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INFLUENCE OF SPLICES ON THE STABILITY BEHAVIOUR OF COLUMNS AND FRAMES

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Abstract. *The paper presents a study on the influence of splice connections on the stability behaviour of compressed steel columns. The column is modelled as two independent prismatic parts connected by a rotational spring at the splice location and rotational and extensional springs at the column ends to represent the effect of the adjacent structure. The general behaviour is characterized using a polynomial Rayleigh-Ritz approximation substituted into the potential energy function, in combination with the Lagrange's method of undetermined multipliers, and based on this model the critical load is found. The load-carrying capacity is analysed with respect to the following variables: (i) location and rotational stiffness of the splice, (ii) change in the column section serial size and (iii) column end-restraints stiffness coefficients. A nonlinear regression model is developed to predict simple relationships between the critical load and the relevant column characteristics.*

1 INTRODUCTION

In structural engineering practice and due to manufacturing, transportation and/or handling restraints, individual steel elements are usually fabricated with a maximum length of 12 meters. During erection of a steel frame and where the element length is insufficient, splices are provided to form a single and longer element. Designers often use the splices for changing cross-sections, in view of a more economical and rational design.

In steelwork construction, column splices are located at a convenient distance for erection and construction above floor beam level and have to be designed (i) to join lengths in line, (ii) to transmit forces and moments between the connected member parts and (iii) to maintain continuity of strength and stiffness through the splice to safeguard the robustness of the structure [1]. Column splices are usually disregarded in determining the distribution of moments and forces in the structure and when the design of the columns itself is being considered, assuming that the splice is providing full continuity in stiffness and strength of the column. This practice is questionable as the splices most times do not provide this continuity. So, the splices may adversely affect the overall frame behaviour, from a stiffness and strength point of view.

Previous research pertaining to the load-carrying capacity of spliced columns includes investigations by Lindner [2], Snijder and Hoenderkamp [3] and Girão Coelho et al. [4,5,6]. Lindner [2] carried out experimental and numerical tests on different column splice types and highlighted the existence of eccentricities at the splice. An adjusted buckling curve for columns having contact splices at column mid-height was later proposed Lindner [2]. Snijder and Hoenderkamp [3] conducted a series of experimental tests to analyse the influence of end plate splices on the load-carrying capacity of slender columns. These tests were used to make design recommendations for column splices. Girão Coelho and co-authors [4,5] further extended this work to produce a relatively simple yet reasonably accurate engineering method for predicting the critical behaviour of spliced columns in steel frames. This paper is a follow-up study to this research.

The current work presents a generalized energy formulation of a framed spliced column in sway and non-sway frames (Figure 1). For analytical modelling, a framed column is represented by means of extensional and rotational restraints at the ends A and B. The splice is modelled as a rotational spring at point C. The potential energy functional of this system uses a Rayleigh-Ritz approximation of the relevant deformation modes of the column. This formulation is presented together with the method of Lagrange multipliers to deal with the constraints at the column splice. Elastic buckling analysis is carried out to find the critical load of the system. The concept of end fixity factor C [7] is successfully applied and the significance of this factor in simplifying the analysis of results is emphasized. Simple relationships between C and the relevant characteristics of the column and splice are derived to a point where the critical load can be readily determined by hand or by computer.

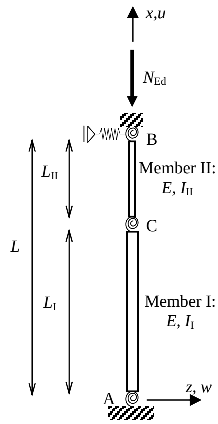


Figure 1: The framed spliced column system.

