Lutosławski's Mid-Century Harmony

Reconceptualizing Chordal Space in the Five Iłłakowicz Songs

José Oliveira Martins

Abstract This article proposes to expand our understanding of Witold Lutosławski's *chordal space* and its relation to formal processes. It argues that the traditional analytical focus on harmonic colors of isolated chords comes at the expense of understanding how chordal interactions shape form. How do individual chords participate in cogent, expressive, and sensuous long-range harmonic processes? The article explores how the arrangement of intervals in twelve-tone chords (especially the ordering, restriction, and modularity of intervals) not only imprint distinct coloristic qualities to single chordal entities but are also implicated in strategies of harmonic progression, relatedness, complementarity, and contrast. These analytical goals prompt a reconceptualization of chordal elements, which are understood as combinations of pitch class and modal quality rather than simply as pitches in register. The analytical focus is on Lutosławski's harmony of the mid- to late fifties, attending in particular to the *Five Iłłakowicz Songs*, the composer's inaugural effort in the use of vertical aggregates. Lutosławski's chordal space in these pieces is interpreted through a series of cyclic graphs and transpositional networks, prompting a novel understanding between twelve-tone chordal harmony and form. The study proposes to contribute to the analytical reappraisal of a set of pieces that anchor an influential harmonic practice in twentieth- and twenty-first-century music.

Keywords Lutosławski's harmony, *Five Iłłakowicz Songs*, chordal space, chordal connections, affinity spaces

1. Introduction

Starting in the mid-1950s, Witold Lutosławski's original work on harmonic aggregates or twelve-tone chords contributed to the rehabilitation of harmony as a creative practice in modern Western art music.[1] Lutosławski's harmony increasingly turned into an anchoring resource for the development of new compositional

1 Lutosławski's efforts faced an adverse avant-garde climate in which the domain of harmony had been marginalized as aesthetically reactionary, and increasingly reduced to a by product of counterpoint and serialization. See Covach 2002: 613–22 for an account on the aesthetic implications of twelve-tone theory (after Schoenberg) for the relations between harmony and melody.

vocabularies, analytic methodologies, and modernist narratives on stylistic influences and continuities.[2] In a posthumously published critical essay, Steven Stucky (2018) reasserts the centrality of harmony for the understanding of Lutosławski's artistic achievements and the assessment of compositional lineages. Stucky's argument for a "continuing story of harmony" largely relies on his interpretation that chordal entities share certain constructive features and functional attributes throughout the long twentieth century.

Locating the origins of Lutosławski's harmony in the music of Debussy, Stucky claims that harmonic formations in this tradition undermine the potential of individual chords to function syntactically, expressing instead primarily coloristic traits, "in the sense that Debussy emancipated the individual chord from its functional, grammatical straightjacket, freeing it to function as something more like *color* than *syntax*" (338).[3] Stucky's account echoes numerous analytical assessments of Lutosławski's harmony that have focused on the properties of individual chords, including the ordered series of pitch intervals between adjacent elements, subset partitions, interval-class pairings, inversional symmetries, and patterned transpositions, and more indirectly, the relationship between chordal entities and the structuring aspects of texture and register. Collectively, this work has given us detailed inventories of intervallic and registral arrangements of twelve-tone chords and rows, which provide a rich resource for the understanding of harmonic color and expressive qualities of chordal space.[4]

Stucky's focus on chord color, however, has left largely unaddressed the question of how chord connections are structured, and how those connections relate to the properties of patterned twelve-tone chords. How might chord-to-chord successions, and other (close or distant) chordal associations, shape harmonic processes often described as cogent and enthralling?

The analytical focus on individual chords and row properties does not entail, however, that commentators have ignored matters of the music's large-scale form, musical motion, and change. Many of Lutosławski's musical processes

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2 The context and tensions posed by Lutosławski's music within the scope of twentieth-century modernism are discussed in a number of sources: see in particular Whittall 2001, Harley 2001, and Thomas 2005.

3 Steven Stucky was certainly aware of the artificiality of a strict separation between chordal color and syntax, a distinction that seems to function more as a suggestive prescription for compositional design: "This is surely an exaggeration . . . but it is a profoundly suggestive idea for later composers, all the same" (2018: 338). Later in the essay, Stucky claimed that often in compositional practice, "the composer needs to figure out the chords." Interestingly (and somewhat paradoxically), Stucky allowed for "alternative syntaxes" to emerge from the music of the second Viennese School (but not from the tradition of Debussy–Lutosławski–Lindberg): "By contrast, Schoenberg's emancipation of the dissonance for something like the opposite purpose—not in order to strip it of syntactical meaning, but to make it available for new, non-tonal syntaxes—is less germane to this particular lineage, since it focuses somewhat less on the specific sound qualities of the combinations it produces" (338). If Stucky allowed for "alternative syntaxes" to emerge in the atonal tradition, why not in the tradition of chordal resonance?

4 Numerous authors have contributed to the current understanding of Lutosławski's harmony. The theoretical formalization of harmony developed in the present study is primarily influenced by the work of John Casken (2001), Martina Homma (1995, 2001), Michael Klein (1999), Iwona Lindstedt (2018), Witold Lutosławski (2007), Jadwiga Paja-Stach (2001), Robert Peck (2003), Charles Bodman Rae (1992, 1994, 2001), Nicholas Reyland (2008), Steven Stucky (1981, 2001, 2018), and Adrian Thomas (2001).

are deemed to be highly dynamic and goal oriented, recruiting a variety of parameters that ignite the analysts' imagination concerning aspects of form, formal characters, and narrative actions or plots. But these analytical writings only indirectly tackle the contribution of harmony to the sense of (continuous and discontinuous) musical motion, so that music's kinetic function is primarily attributed to the active roles of texture, density, and macrorhythm.[5]

This article proposes to further expand our understanding of Lutosławski's *chordal space*.[6] It examines Lutosławski's harmony of the mid- to late fifties, focusing primarily on the *Five Iłłakowicz Songs* (1956–57), but also probing movements of *Musique funèbre* (1954–58) and *Three Postludes* (1958–60). I focus on the interaction of particular twelve-tone chord arrangements, attending especially to the combination of interval patterns, registral deployments, and temporal orderings.[7] These processes are modeled using cyclic graphs of non-octave-repeating interval patterns and two-dimensional lattices or networks of combined cycles that I introduced in earlier writings (see primarily Martins 2011, 2015, 2020).

The conceptualization of chords in this system requires that we consider note elements as a combination of pitch class and cyclic position (or *modal quality*, as addressed below), overstepping the notion of chords as pitches fixed in register (often interpreted as a feature of Lutosławski's music). As a consequence, the representation of note elements in the present model displays the registral ordering of intervals associated with sound quality (color), while also promoting a flexible framework for intra- and interchordal relations implicated in strategies of harmonic progression, relatedness, continuity, and contrast. As a larger goal, this study proposes to contribute to the analytical reappraisal of a set of pieces that anchor an influential harmonic practice in twentieth- and twenty-first-century music.

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5 Large-scale processes in the music of Lutosławski have been approached through various analytical strategies, such as the assessment of macrorhythm and form (Stucky 1981; Rae 1994; Cowie 1977; Harley 2001), and readings based on the notions of music narrativity, plot, and drama (Reyland 2007, 2008; Casken 2001). In addition, studies on musical texture and density have greatly contributed to our understanding of kinetic processes in the music of Lutosławski (Klein 1999; Rust 2004).

Lutosławski discusses the role of harmony in what he considers to be *static* (or narrative) and *dynamic* (or lively) characters in his music, where "character" refers to "the relation of a particular section to the form as a whole" (1962b: 13). See also Lutosławski 1976 and 1985. An instance of static character is the (refrain) *ad libitum* sections in *Jeux vénitiens*, and conversely, the "Apogeum" section of *Musique funèbre* displays a dynamic character.

6 Lutosławski discusses how the listener might make sense of large-scale forms in his own music, using the strategies of anticipation and memory for processing musical parameters (see, for instance, Lutosławski 1967). In a different publication, the composer considers that one of these parameters, harmony, takes on a crucial role in the process of form formation: "I must say I find this harmonic aspect of sound material [vertical sound aggregations and their variants] one of the most important in constructing large forms" (Lutosławski 1962b: 18).

7 The issue of harmonic motion in the music of Lutosławski is addressed by Charles Bodman Rae (1994: 56), who claims that "Lutosławski establishes a sense of progression between harmonies. . . . One can note that the absence of conventional, tonal functions of harmony has required the composer to explore a wide variety of unconventional means of moving from one chord to another." Later, however, Rae (1994: 62–63) points primarily to linear (not harmonic) means for the music's horizontal organization: "Lutosławski's methods of organizing pitch do not apply solely to the vertical dimension of twelve-tone chords and chord-aggregates. He also employs various methods of organizing the horizontal dimension, both as featured melodic lines and as underlying linear patterns."

The present article proposes a way of exploring the horizontal dimension through harmonic (not only linear) nontonal processes. This approach can be seen as complementary to above-mentioned analytical efforts that assess how texture and macrorhythm might contribute to our understanding of Lutosławski's large-scale form and sense of musical action.

2. On Lutosławski's Harmony

2.1 The Expressive Possibilities of Twelve-Tone Chords (the Composer's View)

Lutosławski's compositional breakthrough of the second half of the 1950s came as result of a slow investigative process toward the renewal of his sound language. After the composition of his neoclassical *First Symphony* (1947), the composer sought new principles of pitch organization, moving away from what he sensed to be "a moribund tonal language" (as referred to by Stucky 1981: 32). In the ensuing decade, and in spite of composing one of his most celebrated works, the *Concerto for Orchestra* (1950–54), Lutosławski struggled to find the appropriate means for harmonic expression: "I wrote as I was able, since I could not yet write as I wished," he would often recollect later.[8] The output of the second half of the 1950s finally brought about a significant shift in compositional premises, prompted by a new understanding of harmony, its sensuous effects, and its expressive possibilities. Lutosławski saw in this abundance of harmonic possibilities, liberated from a strict tonal framework, a response to the contemporary skepticism about the role of harmony. In a program note drafted to the 1960 Warsaw Autumn Festival, he frames his enthusiasm for the aesthetic implications of harmony, namely, its ability to sustain a historically pertinent contemporary practice.

> We often hear today that the role of harmony is finished. This is the result of the enormous influence exerted on today's music by the Schoenbergian concept of a sound world based, among other things, on the elimination of man's purely sensuous reaction to sounds and their connections. Personally, I could never write even the simplest work while disregarding this particular, sensuous reaction to the vertical or horizontal arrangements of sounds. Therefore I distrust the statement about the decline of harmony as an element of musical substance. Moreover, I even think that only now can we really start to embrace the whole abundance of the harmonic possibilities contained within the twelve-tone scale; especially now, when we are getting rid of the limitations of tonal thinking forever.

> I was particular interested in these issues, which are so unpopular today [1960], when I was writing the *Five Iłłakowicz Songs*. Therefore, I tried to put everything else aside in this work and to concentrate only on the expressive and coloristic possibilities of the twelve tones in their diverse variants.[9] (Lutosławski 1960: 295)

Despite the composer's interest in the dormant potentialities of the aggregate, he rejected dodecaphonic principles as "intellectual" attitudes toward

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8 The quote is from Pilarsky 1958: 2–3, cited in Stucky 1981: 59. Lutosławski made similar pronouncements on several occasions. For instance, "In spite of the fact that [the *Concerto for Orchestra*] is probably the most often performed piece of mine, I always think of it as a marginal work: I composed it as I was then able to, and not as I should really like. That is why there is such a difference between the *Concerto for Orchestra* and for example, *Five Songs*" (Lutosławski 1993: 98). See also Nikolska 1994: 124.

9 Lutosławski's note draft is slightly different from the actual published booklet note for the fourth Warsaw Autumn Festival in 1960 (Lutosławski 1960: 109–10, translated from the French by Steven Stucky [1981: 65–66]). For instance, the published note emphasizes the notion of a new understanding of harmony: "I believe that it is only now that we are liberating ourselves from the conventions of the tonal system that we can comprehend all the wealth of harmonic possibilities available in the chromatic scale." The earlier draft, however, emphasizes the diminishing importance of harmony due to the overwhelming influence of Schoenbergian serialism.

musical knowledge, claiming they relied on a quantitative approach to musical intervals. Seeking to cast a progressive tone to his practice, the composer affiliated his harmonic interests in the "empirical tradition" of Debussy, Bartók, and Messiaen, which he saw as engaging with the sensuous exploration of the qualitative differences between intervals.[10] Though acknowledging a certain correlation between arithmetical and psychological characterizations of intervals, the composer clearly valued the ability to discriminate the effects of different intervals heard in "simultaneous combinations of sounds and sound sequences":

> It is true that the quality of the interval does correspond in a certain sense to the degree of complexity of the relations between the arithmetical number of vibrations of its tones. But above all, the qualitative differences between the intervals correspond to the different reactions of our ears to these intervals, in that we consciously register certain individual traits of each separate interval irrespective of its range. (Lutosławski 1962a: 23–24)

In short, Lutosławski grounded his sound language on what we might call a phenomenology of *interval quality*: being sensitive to the "different psychological impulses" or "reactions" the different qualities of intervals have on the listener.

