JOSÉ ALBERTO SOARES DA FONSECA

RISK PREMIUMS IN THE PORTUGUESE TREASURY
BILLS INTEREST RATES FROM 1990 TO 1998:
AN ARCH-M APPROACH

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Risk Premiums in the Portuguese Treasury Bills

Interest Rates from 1990 to 1998:
An ARCH-M Approach

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ABSTRACT

One central subject in the literature on the term structure of interest rates is the empirical evidence about risk premiums and their stochastic processes. The traditional theory of the term structure accepted that risk premiums were zero or monotonically increasing with bonds’ maturity.

The new dynamic approach based on intertemporal equilibrium developed by Vasicek (1977), Cox, Ingersoll and Ross (1985) and others, has demonstrated that risk premiums are not proportional to bonds’ maturity, but to the stochastic risk measure of the bond. However, in some of those models it is accepted that the market price of risk is constant over time. A significant change in the theoretical analysis of risk premiums has occurred with the study of Longstaff and Schwartz (1992), according to which the term structure is explained by the stochastic processes followed by a short-term interest rate and its volatility.

One of the main objectives of the present research is to study the (non) stationarity of the series of the Portuguese Treasury bills rates during the 1990s, and the influence on their autoregressive process, of ERM crises, and the convergence criteria for stage 3 of EMU. The other objective is to determine the existence of forward and return premiums and their time variability. An ARCH-M(1) model has revealed to present good explanatory power about the process followed by those premiums.
Introduction

The shape of the yield curve, or term structure of interest rates, is determined by expectations about future interest rates and by risk premiums. Expectations on the future interest rates can be obtained from the stochastic processes followed by current rates. Three types of risk premiums can be measured in the term structure of interest rates: the forward rate premiums, the holding return premiums and the yield-to-maturity premiums. Forward and holding return premiums in Portuguese Treasury bills rates are the subject of this paper, which is divided in two sections. The first section is consecrated to the exposition of the three types of premiums and to their relations. The second section is dedicated to the empirical analysis of the Portuguese Treasury bills interest rates. The first part of the empirical analysis consists in the study of the stationarity of the series. The second part consists in extracting forward and holding return premiums from the interest rate series and in modelling their stochastic processes.

I. Expectations and Risk Premiums in the Term Structure of Interest Rates

Let us represent by \( P_{t,n} \) the price, at time \( t \), of a zero coupon bond, with a maturity equal to \( n \) periods, and a final value of 1. With a continuous time interest compounding, the yield-to-maturity of this bond is defined by:

\[
R_{t,n} = -\frac{\ln P_{t,n}}{n}
\]

(1)

The return from holding this long-term bond between \( t \) and \( t+1 \) is:

\[
h(n)_{t,t+1} = \ln P_{t+1,n-1} - \ln P_{t,n}
\]

(2)

The expected return from holding, between dates \( t \) and \( t+1 \), a zero coupon bond, with an initial maturity \( n \), \( E[h(n)_{t,t+1}] \), is equal to the riskless short-term interest rate, \( r \), plus a holding return premium, \( \phi(n) \):
According to the common dynamic models of bond evaluation, as those of Vasicek (1977) and Cox, Ingersoll and Ross (1985), the return premium depends on the bond’s measure of risk (stochastic duration) and on the market price of risk.

Replacing in equation (2) \( \ln(P_{t,n}) \) and \( \ln(P_{t+1,n-1}) \) according to the definition of yield-to-maturity given in equation (1), the expected holding period return can be represented as a function of the yield-to-maturity:

\[
E(R_{t+1,n-1}) - R_{t,n} = \frac{1}{n-1} (R_{t,n} - r_t) - \frac{1}{n-1} \phi(n)_t
\]  

Solving equation (4) forward, between \( t \) and \( t+n-1 \), the yield-to-maturity, \( R_{t,n} \) can be represented as the sum of the expected values of the short term interest rate plus the yield-to-maturity premium, \( \Pi(n) \):

\[
R_{t,n} = \frac{1}{n} \sum_{i=1}^{n} E_t (r_{t+i-1}) + \Pi(n)_t
\]  