2 ENERGY FORMULATION

2.1 Bending and axial strain energy

The deformation of a prismatic member under the action of loads is characterized by axial elongation (mode 1) and bending deformations (mode 2). In the context of a simplified Generalized Beam Theory (GBT) strategy [8,9], the displacement functions are assumed to be as follows:

$$\text{Axial displacements: } u(x,y,z)=\sum_{i=1}^{2} {}^{i}u(y,z)\times {}^{i}f_u(x) \text{ and } \boldsymbol{u}=\begin{bmatrix} {}^{1}u & {}^{2}u \end{bmatrix}=\begin{bmatrix} 1 & -z_1 \\ \vdots & \vdots \\ 1 & -z_n \end{bmatrix} \tag{1}$$

$$\text{Transverse displacements: } w(x,y,z)=\sum_{i=1}^{2} {}^{i}w(y,z)\times {}^{i}f_w(x) \text{ and } \boldsymbol{w}=\begin{bmatrix} {}^{1}w & {}^{2}w \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix} \tag{2}$$

whereby z_j is the distance between point j and the neutral axis, ${}^{i}u(y, z)$ and ${}^{i}w(y, z)$ are pre-established modal displacement patterns defined along the member cross-section, and ${}^{i}f_u(x)$ and ${}^{i}f_w(x)$ are modal amplitude functions for warping and transverse displacements, respectively. For any mode of deformation the amplitude functions for axial and transverse displacements are related in the form [8,9]:

$$ {}^{i}f_u\left(x\right)=\frac{d\left[{}^{i}f_w\left(x\right)\right]}{dx}={}^{i}f_w'\left(x\right)={}^{i}f' \tag{3}$$

The extensional strain of a column segment of length dx is readily defined as:

$$\varepsilon_x=\sqrt{\left(1+\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2}-1 \tag{4}$$

Expansion in Taylor series, neglecting higher-order terms, yields the following kinematic relation:

$$\varepsilon_x\approx\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2=\sum_{k=1}^{2}\left({}^{k}u\,{}^{k}f''\right)+\frac{1}{2}\sum_{k=1}^{2}\sum_{l=1}^{2}\left({}^{k}w\,{}^{l}w\,{}^{k}f'\,{}^{l}f'\right) \tag{5}$$

From Hooke's law (constitutive relation), the longitudinal stress is then given by:

$$\sigma_x=E\varepsilon_x=E\left[\sum_{i=1}^{2}\left({}^{i}u\,{}^{i}f''\right)+\frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\left({}^{i}w\,{}^{j}w\,{}^{i}f'\,{}^{j}f'\right)\right] \tag{6}$$

The internal strain energy of the member, U_{m}, is then equal to [5]:

$$U_{\mathrm{m}}=\int_{\Omega}\frac{1}{2}\sigma_x\,\varepsilon_x\,d\Omega=\frac{EA}{2}\int_{L}\left({}^{1}f''\right)^2dx+\frac{EI}{2}\int_{L}\left({}^{2}f''\right)^2dx+\frac{EA}{2}\int_{L}\left({}^{2}f'\right)^2\,{}^{1}f''dx+\frac{EA}{8}\int_{L}\left({}^{2}f'\right)^4dx \tag{7}$$

where Ω denotes the member's volume, A the cross sectional area and I the moment of inertia.

2.2 Strain energy stored in the springs

The energy stored in the linear springs (Figure 1) is given by the following expressions:

$$\text{Rotational spring at end A: } U_{\theta\mathrm{a}}=\frac{1}{2}K_{\theta\mathrm{a}}\,\theta_{\mathrm{a}}^2=\frac{EI_{\mathrm{II}}}{2L}k_{\theta\mathrm{a}}\left({}^{2}f_{\mathrm{I}}'\Big|_{x=0}\right)^2 \tag{8}$$

$$\text{Rotational spring at end B: } U_{\theta\mathrm{b}}=\frac{1}{2}K_{\theta\mathrm{b}}\,\theta_{\mathrm{b}}^2=\frac{EI_{\mathrm{II}}}{2L}k_{\theta\mathrm{b}}\left({}^{2}f_{\mathrm{II}}'\Big|_{x=L_{\mathrm{II}}}\right)^2 \tag{9}$$