Given that every aggregate chord contains the twelve pitch classes, it is the registral distribution of the chords' pitch intervals (from low to high register) that differentiates between chord qualities (or colors).[11] The composer investigated the differentiation of twelve-tone sound qualities through a restricted use of pitch-adjacent intervals in the chords' harmonic (vertical) deployments. By concentrating on the qualities of harmonic aggregates, the composer argued that the fewer distinct intervals between adjacent pitches of the aggregate's registral distribution, the more characteristic and differentiated the resulting harmony became:[12] "I soon abandoned the work on scales, which brought on interesting

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10 Lutosławski's self-affiliation with the "empirical tradition" of composers such as Debussy, Bartók, and Varèse (which he sees as more important and having a greater impact on the future than the dodecaphonic system of Schoenberg) is discussed in multiple occasions throughout his life (see, e.g., Lutosławski 1962a or 1983). The duality of musical responses based on the quantification vs. qualification of interval properties resembles Josef Matthias Hauer's conception of intervals as "sacred or mental" vs. "sensual or material" (Covach 2002: 604–7), and more generally invokes the conflict about the "essence" of music through mind vs. senses (referring to the philosophical conflict between the positions of Plato vs. Aristotle), see Bonds 2014, esp. chap. 2, "Isomorphic Resonance" (30–38).

11 The chordal registral succession of intervals between adjacent pitches can be thought to correspond, in twelve-tone theory, to the notion of *adjacent ordered intervals* in a twelve-tone row, also called the *Adjacency Interval Series* (AIS). Twelve-tone chords belonging to the same chord-class if they present the same AIS.

12 The interval restriction of Lutosławski's twelve-tone chords has been a compositional strategy cultivated throughout the composer's output (starting in the mid-fifties) and has been amply examined by a number of Lutosławski scholars. Martina Homma (2001) examines Lutosławski's precompositional studies in twelve-tone rows (many of which were not directly used in actual compositions), which show that the composer systematically explored a number of restricted interval combinations, including the "elementary" alternation of two intervals (between adjacent notes in the series), as well as the development of layered strategies for the structuring of vertical and horizontal dimensions of music. Charles Bodman Rae (1994: 49–74) has rather resorted to the notion of interval-classes (ics: 1, 2, 3, 4, 5, 6) as a generalized scheme for the measurement of intervals between adjacent tones in twelve-tone chords, and posited the preference for the combination of certain interval-class pairings (i.e., the restricted use of two ics between neighboring notes: 1/6, 2/5, 3/4), especially in the context of horizontal (melodic) materials as a means of achieving linear coherence. For a general discussion of intervallic restriction as a principle of twelve-tone chord construction in the music of Lutosławski, see Stucky 2001: 151–52.

results and I concentrated on harmony, I began working on twelve-tone chords. I found a great variety of harmony among them when following a simple rule: the fewer kinds of intervals between the neighboring notes of the chord, the more characteristic the result is" (Lutosławski 1993: 96).

While the twelve-tone harmonies are conceptually simultaneous, the actual compositional deployment of twelve-tone harmonies varies greatly. Rarely are the twelve tones attacked simultaneously; more often, the aggregate is temporally built up through arpeggiation or by articulating various subsets, at times reiterated throughout the duration of the chord. In this article, representations of twelve-tone chords in staff notation often adopt an arpeggiated format for its reading simplicity.

2.2 Reconsidering Stucky's Principles of Lutosławskian Harmony

Stucky (2018: 339) claims that Lutosławski's twelve-tone chords[13] "are not pitch-class sets, they are pitch fields, with each note fixed in register for the life of the chord."[14] While chordal notes in Lutosławski tend to retain their registral placement, there are a number of registral relations that complicate this view. Let us consider three increasingly complex registral relations in the song "Rycerze," the second of the five songs. Its first two twelve-tone chords combine the piano part and the voice such that intrachordal octave relations suggest a flexible treatment of displaced and fixed notes (or interval segments) in register. In the first twelve-tone chord (Figure 1a), two individual pitches in the piano are displaced by one octave in the vocal-melody segment D♭–E♭4–F4–A♭4–B♭4–C, which mostly reiterates the order and register of pitches in the piano part. The displaced pitches are B♭4 (m. 164) and E♭4 (m. 166). These appear an octave higher than B♭3 and E♭3 in the piano part (also, the A4 in the piano at the end of m. 168 finally "corrects" the register of the low bass A1 retained since the beginning of the song). Note that the registral adjustment of B♭ and E♭ allows for a smoother voice leading of the vocal melody by partially reconfiguring the pitch order of the corresponding chordal segment. The resulting octave relations between some notes

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13 Stucky's (2018: 339–41) harmonic characterization refers to Bartók's *Mikrokosmos* No. 143, "Divided Arpeggios," mm. 1–16, which he sees as exemplifying a synthesis of Lutosławskian harmony, starting in the 1950s. His analysis refers to the character of chords starting at m. 6. For complementary and in-depth analytical perspectives on the complex intervallic arrangements of chords and the resultant musical processes explored in this piece, see Gollin 2007 and Martins 2015.

14 The notion of *pitch field* was first defined by Paul Nauert (2003: 181) as "an unordered collection of pitches [n.b. pitches, modeled by integers—not pitch classes modeled by integers mod 12] . . . kept in circulation to the exclusion of other pitches. . . . Like a chord, a pitch field possesses a characteristic harmonic sonority to which all of its constituent pitches contribute, and we assume this sonority will color to some degree any music based on the field." Once the activity of a pitch field is analytically reduced to its chordal (ordered) representation in register (as most analysts do), then we can think of a twelve-tone chord as a pitch-segment (Pseg) of twelve pitches (at times more or less than twelve pitches), in which the order of pitches is given by their registral placement. The interval content of a Pseg that is relevant for this discussion is given by the interval succession between registrally adjacent pitches. Robert Morris (1987: 36–41) formalizes the notion of Pseg and Interval Succession of a Pseg, where the latter can be thought as the output of a function that yields the intervals between adjacent pitches in the segment. When generalized, the function INT of a Pseg is the Interval Succession between successive—not necessarily adjacent—pitches in a Pseg (40).

of the vocal melody and accompaniment can thus be heard as isolated registral displacements of the "fixed" chordal notes.

In the second twelve-tone chord, an entire intrachordal segment is displaced (Figure 1b) by the voice, which builds the segment B♭3–D♭4–F4–A4–C5, duplicating (at the lower octave) the corresponding pitches and interval adjacencies in the piano twelve-tone chord. We can think of the voice as a local "shadow" to the fixed chord in the piano, where perfect octaves are perceptually suggestive of the functional equivalence of pitch classes.[15]

A third instance of pitch displacements occurs later in the song, where pitch segments and interval adjacencies are replicated in neighboring chords, contributing to chordal connection. Consider the sequence of chords in Figure 1c (mm. 174–79). This sequence of chords retains a series of four-note segments (differentiated by color), resulting in their gradual registral repositioning. As highlighted in the figure, the chord 3 segments C♯6–E6–G♯6–B6 (rh in blue) and G4–B♭4–D5–F5 (lh in red) are presented in chord 4 in different registers and different chordal positions: C♯4–E4–G♯4–B4 (lh) and G5–B♭5–D6–F6 (rh). Similarly, the progression preceding chord 6 retains other recontextualizations of four-note segments (and six-note segments in the passage from chords 5 to 6). This article is principally concerned with how the gradual or contrasting recontextualization of pitch segments in various chords and registers frame regulatory relations underlying harmonic processes.

2.3 Reconceptualizing Lutosławski's Chordal Space

These degrees of registral flexibility suggest that chordal notes combine the properties of both pitch class and order position, beyond the strict conception of pitch space. To mediate the analysis of intrachordal formations and interchordal relations, I apply a set of cyclic constructs developed in my earlier work (primarily in Martins 2015). These spaces specify "constructive" features and "navigation modes" in a closed system of pitch relations. A number of these cycles will be used here to map both chordal construction and interchordal connections between *same* chord-classes. Relations between *different* chord-classes will be later modeled by other constructs (*transpositional networks*) that combine various cyclic spaces.

Figure 2 is an example of an affinity space. The space is generated by repeated application of the interval segment 3–3–1, forming a thirty-six-element cycle. Each interval segment forms a *module*, whose boundaries are indicated by lines in the figure. The three order positions in each module form a clockwise set of *modal qualities*, mq(0)–mq(1)–mq(2). Each pitch class appears once in each of the three modal positions. Each of the thirty-six elements is

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15 This perspective is resonant with Jonathan Bernard's (2003) notion of "zones of impingement." Bernard proposes a more flexible approach to the analytical use of the notions of pitch and pitch-class in some instances in the music of Bartók, suggesting that a hybrid relation between these concepts is more analytically productive at times.
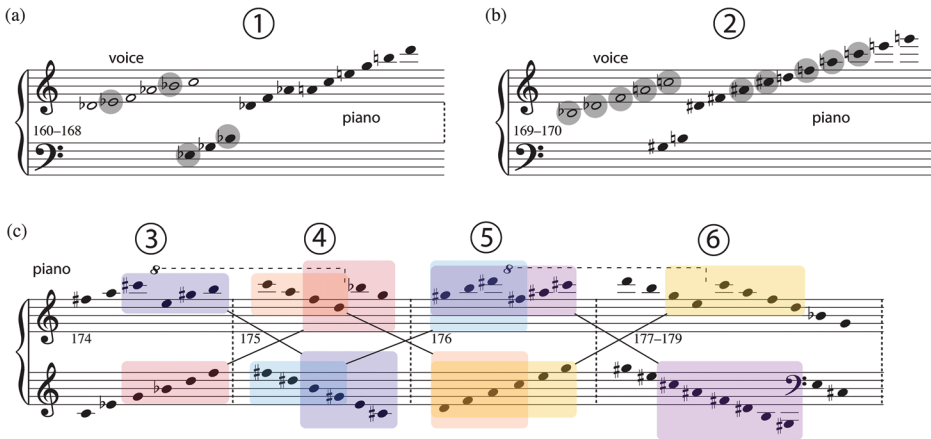
Figure 1. Gradual flexible approach to "fixed register" of twelve-tone chords in "Rycerze," from *Iłłakowicz Songs*: (a) octave displacement of single notes, chord 1; (b) displacement of intrachordal segments, chord 2; (c) interchordal replication of segments.
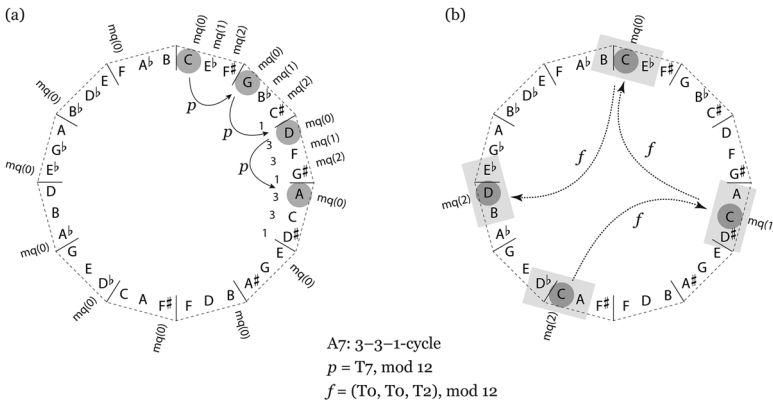

Figure 2. Properties of an affinity space, A_7 = 3–3–1-cycle: (a) operation *p* (transpose); (b) operation *f* (transform).

defined as a unique ordered pair $(pc(x), mq(y))$, where x ranges from $0 \leq x \leq 11$ and y ranges from $0 \leq y \leq 2$. For instance, $(C, 0)$ refers to pc C in position $mq(0)$. Figure 2a indicates the clockwise ordering of mqs in a few adjacent modules, and the twelve occurrences of $mq(0)$ around the cycle.

    This space is conceived as a closed cycle, or an extended scalar space, that produces consistent (non-octave) interval *affinities* at among T_7-related pcs: T_7 $(pc(x), mq(y)) = (pc(x +7), mq(y))$. T_7-related pcs in adjacent modules share the same local interval adjacency (i.e., the same modal quality). My earlier work refers to them as *affinity spaces*, drawing from the medieval term in reference to the transfer (recurrence) of interval segments to different parts of the

scale, as addressed above. In work developed simultaneously but independently, Edward Gollin refers to these constructs as *multiaggregate cycles*. Gollin's work explores some of the properties of the cyclic structure of the constructs, including the multiple appearance of aggregates and the relative distribution of (the same) pc occurrences. In this article, I retain the term *affinity spaces* and explore their constructive features and navigation modes (generalizable from this example) to describe a number of similar cyclic constructs relevant to the analysis of Lutosławski's work.[16] For this specific construct, I use the label A_7: *3–3–1-cycle* (where A stands for affinities between $T_{\pm 7}$-related pitch classes).[17]

The twelve occurrences of each modal quality, and three occurrences of each pitch class, induce two privileged navigation modes. Connecting the twelve periodic occurrences of a given modal quality, the clockwise arrows inside the cycle in Figure 2a illustrate a sequence of shifts from $(C, 0) \to (G, 0) \to (D, 0) \to (A, 0)$. Each shift maps $[\mathrm{pc}(x), \mathrm{mq}(y)]$ to $[T_7(x), \mathrm{mq}(y)]$. I refer to this action or operation as *p*, where *p* stands for "trans*p*ose." Since T_7 is of order 12, the repeated iteration of *p* can range from *p* to p^{12}, and the inverse operation is $p^{-1} = T_5$, sending a given pc(x) to $T_5(x)$ in a counterclockwise adjacent module.[18]

A second operation models space motion between identical pitch classes in the cycle. In Figure 2b, the shifts between boxed elements $(C, 2) \to (C, 1) \to (C, 0)$ consistently span 10 counterclockwise steps (or −10 steps). The continuation of this procedure yields a pitch change, mapping $(C, 0) \to (D, 2)$. In the case of pc C retention, the modal quality consistently decreases the order position: $\mathrm{mq}(2) \to \mathrm{mq}(1) \to \mathrm{mq}(0)$. In the case of pc change (from C to D), the modal quality shifts from $\mathrm{mq}(0) \to \mathrm{mq}(2)$ (back to the highest order position). I refer to all these order position shifts as *f*, where *f* stands for "transform." Both cases (pc retention and pc change) involve a similar *transformation* of local order positions in the cycle.