The relation between the yield-to-maturity premium and the holding return premiums is:

\[
\Pi(n)_t = \frac{1}{n} \sum_{i=1}^{n} E_t (\phi(n - i + 1))_{t+i-1}
\]  

The forward interest rate referred to date \( t+n \) in the future is:

\[
f(t, t+n) = \ln \left( \frac{P_{t,n}}{P_{t, n+1}} \right)
\]  

and the forward premium is the difference between the forward rate and the corresponding expected short-term rate:

\[
L(n)_{t,t+n} = f(t, t+n) - E_t (r_{t+n})
\]
According to the definition of yield-to-maturity, given by (1), the following representation can be given to the forward rate:

\[ f(t, t + n) = (n + 1) R_{t, t+n} - n R_{t, n} \]  

(9).

From equation (5), (6) and (8), the following relation results between the forward premium and the holding return premiums:

\[ L(n)_{t, t+n} = \sum_{i=1}^{n} E_t \left[ \phi(n - i + 1)_{t, t+n} - \phi(n - i)_{t, t+n} \right] \]  

(10).

The possibility of all types of risk premiums being equal to zero is accepted by the traditional expectations hypothesis. However, Cox, Ingersoll and Ross (1981) have demonstrated that zero risk premiums only can be observed when the local expectations hypothesis occurs, which means that at any moment t, the expected holding return from any bond is equal to the riskless interest rate. According to this hypothesis, the holding return premiums, \( \phi(n)_t \), are zero for all maturities. This hypothesis does not imply, however, the nullity of yield-to-maturity premiums nor of forward premiums.

Often, in the term structure literature, it as has been accepted that the holding return premiums should be monotonically increasing with maturity. The results of empirical studies have not confirmed this assumption, and, for example, Fama (1984) and McCulloch (1987) have found non-regularly increasing premiums. The continuous-time approach to equilibrium in the term structure proposed by Vasicek (1977) and Cox, Ingersoll and Ross (1985) gives, as we have referred up, the explanation of the non-increasing return premiums.

II. Empirical analysis of Portuguese Treasury bills rates

The present research is devoted to the analysis of the stochastic processes followed by the Treasury bills interest rates and their risk premiums in Portugal, in the period between January 1990 and April 1998. Treasury bills in Portugal have been
issued between 1985 and 1998 with maturities of 91, 182 and 364 days. Only the first two of these maturities will be the used in our tests, since the nature of the data does not allow the calculation of risk premiums for the 364 days interest rates.

A previous research (Fonseca (1994)) has been devoted to the study of the same interest rates, during the period between 1985 and 1991. That research was based on Vasicek’s model, which supposes that risk premiums remain constant over time. A significant change in the theoretical analysis of risk premiums has occurred with the study of Longstaff and Schwartz (1992), according to which the term structure is explained by the stochastics processes followed by a short-term interest rate and its volatility.

**II.1. Presentation of the data**

The data series used in the present research consist of monthly averages of the interest rates settled at the auctions of Portuguese Treasury bills between January 1990 and April 1998. Those series are published by the Banco de Portugal.

The present study covers a period in which the evolution of the interest rates in Portugal was dominated by the participation of the escudo in the exchange rate mechanism of the EMS (since March 1992) and by the conditions for the participation in stage three of the European Monetary Union. During the period covered in this study also occurred a crisis in currency markets (during the spring of 1993). This crisis caused a significant increase in the volatility of interest rates and also a structural break in the series, as our tests demonstrate (see the graph of the interest rate series in Annex 1).