$$\text{Rotational spring at splice: } U_{\theta\mathrm{c}}=\frac{1}{2}K_{\theta\mathrm{c}}\,\theta_{\mathrm{c}}^2=\frac{EI_{\mathrm{II}}}{2L}k_{\theta\mathrm{c}}\left({}^{2}f_{\mathrm{II}}'\Big|_{x=0}-{}^{2}f_{\mathrm{I}}'\Big|_{x=L_{\mathrm{I}}}\right)^2 \tag{10}$$

$$\text{Extensional spring at end B: } U_{\Delta\mathrm{b}}=\frac{1}{2}K_{\Delta\mathrm{b}}\,\Delta_{\mathrm{b}}^2=\frac{EI_{\mathrm{II}}}{2L^3}k_{\Delta\mathrm{b}}\left({}^{2}f_{\mathrm{II}}\Big|_{x=L_{\mathrm{II}}}\right)^2 \tag{11}$$

where K_θ and K_Δ are rotational and extensional spring constants, respectively, and k are spring coefficients that are defined in non-dimensional form.

2.3 Work done by load

The final component of energy to be identified is the work done by the load. For a centrally loaded column, the potential energy of the external loading is given by:

$$\Pi=-N_{\mathrm{Ed}}\times{}^{1}f_{\mathrm{II}}'\Big|_{x=L_{\mathrm{II}}} \tag{12}$$

2.4 Potential energy functional

The total potential energy of the complete structure is a summation of U_{m} (for each individual member I and II), $U_{\theta\mathrm{a}}$, $U_{\theta\mathrm{b}}$, $U_{\theta\mathrm{c}}$, $U_{\Delta\mathrm{b}}$ minus Π:

$$V=U_{\mathrm{m,I}}+U_{\mathrm{m,II}}+U_{\theta\mathrm{a}}+U_{\theta\mathrm{b}}+U_{\theta\mathrm{c}}+U_{\Delta\mathrm{b}}-\Pi \tag{13}$$

This functional is subjected to the following kinematic constraints that ensure continuity at the splice:

$$u_{\mathrm{I},C} - u_{\mathrm{II},C} = 0 \quad \Leftrightarrow \quad {}^1 f_{\mathrm{I}}'\big|_{x=L_{\mathrm{I}}} - {}^1 f_{\mathrm{II}}'\big|_{x=0} = 0 \quad \left(G_1\left({}^i a_j, N_{\mathrm{Ed}}\right) = 0 \right)$$
$$w_{\mathrm{I},C} - w_{\mathrm{II},C} = 0 \quad \Leftrightarrow \quad {}^2 f_{\mathrm{I}}\big|_{x=L_{\mathrm{I}}} - {}^2 f_{\mathrm{II}}\big|_{x=0} = 0 \quad \left(G_2\left({}^i a_j, N_{\mathrm{Ed}}\right) = 0 \right) \tag{14}$$

We now wish to find a stationary value of a functional subjected to some subsidiary conditions or constraints $G_k({}^i a_j, N_{\mathrm{Ed}})$. The problem is easily tackled by using the approach proposed by Lagrange [10]. The technique is to form a modified potential energy expression:

$$\overline{V}\left({}^i a_j, \lambda_k, N_{\mathrm{Ed}}\right) = V\left({}^i a_j, N_{\mathrm{Ed}}\right) + \sum_{k=1}^{m} \lambda_k G_k\left({}^i a_j, N_{\mathrm{Ed}}\right) \tag{15}$$

where λ_k are the Lagrange multipliers. For the spliced column, the modified potential energy functional may be written as:

$$\overline{V} = U_{\mathrm{m}} + U_{\theta\mathrm{a}} + U_{\theta\mathrm{b}} + U_{\theta\mathrm{c}} + U_{\Delta\mathrm{b}} - \Pi + \lambda_1\left({}^1 f_{\mathrm{I}}'\big|_{x=L_{\mathrm{I}}} - {}^1 f_{\mathrm{II}}'\big|_{x=0}\right) + \lambda_2\left({}^2 f_{\mathrm{I}}\big|_{x=L_{\mathrm{I}}} - {}^1 f_{\mathrm{II}}\big|_{x=0}\right) \tag{16}$$