For instance, consider the sequence of three-note segments highlighted in Figure 2b, which draws from local segments surrounding the note transformations outlined above (darker boxes). In this sequence (A–**C**–*D*♭ –> A–**C**–D# –> B–**C**–E♭ –> B–**D**–*E*♭), *f* models not only the transformation between central elements (in bold) in each segment, but also the mapping between first elements, as well as that of third elements. In each transformation, the notes at mq(0) (in italic) shift to the notes at mq(2) (underlined), a whole step higher. From a local

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16 In the article, I also adapt Gollin's intervallic label for designating a multiaggregate cycle.

17 An *affinity space* can be thought of as a conceptual construct based on non-octave iterations of an interval segment, which results in a system of interlocked combinations of interval cycles. Each element in the cycle can be characterized by the combination (ordered pair) of a pitch class and a modal quality (pc, mq). Affinity-space constructs have the structure of cyclic groups of order n, where n is the cardinality of the space. See Martins 2015: esp. 277–80. These spaces share structuring features of constructs such as George Perle's (1996) notion of difference alignments, and Gollin's (2007: 146) notion of multiaggregate cycles: "a compound interval cycle that covers (or runs through the tones of) more than one aggregate." For the theoretical framework of compound and multiaggregate cycles, see Gollin 2007, 146–48. The application of *affinities* to the current modeling of scalar-like relations recovers some aspects of medieval theory and terminology for non-octave relations; see Pesce 1986, 1987.

18 The operations *transposition* and *transformation* within affinity spaces are formalized in Martins 2015: 279–81.

scalar perspective, the transformation yielding the pc change (from C to D) also retains the same neighboring notes B and E♭, and that is always the case for the action f involving $mq(0) \rightarrow mq(2)$ on this space. Because the pitch change induced by f is T_2, and 2 is a divisor of 12, f can be thought to partition the thirty-six-element cycle into two cocycles of order 18. This operation can be formalized as $f = (T_0, T_0, T_2)$, mod 12. In short, either the T_0-related pcs are surrounded by different pc-neighbors in distinct parts of the space, or T_2-related pcs are neighbored by the same pair of pcs.[19]

3. The Harmonic Space of the *Five Iłłakowicz Songs*

We now explore how affinity spaces apply to Lutosławski's *Five Songs*, composed in 1956–57, for female voice and piano, setting texts for children by the Polish poet Kazimiera Iłłakowicz. The simple but strongly contrasting imagery of the texts became a fertile ground for investigating the relation between rich harmonies and their sensuous and coloristic implications.[20] While the intervallic arrangements of chords in the *Iłłakowicz* songs are well understood and are considered the composer's seminal work in the verticalization of twelve-tone harmonies (and partitions), it is still not clear how relations between individual chordal arrangements might lead to a better understanding of the large-scale harmonic processes in the songs, including the effects of harmonic continuity, discontinuity, complementarity, and contrast.

I will use the frameworks of affinity spaces and transpositional networks to negotiate aspects of chordal construction and connection throughout the songs. A methodological point: twelve-tone chords occur mostly as vertical entities in the *Five Songs*, and the register distributions (or at times, temporal order) within individual chords are represented by mappings onto a clockwise direction of the affinity cycle.[21] The analytical commentary in the article examines aspects of harmonic transformation and interval reversal in "Morze" (song no. 1); harmonic continuity, complementarity, contrast, and contraction in "Rycerze" (song no. 2); harmonic continuity, completion, and voice leading in "Zima" (song no. 3); harmonic recontextualization and derivation in "Wiatr" (song no. 4); and aspects of harmonic contrast and integration in "Dzwony cerkiewne" (song no. 5).

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19 The specific physiognomy of a given affinity space can vary considerably and depends on the intervals constituting the periodic interval segment. In any case, the formula $A_z = a–b–c \ldots$-cycle structures the modular unit of the space, where $z = (a+b+c+\ldots)$, mod 12. For a more mathematically rigorous presentation, see Martins 2009.

20 The *Five Iłłakowicz Songs* were later transcribed for female voice and instrumental ensemble (1958). Lutosławski (1960: 295) states he chose to "limit [his] field of creative action," in which he used a "simple, homophonic texture which does not go beyond traditional rhythm," while finding "a lot of the kind of satisfaction that generally comes from writing music" inspired by the "particular poetic charm of Iłłakowicz's *Children Rhymes*." The measure numbers run continuously through the five songs.

21 The compositional deployment in register and time of (vertical) twelve-tone chords can be characterized both by the notion of *macroharmony* as "the total collection of notes used over small stretches of musical time" (Tymoczko 2011: 15–17) and *pitch field* (Nauert 2003). While a macroharmony is a pitch-class set, a pitch field is a set of pitches fixed in register.

Example 1. Lutosławski's "Morze," *Iłłakowicz Songs*: mm. 20–23. The two lists of pitch classes represent the registral ordering of pitches, from lowest to highest.

① F♯–C♯–D–F–G♯–A–C–D♯–E–G–A♯–B ‖ ② B–E–G♯–D–F–A–C–E♭–G♭–B♭–D♭–G

3.1 "Morze" (The Sea)

The text of the first song portrays a fragile and strongly suggestive scenery in the sea, where feathers of geese and ducks float and swim toward waves and are affected by tides. Reflecting that imagery, the opening twelve-tone chord of "Morze" is gradually built up through a counterpoint of asynchronous undulating gestures in the piano part and voice (mm. 1–21). The passage culminates in a rich texture of the complete twelve-tone chord in mm. 20–21 (shown in Example 1), combining right and left hands in the piano part. The excerpted passage is preceded by an alternation of C♯ and D in the piano, gradually expanding to a superimposition of two layers. Figure 3a combines the piano lh, a five-note pentatonic line of fifths (F♯–C♯–G♯–D♯–A♯), with the piano rh, a seven-note collection of diatonic thirds (D–F–A–C–E–G–B), registrally arranged in a recurrent 3–4 pattern. (The climatic point of the passage reaches the low F♯3 in the bass and B5 in the treble on each of the segments.) These two segments are registrally coordinated and can be conceived to project the recurrent 3–3–1 pattern of Figure 3b, from which the undulating gestures of the voice derive the melodic intervals.[22]

Complementary to the long opening chord stands the closing chord of the song (mm. 47–58), which returns to the gestural swelling of the opening chord, though reversing the amplitude of its waves, gradually decreasing its undulating activity until coming to a rest on the alternation of D and E♯.[23] Similarly, the closing chord superimposes a pentachordal segment of fifths G–D–A–E–B with the recurrent 3–4 diatonic thirds pattern, E♭–G♭–B♭–D♭–F–A♭–C. Together, these

22 Measure 21 combines ceaselessly the ending of chord 1 (up to F♯ in the left hand) and the beginning of chord 2. And the voice retains the pattern of chord 1 up to measure 23 (which also coincides with chord 2 at that register, except for A♭4).

23 Between the opening and closing chords, the music passes through seven shorter-duration twelve-tone chords (plus a pentachord), which emphasize upward arpeggiation and intervallic symmetry over modular recurrence. For a reduction of this chord progression, see Rae 1994: 58–59.
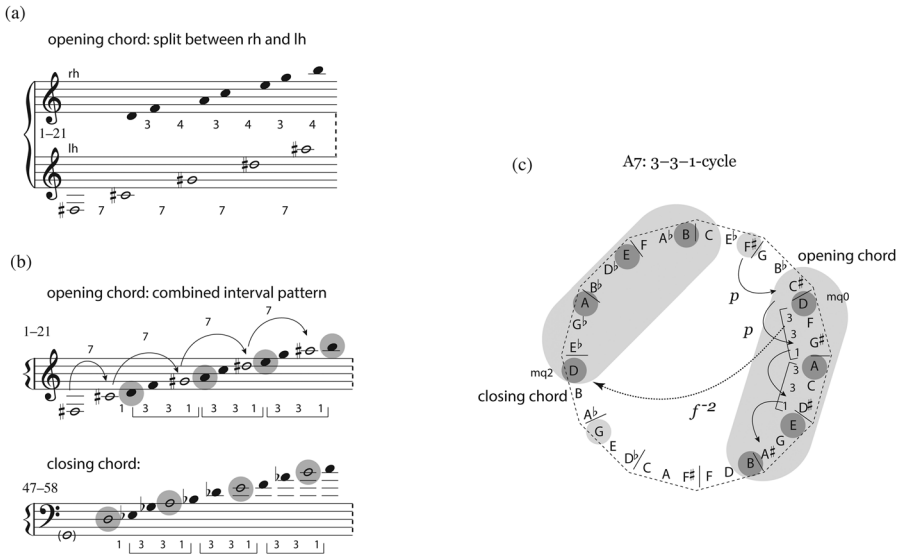
Figure 3. Opening and closing chords in "Morze": (a) pentatonic and diatonic segments in separate hands; (b) combined hands in opening and closing chords; (c) affinity space representation, A7: 3–3–1-cycle and the mapping of opening and closing chords.

segments also combine into a recurrent interval pattern 3–3–1, transposing the opening chord by T_1, mod 12 (T_{-11} in pitch space).

Given the intervallic recursion of both *opening* and *closing* chords, we can posit the display of affinities for the space A7: 3–3–1-cycle, as suggested in Figure 3c, whose abstract properties and operations were already discussed above in reference to Figure 2. Therefore, the opening and closing chords both motivate and are inscribed in the cycle. While the two chords are mapped into unique (non-overlapping) locations of the affinity space, the segment of fifths D–A–E–B (highlighted in the figure) is shared by both chord mappings (notes within the segment are *p* related). The (octave and sixth) range of the fifth segments from D to B also corresponds to the lower and upper limits of the corresponding undulating gestures in the two chords. The modal qualities involved, however, are distinct: D–A–E–B has mq(0) in the opening chord and mq(2) in the closing chord (in addition, the fifth segments are played by different hands and in different registers). This interchordal relationship is modeled by the transformation f^{-2} (from mq(0) to mq(2)), which changes the intervallic context for the shared fifth segment from opening to closing chords. The change of interval context for the shared pc D is brought into focus by the very opening and closing measures of the song (Example 2, mm. 1–4 and mm. 54–58). C♯–D and D–E♭ initiate and terminate the undulating motion, reversing the semitonal association for D, and the transformation of its modal quality. The reconfiguration of modal qualities as a result of the pitch transformation is matched by the reversal of the overall gestural motion from opening to closing chords. The trajectory of

Example 2. Lutosławski's "Morze," from *Iłłakowicz Songs*: opening and closing measures of the song (mm. 1–4 and mm. 54–58); transformation of modal quality (f^{-2}) and reversal of semitonal association for the common pc D, from C♯–D to D–E♭.

gestural expansion and recoil is musically suggestive of a tide's flow and ebb. This formal process is initiated by a semitonal oscillation that grows into ever larger undulating gestures or waves and comes to an end by folding their registral amplitude until it rests again on a single semitone.[24]

3.2 "Rycerze" (Knights)

The text imagery and musical processes in "Rycerze," the fourth song in the set, are divided into two parts. First the knights set out boldly to battle, cast in an affirmative music, with busy galloping diatonic arpeggios and seventh chords (mm. 160–80); then they return home in silence wounded from fighting, which is cast in tragic diminished-seventh chords, at first long and loud, but becoming thinner

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24 Two additional aspects contribute to (but also somewhat complicate) the metaphorical reading of the gestural reversal of opening and closing chords as flow and ebb. First, this reversal is reinforced by the hand switch between white and black keys from opening to closing chords. This switch ingeniously allows the ambitus of the opening rh white key heptachord D4–B5 to be reconfigured within the exact same ambitus of the closing white key tetrachord an octave lower as D3–B4 (a white key pentachord is completed with the final G1, in the "wrong" lower register). Also, given that the opening and closing chords are not laid out strictly in the same register (such that the starting focal D4 is later reconfigured as D3), this registral shift might indicate that there's also a directionality to the piece's overall reversal gesture. We can read this lack of symmetry as perhaps suggesting that the "place" where the song begins is not strictly the same as where it ends, but rather alludes to organic variable shifts one can expect in flows and ebbs.