**II.1. The analysis of the stationarity of the series**

Non stationary series have a long memory, meaning that the effect of a shock remains over time. Our test began with the estimation of the Augmented Dickey-Fuller (ADF) Z statistic on the following three types of regressions:

\[ \Delta r_t = \gamma_0 r_{t-1} + \sum_{i=1}^{P} \beta_i \Delta r_{t-i} + \epsilon_t \]  

(11)
\[
\Delta r_t = a_0 + \gamma r_{t-1} + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \varepsilon_t \tag{12},
\]

and

\[
\Delta r_t = a_0 + a_1 t + \gamma r_{t-1} + \sum_{i=1}^{p} \Delta \beta_i r_{t-i} + \varepsilon_t \tag{13}.
\]

The lagged differences of the dependent variable in the regressions have been included to ensure that the error terms are white noise. The number of lagged differences in each regression has been determined by the Akaike-Schwartz criterion. In equation (12), together with the ADF Z statistic, we have tested the joint hypothesis of a unit root and no constant, \( a_0 = \gamma = 0 \). In the same equation, we have done a test to determine the significance of the constant term. In equation (13), together with the ADF Z statistic, we have tested the joint hypothesis of a unit root, no constant and no trend, \( a_0 = a_1 = \gamma = 0 \). The results of these tests are presented in Table I for equation (11), in Table II for equation (12) and in Table III for equation (13). The critical values of the ADF Z statistic for the 1%, 5% and 10% significance levels are also presented in the tables.

### Table I – ADF tests on equation (11)

<table>
<thead>
<tr>
<th><strong>91 days interest rate</strong></th>
<th><strong>182 days interest rate</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>* ADF Z test with 5 lags:</td>
<td>* ADF Z test with 2 lags:</td>
</tr>
<tr>
<td>-0.9095</td>
<td>-0.9804</td>
</tr>
<tr>
<td>* 1% 5% 10%</td>
<td>* 1% 5% 10%</td>
</tr>
<tr>
<td>-13.3 -7.9 -5.6</td>
<td>-13.3 -7.9 -5.6</td>
</tr>
</tbody>
</table>

The results of the model without constant and without trend (Table I) lead to the non rejection of the null hypothesis of a unit root in the two interest rate series. The tests on the models with constant (equation (12)), and with constant and trend (equation (13)) have been conducted as a consequence of the result of the tests on the first model.
Table II – ADF tests on equation (12)

<table>
<thead>
<tr>
<th>91 days interest rate</th>
<th>182 days interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ADF Z test with 5 lags:</strong></td>
<td><strong>ADF Z test with 2 lags:</strong></td>
</tr>
<tr>
<td></td>
<td>0.1267</td>
</tr>
<tr>
<td>*</td>
<td>1%</td>
</tr>
<tr>
<td>*</td>
<td>-19.8</td>
</tr>
<tr>
<td>Coefficient and T-Statistic on the Constant:</td>
<td>Coefficient and T-Statistic on the Constant:</td>
</tr>
<tr>
<td>*</td>
<td>-0.16376</td>
</tr>
<tr>
<td>Joint test of a unit root and no constant:</td>
<td>Joint test of a unit root and no constant:</td>
</tr>
<tr>
<td></td>
<td>2.4137</td>
</tr>
<tr>
<td>*</td>
<td>1%</td>
</tr>
<tr>
<td>*</td>
<td>6.70</td>
</tr>
</tbody>
</table>

The results for the ADF Z statistic lead to the non-rejection of the null hypothesis of a unit root in both series of interest rates. These results are confirmed by the low value of the T-statistic of the constant in the two cases. The results of the joint tests of a unit root and no constant show, additionally, that the restricted regression is not binding for levels between 1% and 10%, in the case of the 91 days interest rate, and for levels between 1% and 5%, in the case of the 182 days interest rate.

