u \quad (9)$$

where, C is a (NS x NC) kinematic transformation matrix. Since these modal deformations are obtained as functions of the linearly independent displacements **u** it follows that such deformations are necessarily compatible and the rotation/displacement relations may be taken as the compatibility conditions.

The yield line theory considers a very simple yield criterion involving solely the normal bending moment and the normal angular discontinuity at every element side.The yield conditions impose limit values to the magnitudes of the total bending moments at the element side.For the i^{th} boundary side, the yield conditions require that:

$$-m_{*i}^- \le m_i \le +m_{*i}^+ \quad (10)$$

where m_{*i}^+ and m_{*i}^- are the magnitudes of the total plastic moments of resistance for positive and negative bending along the i^{th} boundary side. If α_i is the angle between the boundary side of length l_i and the y axis,

$$m_{*i}^+ = l_i(\cos^2 \alpha_i)\, m_x^+ + l_i(\sin^2 \alpha_i)\, m_y^+ \quad (11a)$$

$$m_{*i}^- = l_i(\cos^2 \alpha_i)\, m_x^- + l_i(\sin^2 \alpha_i)\, m_y^- \quad (11b)$$

The mechanism deformation can only take place at the element sides where the normal bending moment reaches one of its limiting values. That is to say the angular discontinuity Θ_i at the element side can only take a positive value Θ_i^+ when m_i is equal to m_{*i}^+ and it can only take a negative value Θ_i^- when m_i is equal to m_{*i}^-. Providing that at the incipient plastic collapse the displacements are small for the plastic analysis to be based on the undeformed geometry the structure previous to the loading, elastoplastic deformations need not to be considered. The plastic behaviour of slabs is not necessarily restricted to the finite element sides. Thus, the finite element modelling leads to a representation which is approximate for the slab.

Identification of the stochastic most relevant mechanism

The problem arises to evaluate the conditional failure probability of the structural system given a certain load event. The program seeks the stochastic most relevant yield-line pattern that can be attained when the yield-lines are confined to the F.E. boundaries. This method thereby automates the yield-line search within the selected F.E. system. For the uncorrelated random variables m_*^+, m_*^-, λ_F and positive $[\mu_{m^+}\,\theta_*^+ + \mu_{m^-}\,\theta_*^- - \mu_F\, u_*]$, the identification of the stochastic most relevant mechanism consists of minimizing the reliability index β,

$$\min \beta = \frac{[\mu_{m^+}\,\theta_*^+ + \mu_{m^-}\,\theta_*^- - \mu_F\, u_*]}{\sqrt{(\sigma_{m^+})^2\,(\theta_*^+)^2 + (\sigma_{m^-})^2\,(\theta_*^-)^2 + (\sigma_F)^2\,(u_*)^2}} \qquad (12)$$

subject to the compatibility relations (9), the linear incidence equations,

$$\theta_*^+ = J_\theta^{\,+}\,\theta^+ \quad ; \quad \theta_*^- = J_\theta^{\,-}\,\theta^- \quad ; \quad u_* = J_u\, u \qquad (13)$$

where the incidence matrices $J_\theta^{\,+}$, $J_\theta^{\,-}$ and J_u are obtained by associating the rotations of the element sides with the rotations of the members represented by the same random variables θ_*^+, θ_*^- and the displacements of the point loads (or in the case of uniformly distributed load, deflections of the triangular elements centroids) linked by the random variable u_*. Sign constraints on the variables need also to be considered:

$$\theta^+ \geq 0\,,\ \theta^- \geq 0\,,\ u \geq 0\,,\ \theta_*^+ \geq 0\,,\ \theta_*^- \geq 0\,,\ u_* \geq 0 \qquad (14)$$

If matrices C_{θ^+}, C_{θ^-} and C_u represent the correlations between the bending moments of resistance and between the loads, respectively, the reliability index is given by,

$$\beta = \frac{[\mu_{m^+}{}^t\,\theta_*^+ + \mu_{m^-}{}^t\,\theta_*^- - \mu_F{}^t\, u_*]}{\sqrt{(\theta_*^+\sigma_{m^+}C_{\theta^+}\sigma_{m^+}\theta_*^+)+(\theta_*^-\sigma_{m^-}C_{\theta^-}\sigma_{m^-}\theta_*^-)+u_*\sigma_F C_u \sigma_F u_*}} \qquad (15)$$

For random variables that are correlated, the original variates may be transformed to a set of uncorrelated variables. The required set of uncorrelated transformed variates can be obtained through an orthogonal transformation. If the probability distribution functions of the random variables are not Gaussian, the Rosenblatt[12] transformation may be used.