    The superposition of white and black keys is explored in a number of pieces by modernist composers in the evocation of imagery related to water, sea or mist. Some of these pieces include Ravel's *Jeux d'eau*, Szymanowski's "La fontaine d'Arethuse" from *Mythes*, Debussy's "Brouillards" from the *Piano Preludes*, book 2, and Bartók's "Boating" from the *Mikrokosmos*. I wish to thank one of the anonymous readers for suggesting some of these pieces and encouraging me to think in this direction.
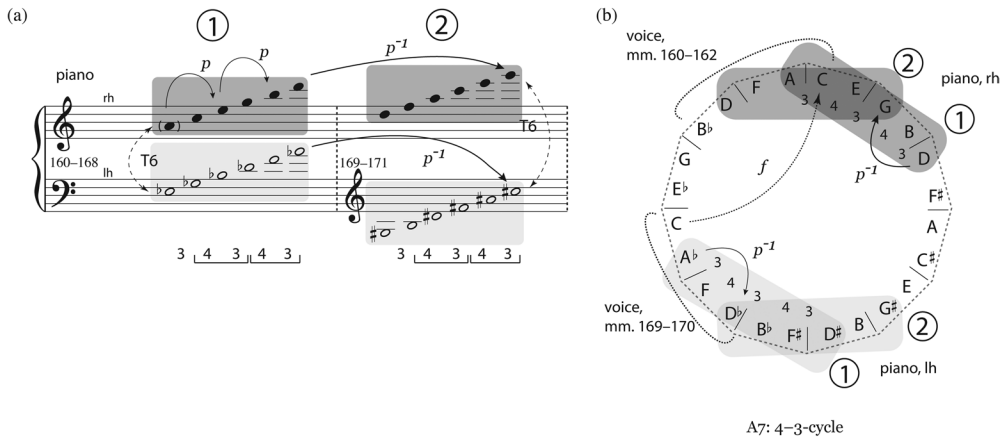
Figure 4. Harmonic space for "Rycerze" (chords 1 and 2, mm. 160–71): (a) complementary rh/lh hexachords for chords 1 and 2; (b) chordal relations via space representation A$_7$: 4–3-cycle.

and dispersed (mm. 181–99). The harmonic contrast between parts, however, is not only of constitutive vocabulary (diatonic vs. diminished chords) but also of distinct pitch systems: the first part suggests and exhausts a single affinity cycle, whereas the second part suggests a number of distinct affinity spaces that are integrated within a single transpositional network.

The harmonic space of Rycerze's first part is constituted by the succession of six twelve-tone chords. Figure 4a shows that the opening chord 1 is partitioned into two superimposed complementary hexachords (marked as filled and open noteheads). Both hexachords are registrally arranged to highlight the alternation of i3 and i4, thus suggesting the construction of a 24-pc space (Figure 4b), which displays affinities structured by the A$_7$: 4–3-cycle.

Chord 2 of the piece is similarly partitioned by two complementary hexachords. Each of the hexachords relates to those of chord 1 by the operation p^{-1}, thereby extending counterclockwise the harmonic region of chord 1 (see brackets in Figure 4b). There is an interesting transformation suggested by the singing voice from chords 1 to 2. While the voice melody emphasizes the interval distribution of underlying twelve-tone chords, the gestural contour projects a transformation between chords captured by the operation f, acting on pc C as suggested in Example 3. As the voice reaches the melodic peak C5 during chord 1 in m. 162, it effectively extends clockwise the position of the left-hand hexachord in the affinity cycle (see dotted bracket in Figure 4b). When C5 is later reached as the melodic peak in m. 169, it occurs within the interval context of chord 2; the arrow f (in the example and figure) connects these two positions of pc C, where the same pc is transformed from mq(1) to mq(0) in the cycle.

Chords 3 through 6 extend the harmonic relations established between chords 1 and 2, conveying a gradual and coherent exploration of the entire affinity space. As already discussed in the introduction of the article, Figure 1c shows

Example 3. Lutosławski's "Rycerze," *Iłłakowicz Songs*: transformation of harmonic/scalar context for the note C♯ in the voice melody (mm. 162, 169).
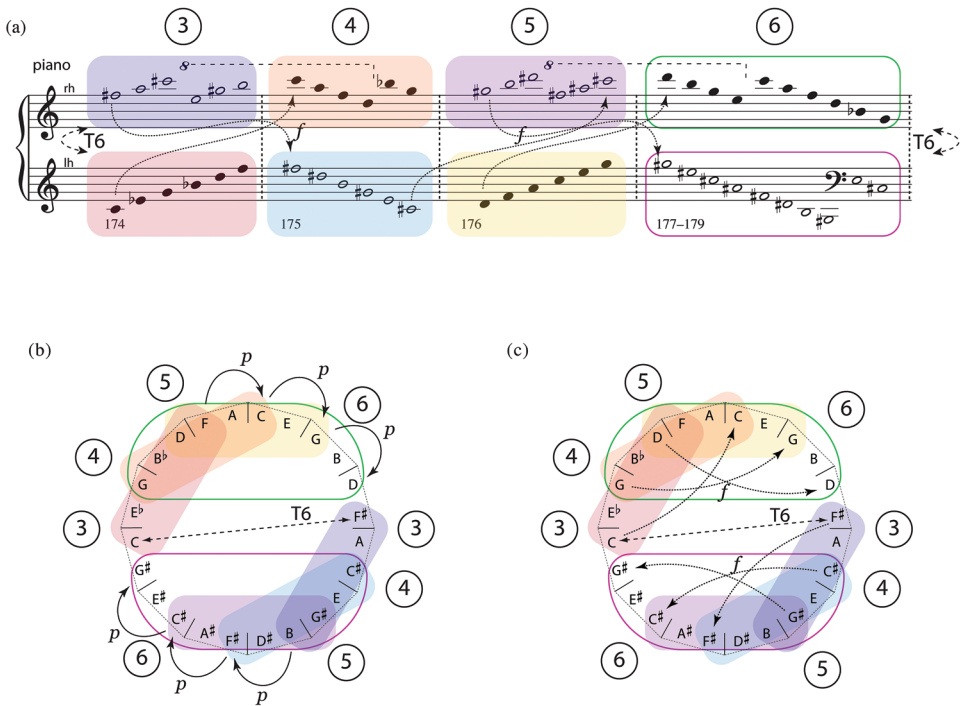
Figure 5. **Harmonic space for "Rycerze," *Iłłakowicz Songs* (chords 3–6, mm. 174–79): (a)
superimposition of right- and left-hand hexachords; (b) chordal segments related by *p*; (c) notes in
different chords related by *f*.**

that adjacent chords in this progression (from 3 to 6) partially overlap tetrachordal
segments, retaining a strong aspect of harmonic continuity despite the shifts in
registral usage. Figure 5a reconfigures this progression, emphasizing that harmonic
continuity engages a voice exchange between right and left hands (solid vs. open
note-heads and also color gradation). This harmonic continuity is further empha-
sized in Figure 5b, which shows that each new chord in the progression relates to
the previous chord by $T_p = T_7$, shifting clockwise each chord's harmonic region in
the space. (The common tetrachordal segments are represented by the overlap-
ping of chordal segments in the cycle). Also, the operation *f* instantiated by the solo
voice between chords 1 and 2 contributes to the sense of continuity of the passage.
The low C3 of chord 3 and the high C5 of chord 4 stand in the same transforma-
tion relation as the two C5s discussed for the voice during chords 1 and 2 (see
Figure 5c). (Similarly, the lower right-hand F♯4 of chord 3 is related by *f* to the high
F♯5 in the left hand of chord 4). These relations are then replicated by each succes-
sive chord up to chord 6, which covers almost the entire affinity space. And taken
together, chords 3–6 partition the affinity space into two complementary halves.

The second part of "Rycerze" speaks of listless horses returning home, trail-
ing the battle wagons, silent and dejected (mm. 181–99). The closing passage
(mm. 194–99, Example 4) is punctuated by five twelve-tone arrangements, each

Example 4. Lutosławski's "Rycerze," Iłłakowicz Songs: closing passage (mm. 194–99).
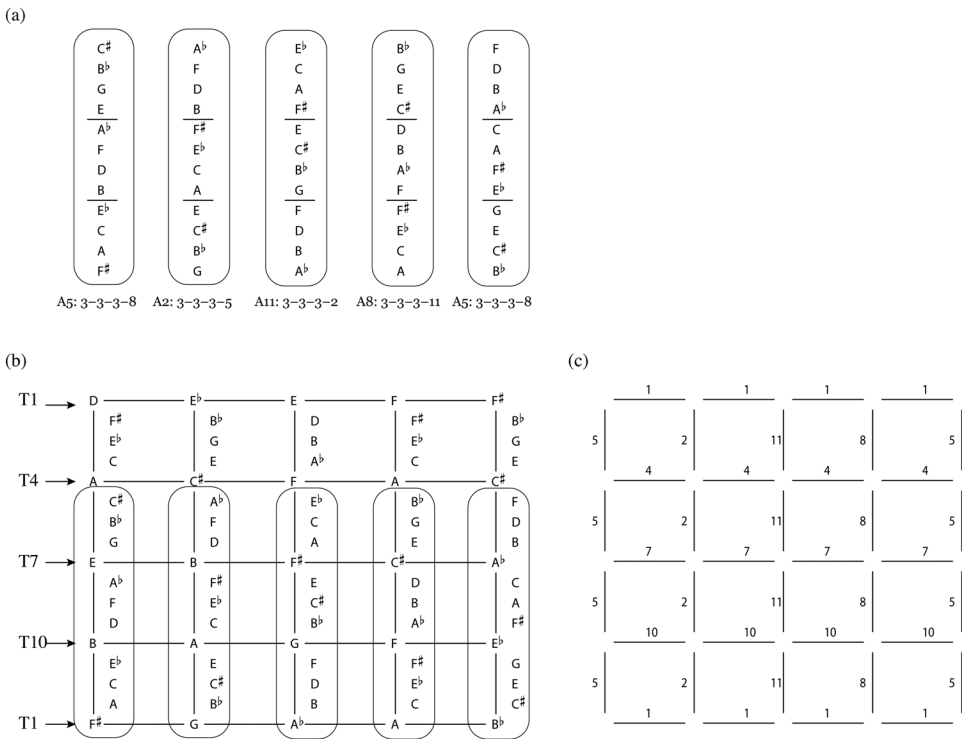
Figure 6. (a) Progression of five twelve-tone chords stacking fully diminished seventh chords; (b) lattice structuring both horizontal and vertical relations of chords; (c) *progressive* transpositional network with degree of variation △ 9 (mod 12).

consisting of three fully diminished seventh chords (i.e., [0369] tetrachords), which gradually overlap each other's registers, tending toward undifferentiated layers of activity, psychologically fitting for the text imagery. I use this musical passage to introduce the notion of transpositional network and its relation to the affinity spaces already explored in a few different variants.[25]

   The order and vertical spacing of arpeggiated fully diminished seventh chords create periodic procedures suggestive of affinity spaces (as harmonic constructs). The interval of recurrence for stacked diminished seventh chords varies in each iteration and suggests the following cycles with varying degrees of affinity: A_5, A_2, A_{11}, A_8, and back to A_5, shown in Figure 6a.[26] In turn, these different affinities indicate a counterpoint of three concurrent (horizontal) strands of fully diminished tetrachords, moving at different transpositional levels.

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25 *Transpositional networks* are formalized in Martins 2011. These structures may assume different designs (called *homogeneous*, *progressive*, and *dynamic*), depending on the formal features structuring the degree of variation between the edges of the transpositional lattice.

26 Rae (1994: 53) notes the importance of the changing intervals spanning between neighboring strands of the chord aggregate in this passage (what is referred to here as the *intervals of affinity* in each chord).

Throughout the passage, the strands progress (from lower to higher register) by T_1, T_{10}, and T_7 (mod 12), respectively, gradually contracting the entire texture in pitch space. Vertical and horizontal relations between tetrachords are configured in the lattice of Figure 6b. This network models the passage's three-part counterpoint, coordinating vertical transpositional levels (within twelve-tone chords) with horizontal transpositional lines (across chords). This coordination of horizontal and vertical transpositional levels underlies a *progressive transpositional network*. The network's lattice shown in Figure 6c highlights the vertical 5–2–11–8 and horizontal 1–10–7–4 transpositional levels up to the point of repetition.

While the transpositional values (connecting fully diminished sevenths) are distinct in vertical and horizontal dimensions, the degree to which they change both intrachordally (up in register) and interchordally (successively in time) remains constant. In other words, the vertical transpositional levels 5–2–11–8 decrease by −3 (or 9, mod 12) and similarly the horizontal values 1–10–7–4 vary by the same transpositional amount. The network thus displays a transpositional degree of variation (Δ) of 9 (mod 12), suggesting the vertical (harmonic) compression of the counterpoint is consistent with its linear progression.[27]

3.3. "Zima" (Winter)

The third song of the set, "Zima," slowly unfolds seven similarly constructed twelve-tone chords from beginning to the end of the song. The chords are registrally structured by A11=4–4–3, which can be thought of as augmented triads separated by minor thirds, as portrayed in the reduction in Figure 7a.[28] The rhythmic and registral grouping in the piano part partially confirms this triadic segmentation of the chord by gradually unfolding T_1-related (mod 12) augmented triads that descend in register while "ascending" in pc-space, and vice versa (Figure 7b). The affinity system is graphed in Figure 7c. Chord 1 extends beyond twelve tones (totaling fifteen pitches), that is, it is registrally bookended by the same augmented triad in different rotations (labeled Aug3). The mapping

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27 The vertical stacking of the actual twelve-tone chords does not use the transpositional level T_4 of the network. Nevertheless, its use would simply duplicate an existing fully diminished seventh chord, which would be redundant for the purpose of twelve-tone chord construction. In addition, the beginning of the second part of the song (mm. 181–93) displays three fully diminished seventh chords (arranged in register as B♯–D♯–F♯–A, G♯–B–D–F, and (E)–G–B♭–D♭) related by T_8, which conforms to the vertical transpositional levels of the network, though the actual pitches do not conform to the network's horizontal transpositional levels. The formula of this progressive transpositional network is discussed in Martins 2011: 132–34. The corresponding formula for this particular lattice is Δdx|y=Δdy|x=9, and Δdx|x=Δdy|y=0, where Δd refers to the degree of variation of transpositional values, attending to their change across axes x and y. Accordingly, the transpositional values in each horizontal line (x axis) move by 9 (mod 12) in every upward shift, and similarly each vertical line (y axis) moves by 9 in every rightward shift. Additionally, transpositional values do not change (degree of variation Δd is 0) along horizontal lines or along vertical lines (i.e., within chords and within contrapuntal lines).