2.5 Critical buckling load

Eq. (16) is a functional representing the total potential energy of the physical system. The advantage of this method lies in the fact that the problem with constraints can be treated in exactly the same manner as though it was free. Thus, for the system to satisfy equilibrium $\overline{V}$ has to be stationary. The calculus of variations is then used to find the stationary point of the functional. Exact solutions can be obtained using the method of eigenvalue analysis. This is not a practical method to solve the characteristic equations of the differential equations. Approximate methods such as the Rayleigh-Ritz method seem a very attractive alternative to an otherwise complex problem. Essentially, in this method, the modes of deformation of the system are defined by means of assumed displacement functions that satisfy the geometric boundary conditions of the system. As a result, the total potential energy reduces from a functional to a gradient potential function that depends on a finite set of discrete generalized coordinates ${}^i a_j$ and λ_k and the scaling factor N_{Ed}. Thus, ordinary calculus can be used to obtain solutions directly.

In the context of the Rayleigh-Ritz method, the amplitude functions are approximated by polynomials in the form:

$${}^i f \approx \sum_{j=1}^{n_i} {}^i a_j \, {}^i \varphi_j \tag{17}$$

These polynomials form a set of coordinate functions that satisfy the kinetic boundary conditions of the problem and are orthonormal functions that enable fast convergence of the method [9]. The coordinate functions are given by:

Member I:

$${}^1\varphi_{1,\mathrm{I}} = \sqrt{5/L_{\mathrm{I}}}\, x^2/L_{\mathrm{I}}$$
$${}^2\varphi_{1,\mathrm{I}} = \sqrt{3/L_{\mathrm{I}}}\, x$$
$${}^2\varphi_{2,\mathrm{I}} = \sqrt{5/L_{\mathrm{I}}}\left(-3x + 4x^2/L_{\mathrm{I}}\right)$$
$${}^2\varphi_{3,\mathrm{I}} = \sqrt{7/L_{\mathrm{I}}}\left(6x - 20x^2/L_{\mathrm{I}} + 15x^3/L_{\mathrm{I}}^2\right)$$
$${}^2\varphi_{4,\mathrm{I}} = \sqrt{9/L_{\mathrm{I}}}\left(-10x + 60x^2/L_{\mathrm{I}} - 105x^3/L_{\mathrm{I}}^2 + 56x^4/L_{\mathrm{I}}^3\right) \tag{18}$$
$${}^2\varphi_{5,\mathrm{I}} = \sqrt{11/L_{\mathrm{I}}}\left(15x - 140x^2/L_{\mathrm{I}} + 420x^3/L_{\mathrm{I}}^2 - 504x^4/L_{\mathrm{I}}^3 + 210x^5/L_{\mathrm{I}}^4\right)$$
$${}^2\varphi_{6,\mathrm{I}} = \sqrt{13/L_{\mathrm{I}}}\left(-21x + 280x^2/L_{\mathrm{I}} - 1260x^3/L_{\mathrm{I}}^2 + 2520x^4/L_{\mathrm{I}}^3 - 2310x^5/L_{\mathrm{I}}^4 + 792x^6/L_{\mathrm{I}}^5\right)$$
$${}^2\varphi_{7,\mathrm{I}} = \sqrt{15/L_{\mathrm{I}}}\left(28x - 504x^2/L_{\mathrm{I}} + 3150x^3/L_{\mathrm{I}}^2 - 9240x^4/L_{\mathrm{I}}^3 + 13860x^5/L_{\mathrm{I}}^4 - 10296x^6/L_{\mathrm{I}}^5 + 3003x^7/L_{\mathrm{I}}^6\right)$$

