

Predicting Transverse Turbulent Diffusivity in Straight Alluvial Rivers

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ABSTRACT

In this work the distribution and mean of transverse turbulent diffusion coefficient in straight alluvial rivers are determined using a vertical turbulent diffusion coefficient and a hydraulic geometry equation relating the depth to the lateral position in the section. The cross-section averaged transverse turbulent diffusion coefficient was found to be in good agreement with 138 sets of experimental data.

1. INTRODUCTION

Turbulent transport of momentum, heat and mass dominates many fluid flow phenomena occurring in physics, fluid mechanics, hydraulic engineering, and environmental sciences (Nezu and Nakagawa, 1993). The capability of turbulent transport in longitudinal, vertical, and transverse directions (denoted by x , z , and y) is often described by turbulent diffusion coefficients or diffusivities for momentum, mass and heat. Of practical importance are the vertical and transverse diffusion coefficients. The vertical diffusion coefficient is relatively easy to determine and its value is thus widely reported in the literature (Fischer et al., 1979; and Rutherford, 1994). However, there is no generally accepted method for estimating the transverse turbulent diffusion coefficient.

The purpose of this paper is to present a simple yet accurate method for estimating the transverse turbulent diffusion coefficient in straight rivers. To be able to derive a theoretical expression for the transverse turbulent diffusion coefficient and its distribution over the flow field in a straight channel, the lateral distribution of the flow depth must be known. The objectives of this study are therefore (1) to determine the cross-sectional channel shape of

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alluvial rivers; (2) to define the cross-sectional distribution of the local flow depth in straight rivers; (3) to establish theoretical expressions representing the spatial distribution and cross-section-averaged value of the transverse turbulent diffusion coefficient; and (4) to clarify different expressions of dimensionless transverse turbulent diffusion coefficients.

2. THEORETICAL BACKGROUND

In terms of the ‘Reynolds analogy’ (Schlichting, 1979), the diffusivities ν_t for momentum and ϵ_t for mass are equal for passive substances in isotropic turbulence (Fischer et al., 1979; and Chang, 1988). The thermal diffusivity is often related directly to the eddy diffusivity ν_t by the turbulent Prandtl number. The eddy diffusivity ν_t is, therefore, enormously important to turbulent transport processes, such as effluent mixing in near-field, sediment transport, turbulence-driven secondary motion, and so on. Consequently, many investigations concerning turbulent transport processes have focused on determining ν_t . The eddy diffusivity ν_t was introduced by Boussinesq in 1877 on the assumption that, in analogy with the viscous stresses in laminar flow, turbulent stresses are proportional to the mean-velocity gradients. Various turbulence models have been developed to determine the distribution of ν_t over flow fields (ASCE, 1988; Nezu and Nakagawa, 1993).

The mixing of pollutants in anisotropic turbulent flow is a common problem in environmental fluid dynamics. The anisotropic turbulence is characterized by three different diffusivities instead of a single ν_t , i.e., vertical turbulent diffusion coefficient ϵ_z , longitudinal turbulent diffusion coefficient ϵ_x , and transverse turbulent diffusion coefficient ϵ_y . From the ‘Reynolds analogy’ and log-law of vertical velocity distribution, and assuming a linear shear stress distribution, $\tau(z) = \tau_{\text{bottom}} (1-z/h)$, ϵ_z can be described by a parabolic distribution:

$$\epsilon_z(\eta) = \nu_t = \kappa h u_* \eta(1-\eta) \quad (1)$$

where: h is the local flow depth; u_* is the corresponding local shear velocity; and $\eta = z/h$ is a non-dimensional depth. Eq. (1) was derived for infinite wide channels with constant flow depth h . For a natural river with variable flow depth across the channel Eq. (1) is assumed to be valid locally. The mean vertical turbulent diffusion coefficient can be expressed as

$$\bar{\epsilon}_z = \frac{1}{6} \kappa u_* h \quad (2)$$

which is found after integration of Eq. (1) over the non-dimensional depth η . The von Karman constant, κ , is approximately equal to 0.4. The theoretical eddy viscosity value of the flow down an infinitely wide inclined plane, as proportional to the friction velocity and the depth and with a proportionality coefficient of $\kappa/6 = 0.07$, was given by Elder (Fischer et al., 1979). Many laboratory experiments and also field experiments have shown that this coefficient is close to 0.1.