28 This 4–4–3-cycle arrangement is discussed in Gollin (2007: 143–44, 153) as a multiaggregate cycle, pertaining to the analysis of the opening of Bartók's *Etude* op. 18/2.
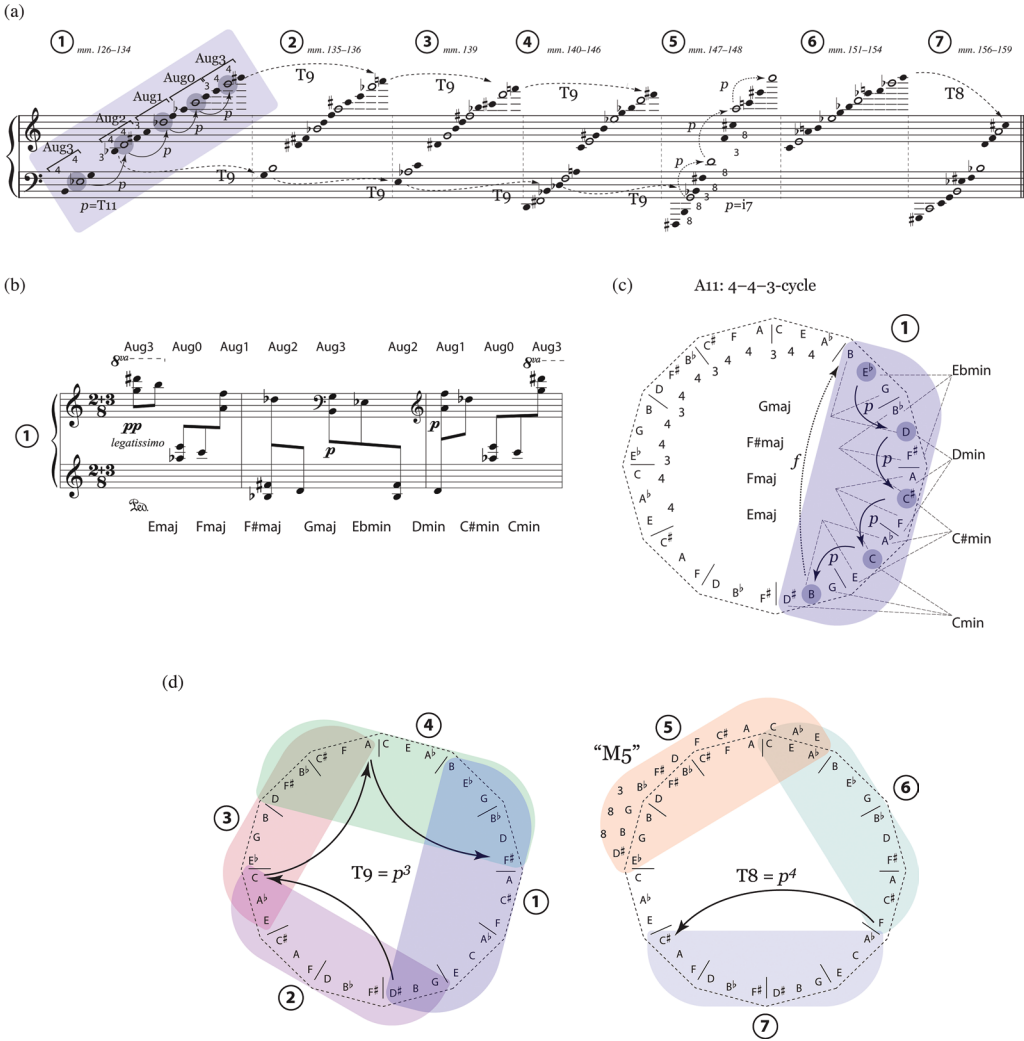
Figure 7. Harmonic space for "Zima": (a) seven twelve-tone chords as stacks of augmented triads; (b) chord 1: local segmentation of augmented and major/minor triads; (c) harmonic space, $A_{11} = 4$–4–3-cycle, suggested by the intervallic disposition of twelve-tone chords; (d) location of all chords in the affinity cycle and operations.

of Aug3 in the affinity cycle is thus assigned to two different locations or local regions (pc B, for instance, is related by f from the high B6 at mq(1) to lower register B2 at mq(0), within chord 1).

The slow unfolding of twelve-tone chords projects a slightly unbalanced rhythmic surface caused by the irregular alignment of superimposed notes in adjacent augmented triads (perhaps suggestive of the irregular patterns of falling snowflakes). The result is a harmonic flow that locally brings out major and

minor as well as augmented triads (Figure 7b).[29] Major and minor triads are projected by taking advantage of the periodicity interval of the space/chord (T_{11}). In other words, the "internal" notes of augmented triads (mq(1), indicated by open note-head and circled notes in Figures 7a and 7b) substitute for other mq(1)-positioned notes in an adjacent (upper or lower) augmented triads, connected by p arrows in the figure. The music thus engages a series of major triads ascending in pc-space when the music descends in register (Emaj, Fmaj, F♯maj, and Gmaj) and conversely a series of minor triads descending in pc-space when the music ascends in register (E♭min, Dmin, C♯min, and Cmin).[30]

The remaining twelve-tone chords unfolded throughout the piece adopt the formation of chord 1 (augmented triads separated by a minor third), except for the climactic chord 5, which redistributes the intervals (Figure 7a). Figure 7d projects the colored locations of all chords and indicates some basic relationships between them. The progression of chords 1, 2, 3, and 4 results in a continuous (and clockwise) exploration of regions in the space, where successive chords overlap by one augmented triad (or two in the case of chord 4). Because affinity-space elements are pitch classes, the overlapping segments between chords refer both to the highest and lowest augmented triads in consecutive chords. For instance, the segment G–B–D♯ in the highest register of chord 1 is contextually assigned to the same space location as the G–B–D♯ in the lowest register of chord 2 (the same occurs in the progression from chords 2 to 3, and 3 to 4). Chord 4 concludes the exploration of the entire space by overlapping two augmented triads with both chords 3 and 1. In the perspective of temporal succession, the first four twelve-tone chords reiterate descents by T_9, mod 12 (T_{-3} in pitch space), which also correspond to the operation of p^3 in the affinity cycle (see arrows connecting the most clockwise positioned notes in each chord representation, which correspond to the highest in register).

In other words, the first four-chord succession brings a certain closure to the musical process by exhausting the proposed affinity space (as a field of harmonic possibilities for this chordal class). The analytical exploration of space exhaustion—via a (p^3/T_9) series that uses up its (four) possibilities—is also suggestive of the textual imagery, which at this point invokes the image of snow falling until the sky is empty and frozen. The space exhaustion is followed by a climactic chord 5, which slows down the rhythmic flow with longer durations and the imagery of a carpeted earth, brimful of snow (one could suggestively add, *as a consequence of the snowfall during the first four chords*).

The remaining twelve-tone chords 5, 6, and 7 nearly reiterate a full exploration of the affinity space, though adopting a different strategy. Chord 5 stacks four augmented triads in open position, such that the chord's recurrent unit

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29 The "stacking" of augmented chords is structurally identical to the opening of Liszt's *Faust Symphony*.

30 The feature of close harmonic oscillation between major and minor is also explored in Lutosławski's works of the early fifties; see Paja-Stach 2001: 271–75.
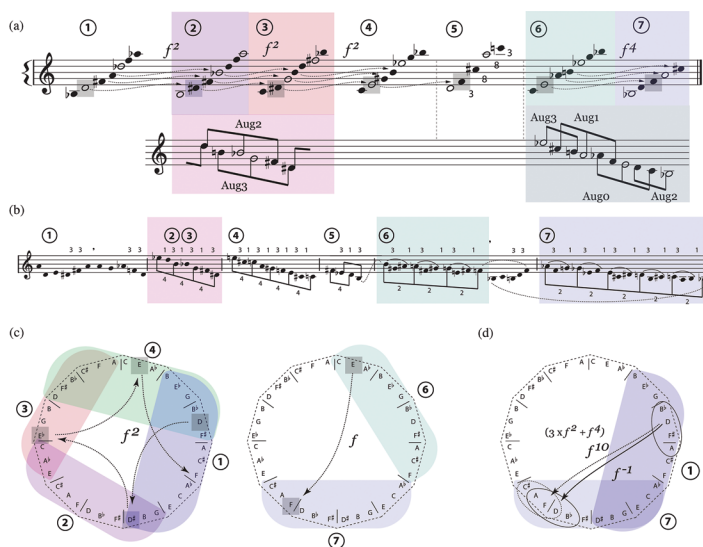
Figure 8. Mid-range chord space and voice interval patterns for "Zima": (a) transformations f^2 and f^4 across twelve-tone chords in the middle register; (b) voice patterns embody the voice-leading relations of adjacent chords; (c) voice leading projects D–D♯–E♭–E in chords 1–4; (d) cumulative voice-leading progression from chords 1 through 7.

becomes 8–8–3, resulting in an overall interval affinity of i7. Although chord 5 displays a different affinity from the piece's remaining twelve-tone chords, the registral order of the four augmented triads (from lowest to highest) is preserved in the affinity cycle by swapping the internal position of two notes of the augmented triad (using the operation M5) (see Figure 7d). Chords 6 and 7, while returning to original affinities (A_{11}), are now registrally split (the text refers to daylight and after sundown) and relate by T_8 (mod 12); in the affinity cycle, they are strictly adjacent and relate by p^4.[31]

The voice melody in "Zima," unlike in other *Iłłakowicz* songs, does not simply (more or less) reiterate the intervallic scheme of underlying twelve-tone chords, but rather tends to move more linearly, exploring the interval gaps between chord tones. I examine first the voice-leading relations between successive chords, then the vocal melody as a result of the chordal voice leading. Figure 8a focuses on voice-leading relations emerging at the treble-clef (voice) range. For instance, starting with D4 in chord 1 (boxed in the figure) and tracing the ensuing voice-leading path throughout the piece, there is a gradual ascent up to the climactic chord 5 (boxed notes D–D♯–D♯–E[−]F). As chord 6 proceeds to chord 7, the voice-leading E–F is reenacted. Figure 8c models and traces the voice-leading path in chords 1 through 4 by f^2 (see arrows inside the cycle). Since the operation f corresponds to −13 steps (from mq2 to mq1, or

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31 It is not clear why the particular space dispositions of chords 6 and 7 might be appropriate for the song's closure. The following analysis concerning the song's melodic patterns and total voice-leading addresses in part aspects of harmonic return.

mq1 to mq0), f^2 corresponds to 10 clockwise steps (or −26 steps), yielding the progression (D, 1) → (D♯, 2) → (Eb♭ 0) → (E, 1) → (F, 2). More generally, voice-leading relations between adjacent triads result in two voices moving by ascending semitone and one voice retaining the same pc. Note that the voice leading E–F from chord 4 to 5 still corresponds to a move of +10 steps in the space from (E, 1) → (F, 2). As discussed above, however, chord 5 brings a new chord type (M5 transform), where pc F is actually located at (F, 1). The E–F voice-leading from chord 6 to 7 retakes (E, 1) which is transformed via f^4 to (F, 2). In other words, the voice leading of the final chord progression does not replay the transformations of chords 1–4, but rather doubles the transformation value such that two voices (in a triad) move by semitone and the other moves by a whole tone.

The scalar patterns of the vocal melody (shown in the lower-level reduction of Figure 8b) embody the voice-leading relations between adjacent chords in the same register. For instance, the progression from chord 2 to chord 3 alternates between intervals i1 and i3, thus being periodic at the major third (i4). The interval alternation 3–1 seems to be modeled after the scalar arrangement (highlighted in the box of Figure 8a) that results from the voice leading connections by f^2 just discussed (i.e., the combination of chords 2 and 3 preserving register).

The voice patterns change during chords 6 and 7, descending by whole tones, though still elaborating on i3 and i1. Similarly, the whole-tone descent is modeled after the scalar pattern (color highlighted in the box of Figure 8a) that results from voice-leading connections between chords 6 and 7, but now as a result of transformations f^4. In particular, the triads Aug3 and Aug1 combine to model the voice patterns during chord 6, and the triads Aug0 and Aug2 combine to model the voice melody during chord 7.