These mathematical programs belong to the class of fractional programming problems. The minimization of β shares its solutions with the quadratic concave maximization:

$$\max 1/\beta^2 = (\sigma_{m^+})^2\,(\theta_*^+)^2 + (\sigma_{m^-})^{\cdot}\,(\theta_*^-)^2 + (\sigma_F)^2\,(u_*)^2 \qquad (16a)$$

subject to,

$$\mu_{m^+}\,\theta_*^+ + \mu_{m^-}\,\theta_*^- - \mu_F\, u_* = 1 \qquad (16b)$$

$$\theta_*^+ = J_\theta^{\,+}\,\theta^+ \quad ; \quad \theta_*^- = J_\theta^{\,-}\,\theta^- \quad ; \quad u_* = J_u\, u \qquad (16c)$$

$$\theta = C\, u \qquad (16d)$$

$$\theta^+ \geq 0\,,\ \theta^- \geq 0\,,\ u \geq 0\,,\ \theta_*^+ \geq 0\,,\ \theta_*^- \geq 0\,,\ u_* \geq 0 \qquad (16e)$$

This problem cannot be solved by convex programming techniques because of the possibility of nonglobal local minima.The global optimum of these programs gives the plastic deformations for the stochastic most important mechanism and the reduced random variables can be evaluated by using,

$$m_*^{+'} = - \sigma_{m^+} \theta_*^+ \beta^2 \quad (17a)$$

$$m_*^{-'} = - \sigma_{m^-} \theta_*^- \beta^2 \quad (17b)$$

$$\lambda_{F}' = \sigma_F u_* \beta^2 \quad (17c)$$

Cutting Plane Based Method for solving Quadratic Convex Maximization

In Konno's[13] cutting plane method it is defined an associated equivalent program with bilinear objective function (BLP). The algorithm operates by keeping one set of variables fixed and partially dualizing the BLP using linear duality theory. A local maximum is computed and a cutting plane generated by exploiting the symmetric nature of BLP. In the next iteration, this procedure either generates a point which is strictly better than the last local maximum found, or generates a cut which is deeper until the convergence criteria to a ε-solution is met. Tui's[14] cut that is devised using local information only becomes shallower as the dimension increases and the results of numerical experiments reported by Zwart[15] were quite disappointing. Konno combined Ritter16 and Tui methods producing a much stronger cut. It is a common contention that cutting plane based methods do not work well when the number of constraints and variables is very large. Nevertheless, the special nature of this problem in which the nonlinearities are restricted to the variables associated with the random variables, the possibility of obtaining very deep cuts and generating sub-optimal solutions corresponding to other dominant modes at a small cost, makes this algorithm very attractive from a computational point of view

Enumeration of other stochatically important mechanisms

Konno's method for bilinear programming partitions the linear domain into basic and nonbasic vectors. The selection of the incoming variable into the basis is based on the information available from the dual linear problem. It is much more efficient than Murty's[17] algorithm that enumerates and ranks all possible vertices (local solutions) without taking into account the explicit form of the objective function which is used to define the domain of BLP's dual problem.

OUTER PROBLEM

Objective Function

The object of reliability-based plastic optimization of reinforced concrete slabs is usually to find the slab thickness and the corresponding amount of reinforcing steel so that a specified reliability level against plastic collapse be provided and an adopted objective function be minimized. The relevance of the adopted objective function is an open subject with important consequences on the optimum solution (Surahman and Rojiani[18]). For a reinforced concrete slab of known plan-form and whose thickness and boundary conditions are also known, the reliability-based design problem reduces to determining the reinforcement details such that the total volume of reinforcement is minimized.

For under-reinforced sections of known lever arm, the area of reinforcement has a linear relation to the plastic moment of resistance. Thus, if orthotropic reinforcement is to be used, and if μ_{mx}^{+i} (μ_{mx}^{-i}) is the plastic moment of resistance per unit length to be allocated to the i^{th} F.E. in the x direction for positive (negative) bending and μ_{my}^{+i} (μ_{my}^{-i}) is the plastic moment of resistance per unit length to be allocated to the i^{th} F.E. in the y direction for positive (negative) bending, then the total volume of reinforcement is given by,

$$V = \rho \, \Sigma_{i=1,FE} \; A_i \left[\frac{\mu_{mx}^{+i}}{a_x^{+i}} + \frac{\mu_{my}^{+i}}{a_y^{+i}} + \frac{\mu_{mx}^{-i}}{a_x^{-i}} + \frac{\mu_{my}^{-i}}{a_y^{-i}} \right] \quad (18)$$

where a_x^{+i}, a_y^{+i} are the appropriate lever arms, A_i is the area of the i^{th} F.E. and ρ is a known proportionality factor. Thus, the total volume of reinforcement can therefore be expressed as the linear function,

$$\text{Min } V = c^T d = a_+^t \, \mu_m^+ + a_-^t \, \mu_m^- \quad (19)$$

where a_+ and a_- vectors of known constants whilst μ_m^+ and μ_m^- are the vectors of design variables: means of the plastic moments of resistance with respect to positive and negative bending, respectively.