A tracer or some pollutant is mixed quite quickly over the full channel depth. From then on the mixing is controlled by the transverse turbulent diffusion until mixing is complete over the full channel width. Much work has been done to determine the transverse turbulent diffusion coefficient for different flow conditions, using both turbulence theory and laboratory experiments. By analogy with the vertical coefficient, in Eq. (2), it has been argued that the transverse mixing coefficient may be related to the friction velocity and a length scale, usually the depth. However, the question of whether it is theoretically possible to predict the value of

the transverse eddy diffusivity without relying on experiments remains open (Rutherford, 1994; Fischer et al., 1979). As a result, a large number of experiments on transverse mixing have been conducted to investigate the factors controlling transverse turbulent mixing.

Using a dimensional analysis method and 71 sets of experimental data in straight rectangular channels, Lau and Krishnappan (1977) have found that: (1) the cross-section averaged dimensionless transverse turbulent diffusion coefficient $E_y/(u_*B)$ decreases with increasing channel width-depth ratio B/H , and increases as the Darcy-Weisbach friction factor f increases; (2) the dominant mechanism in transverse spreading is the secondary circulation which is governed by B/H in a rectangular channel; (3) it is better to use the dimensionless turbulent diffusion coefficient $E_y/(u_*B)$ than $E_y/(u_*H)$. Here u_* is the cross-sectional mean value of shear velocity. Webel and Schatzmann (1984) carried out a systematic experimental study and found that $E_y/(u_*H)$ has a constant value of 0.13, which agrees quite well with $E_y/(u_*H) = 0.17$, measured in a straight section of the River Rhine near Karlsruhe in Germany. Based on the results of a total of 75 separate experiments in straight channels, Fischer et al. (1979) suggested that $E_y = 0.15u_*H$. Rutherford (1994) has summarized the results from a number of studies and found that $E_y/(u_*H)$ lies in the range of 0.10 to 0.26. The ratio between the transverse and vertical cross-section averaged diffusion coefficients appears to be $E_y/E_z \approx 2-3$. It should be pointed out that the notations E_x , E_y and E_z are the cross-section averaged diffusion coefficients and $\bar{\epsilon}_x$, $\bar{\epsilon}_y$, and $\bar{\epsilon}_z$ are the depth averaged diffusion coefficients, corresponding to the local diffusion coefficients ϵ_x , ϵ_y , and ϵ_z , respectively.

3. TRANSVERSE DISTRIBUTION OF LOCAL FLOW DEPTH

The cross-sectional shape of a natural river and the lateral distribution of flow depth h influence both the velocity distribution and the transverse mixing of river pollutants. The cross-sectional shapes of straight alluvial channels have been the subject of many investigations, as reported by the ASCE Task Committee (1998). The generalized typical cross-section of a river channel is symmetrical, as shown in Fig. 1, and is constant along the river. The local flow depth $h(y)$ varies with the lateral distance y from the channel centre, where $y=0$. B is the full width of the symmetrical channel and b is half the width, $b=B/2$. The maximum depth $h=H_c$ is in the channel centre where $y=0$. Three types of equations have been proposed to describe the cross-sectional shape of straight channels: the cosine, the exponential and the parabolic cross-sectional shape. The cosine and parabolic shapes are very similar. The most commonly assumed distribution is the parabolic one.