Figure 8d shows that the voice-leading progression from chords 1 through 7 adds up to f^{10} (three times f^2 plus f^4). Chords 1 and 7 are the only two chords that embed A4 and Bb3, the first and last pitches of the singing voice in the piece. Therefore, we can think of chord 7 as a partial return home: partial because chords 1 and 7 have two mismatched pitches in their common register (i.e., chord 1: B–Eb–*G*–Bb–D–*F♯*–A–Db vs. chord 7: B–Eb–*F♯*–Bb–D–*F*–A–C♯), a relationship captured by f^{-1} in the space. One of these mismatches occurs between F♯4 and F4—a chromatic inflection that is particularly emphasized by the singing melodic voice during chord 1.

3.4 "Wiatr" (The Wind)

The second song of the cycle, "Wiatr," talks about the wind's capricious nature, blowing down railings, tearing up paths, and pulling down roof tiles and wires. The music explores this playful imagery by displaying rhythmic irregularities, metric changes, and the fast alternation of a variety of tetrachordal partitions of the aggregate. The coloristic aspects of twelve-tone chords in the piece are explored through the effect of various simultaneous close-position tetrachords in

the piano partitioning and exhausting the aggregate. Three different tetrachordal partitions are explored throughout the song using set-classes [0235], [0123], and [0257], which we can label *diatonic*, *chromatic*, and *pentatonic*, respectively. While each of the set-class partitions creates a distinct harmonic color, all three have in common the combination of a pair of whole-tone dyads (i2) at different transpositional levels. I take this feature as crucial to construct a *network of harmonic relations* between the different chord types. An additional partition of the aggregate (stackings of fully diminished seventh chords, set-class [0369]) appears at climatic passages toward the end of the piece (mm. 101–7 and 111–14), and closes off the song in a single verticality (m. 125), as a final "wind gust."

The passage in Example 5 (mm. 90–103) displays the four tetrachordal partitions (using distinct highlighting colors) leading to the song's climatic moment: [0257] in mm. 90–92, [0235] in mm. 92–95, [0123] in mm. 96–100, and [0369] in mm. 101–7. Each twelve-tone chord in the song is the result of the combination of three tetrachords played in close position and immediate succession. Except for the case of the [0369] partition, all other tetrachordal partitions are transpositionally combined to take advantage of the absence of ic4 in their respective set-classes, to produce the full aggregate via transposition by T4/T8.[32]

Based on the (registral and temporal) ordered relations between tetrachords, I propose a structural arrangement that maximizes whole-tone dyads between the three partitions that make up the body of the song: [0235], [0123], and [0257]. However, there is a marked distinction in how the network of harmonic relations proposed here sustains the modeling of twelve-tone chords for the piece. Unlike in most analyses offered in the article, the modeling of chordal space operating through this network does not directly correspond to specific chordal connections or associations that appear in the piece's chordal succession.[33] Rather, the vertical dimension of the network models the specific chordal constructions, corresponding to the various tetrachordal stackings throughout the piece. Therefore, the horizontal dimension of the network represents not specific chordal successions, but rather chordal connections abstractly related via their common local interval structure.

Figure 9a displays six twelve-tone chords (distributed by the six columns), which capture both the order of succession and register of the partitions used in the piece. Columns 1 and 2 arrange [0235] contiguous tetrachords, related

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32 These four tetrachordal set classes omit interval-class 4, thereby yielding the full aggregate when recursively transposed by T_4 or T_8. Only seven tetrachords have this property. Tetrachords in Example 5 that compose an aggregate do not registrally overlap, and the T_4 relation spanning between superimposed tetrachords actually corresponds to i16 in pitch space. Elsewhere in the song, tetrachordal stacking produces a slight register overlap between distinct tetrachords. For instance, the first chord of the piece (not shown here) registrally deploys T_4-related [0235] tetrachords (i4 in pitch space), C–D–E♭–F | E–F♯–G–A | G♯–A♯–B–C♯. Interestingly, the combination of tetrachords via T_4 throughout the first chord (first phrase, mm. 61–68) coordinates both their ascending register C–E–C♯, but also their temporal order of appearance E–G♯–C. However, the chordal rotation in mm. 69–73 that is, G♯–A♯–B–C♯ | C–D–E♭–F | E–F♯–G–A, retains a T_4 relation via register G♯–C–E, but a T_8 relation via temporal order C–G♯–E. The network of harmonic relations developed here explores both T_4 and T_8 relation between tetrachords.

33 The same principle applies to the case of the network constructed for song no. 5, ""Dzwony cerkiewne," discussed below.

Example 5. Lutosławski's "Wiatr," *Iłłakowicz Songs*: **tetrachordal partitions in mm. 90–103: [0257], [0235], [0123], and [0369].**

by T_4 and T_8 respectively; columns 3 and 4 do the same for [0123] tetrachords; and similarly, columns 5 and 6 arrange [0257] tetrachords by T_4 and T_8. The six columns form a closed network. (Note, however, that a complete system of tetrachordal relations comprises four transpositions of such closed network, i.e., twenty-four twelve-tone chords organized into four networks.)

Figure 9b shows intervallic arrangements for the six twelve-tone chords, where each column suggests an affinity space. Columns 1 through 6 alternate affinities, forming the cycles: A_4: 2–1–2–11, A_8: 2–1–2–3, A_4: 2–9–2–3, A_8: 2–9–2–7, A_4: 2–5–2–7, and A_8: 2–5–2–11-cycle. The arrangement of intervals in each column shows that it inscribes not one, but rather two tetrachordal partitions in adjacent positions, combining two whole-tone dyads (i2) with one of the odd intervals. For instance, the first column stacks tetrachordal partitions 2–1–2 and 2–11–2 (see the partially overlapping boxes in each column). In other words, each of the tetrachordal
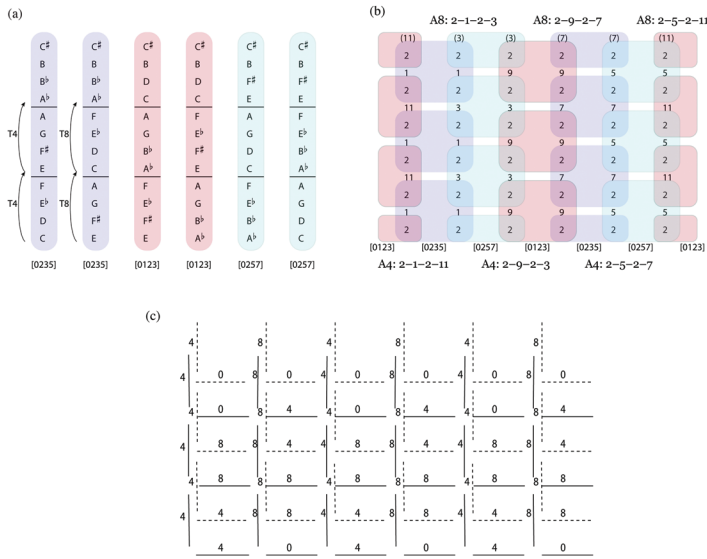
Figure 9. A closed network for "Wiatr," *Iłłakowicz Songs*: (a) six twelve-tone chords formed by contiguous tetrachords ([0235], [0123], and [0257]) related by T_4 and T_8; (b) intervallic arrangements of twelve-tone chords suggest affinity cycles, where each inscribes two tetrachordal partitions; (c) network of transpositional levels between whole-tone dyads.

partitions used in the piece in close position also *indirectly* inscribes another partition via *register* relations between tetrachords. In addition, the arrangement of columns emphasizes the display of common interval segments between columns, promoting a "horizontal" continuity of the network system. Figure 9c reduces the intervallic arrangement to a network of transpositional levels for whole-tone dyads (i.e., nodes in the network represent a whole-tone dyads), where the vertical values represent intraspace affinities and horizontal values represent interspace transpositional relations. As noted earlier, because the representations of Figures 9b and 9c focus on transpositional levels and not on specific pitch classes, they afford a system of four cyclic realizations (transpositions) of the six twelve-tone chords of Figure 9a. In addition, Figure 9c systematizes both the intrachordal transpositional relations within twelve-tone chords and interchordal relations across chords, highlighting the network's (vertical and horizontal) intervallic consistency.

The tetrachordal partitions of [0369] seem at first to be excluded from this closed network of whole-tone dyads. But the registral display of verticalized twelve-tone chords (as the stacking of [0369]s) reveals a closer relation with the remaining partitions. Figure 10 diagrams the two twelve-tone chords that combine [0369] partitions used in the song (mm. 111–14, 125, the climactic passage and final chord, resp.). In these verticalizations, the strict registral order of intervals in both chords also inscribes the remaining tetrachords in adjacent interval segments. This invites the interpretation that the "exceptional" wind gusts of combined diminished-seventh chords combine other wind types as expressed by the tetrachordal partitions.

Climatic passage   Final chord
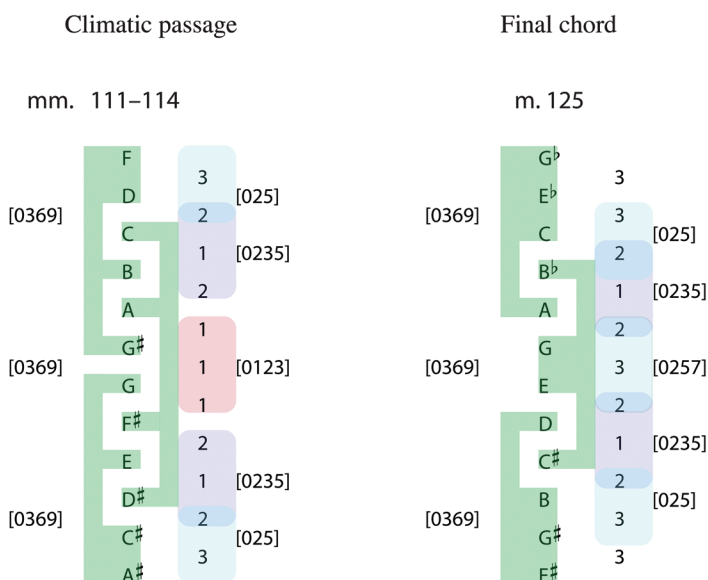
mm. 111–114   m. 125

Figure 10. The two twelve-tone chords that combine [0369] partitions in the song (climatic passage and final chord) and other "whole-tone" dyad tetrachords.

3.5 "Dzwony cerkiewne" (Church Bells)

The fifth and final song in the set is structured by two contrasting twelve-tone chords that sustain a two-part harmonic division of the piece (mm. 200–224 and 225–60, resp.). The expressive power of the song rests largely on the harmonic quality of the tetrachordal partitions in the first part, which have strong tonal resonances, and the trichordal partitions in the second part, which project a contrasting atonal character. These partitions are reinforced by sensuous and coloristic character, emphasized by the markings *soave* and *rude*, and mirror the textual imagery of the two-part division: "We like the bells pealing, . . . when joy leaps above the roof tops," which contrasts with "but then we also like the sound of the church bells when they are angry."

The question of harmonic drive or forward progress is perhaps less pressing here, since each twelve-tone chord is sustained for a considerable time span, and, as a result, our attention is perhaps more focused on intra-aggregate relations between the partitions. Nonetheless, the large-scale relations emerging from the juxtaposition of first and second parts, and in particular the relations between tetrachordal and trichordal segmentations, are not well understood and seem irreconcilable. The score of Example 6 shows the ending of the first part and the beginning of the second, mm. 220–29, highlighting in different colors tetrachordal and trichordal partitions of the aggregate.

The arrangement of intervals in the twelve-tone chords shown in Figure 11 exposes their dissimilar interval character, periodic features, and range. The

Example 6. Lutosławski's "Dzwony cerkiewne," *Iłłakowicz Songs*: end of the first and beginning of the second parts, mm. 220–29.

First part: 'bells pealing with joy'

Second part: 'bells are angry'

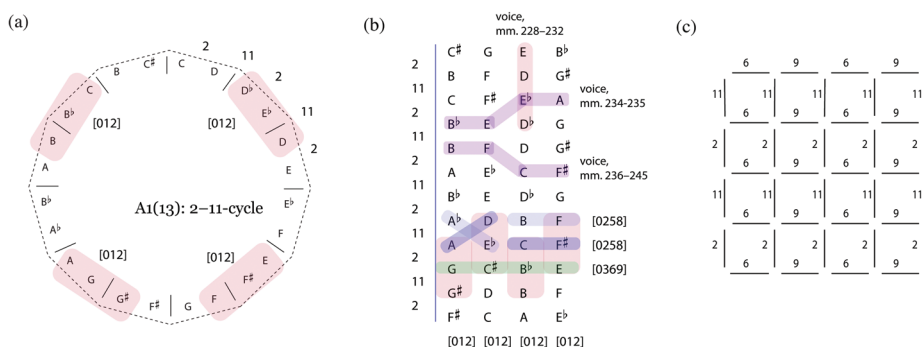**Figure 11. Contrasting twelve-tone chords in "Dzwony cerkiewne."**

Figure 12.   (a) affinity space A₁(₁₃): 2–11-cycle (or 11–2); (b) rotations of affinity cycle display a clustering of tetrachordal and trichordal partitions; (c) transpositional network.

tetrachordal partitions in the first part are (1) the fully diminished seventh chord G–B♭–C♯–E [0369] in the upper register of the piano (expressed as a combination of tritones E–B♭ and C♯–G), (2) the half diminished seventh chord E♭–F–A♭–B [0258] in the middle register (lighter blue), and (3) the dominant seventh chord D–F♯–A–C [0258] in lower register (darker blue). The upper register tetrachord opens the song and is carried out throughout the section continuously, whereas the half-diminished and dominant seventh chords are interspaced by quasi-periodic durations.[34] In the second part, the twelve-tone chord is partitioned into four "chromatic" [012] trichords expressed as 2–11 and 11–2 (i.e., D♭–E♭–D, G♯–G–A, E–F♯–F, and B–B♭–C), occupying almost the full acoustic range of the piano.