Technological and serviceability constraints

Frequently, it is convenient to establish, "a priori", relationships between the reinforcement at various elements in order to satisfy requirements and simplify the detailing arrangements. These are termed technological constraints,

$$T^+ \, \mu_m^+ + T^- \, \mu_m^- = 0 \quad (20)$$

where T^+ and T^- are the appropriate technological matrices. If the reinforcement is selected only with regard to plastic collapse and without reference to the slab's behaviour at working load, then inevitably the design may be unsatisfactory with respect to serviceability requirements. However safety against loss of serviceability, which is defined by the formation of the first yield line can be checked by means of a simplified probabilistic procedure. Thus, for practical design, additional constraints will be required and these serviceability constraints may be most readily included in the form of lower bounds on the design variables.

Reliability Constraints

By fixing the design variables, the inner problem gives the yield line rotations Θ_* and nodal displacements u_* associated with the stochastic most important mechanism and other relevant modes. Clearly, the reliability analysis for another set of design variables (but the same mechanism) would give proportional yield line rotations and nodal displacements. For a prespecified reliability index β_S, the single mode probability constraints defined by m collapse modes will be satisfied if (21),

$$\mu_{m^+} \Theta_*{}^+{}_k + \mu_{m^-} \Theta_*{}^-{}_k - \mu_F \, u_{*k} \geq \beta_* \left[\sqrt{\sigma_{m^+}{}^2 \, \Theta_*{}^+{}_k{}^2 + \sigma_{m^-}{}^2 \, \Theta_*{}^-{}_k{}^2 + \sigma_F{}^2 \, u_{*k}{}^2}\right]$$

where $k=1,..,m$ .It can be shown that these constraints are convex with respect to the design variables $d^t = [\, \mu_m^{+\,t} \;\; \mu_m^- \,]$. Since the convex approximation of the multi-mode constraint (7) is convex, the outer problem can be solved by any convex programming technique. It has a linear objective function (19) and the domain is defined by linear (20) and nonlinear constraints (7),(21). These nonlinear constraints could be linearized by creating a first-order Taylor series expansion and the linearized problem could be solved by using linear programming. The design is updated and a new iteration performed repeating the process until convergence is achieved.

Since these move limits are critical, the procedure can be made much more efficient by using either Sequential Quadratic Programming or Sequential Convex Programming (Vanderplaats[19]).

NUMERICAL EXAMPLE

The reinforcement in the top and bottom layers inside distinct design regions of a uniformly loaded clamped square slab (l=10 m) is required. The isotropic symmetry of the material properties and the symmetric features of the slab geometry, reinforcement and loading allow for the consideration of one only quarter of the slab. In this example, 3 design variables are considered for each type of reinforcement, as represented in Fig.2, in the total of 12 random bending moments of resistance. It is assumed that random loading (μ_F=5 kN/m^2 ; Ω_F = 0.3) and the design variables (Ω_{m^+}=Ω_{m^-}=0.05) are Gaussian and statistically independent. The discretization of the selected quarter of the slab is achieved using 16 finite elements (Fig.3) in such a way that the interelement sides are located along lines of reinforcement transition, that is along the lines in between distinct regions. Cornell bounds are used to evaluate the multi-mode reliability constraint, p_o = 3.65x10^{-3} and the single mode maximum probability of failure is 1.x10^{-3} (β_s = 3.090). The following technological constraints enforcing relationships between design variables are introduced to avoid an unacceptable layout of the steel reinforcement,

$$d_1 = d_4 \;;\; d_2 = d_5 \;;\; d_3 = d_6 \;;\; d_2 = d_8 \;;\; d_7 = d_{10} \;;\; d_8 = d_{11} \;;\; d_9 = d_{12} \;;\; d_2 = 3/4\, d_7$$

The optimal solution (in kNm/m) was obtained by considering the contribution of five major mechanisms with reliability indices 3.090, 3.0937, 3.0941 and 3.422 (2), respectively:

$$d_1 = d_4 = 13.43 \quad d_2 = d_5 = d_8 = d_{11} = 20.15 \quad d_3 = d_6 = 15.11 \quad d_7 = d_{10} = 26.87 \quad d_9 = d_{12} = 0.$$