Member II:

$$
\begin{aligned}
&{}^{1}\varphi_{1,\mathrm{II}} = x\big/\sqrt{L_{\mathrm{II}}}\\
&{}^{1}\varphi_{2,\mathrm{II}} = \sqrt{3/L_{\mathrm{II}}}\left(-1+x/L_{\mathrm{II}}\right)\\
&{}^{2}\varphi_{1,\mathrm{II}} = \sqrt{L_{\mathrm{II}}}\\
&{}^{2}\varphi_{2,\mathrm{II}} = \sqrt{3/L_{\mathrm{II}}}\left(-L_{\mathrm{II}}+2x\right)\\
&{}^{2}\varphi_{3,\mathrm{II}} = \sqrt{5/L_{\mathrm{II}}}\left(L_{\mathrm{II}}-6x+6x^2/L_{\mathrm{II}}\right)\\
&{}^{2}\varphi_{4,\mathrm{II}} = \sqrt{7/L_{\mathrm{II}}}\left(-L_{\mathrm{II}}+12x-30x^2/L_{\mathrm{II}}+20x^3/L_{\mathrm{II}}{}^2\right)\\
&{}^{2}\varphi_{5,\mathrm{II}} = \sqrt{9/L_{\mathrm{II}}}\left(L_{\mathrm{II}}-20x+90x^2/L_{\mathrm{II}}-140x^3/L_{\mathrm{II}}{}^2+70x^4/L_{\mathrm{II}}{}^3\right)\\
&{}^{2}\varphi_{6,\mathrm{II}} = \sqrt{11/L_{\mathrm{II}}}\left(-L_{\mathrm{II}}+30x-210x^2/L_{\mathrm{II}}+560x^3/L_{\mathrm{II}}{}^2-630x^4/L_{\mathrm{II}}{}^3+252x^5/L_{\mathrm{II}}{}^4\right)\\
&{}^{2}\varphi_{7,\mathrm{II}} = \sqrt{15/L_{\mathrm{II}}}\left(L_{\mathrm{II}}-42x+420x^2/L_{\mathrm{II}}-1680x^3/L_{\mathrm{II}}{}^2+3150x^4/L_{\mathrm{II}}{}^3-2772x^5/L_{\mathrm{II}}{}^4+924x^6/L_{\mathrm{II}}{}^5\right)
\end{aligned}
\tag{19}
$$

Equilibrium of the system is obtained by rendering stationary the total potential function with respect to the generalized coordinates ${}^{i}a_j$ and λ_k. The solution that emerges from the unloaded state, the fundamental path (FP) is a function of N_{Ed}. A sliding set of incremental coordinates ${}^{i}q_j$ and q_k is then defined by the following equations [11]:

$$
{}^{i}a_j = {}^{i}a_{j\,\mathrm{FP}}\left(N_{\mathrm{Ed}}\right) + {}^{i}q_j \quad \text{and} \quad \lambda_k = \lambda_{k\,\mathrm{FP}}\left(N_{\mathrm{Ed}}\right) + q_k \tag{20}
$$

A new energy function W is now introduced [11]:

$$
W\left({}^{i}q_j, q_k, N_{\mathrm{Ed}}\right) = \overline{V}\left({}^{i}a_{j\,\mathrm{FP}} + {}^{i}q_j, \lambda_{k\,\mathrm{FP}} + q_k, N_{\mathrm{Ed}}\right) \tag{21}
$$

A global numbering for coordinates ${}^{i}q_j$ and q_k (q_l) can now be adopted. The equilibrium and stability conditions hold good for this transformed energy function W. In this new $N_{\mathrm{Ed}} - q_l$ space, the fundamental path is defined trivially by $q_l = 0$.The critical points along the fundamental path are now those points that render zero the determinant of the total potential energy Hessian matrix along the fundamental path:

$$
\mathbf{H}_{\mathrm{FP}} = \mathbf{H}_{\mathrm{FP},0} + N_{\mathrm{Ed}}\ \mathbf{H}_{\mathrm{FP},1} \tag{22}
$$

The relevant states of critical equilibrium are identified via a local linear eigenvalue equation $\mathbf{H}_{\mathrm{FP}}\,\mathbf{q} = 0$, $\mathbf{q}$ representing the local eigenvector [11]. By substituting the forms in Eq. (22), we thus obtain the critical state identity:

$$
\mathbf{H}_{\mathrm{FP}}\mathbf{q} = \left(\mathbf{H}_{\mathrm{FP},0} + N_{\mathrm{Ed}}\mathbf{H}_{\mathrm{FP},1}\right)\mathbf{q} = 0 \tag{23}
$$