The interval disposition of the contrasting twelve-tone chords seems at first irreconcilable. The systematicity and interval periodicity of the harmony in the second part do not find a corresponding treatment in the first part. The contrasting character of twelve-tone chords, however, is built on complementary aspects of the interval arrangement of a network system. Figure 12a depicts the affinity space A₁(₁₃): 2–11-cycle (or 11–2), where the trichordal partitions of the second part's twelve-tone chord are highlighted (in red) in discontinuous regions of the space.

This space is reconfigured in Figure 12b, where the columns indicate various rotations of the affinity cycle, so that the trichordal partitions are vertically highlighted (in red) and now appear clustered horizontally in the network. This arrangement brings out close horizontal relations (green and darker blue ovals), which correspond to the tetrachordal partitions active in the first part of the song. Each of the three tetrachords uses one note of each of the second part's four trichords. The horizontal segment (in green) G–C♯–B♭–E [0369] corresponds to the tritone entanglement of the upper-register tetrachord, and the

34 The mid- and lower-register chords and notes B and C are interspaced by 7 or 6 quarter notes. The exact harmonic qualities of these remaining tetrachords depend on the associations the listener/analyst brings between the notes B (pc 11) and C (pc 0), which appear isolated both in the piano and the voice) and the trichords E♭–F–A♭ and A–D–F♯. The possible associations/segmentations are thus: E♭–F–A♭–B (half-diminished seventh or [0258]) and A–D–F♯C} (dominant-seventh or [0258]); or else E♭–F–A♭–C and A–D–F♯–B (both [0358] or minor-minor seventh chords). Besides the mentioned emphasis on B and C, the voice moves mostly chromatically, thus moving between the different tetrachordal partitions.

remaining middle- and lower-register tetrachords (E–F–A♭–B in lighter blue and A–D–F♯–C in darker blue, both [0258]) are formed by close-by positions in the network (the notes A♭ and B are pc replications graphically used for their proximity). Other prominent tetrachords explored by the singing voice in the second part of the song are also highlighted in certain contiguous regions of the system in Figure 12b. The [0123] chromatic tetrachord C♯–D♭–D–E is vertically aligned in red (mm. 228–32); and the [0167] tetrachords E–A–B♭–E♭ (mm. 234–35, 246–52) and F♯–B–C–F (mm. 236–45) also explore close by regions (in purple) in the space.[35] Finally, Figure 12c presents a reduction of the transpositional structure of the network, which displays a consistent alternation of transpositional levels both in the horizontal and vertical dimensions, which characterize the first and second parts, respectively, thereby suggesting their integration.

4. Other Representations of Harmonic Space

4.1 *Musique funèbre*'s "Prologue" and the Linearization of Twelve-Tone Rows

Lutosławski adopted some strict dodecaphonic procedures in *Musique funèbre* (1958) with the purpose of exploring harmony through contrapuntal means. Lutosławski insisted that his treatment of twelve-tone rows is substantially different from that of Second Viennese composers. In an interview to Bálint Varga in the 1970s, he explained that "Schoenberg's principles were among other things intended to replace functional harmony. I have never been interested in that goal. The use of a row had to serve a difficult purpose: to create a special kind of harmony. . . . In reality, then, *Funeral Music* has very little to do with twelve-tone music" (Varga 1976: 11). In *Musique funèbre*, a piece for string orchestra written in the memory of Béla Bartók, the compositional play of the outer sections ("Prologue" and "Epilogue") is structured by a sharp restriction of row intervals, rhythmic values, and a canonic imitation scheme, which limits the possibilities for resulting horizontal and vertical intervals. Dodecaphonic and imitative procedures in these sections act primarily as ways to systematize melodic and harmonic qualities, producing what Lutosławski refers to as an "atmosphere of open sonority":

> [*Musique funèbre*] is the only piece where I have used a twelve-tone row methodically—in the outer two sections. . . . But what matters in these two sections is the vertical result of using the row. . . . Used canonically it gives certain harmonic results, which—containing neither third nor sixth—produces a certain atmosphere of *open sonority*, which corresponds particularly to the title of the piece. (Couchoud 1981: 88; qtd. in Rae 1994: 67)

In "Prologue," twelve-tone rows are thematically linearized, strictly alternating tritones and semitones.[36] Rows are worked out contrapuntally in a gradual and

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35 In mm. 228–232, the range of the voice tetrachord (D♯–D) retains the characteristic i11 of the affinity space and network, and similarly in mm. 234–244 (E–E♭, and F♯–F) with the two characteristic Bartókian [0167] sonorities.

36 Linearized twelve-tone rows in the "Prologue" and "Epilogue" display a limited registral ambitus as opposed to the characteristic use of twelve-tone chords as (vertical) harmonic entities stretched across the register.

Example 7. Lutosławski's "Prologue," *Musique funèbre* (mm. 1–8): opening canon of prime-row statements at T_6 (P_5 and P_{11}), followed by inversion statements also at T_6 (I_{11} and I_5).

cumulative entanglement of tritone-related canonical entries (alternating sections built on transposed rows with sections of inverted rows) toward two climatic moments, resembling the overall textural and formal layout of the first movement of Bartók's *Music for Strings Percussion and Celesta*.[37] As shown in Example 7, the movement's opening two-part canon illustrates the "methodical" use of twelve-tone rows. Each pair of tritone-related rows—P_5 and P_{11}—reiterates melodically the alternation of i6 and i11.[38] The complete prime statements of the series are immediately followed by canonical entries of their inversions I_{11} and I_5, alternating melodic intervals i6 and i1, and producing i0, i5 (and i6) as vertical intervals. Once inversion statements are completed, the parts return to prime statements, and so on, alternating gradually denser canonical passages of prime and inversion row sections throughout the movement.[39]

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37 Lutosławski (1968: 113) explicitly states he has not "sought inspiration amongst Bartók's own music, and any eventual resemblances that may appear in *Musique funèbre* are unintentional. And if these resemblances do really exist, then this proves once again the undeniable fact that studying the works of Bartók has been one of the fundamental lessons to be taken by the majority of composers of my generation." The resemblances between the "Prologue" of *Musique funèbre* and the first movement of Bartók's *Music for Strings, Percussion and Celesta* are numerous, such as the canonic texture and overall formal layout (gradual buildup toward and from a climax), and chromatic themes framed by the tetrachord [0167] (four-note segments of the row in the "Prologue" and as registral boundaries in the first movement of *MSPC*). In comparing these pieces, Charles Bodman Rae (1994: 66–67) points out that the canonical technique conforms to an established convention of memorial tributes, such as Stravinsky's *In memoriam Dylan Thomas* (Dirge-Canons and Song 1954). Also, Joe Brumbeloe (1983) demonstrates that both the 24-cycle created by row pairings and the formal layout of the piece are strongly structured by symmetrical procedures. Given the interval series of the row, prime forms are duplicated by retrograde inversions, and inverted forms are duplicated by retrograde forms.

38 The labeling of the rows follows standard practice, using P and I to stand for *Prime* and *Inversion*, respectively, whereas the numeral refers to the pc starting the row. Also, the labeling of intervals (i) refers to ordered pc-intervals.

39 The canonical technique of successive entries unfolds an isorhythmic pattern (*talea*) of seventeen durations (during twenty-three beats). For an analysis of the prologue's formal processes, see Stucky 1981: 70–72.
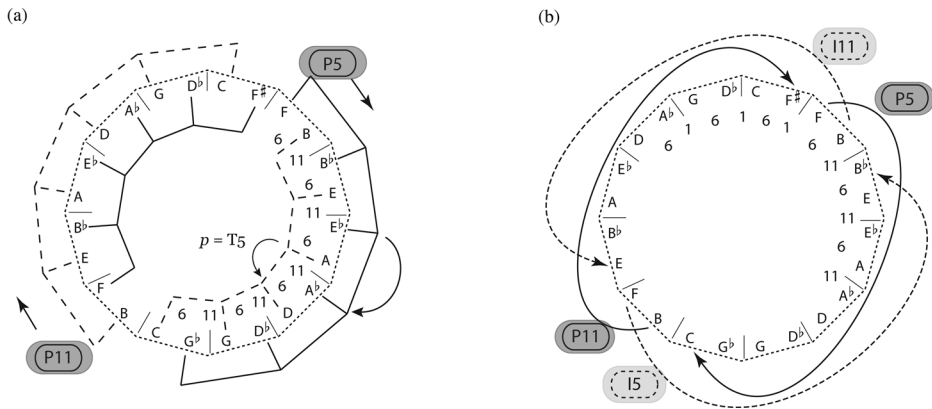
Figure 13. A 24-pc construct for "Prologue," structured as A$_5$: 6–11-cycle): (a) interlocked and complementary "lines of fourths" in each twelve-tone row; (b) prime and inversion rows explore the entire space (clockwise and counterclockwise, resp.).

Figure 13a introduces a 24-pc construct that models the opening canon of prime-row statements, as well as all row entries throughout the movement. Linearized P rows are modeled by a clockwise direction, and I rows by a counterclockwise direction. Structured as an A$_5$: 6–11-cycle, the construct captures aspects of the row's internal periodicity and the harmonic (vertical) space resulting from combined contrapuntal rows.[40] Given the consistent alternation of i6 and i11, each prime row can be conceived as the interlock of two complementary hexachords as "lines-of-fourths": F–B♭–E♭–A♭–D♭–G♭ and B–E–A–D–G–C, marked by solid and dashed lines. Both P$_5$ and P$_{11}$ use the same two hexachords but swap their respective order. There is an interesting recursive relationship of complementarity in this combination of rows: each linearized twelve-tone chord is made up of two complementary lines of fourths, and the entire cycle is made up of two complementary P$_5$ and P$_{11}$ statements. Also, the same recursive construct can be used to model the combination of inversions I$_5$ and I$_{11}$ (see Figure 13b), by allowing temporal order of the row elements to correspond to counterclockwise direction in the cycle, transforming i11 into i1.[41] As with the prime statements, the combination of tritone-related inversions exhausts the affinity space.

The canonical relation between rows produces the vertical "open sonorities" i0 and i7 as result of their nearly *punctus/contrapunctus* alignment (i6 "suspensions" are also occasionally interspaced). The affinity cycle also allows us to reconcile the row's alternation of intervals 6–11 with the vertical alignment, that is, intra- and interrow relations. These features are formalized as the two opera-

40 Robert W. Peck (2003: 11–13) analyzes the piece's canons as instancing of mapping the musical materials to the nodes of a Klein-bottle graph, where the alternation of (the operators) T$_{11}$ and T$_6$ creates a cyclic group of order 24.

41 Given the row's interval symmetry (using the Adjacency Interval Series), P also corresponds to RI and I to R, specifically P$_5$=R$_{10}$; P$_{11}$=RI$_6$; I$_5$=R$_{10}$; I$_{11}$=R$_4$.

tions of the space: $p=T_5 \bmod 12$ for the row's periodicity and $f=(T_0, T_7 \bmod 12)$ for the vertical alignment of rows. Example 8a illustrates that canonical entries of prime rows mostly align (see ovals from P_5 to P_{11}) at i0 and i7. The framing of these vertical "open sonorities" involves the f transformation between distinct positions mq(0) and mq(1) in the affinity cycle of Figure 14a. The alignments between different rows are modeled by either connecting a given pc(x) in mq(1) to $T_0(x)$ in mq(0) at 11 clockwise steps in the cycle (solid arrows), or a given pc(x) in mq(0) to $T_7(x)$ in mq(1), also 11 clockwise steps away (dotted arrows).[42] Conversely, the resulting vertical intervals from I_{11} to I_5 are modeled by $f^{-1}=(T_0, T_5)$, which corresponds to moving 11 counterclockwise steps in the cycle.

The movement proceeds from the opening canon to the first climactic point by slowly adding successive canonical parts. The canonical piling up is carefully constructed to retain vertical relationships of "open sonorities." As shown in Example 8b, mm. 33–37 introduce an eight-part canon right after the first climactic point. Selecting a "vertical slice" from bottom to top (see ovals in m. 35) yields a harmony of i0s (octaves/unisons) and i5s in adjacent voices (segment B–B–E–E–A–A–D–D), which extends the notion of open sonorities further. Figure 14b traces this vertical slice from top to bottom (D–D–A–A–E–E–B–B, following the order of canonical entries), resulting in a series of f transformations, alternating pc retention $T_0(x)$ and the spanning of $T_7(x)$.[43]

4.2 "Métamorphoses" and the Coordination of Diatonic Scales and Twelve-Tone Rows

"Métamorphoses," the second movement of *Musique funèbre*, explores the combination of a modal/diatonic perspective within the larger framework of twelve-tone chords. The movement progresses by a series of "episodes" in which "Locrian" segments are cumulatively added to each note of a twelve-tone row framework.[44] Figure 15a shows a reduction of episodes corresponding to Métamorphoses 1 and 6. In Métamorphose 1, every note of the twelve-tone row used in the prologue (connected by a stem) is coupled with a single note one semitone away. Each subsequent Métamorphose transposes by T_5 the twelve-tone row framework and expands the number of interpolated notes surrounding each note of the row, progressively filling in Locrian modes. By Métamorphose 6, Locrian segments encompass six notes. The first four segments of these are highlighted in the figure.[45]

42 Either the T_0-related pcs (same pc) are surrounded by different pc-neighbors in distinct parts of the space, or T_7-related pcs are neighbored by the same pair of pcs.