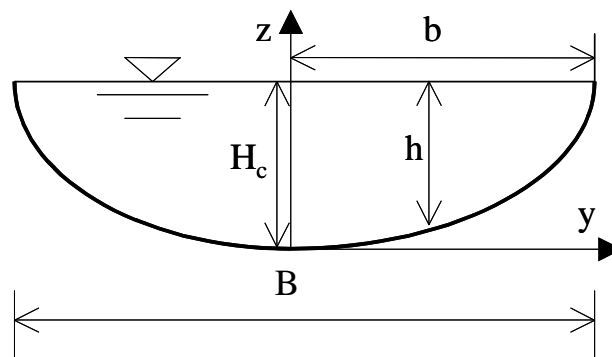


Figure 1 - Coordinate system and generalized channel profile

The parabolic transverse profile proposed by Mironenko et al. (1984) and Cao and Knight (1997) lead to the equation:

$$\frac{h(y)}{H_c} = 1 - \left(\frac{y}{b} \right)^2 \quad (3)$$

Channel shape equations, such as Eq. (3), are only applicable to canals or to the bank regions of straight rivers. To describe the cross sectional shape of natural alluvial rivers, the channels are usually generalized with a flat-bed region and two curving bank regions (Vigilar and Diplas, 1997). The width of the flat-bed region is determined numerically. This means that available channel shape equations cannot be used directly to simulate the cross-sectional shape of natural rivers.

The stable channel morphology of a natural alluvial river is described by its hydraulic geometry, referring to the interrelations among water discharge, channel width, flow depth, velocity, etc. The hydraulic geometry of rivers is distinguished between at-a-station hydraulic geometry and downstream hydraulic geometry. At a river cross-section, the water surface width B and mean flow depth H vary with flow discharge Q . Leopold and Maddock (1953), Chang (1988), Richards (1982), and Chien et al. (1987) are among authors who describe these variables as power functions of the discharge:

$$B = aQ^\delta \quad (4)$$

$$H = dQ^\theta \quad (5)$$

where a , d , δ , and θ are numerical constants. The average values of the exponents δ and θ have been obtained by Chien et al. (1987) for 374 river cross-sections, representing a large variety of rivers from all over the world. These averages are $\delta = 0.14$ and $\theta = 0.43$. The channel shape with $\delta = 0.14$ and $\theta = 0.43$ corresponds to the rivers occurring most frequently in nature (Park, 1977), and thus is the most stable channel shape (Deng and Singh, 1999). It is logical to define a channel shape parameter, $\beta = \theta/\delta$. $\beta \approx 3.07$ for stable rivers in dynamic equilibrium. However, most natural rivers are not in the dynamic equilibrium state and therefore their channel shape parameter β should be a variable rather than a constant value. Eqs. (4) and (5) lead to the following at-a-station hydraulic relation between surface flow width and mean depth:

$$B = eH^{1/\beta} \quad (6)$$

where e is a constant. The generalized parabolic curve is $h = H_c - py^q$, which with $y = 0$ at the centreline and $y = b$ at the shore, means that $H_c = pb^q$. The mean depth in a cross-section is then $H_c q/(q+1)$ or $pb^q[q/(q+1)]$. To satisfy the relationship of Eq. (6) q must be equal to β . With β as the only parameter, the dimensionless depth variation in a cross section is given by Eq. 7. This expression originates from the at-a-station hydraulic geometry equations and is supported by 374 sets of alluvial river data.

$$\frac{h(y)}{H_c} = 1 - \left(\frac{y}{b} \right)^\beta \quad (7a)$$

or

$$\frac{h(\xi)}{H_c} = 1 - \xi^\beta \quad (7b)$$

where a dimensionless lateral coordinate $\xi = y/b$ is introduced. The coordinate system is shown in Fig. 1. Eq. (7a) reduces to Eq. (3) when $\beta = 2$. In general, the value of the parameter

β is larger than 2 and should be closely associated with the width-depth ratio B/H for natural alluvial rivers. Eq. (3) with $\beta = 2$ is found valid by Cao and Knight (1997) under the condition that $B/H=8$ or $\ln(B/H)\approx 2.08$. Moreover, using the model of plane geometry of river meandering proposed by Chang (1988), it is found that $B/H = 21$ or $\ln(B/H) \approx 3.04$ for stable straight rivers with an arc angle close to zero. As mentioned earlier, the mean channel shape parameter of straight stable rivers is $\beta \approx 3.07$. Thus, $\beta = \ln(B/H)$ when $B/H = 8$ and when 21. On these grounds, it is inferred that a functional relationship exists between the channel shape parameter β and the channel width-depth ratio B/H :

$$\beta = \ln(B/H) \quad (8)$$

From Eq. (7b), the cross-section averaged flow depth is obtained as

$$H = H_c \int_0^1 (1 - \xi^\beta) d\xi = \frac{\beta}{\beta + 1} H_c \quad (9)$$