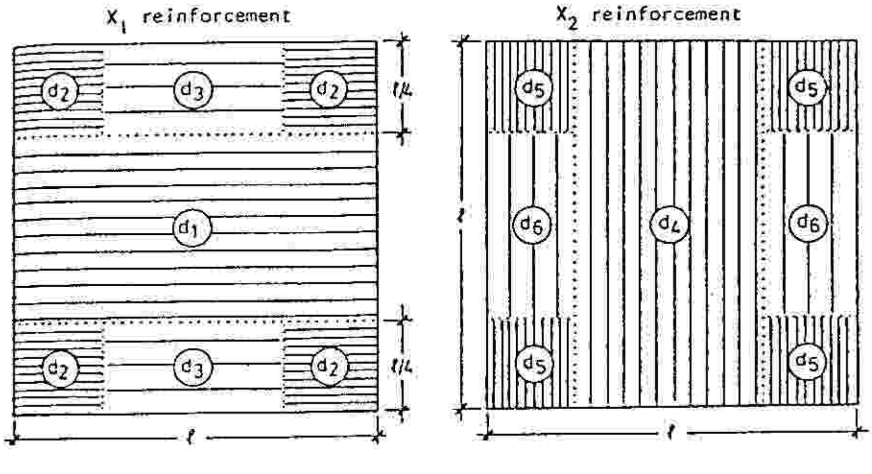

Figure 2 Design regions for the bottom steel reinforcement

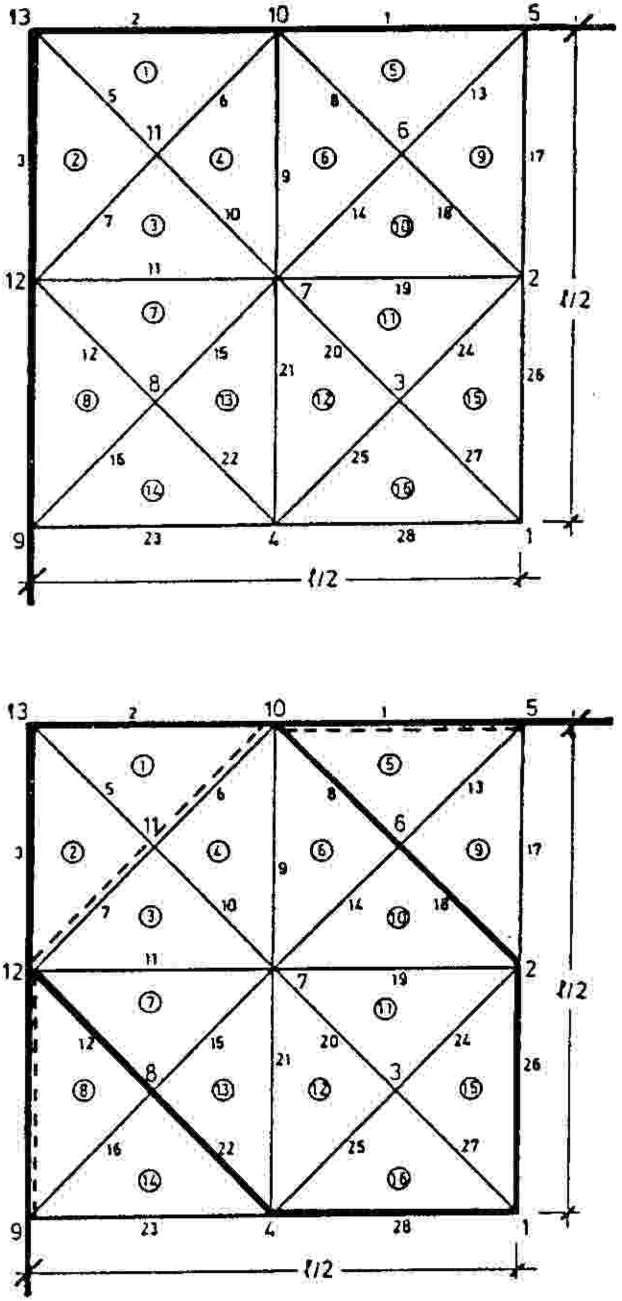

Figure 3 Finite element discretization and stochastic most relevant mechanism

CONCLUSIONS

An integrated form of yield line theory incorporating finite elements and parametric optimal design is developed and applied to design reinforced concrete slabs for a given reliability. The solution method can be divided in two alternating subprocedures: a) an optimization of the nonconvex fractional program giving the reliability index of the stochastic most important mechanism (and is able to enumerate the remaining relevant local solutions); b) an optimization of the convex outer problem that includes the cost function. The proposed technique is illustrated on a reinforced concrete slab. It provides a powerful tool to obtain a practical optimum solution offering the potential for application within a reliability-based code context which is an important goal in structural design.

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