43 We could also model the vertical moment by a series of $\pm f$ or $|f|=(ic0, ic5)$ to avoid imposing an interval direction to the vertical harmony.

44 Each episode in "Métamorphoses" adds a note to a Locrian segment until the mode 1–2–2–1–2–2 reaches completion. For a detailed account of the combination of Locrian segments and the twelve-tone row framework in each episode, see Rae 1994: 67–69.

45 Episodes 3 and 4 in "Métamorphoses" have four notes each. Given the interval segment 1–2–2–1, Métamorphose 4 completes it by using one note of the neighboring segment (thereby standing for a five-note segment). For that reason, episode 6 reaches six notes per segment.

Example 8. Lutosławski's "Prologue," *Musique funèbre*: **vertical "open sonorities": (a) mm. 1–4; (b) mm. 33–37.**
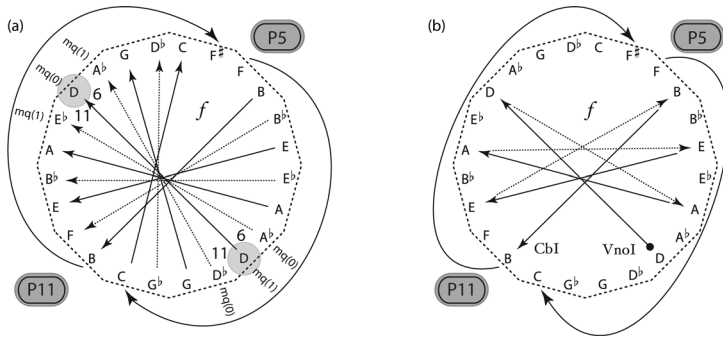
Figure 14. Transformation *f* = (i0, i7), mod 12. The transformation *f* either shifts a given pc(x) to T₀(x) located 11 clockwise steps away, in a different local context (solid lines), or it shifts a given pc(x) to T₇(x), 11 clockwise steps away, which neighbors the same pair of pcs (dotted lines): (a) mm. 1–4; (b) mm. 33–37.

Figure 15. Pitch reduction of "Metamorphoses" (second movement of *Musique funèbre*): (a) in sections (Métamorphoses) 1 and 6, P-rows are integrated within Locrian segments; (b) integration of P-row space within an affinity cycle A₅: 2–2–1-cycle, which embeds Locrian segments.

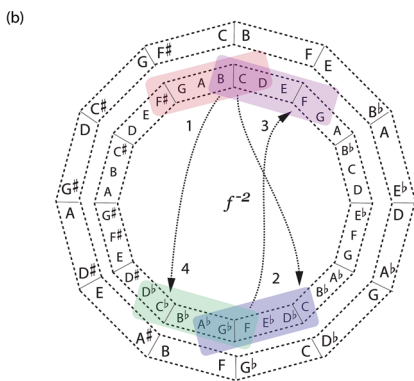Figure 15b suggests a way of integrating the Locrian segments within the twelve-tone row framework. The figure shows two coordinated pitch spaces: the external circle retains the 24-pc space I have used to model the prologue, whereas the internal circle presents a 36-pc space, whose affinity is structured by $A_5 = 2–2–1$-cycle.[46] The two spaces are coordinated by reversing the order of their constituent semitones. All Locrian segments are efficiently embedded in the 36-pc space. The figure highlights the location of the first four Locrian segments in the episode. Adjacent Locrian segments in the music are located in quasi-opposite locations in the affinity space; the harmonic continuity between contrasting segments thus relies in part on common-tone relations. Arrows in Figures 15a and 15b connect some of the common tones between adjacent Locrian segments via f^2. These common tones often appear in the score as sustained notes across successive sections and characterize the change of interval context for a given pc in successive Locrian segments.

4.3 "Postludium 1" from *Three Postludes* and Dynamic Transpositional Networks

Finally, I now turn to aspects of harmonic construction and process for "Postludium 1," composed between 1958 and 1960, and published as the first of *Three Postludes*. The harmonic setup of the piece observes many of the features regarding chord construction and isorhythmic structure characteristic of earlier pieces of the period. The piece, however, makes use mostly of vertical hexachords that engage a variety of recurrent patterns, resulting in a complex but highly directed chordal space.[47]

Following Stucky (1981: 80–83), I divide the piece into three sections as suggested in Figure 16a. Section 2 presents us with the most complex chord progression, grounded on a gradual contraction of intervals from chord 3 up to chord 7. After the crescendo to triple forte during chord 7, the harmonic contraction is temporarily halted by the ethereal chord 8, in triple piano, which returns to the more consonant sonority (stacked fourths) that characterizes chord 2 in section 1, where the sudden *pianissimo* helps evoke the image of a wishful return. This return, however, is ultimately denied by the climatic chord 10 in triple forte, occurring nearly at the point of Golden Section (Rae 1994: 72–73), which further extends the contraction of intervals to a semitonal cluster.[48]

The harmonic progression of section 2 is structured by a network (Figure 16b) not yet discussed in the article, a *dynamic transpositional network*. This kind of network is modeled by a lattice of pc-intervals that gradually contracts (or expands) every successive interval by a constant variation, which in the present case

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46 Martins 2006 refers to the affinity cycle A_5: <2, 2, 1> as *Guidonian space*, as it embeds Guidonian hexachords in neighboring modules.

47 Stucky (1981: 72) considers the postludes compositionally problematic, failing to address the complex issues of "time and rhythm, polyphony and form" that were eventually reached with *Jeux Vénitiens*. Being aware of these limitations, Lutosławski gave up on the originally planned four-movement scheme.

48 The passage is also discussed in Stucky 2001, 156–58.

(a)

Section I | Section II | interval contraction | fff ppp failed climax | fff | Section III climax

bb. 41   44   46   48   50   54   56   61

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10)

p = 2

A2: 7–7   A2: 5–9   A1: 7–6   A1: 6–7   A1: 5–8   A1: 4–9   A1: 3–10   A2: 5–9   A1: 3–10

p = 1   (*p* = 2)   *p* = 1
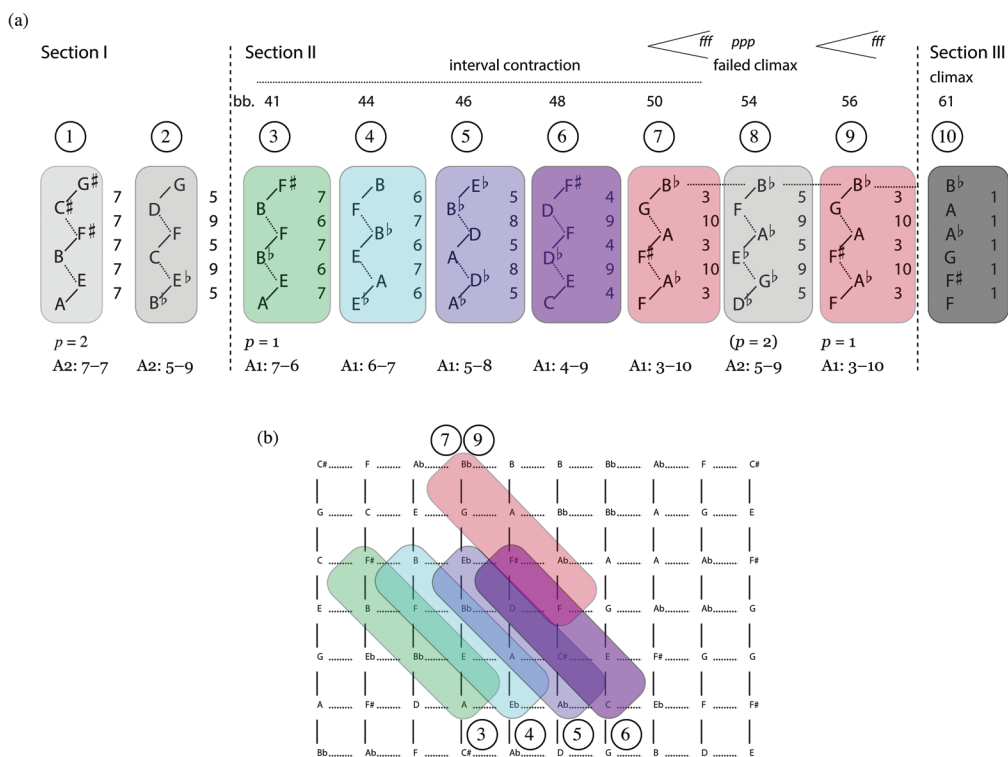
(b)

(7)(9)

(3) (4) (5) (6)

Figure 16. (a) Harmonic progression of hexachords in "Postludium I"; (b) the lattice represents a *dynamic transpositional network.*

is 11 (mod 12). In other words, comparing a given directed pc-interval i(x) (represented in the figure by solid or dotted dashes between pcs) with a parallel and adjacent interval in a (right or up) direction always yields a i(x − 1) (mod 12).[49] Given this network structure, each chord in the figure is constructed by moving northwest in zig-zag segments. One of the interesting features of this interval arrangement is that the sum of two consecutive zig-zag intervals is always i11, mod 12. This interval corresponds to the *transposition* (*p*) characterizing the affinity of chords 3–7, though the internal arrangement of hexachords varies. The entire network is thus not homogenous since the space gradually contracts toward the right of the network. The progression from chords 3–7 thus moves toward this space contraction by using some of the common tones between successive chords. In addition, chord 7 (considered as a pc-set) creates the first semitonal cluster. Figure 16a shows that chords 1 and 2 as well as chord 8 use the interval of *transposition p*=i2 to structure their affinities, thus belonging to a different subspace. However, chord 10, a semitonal cluster, stands as an appropriate progression from chord 7, reinterpreting its notes in the low register, while the upper register extends to the entire aggregate.

49 Martins (2011: 136–37) defines this dynamic T-net as Δdx|x=Δdy|y=Δdx|y=Δdy|x=11.

Concluding Thoughts on Lutosławski's Harmony

Despite the fascination Lutosławski's harmony has exerted on scholars, composers, and audiences since the mid-fifties of the past century, the scope of analytical studies characterizing the harmonic organization of this music has been somewhat restricted by the overpowering emphasis on the examination of individual (twelve-tone) chords and rows, especially their strict characterization as pitch-entities fixed in register. This focus has been implicitly justified by the idea that Lutosławski's chords function primarily as color entities over their capacity of conveying harmonic syntax. Also, the focus on harmonic color and its association with the specific distribution of intervals (and groups of intervals) in register has left largely unaddressed the multiple instances in which considerations about pitch and pitch-interval patterns interact with and give rise to intra- and interchordal connections and, more generally, the sense of motion in chordal space. While I make no explicit claims about the activation of a syntactical regulatory system, I expand the traditional harmonic focus on chordal construction to model aspects of chordal connection and association, thereby assessing formal aspects of continuity, contrast, complementarity, and so on.

I have explored how Lutosławski's chordal space is activated by the connecting potential of individual chords, harmonic change, and other interchordal relations. I have argued that the harmony's potential for chordal progression and association requires a reconceptualization of chordal elements, in which they are better understood in terms of a combination of pitch class and modal quality, rather than simply as pitches fixed in register. Chordal space, therefore, becomes not simply a succession of pitch fields, but rather a dynamic hybrid space, where both registral positions and the recontextualization of pitches and intervals across chords impact the estimation of larger harmonic processes and the surmise of strategies for harmonic continuity, contrast, complementarity, and other relations.

This analytical regime was probed in Lutosławski's harmony of the mid- to late fifties, in particular the *Five Iłłakowicz Songs*, which inaugurated the composer's work with vertical aggregates and initiates his mature period. The harmonic possibilities for chordal classes are constituted as a set of suggestive graphic representations, consisting of cyclic groups (affinity spaces) of non-octave-repeating interval patterns and various types of two-dimensional lattices (transpositional networks) that coordinate those cycles. The principles of construction and operation that structure these representations comprise a theoretical system of posttonal space that can be interpreted as an *esthesic structure*, which mediates the emergent harmonic relations for Lutosławski's chordal space.[50] By showing that chordal construction is reconciled with chordal progression in this music, the article proposes to reappraise a set of transformative pieces of the mid- to late fifties that anchor an influential harmonic practice in twentieth-century music.

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50 The notion of considering a music-analytic methodology as an esthesic structure (in contrast to the music's *immanent structure*) is developed in Rings 2011: 35–40. Of course, the framework reference for these distinctions is the tripartite semiological division (poietic, neutral, and esthesic levels) advanced by Jean-Jacques Nattiez (1990), who borrowed these terms from semiologist Jean Molino.

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José Oliveira Martins is a music theorist and professor in the Faculty of Arts and Humanities and Director of the Center for Interdisciplinary Studies (CEIS20) of the University of Coimbra. He is the current president of the Portuguese Society for Music Research. ⸎