4. TRANSVERSE TURBULENT DIFFUSIVITY IN STRAIGHT ALLUVIAL RIVERS

Since the lateral as well as the vertical turbulent diffusivities can be related to a length and a velocity scale, it is assumed that the turbulent diffusivities in the vertical and lateral directions are related to each other by a parameter γ , such that

$$\varepsilon_z = \gamma \varepsilon_y \quad (10)$$

Combining equations (1) and (10) yields

$$\varepsilon_y(\xi, \eta) = \kappa u_* \eta (1 - \eta) / \gamma \quad (11)$$

Using Eqs. (7), (9), and $u_* = (ghS)^{1/2}$, and the relationship between the friction velocity at the centreline and the centreline depth $u_{* \text{centreline}} = (g S H_c)^{1/2}$, replacing the local friction velocity $u_{* \text{local}}$ with the cross-sectional mean value \bar{u}_* , $u_{* \text{local}} = u_{* \text{centreline}} (1 - \xi^\beta)^{1/2} = \bar{u}_* (\beta + 1) / \beta (1 - \xi^\beta)^{1/2}$, the spatial distribution of transverse turbulent diffusion coefficient is expressed as follows:

$$\varepsilon_y(\xi, \eta) = \frac{\kappa}{\gamma} \left(\frac{\beta + 1}{\beta} \right)^{3/2} (1 - \xi^\beta)^{3/2} H \bar{u}_* (1 - \eta) \eta \quad (12)$$

Integrating Eq. (12) over the vertical η one obtains:

$$\bar{\varepsilon}_y(\xi) = \frac{1}{6} \frac{\kappa}{\gamma} \left(\frac{\beta + 1}{\beta} \right)^{3/2} (1 - \xi^\beta)^{3/2} H \bar{u}_* \quad (13)$$

Eq. (13) can be recast as

$$\bar{\varepsilon}_y(\xi) = \frac{1}{6} \frac{C}{\gamma} \kappa H \bar{u}_* \quad (14)$$

where $C = [(\beta+1)/\beta]^{3/2}(1-\xi^\beta)^{3/2}$. The variation of parameter C with channel shape parameter β is shown in Fig. 2 and Fig. 3. Fig. 2 demonstrates that the parameter C depends on both the channel shape parameter β and the lateral distance ξ from the channel centre.