This analysis yields the critical buckling load of the spliced column, N_{cr} that can be expressed in terms of an end fixity factor C [7]:

$$
N_{\mathrm{cr}} = C\pi^2 EI_{\mathrm{II}}\big/L^2 \tag{24}
$$

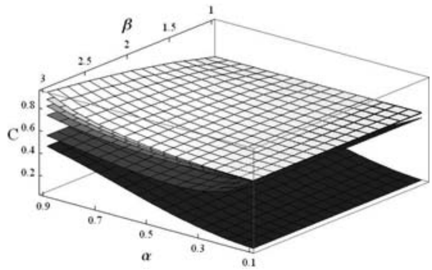
3 NUMERICAL RESULTS

The purpose of this numerical study is to ascertain the effect of the following variables on the general equilibrium response: (i) splice location ($L_{\mathrm{I}} = \alpha L$), (ii) ratio between second moment of area of lower and upper column members ($\beta = I_{\mathrm{I}}/I_{\mathrm{II}}$), (iii) splice rotational stiffness ($k_{\theta\mathrm{c}}$) and (iv) end-restraints stiffness coefficients ($k_{\theta\mathrm{a}}$, $k_{\theta\mathrm{b}}$ and $k_{\Delta\mathrm{b}}$). Results are independent from the column length L. These properties are varied parametrically as shown in Table 1.

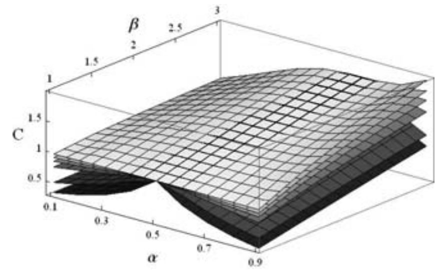
Table 1: Parameters for regression analysis

Parameter	Range of parameter selected
Splice location α	0.1 to 0.9, i = 0.2
Ratio between second moment of area β	1 to 3, i = 0.5
Non-dimensional stiffness coefficients	→0, 0.25, 0.5, 1, 3, 5, 7.5,10, 15, 20, 35, 50, 75, 100

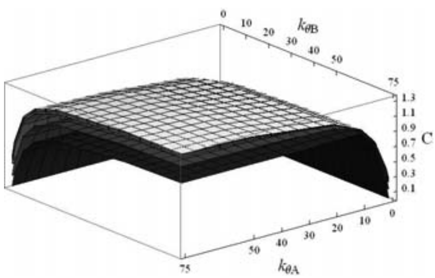
These results form a comprehensive analytical database. We can now generate a continuous function that approximates the values of the end fixity factor within the domain of analyses and with a minimum error. The analysis results are then used to develop a multiple regression model to approximate the end fixity factor $C \equiv C_{\text{fit}}(\alpha, \beta, k_{\theta a}, k_{\theta b}, k_{\theta c}, k_{\Delta b})$ from the data in the database, by means of piecewise approximations. In developing the regression model, the relationship between the dependent variable and each independent variable is studied separately, while all other independent variables are kept constant. The dependent variable is approximated by a continuous function that is linear in terms of a set of regression coefficients, which are determined by enforcing the method of least squares that minimizes the sum of the squares of the residuals. Approximating (or coordinate) functions are then selected for each independent variable. The multiple regression model is formed as the product of the individual coordinate functions.