Table III – ADF tests on equation (13)

<table>
<thead>
<tr>
<th>91 days interest rate</th>
<th>182 days interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ADF Z test with 5 lags:</strong></td>
<td><strong>ADF Z test with 2 lags:</strong></td>
</tr>
<tr>
<td></td>
<td>-21.6938</td>
</tr>
<tr>
<td>*</td>
<td>1%</td>
</tr>
<tr>
<td>*</td>
<td>-27.4</td>
</tr>
<tr>
<td>Coefficient and T-Statistic on the Constant:</td>
<td>Coefficient and T-Statistic on the Constant:</td>
</tr>
<tr>
<td>*</td>
<td>4.10049</td>
</tr>
<tr>
<td>Coefficient and T-Statistic on the Linear Trend:</td>
<td>Coefficient and T-Statistic on the Linear Trend:</td>
</tr>
<tr>
<td>*</td>
<td>-0.03133</td>
</tr>
<tr>
<td>Joint test of a unit root and no linear trend:</td>
<td>Joint test of a unit root and no linear trend:</td>
</tr>
<tr>
<td>5.5170</td>
<td></td>
</tr>
<tr>
<td>*</td>
<td>1%</td>
</tr>
<tr>
<td>*</td>
<td>8.73</td>
</tr>
</tbody>
</table>

The results of ADF tests lead to the rejection of the null hypothesis of a unit root in the model with trend (equation (13)), in the case of the 91 days interest rate. However, these results, and the value of the T-statistic on the trend’s coefficient, are not in accordance with the results of the tests on the joint hypothesis of a unit root, no constant and no trend, which lead to the conclusion that the restricted equation is not
The contradiction between the results of different tests, in the case of the 91 days interest rate, and the quite evident non-stationarity of the 182 rate in the three models, lead us to test the hypothesis of a structural break in the series. For that we have used a research procedure, proposed by Perron (1997), to select the moment of the structural break. This procedure tests the presence of a unit root together with the presence of a change (or several changes) in the trend function. The first model considered in Perron’s procedure allows a change in the intercept under the null alternative and the null hypotheses of a unit root. In the case under analysis in the present research, this model is represented by the following equation:

\[ r_t = \mu + \theta DU_t + \beta t + \delta D(T_b) + \alpha r_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta r_{t-i} + \varepsilon_t \]  

(14),

where \( T_b +1 \) denotes the time at which occurs the change in the trend function, being \( D(T_b) = 1 \) at \( t = T_b + 1 \). The dummy variable \( DU_t \) ( \( t > T_b \)), represents the change in the trend intercept, that results from the innovation that begins at \( T_b \).

The second model considered in Perron (1997) allows both a change in the intercept and in the slope, at time \( T_b +1 \). In the case under analysis this model is represented by the following equation:

\[ r_t = \mu + \theta DU_t + \beta t + \gamma DT_t + \delta D(T_b) + \alpha r_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta r_{t-i} + \varepsilon_t \]  

(15).

Where \( DT_t \) represents the change in the slope.

Stationary series for the 91 days and the 182 days interest rates resulted from the implementation of the first of these models. This means that the structural break (occurred at March 1993) is the result of a change in the intercept of the trend function. The second model, by the contrary, did not generate a stationary series for any of the maturities. According to these results the hypotheses of a change in the slope of the trend function can be excluded. In Tables IV and V are presented the results of the tests on the first model, for the 91 days and 182 days interest rates, respectively.
Table IV: Test of structural break on the 91 days interest rate, using the model represented in equation (14)

<table>
<thead>
<tr>
<th>Break date TB = 1993:03</th>
<th>Statistic T (α=1) = -5.88068</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical values at 1%</td>
<td>-5.68</td>
</tr>
<tr>
<td>5%</td>
<td>-5.05</td>
</tr>
<tr>
<td>10%</td>
<td>-4.77</td>
</tr>
<tr>
<td>50%</td>
<td>-3.71</td>
</tr>
<tr>
<td>90%</td>
<td>-2.40</td>
</tr>
<tr>
<td>95%</td>
<td>-1.88</td>
</tr>
<tr>
<td>99%</td>
<td>-0.34</td>
</tr>
<tr>
<td>For 100 obs.</td>
<td></td>
</tr>
</tbody>
</table>