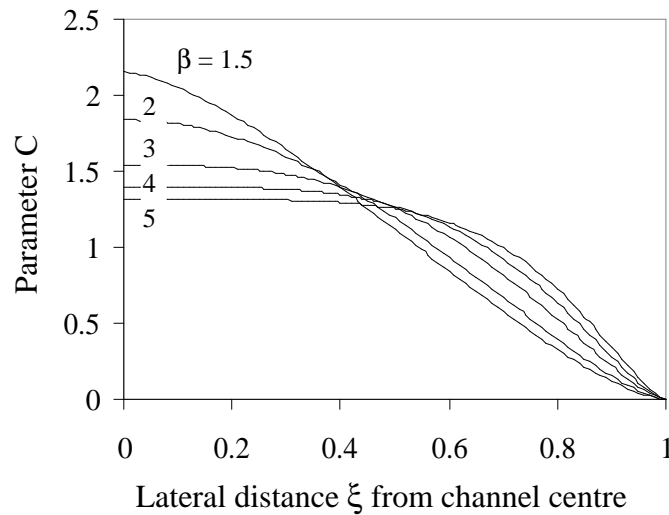


Figure 2 - Variation of parameter C with β

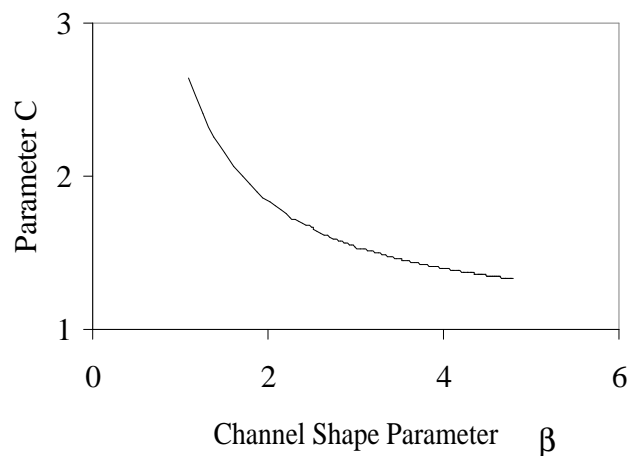


Figure 3 - Variation of parameter C with β in channel centre

Parameter C varies dramatically with β and is determined only by β in the channel centre, as shown in Fig. 3. However, the integration of C from $\xi = 0$ to 1 indicates that the mean value of C changes in a narrow range from 1.10 to 1.04 when β varies from 1.5 to 6.0. It should be noted that $\bar{C}=1.04 - 1.10$ is the cross-sectional average of C values for $\beta=1.5 - 6.0$ instead of the C value shown in Fig. 3. Thus, for simplicity, the parameter C can be taken as a constant value of 1.06. It means that channel width-depth ratio has a significant influence on the distribution of lateral turbulent diffusion coefficient, especially in the centre area of a channel with small width-depth ratio, but little on the cross-sectional average value of ϵ_y . It is therefore expected that the influence of β on the distribution of sediment and pollutants in rivers increases with decreasing channel width-depth ratio.

Based on the experimental data published by Rutherford (1994), it is found that $\gamma=1/2.26$. Substitution of $C=1.06$, $\kappa=0.4$ and $\gamma=1/2.26$ into Eq. (14) yields the cross-section averaged transverse turbulent diffusion coefficient for natural rivers:

$$\frac{E_y}{H\overline{u_*}} = 0.16 \quad (15)$$

or

$$\frac{E_y}{B\overline{u_*}} = \frac{0.16}{B/H} \quad (16)$$

It is apparent that Eqs. (15) and (16) are equivalent and can be transformed mutually by B/H . The accuracy of Eqs. (15) and (16) is verified by means of 138 sets of experimental data on transverse turbulent diffusion collected by Rutherford (1994). Fig. 4 demonstrates that the transverse turbulent diffusion coefficient predicted by Eq. (16) agrees well with that given by the 138 sets of measured data. Therefore, Eqs.(15) and (16) can be used to describe the section averaged lateral turbulent diffusion coefficient in straight natural rivers.

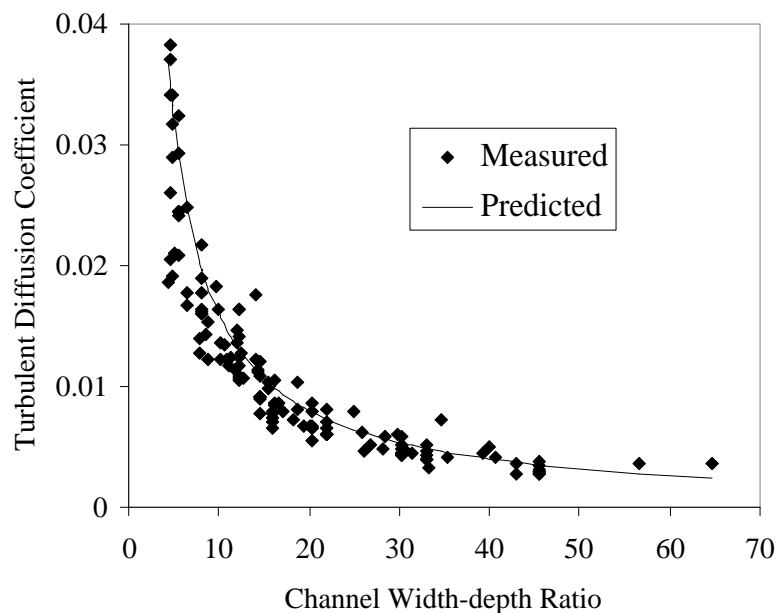


Figure 4 – Variation of turbulent diffusion coefficient with width-depth ratio

5. CONCLUSIONS

The following conclusions are drawn from this study:

