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ENTROPY BASED SHAPE OPTIMIZATION OF ARCH DAMS

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ABSTRACT

In this paper it is described a new methodology for finding the optimal shape of arch dams. Several finite element models of a dam are tested in order to make statements about the validity of the analyser. Next, it is described a semi-analytical procedure for generating the sensitivity analysis information. In the shape optimization phase an unconstrained non-linear function is minimized by means of entropy maximization.

INTRODUCTION

In shape optimization it is intended to find the exterior and interior boundaries of the structure in an automatic way. Some inconsistent results were found on a literature survey on three-dimensional shape optimization of doubly curved arch-dams [1]. In related recent work, polynomial shape functions were optimized by a sequential linear programming technique associated with either a 8 node [2] or a 20 node [3] isoparametric element. In this work, several finite element models of a dam are tested in order to make statements about the validity of the analyser. The shape optimization phase consists of minimizing a whole set of goals such as weight, stresses at all points of the finite element mesh and nodal displacements. Entropy maximization methods, recently developped in constrained non-linear programming are employed as a means of replacing this multi-objective problem by the minimization of an unconstrained non-linear function.

FINITE ELEMENT ANALYSER

We shall confine our attention here to static, linear behaviour of the structure. The structural response R is related to the nodal displacement vector u by,

$$R = Q^t u \tag{1}$$

where Q is the virtual load vector and u satisfies:

$$K u = P \tag{2}$$

in which K is the global stiffness matrix of the structure and P is the external load vector. Both K and P are functions of the design variables a.

Table 1. Finite elements results for Alvito dam
(mesh made of 20 node brick elements and the loading condition consists of water pressure and self-weight)

	Nº elements	CPU time (sec)
A : Finite element mesh without foundation interaction	9	225
B :	16	458
C : (Dam clamped on a rigid valley)	25	825
D : Finite element mesh with foundation interaction (complete dam-foundation system)	9 + 14	1122
E : (dam foundation interaction on the downstream side)	9 + 8	786

The major part of the cost of the analysis is in the solution of the banded system of equations (2). The technique used in this work is based on the Gaussian elimination method using the frontal philosophy. The facility of solution with multiple right hand sides in this method is particularly useful in the context of calculating design derivatives.

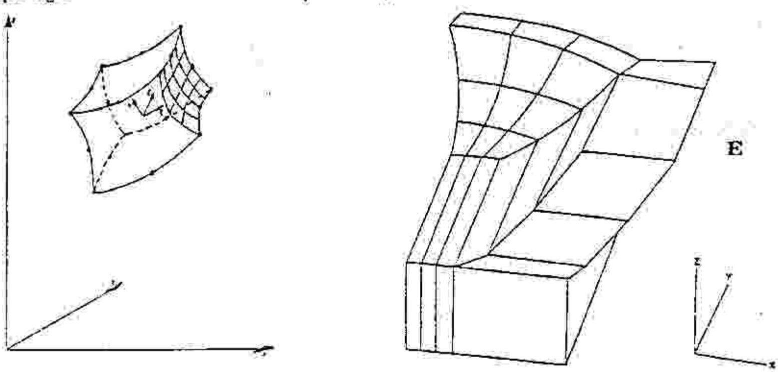

Figure 1. Mesh corresponding to model E

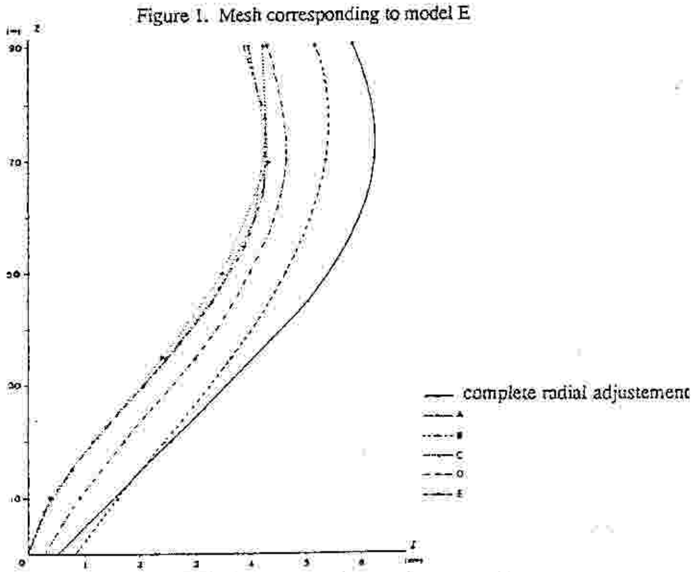

Figure 2 Normal displacements at the central crown cantilever

From the results of Fig. 2 it may be concluded that no advantage can be gained from using more than 9 elements to model the concrete dam. Model E prooved to be closer to the observed behaviour. Models A and C were also tested for the 8 node brick element. Although the CPU was reduced to 137 and 152 sec, the errors involved in evaluating the normal displacements of the crest profile were 163% and 15%, respectively.