From 1990:01 to 1998:04 search limited from 1991:04 to 1997:01
Number of lags retained: 9

Explained variable: $R_{t,91}$

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>0.09378</td>
</tr>
<tr>
<td>DU</td>
<td>-0.01008</td>
</tr>
<tr>
<td>D(Tb)</td>
<td>0.02429</td>
</tr>
<tr>
<td>Trend</td>
<td>-0.0006173</td>
</tr>
<tr>
<td>$R_{t-1,91}$</td>
<td>0.52790</td>
</tr>
</tbody>
</table>

According to the results presented on Table IV, the 91 days interest rate series is stationary ($\alpha$ being statistically different from 1) with a trend and a structural break on the intercept that occurred at March 1993. That month has been a period of crisis in the ERM. The changes in central exchanges rates and in the bands of floatation that followed that crisis explains the breakdown in the level of interest rates.

The results of the tests on the model of equation (14) applied the 182 days interest rates series (Table V) also confirm the presence of stationarity around a trend, and a break in the intercept of the trend function in March 1993.

---

1 The same data for the structure break has been obtained with a different procedure used in a previous version of this paper. That procedure has consisted in the regression of the interest rate series, on a dummy variable (representing the structure break) and the trend. We have used a research procedure to select the date of the structural break which has consisted in maximizing the T statistic of the coefficient of the dummy variable.
Table V: Test of structural break on the 182 days interest rate, using the model represented in equation (14)

<table>
<thead>
<tr>
<th>break date TB = 1993:03</th>
<th>statistic t(α=1) =</th>
<th>-5.42104</th>
</tr>
</thead>
<tbody>
<tr>
<td>critical values at 1%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>for 100 obs.</td>
<td>-5.68</td>
<td>-5.05</td>
</tr>
</tbody>
</table>

from 1990:01 to 1998:04 search limited from 1991:04 to 1997:01
number of lags retained : 9

explained variable : $R_{t,182}$

<table>
<thead>
<tr>
<th>coefficient</th>
<th>student</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>0.06769</td>
</tr>
<tr>
<td>DU</td>
<td>-0.00667</td>
</tr>
<tr>
<td>D(Tb)</td>
<td>0.02247</td>
</tr>
<tr>
<td>TIME</td>
<td>-0.00045288</td>
</tr>
<tr>
<td>$R_{t+1,182}$</td>
<td>0.66646</td>
</tr>
</tbody>
</table>

The same kind of results, as those that have been observed for the 91 days interest rate, have been obtained for the 182 and 364 days interest rates, where the T statistic on the dummy variable also confirms the occurrence of a structure break. In both cases, the series corrected from the structural break become stationary, as the results of the Perron test show.

II.2. Forward rates premiums

As in Part I, in the analysis of risk premiums, only the interest rates with terms of 91 and 182 days have been used. The lack of an intermediate maturity between 182 and 364 days does not allow the calculation of forward 91 days rates for a future longer than three months. Forward 91 days interest rates, on month $t$, referred to month $t+3$, were calculated every month through the current rates of 91 and 182 days:
\[ f(t, t+3) = \ln \left( \frac{P_{t,3}}{P_{t,6}} \right) \] (16).

Forward risk premiums, \( L_{t,t+3} \) are equal to the difference between the forward interest rate \( f(t, t+3) \) and the 91 days interest rate expected on month \( t \) to be observed on month \( t+3 \):

\[ L_{t,t+3} = f(t, t+3) - E_t \left( r_{t+3,91d} \right) \] (17).

Although risk premiums are defined *ex ante*, often they have to be approximated in empirical analysis by *ex post* data. In fact, actual premiums shall not be quite different from their expectations. Under this assumption, *ex post* premiums, defined as:

\[ L_{t,t+3} = f(t, t+3) - r_{t+3,91d} \] (18),

have been used in the subsequent analysis.