- The cross-sectional channel shape of straight alluvial rivers can be described by the power law equation (7).
- Two expressions for $E_y/B\overline{u_*}$ and $E_y/H\overline{u_*}$ are equivalent and can be transformed mutually by B/H . The dimensionless transverse turbulent diffusion coefficient $E_y/H\overline{u_*}$ can be approximately regarded as a constant with a value of 0.16 for straight natural rivers. The ratio between the transverse and vertical turbulent diffusion coefficients is $E_y/E_z = 2.26$.
- Spatial distributions of dimensionless turbulent diffusion coefficient can be described by Eqs. (12) and (13). The channel shape parameter β has a significant influence on the distribution of transverse turbulent diffusion coefficient in natural rivers, and the influence

increases with decreasing channel shape parameter β . Nevertheless, the cross-section averaged transverse turbulent diffusion coefficient is somewhat independent on β .

6. ACKNOWLEDGMENTS

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7. REFERENCES

ASCE Task Committee on Hydraulics, Bank Mechanics and Modeling of River Width Adjustment. River width adjustment, I: Processes and Mechanisms, *Journal of Hydraulic Engineering*, ASCE, **124**(9), 881-902 (1998).

ASCE Task Committee on Turbulence Models in Hydraulic Computations, Turbulence modeling of surface water flow and transport, Parts I-III, *Journal of Hydraulic Engineering*, ASCE, **114**(9), 970-1051 (1988).

Cao, S.Y. and Knight, D.W., Entropy-based design approach of threshold alluvial channels, *Journal of Hydraulic Research*, IAHR, **35**(4), 505-524 (1997).

Chang, H.H., *Fluvial processes in river engineering*, John Wiley & Sons, Inc., New York, USA (1988).

Chien, N., Zhang, R. and Zhou, Z.D., *Fluvial processes*, Science Press, Beijing, China, 356-365 (1987).

Deng, Z.Q. and Singh, V.P., Mechanism and conditions for change in channel pattern, *Journal of Hydraulic Research*, IAHR, **37**(4), 465-478 (1999).

Fischer, H.B., List, E.J., Kon, R.C.Y., Imberger, J. and Brooks, N.H., *Mixing in inland and coastal waters*, Academic Press, New York, U.S.A., 1-145 (1979).

Lau, Y.L. and Krishnappan, B.G., Transverse dispersion in rectangular channels, *Journal of the Hydraulics Division*, ASCE, **103**(10), 1173-1189 (1977).

Leopold, L. B. and Maddock, T. J., Hydraulic geometry of stream channels and some physiographic implications, *U. S. Geological Survey Professional Paper 252*, 55p. (1953).

Mironenko, A.P., Willardson, L.S. and Jenab, S.A., Parabolic canal design and analysis, *Journal of Irrigation and Drainage Engineering*, ASCE, **110**(2), 241-246 (1984).

Nezu, I. and Nakagawa, H., *Turbulence in open-channel flows*, A.A.Balkema, Rotterdam, Netherlands (1993).

Park, C.C., World-wide variations in hydraulic geometry exponents of stream channels: an analysis and some observations, *Journal of Hydrology*, **33**(6), 133-146 (1977).

Richards, K., *Rivers: form and process in alluvial channels*, Methuen & Co. Ltd., London, 148-179 (1982).

Rutherford, J.C., *River mixing*. John Wiley & Sons, Chichester, England (1994).

Schlichting, H., *Boundary-layer theory* (Seventh Edition), McGraw-Hill Book Company, U.S.A., 706-712 (1979).

Vigilar, G. G. and Diplas, P., Stable channels with mobile bed: formulation and numerical solution, *Journal of Hydraulic Engineering*, ASCE, **123**(3), 189-199 (1997).

Webel, G. and Schatzmann, M., Transverse mixing in open channel flow, *Journal of Hydraulic Engineering*, ASCE, **110**(4), 423-435 (1984).

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