SHAPES

For the purpose of optimization, the shape of the dam is usually described by two polynomial expressions, one for the mid surface and the other for the thickness variation along the mid-surface. The design variables are therefore the coefficients a_i of the polynomials:

$$x_{mid} = a_1 y^2 + a_2 y^2 z - a_3 z + a_4 z^2 \quad (3)$$

$$t = a_5 + a_6 y^2 z + a_7 z \quad (4)$$

The objective is to minimize the volume of the concrete subject to the following constraints: Limits on the principal stresses (compressive stresses on the crown section on the abutment and tensile stresses on the crown section); limits on the angles of the cross section to control overhanging; limits on the amount by which the shape variables are allowed to change. The corresponding mathematical programming problem can be solved by sequential linear programming [2,3]. All functions are approximated by first order Taylor series at the current design a^o, yielding a LP problem in the design change Δa:

$$\text{Min} \quad F(a^o) + \Sigma_{i=1,N} (\partial F/\partial a_i)^o \Delta a_i \quad (5a)$$

$$\text{st} \quad \Sigma_{i=1,N} (\partial R_j/\partial a_i)^o \Delta a_i \le c_j - R_j(a^o) \qquad j = 1,...M \quad (5b)$$

SEMI-ANALYTIC METHOD FOR SENSITIVITY CALCULATION

The formulas for the analytic method of sensitivity calculation [2,3] are:

$$\frac{\partial R}{\partial a_i} = \frac{\partial Q^t}{\partial a_i} u + Q^t \frac{\partial u}{\partial a_i} \quad (6)$$

$$\frac{\partial u}{\partial a_i} = -K^{-1} \frac{\partial K}{\partial a_i} u + K^{-1} \frac{\partial P}{\partial a_i} \quad (7)$$

Q, U and K are all functions of a and the obtaining of the expresssions for $\partial K/\partial a_i$ and $\partial Q/\partial a_i$ involves lengthy derivation. The semi-analytic method of sensitivity analysis consists of the following steps:

1 - Given a proper step length vector $\Delta a = (0, 0,..., \Delta a_i,...,0)$, the difference approximation of pseudo-load vector Q_p is:

$$Q_p = \Sigma_{e\in E} [-K_e(a+\Delta a) u + Ke(a) u + P_e(a+\Delta a) - P_e(a)] / \Delta a_i \quad (8)$$

where subscript e denotes the quantity of the eth element and E is the set of elements related to the design variable a

2. - Solve $\partial u/\partial a_i$ from,

$$\partial u/\partial a_i = K^{-1} Q_p \quad (9)$$

3. - Determine the first-order approximation of displacement at design $a+\Delta a$,

$$u(a+\Delta a) \cong u(a) + \partial u/\partial a_i \, \Delta a_i \quad (10)$$

4.- Obtain the sensitivity of state variable by local difference:

$$\partial R/\partial a_i \cong [R(a+\Delta a, u+\Delta u) - R(a,u)] / \Delta a_i \quad (11)$$

ENTROPY BASED SHAPE OPTIMIZATION

The maximum entropy formalism is concerned with establishing what logical unbiased inferences can be drawn from available information [4]. In this work it is intended to minimize a whole set of goals such as weight, stresses at all points of a finite element mesh and nodal displacements by shape optimization. All goals need to be formulated in the normalized form. For instance, the upper limits on a stress become:

$$\sigma(a)/\sigma^U - 1 \leq 0 \qquad \leftrightarrow \qquad g(a) \leq 0 \tag{12}$$

Then, the following problem needs to be solved,

$$\mathrm{Min}_a \; F = [g_1(a), g_2(a), \ldots, g_{M+1}(a)] \tag{13a}$$

$$\mathrm{st} \quad a^L \leq a \leq a^U \tag{13b}$$

It can be reformulated in the minimax form,

$$\mathrm{Min}_a \; \mathrm{Max}_j \; F = [g_1(a), g_2(a), \ldots, g_{M+1}(a)] \tag{14a}$$

$$\mathrm{st} \quad a^L \leq a \leq a^U \tag{14b}$$

The solution a* of the minimax problem is also the solution of the convex scalar optimization problem:

$$\mathrm{Min}_{a,\, \rho \to \text{ large}} \quad F = 1/\rho \;\ln \Sigma_{j=1,M+1} \; \exp[\, \rho \, g_j(a)] \tag{15}$$

Linearizing all g(a) using the sensitivities information leads to:

$$\mathrm{Min}_{\Delta a} \quad \underline{F} = 1/\rho \;\ln \Sigma_{j=1,M+1} \; \exp\{\rho \, [g_j(a^o) + \Sigma_{i=1,N} \left(\frac{\partial g_j}{\partial a_i}\right)^o \Delta \; a_i]\} \tag{16}$$

$$\rho \to \text{ large}$$

$\underline{F}$ is a nonlinear function of Δ a and the optimal solution can be found by an unconstrained minimization algorithm, such as Hooke and Jeeves.

CONCLUSIONS

We conclude that the 20 node isoparametric element brick is an efficient element. When using such an element, a fairly coarse idealization not only gives reliable results but also provides a good representation for curved surfaces.

The efficiency of the method described to calculate the sensitities is by no means lower than that of the analytic met[illegible] because the computational effort involved in calculating $K_e(a+\Delta a)\,u - K_e(a)\,u$ and $R[a+\Delta a, U(a+\Delta a)] - R(a,u)$ is often no more than that in evaluating $(\partial K/\partial a_i)\,u$ and $(\partial Q^t/\partial a_i)\,u + Q^t(\partial u/\partial a_i)$.

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