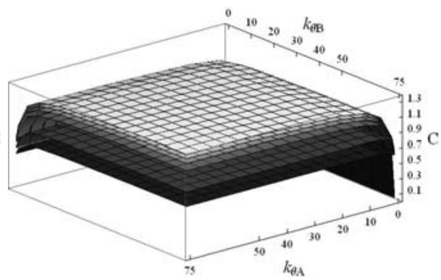
$k_{\theta a} = 0.01$, $k_{\theta b} = 100$, $k_{\Delta b} = 5$
$k_{\theta c} = 0.01, 1, 5, 10, 20, 100$



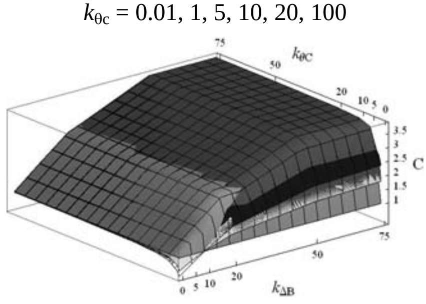
$k_{\theta a} = 100$, $k_{\theta b} = 100$, $k_{\Delta b} = 0.01$
$k_{\theta c} = 0.01, 1, 5, 10, 20, 100$



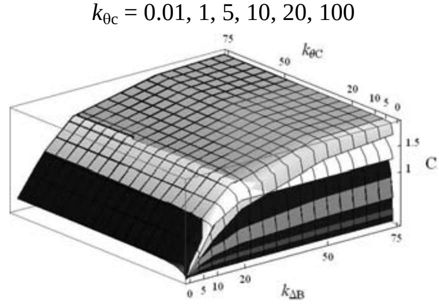
$\beta = 2$, $\alpha = 0.5$, $k_{\Delta b} = 0.01$
$k_{\theta c} = 0.01, 1, 5, 10, 20, 100$



$\beta = 1$, $\alpha = 0.1$, $k_{\Delta b} = 5$
$k_{\theta c} = 0.01, 1, 5, 10, 20, 100$



$\beta = 1$, $k_{\theta a} = 100$, $k_{\theta b} = 100$
$\alpha = 0.1, 0.3, 0.5, 0.7, 0.9$



$\beta = 1$, $k_{\theta a} = 0.01$, $k_{\theta b} = 100$
$\alpha = 0.1, 0.3, 0.5, 0.7, 0.9$

Figure 2: Variation of *C* with the relevant properties.

Forming the regression model as a product allows the effect of each independent variable to be examined separately and facilitates the process of selecting suitable coordinate functions for the individual independent variables. Some key results are illustrated graphically in Figure 2. The graphics suggest that:

1. The response C *vs.* α can be approximated by a quadratic function.

2. Typical C *vs.* β behaviour is characterized by a monotonic increasing function that can be generally approximated by a simple linear relationship.

3. The degree of rotational and extensional end restraint is an essential parameter for the computation of C. The shape of the curves C *vs.* k ($k \equiv k_{\theta a}$, $k_{\theta b}$), C *vs.* $k_{\theta c}$ and C *vs.* $k_{\Delta b}$ is best described by an arctangent function.

The end fixity factor is then predicted by means of an expression in the form:

$$C_{\text{fit}}\left(\alpha,\beta,k_{\Delta b},k_{\theta a},k_{\theta b},k_{\theta c}\right)=C_1\left(1+C_2\alpha+C_3\alpha^2\right)\left(1+C_4\beta\right)\left[1+C_5\arctan\left(\frac{k_{\Delta b}}{C_6}\right)\right]\times$$
$$\times\left[1+C_7\arctan\left(\frac{k_{\theta a}}{C_8}\right)\right]\left[1+C_9\arctan\left(\frac{k_{\theta b}}{C_{10}}\right)\right]\left[1+C_{11}\arctan\left(\frac{k_{\theta c}}{C_{12}}\right)\right] \tag{25}$$

where C_i are regression coefficients.

As expected in developing a predictive regression model, many models were tried, analysed and assessed for accuracy and effectiveness. The final model presented here evolved out of several attempts to develop conventional (nonlinear) regression models by means of simple mathematical functions. The overall character of the response is well captured and the number of regression coefficients is kept small in order to provide a compact procedure for the simplified method. The accuracy of the model is measured by means of the R-Squared value (R^2). The R-Squared gives the fraction of the variation of the response that is predicted by the model. A good model fit yields values of R-squared close to unity.