The first step consisted in the statistical analysis of the forward premiums series:

**Table VI - Statistics on the Forward Premiums**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>Signif Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Mean</td>
<td>0.00200027253</td>
<td></td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.00297718890</td>
<td>0.000302</td>
</tr>
<tr>
<td>t-Statistic</td>
<td>6.61711</td>
<td>6.61711</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.60209</td>
<td>0.01715283</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.25818</td>
<td>0.00000000</td>
</tr>
<tr>
<td>Variance</td>
<td>8.863654e-006</td>
<td></td>
</tr>
<tr>
<td>SE of Sample Mean</td>
<td>0.0000302</td>
<td></td>
</tr>
<tr>
<td>Signif Level (Mean=0)</td>
<td>0.00000000</td>
<td></td>
</tr>
<tr>
<td>Signif Level (Sk=0)</td>
<td>0.01715283</td>
<td></td>
</tr>
<tr>
<td>Signif Level (Ku=0)</td>
<td>0.00000000</td>
<td></td>
</tr>
</tbody>
</table>

According to the statistical analysis, the forward premiums do not follow a normal distribution, as it is shown by the positive value of the skewness, and by the excess kurtosis parameter, higher than 3. The non-normality of the series is probably the result of the very high volatility of those premiums during the spring of 1993, as is shown in the graph of the respective series, presented in Annex 2.

The researches of Engle and Ng (1993), Bekaert, Hodrick and Marshall (1997) and Lardic and Mignon (1999) lead to the conclusion that the variance of the forward premium has important implications on interest rates curves. These authors also concluded that the most adequate models to explain the time variability of risk premiums are of ARCH or GARCH type, because periods of high volatility of the premiums are, in general, followed by other periods of high volatility.
Using a maximum-likelihood procedure with the Program RATS, we estimated ARCH-M models of several orders on the series of the forward premiums. Before the estimation of the ARCH-M model, we have estimated the Autocorrelation Functions and the Partial Autocorrelation Functions, which decline very rapidly, as is shown in the graph presented in Annex 3. On the other side, the results of the estimation of the ARCH-M models lead to the conclusion the most adequate is the ARCH-M(1):

\[ L_{t,t+3} = \beta + \delta h_t + \epsilon_t \]  \hspace{1cm} (19),

where

\[ h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 \]  \hspace{1cm} (20).

The results of the estimation of that model are presented in Table VII.

**Table VII- ARCH-M(1) Model on the Forward Premiums**

<table>
<thead>
<tr>
<th>Monthly Data From 1990:04 To 1998:04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usable Observations</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
</tr>
<tr>
<td>Function Value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff</th>
<th>Std Error</th>
<th>T-Stat</th>
<th>Signif</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\beta$</td>
<td>0.000956200</td>
<td>0.000231624</td>
<td>4.12824</td>
<td>0.00003655</td>
</tr>
<tr>
<td>2. $\delta$</td>
<td>60.506629850</td>
<td>23.205820172</td>
<td>2.60739</td>
<td>0.00912353</td>
</tr>
<tr>
<td>3. $\alpha_0$</td>
<td>0.000002727</td>
<td>0.000000731</td>
<td>3.73195</td>
<td>0.00019001</td>
</tr>
<tr>
<td>4. $\alpha_1$</td>
<td>0.856972356</td>
<td>0.315454128</td>
<td>2.71663</td>
<td>0.00659502</td>
</tr>
</tbody>
</table>

ARCH-M models of order superior to one did not present any improvement on the value of the maximum-likelihood function, and presented low values of the T statistics on the coefficients.

**II.2. Holding return premiums**

Holding return premiums have often been central in term structure analysis. Fama (1984) conducted empirical tests on those premiums, which lead to the conclusion that, contrary to what was admitted in the traditional theory of the term structure, those premiums were not monotonically increasing with the maturity of interest rates. As we have mentioned before, these results can be explained by the fact that the bonds’ risk measure is not a scalar function of the maturity.
The return from buying a 182 days Treasury bill, and holding it for 91 days is:

\[ h(182d)_{t,+3} = \ln \left( \frac{P_{t+3,3}}{P_{t,6}} \right) \]  

(21),

The actual return premium from this investment is:

\[ \phi (182d)_t = h(182d)_{t,+3} - r_{91} \]  

(22).