Nonlinear regression analysis is performed with the Mathematica software [12]. Regression coefficients are determined for the spliced column using piecewise approximations depending on the nature of the segments that comprise the above relationship. The domain of analyses of the spring coefficients is divided into three intervals: G1 for $k \in]0,3]$, G2 for $k \in]3,15]$ and G3 for $k \in]15,100]$. Table 2 sets out the computed regression coefficients and values for the R-Squared factor are also given.

4 CONCLUDING REMARKS AND SCOPE FOR FURTHER WORK

The paper has presented an application of the total potential energy method to the buckling behaviour of a spliced column in sway and non-sway frames. This is a variational problem, i.e. the finding of thestationary point of a functional, with additional conditions at the splice that is solved by means of the Lagrange's method of undetermined multipliers. The Rayleigh-Ritz procedure has been used to reduce this variational problem with constraints to a mere differentiation that form a set of algebraic equations of equilibrium. These equations are solved by using an algebraic manipulator [12] and the critical load is calculated. The buckling response was found to be particularly sensitive to the following variables: (i) splice location, (ii) ratio between second moment of area of lower and upper column members, (iii) splice rotational stiffness and (iv) end-restraints stiffness coefficients. The significance of each of these variables has been assessed. A parametric study was devised and the results were then used to develop regression equations for predicting the critical load of the system via the concept of end fixity factor C.

The work outlined above affords some basis to produce design guidance on column splices. The authors attempt to set up sound design criteria regarding the requirements for stiffness and strength of column splices. Experimental and numerical finite element studies focusing on the buckling response are also necessary in order to validate the predictive expressions.

It should be noted that the investigated configuration was rather limited to a particular case. The derivation has been carried for two-dimensional frames and only uniaxial bending behaviour has been considered. Future work will incorporate bi-axial bending and torsion effects in the design of splices. In addition to this, the influence of splices on the overall stability behaviour of frames will also be investigated.

Table 2: Parameter table.

Range	$k_{\theta a} \in G1$, $k_{\theta b} \in G1$	$k_{\theta a} \in G1$, $k_{\theta b} \in G2$	$k_{\theta a} \in G1$, $k_{\theta b} \in G3$	$k_{\theta a} \in G2$, $k_{\theta b} \in G1$	$k_{\theta a} \in G2$, $k_{\theta b} \in G2$	$k_{\theta a} \in G2$, $k_{\theta b} \in G3$	$k_{\theta a} \in G3$, $k_{\theta b} \in G1$	$k_{\theta a} \in G3$, $k_{\theta b} \in G2$	$k_{\theta a} \in G3$, $k_{\theta b} \in G3$
C_1	0.0188	0.0536	0.0621	0.0605	0.0890	0.1005	0.0685	0.0865	0.0995
C_2	-1.0746	-0.8905	-0.5538	-0.9201	-0.6922	-0.4441	-0.6239	-0.3152	-0.0097
C_3	1.6481	1.5441	1.2462	1.2899	1.1190	0.8795	1.0100	0.7906	0.4993
C_4	0.2417	0.2390	0.2456	0.1874	0.1860	0.1934	0.2202	0.2304	0.2437
C_5	8.5978	4.9043	4.7424	3.6457	2.7180	2.7107	3.2346	2.4620	2.4769
C_6	8.1153	13.0053	16.9895	11.4866	15.2379	19.5321	16.3117	20.9021	26.3750
C_7	0.2736	0.3294	0.3289	0.4854	0.6565	0.7675	0.5434	0.6613	0.7640
C_8	1.5	1.5	1.5	3.0	3.0	3.0	5.0	5.0	5.0
C_9	0.3438	0.4419	0.3667	0.4213	0.5721	0.3987	0.4304	0.6923	0.4497
C_{10}	1.5	3.0	5.0	1.5	3.0	5.0	1.5	3.0	5.0
C_{11}	1.5929	0.6725	0.6498	0.9489	0.5607	0.5687	0.8628	0.5690	0.5945
C_{12}	1.3821	1.6983	1.8891	1.5744	1.9524	2.3116	1.7536	2.1201	2.6502
R^2	0.9043	0.8986	0.9115	0.9115	0.9311	0.9447	0.9057	0.9381	0.8874

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