The relation between forward and holding return premiums which we have presented previously\(^2\), suggests that the autoregressive process of the second type of premiums must not be very different from the autoregressive process that we have found for forward premiums. The graph presented in Annex 4 is presented shows the differences between the holding return premiums and the forward premiums, which show that they are very close to each other. We began the analysis of the holding premium series by its statistical analysis (Table VIII).

### Table VIII- Statistics on Series of Holding Return Premiums

<table>
<thead>
<tr>
<th></th>
<th>Sample Mean</th>
<th>Variance</th>
<th>SE of Sample Mean</th>
<th>Signif Level (Mean=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Mean</td>
<td>0.00200601213</td>
<td>8.907586e-006</td>
<td>0.000303</td>
<td>0.000000000</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.00298455797</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-Statistic</td>
<td>6.61972</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>0.63039</td>
<td></td>
<td>0.01258001</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.28804</td>
<td></td>
<td>0.000000000</td>
<td></td>
</tr>
</tbody>
</table>

The mean of the holding return premiums is only slightly different from the mean of the forward premiums, and they also exhibit positive asymmetry and excess kurtosis. We have used the same procedure as with forward premiums, to estimate the autoregressive process of return premiums, and the result has also been an ARCH-M(1) model. The coefficients, standard errors, T statistics, and significance levels, of this model are presented in Table IX.

---

\(^2\) See equation (9).
Table IX- ARCH-M(1) Model on the Holding Return Premiums

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff</th>
<th>Std Error</th>
<th>T-Stat</th>
<th>Signif</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 β</td>
<td>0.000954662</td>
<td>0.000228440</td>
<td>4.17905</td>
<td>0.00002927</td>
</tr>
<tr>
<td>2 δ</td>
<td>60.156257143</td>
<td>22.966902762</td>
<td>2.61926</td>
<td>0.00881211</td>
</tr>
<tr>
<td>3 α0</td>
<td>0.000002710</td>
<td>0.000000774</td>
<td>3.50022</td>
<td>0.00046487</td>
</tr>
<tr>
<td>4 α1</td>
<td>0.866852185</td>
<td>0.318625176</td>
<td>2.72060</td>
<td>0.00651632</td>
</tr>
</tbody>
</table>

Although, the statistical analysis and the estimated ARCH-M(1) models show a
great proximity between forward premiums and holding return premiums in the case of
Portuguese Treasury bills rates, it is not possible to conclude that this is a general rule.
In the case under analysis, that proximity must be explained mainly by the fact that
both premiums are referred to the same time horizon.

Conclusion

The main questions about premiums in the term structure of interest rates concern
their relation with maturity and their time variability. The traditional theory of the term
structure of interest rates accepted that all types of risk premiums were monotonically
increasing with maturity. Modern intertemporal approaches defend, by the contrary,
that the equilibrium in the term structure implies that risk premiums must be
proportional to bonds’ risk measures which are not equal to their maturity.

The research underlying this paper has been dedicated to the question of the
time variability of the risk premiums in the term structure. Using interest rates data
from the auctions of Portuguese Treasury bills, we have obtained strong evidence
confirming the variability of the risk premiums contained on those interest rates. Our
tests have highlighted the structural break in Portuguese Treasury bills rates that
occurred as a result of the crisis in currences of March 1993, and of the subsequent
changes in the ERM. We also concluded that an ARCH-M(1) model is the best
approach to the stochastic processes followed by forward premiums and holding return
premiums in Portuguese Treasury bills interest rates.
Annex 1

Portuguese Treasury Bills Interest Rates

Annex 2

Forward Risk Premiums
References


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