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A FIEDLER'S TYPE CHARACTERIZATION OF BAND MATRICES

AMÉRICO BENTO AND ANTÓNIO LEAL DUARTE

Abstract: Let $\mathbb{K}$ be a field and p an integer positive number. We denote by $\mathcal{B}_n^p(\mathbb{K})$ the set of n-by-n symmetric band matrices of bandwidth $2p-1$, i.e., if $A = [a_{ij}] \in \mathcal{B}_n^p(\mathbb{K})$ then $a_{ij} = 0$ if $|i-j| > p-1$. Let $\widehat{\mathcal{B}}_n^p(\mathbb{K})$ be the set of matrices from $\mathcal{B}_n^p(\mathbb{K})$ in which the entries (i,j), $|i-j| = p-1$, are different from zero.

Let A be a n-by-n symmetric matrix with entries from $\mathbb{K}$; and p such that $3 \leqslant p \leqslant n$. We will show that: $\operatorname{rank}(A+B) \geqslant n-p+1$, for every $B \in \mathcal{B}_n^{p-1}(\mathbb{K})$, if and only if $A \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$.

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1. Introduction

Miroslav Fiedler, in [2], (see also [5] for a different proof) characterizes real symmetric irreducible tridiagonal matrices (up to permutational similarities) in the following way:

Theorem 1. *(Fiedler's characterization of tridiagonal matrices) Let A be a n-by-n real symmetric matrix. Then, $\operatorname{rank}(A+D) \geqslant n-1$, for every real diagonal D, if and only if A is irreducible and tridiagonalizable by a permutational similarity.*

In this work we answer the following question: *what happens if instead a diagonal matrix D, we consider a symmetric band matrix of a fixed bandwidth?*

More precisely, given integers n and p, with $3 \leqslant p \leqslant n$, we want to know which are the n-by-n symmetric matrices A, with elements in an arbitrary field $\mathbb{K}$, that satisfy the relation

$$\operatorname{rank}(A+B) \geqslant n-(p-1). \tag{1}$$

for any n-by-n symmetric band matrix $B = [b_{ij}]$ with elements in $\mathbb{K}$ and bandwidth $2p-3$ (that is such that $b_{ij} = 0$ if $|i-j| \geqslant p-1$).

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It is not difficult to see that the symmetric band matrices $A = [a_{ij}]$ of bandwidth $2p-1$ and such that $a_{ij} \neq 0$ if $|i-j| = p-1$ satisfy the relation (1) (see below the first part of the proof of Theorem 6). We show that these matrices are the *unique matrices* satisfying (1). In fact, our result is:

Theorem 2. *Let p, n be integers such that $3 \leqslant p \leqslant n$ and $A = [a_{ij}]$ a n-by-n symmetric matrix with entries from the field $\mathbb{K}$. Then, we have: rank $(A+B) \geqslant n-(p-1)$, for every symmetric band matrix B of bandwidth $2p-3$, if and only if A is a band matrix of bandwidth $2p-1$ with the entries (i,j) such that $|i-j| = p-1$ different from zero.*

We note that the case $p = 2$ and $\mathbb{K} = \mathbb{R}$ correspond to Fiedler's characterization of tridiagonal matrices (Theorem 1); the case $p = 2$ and $\mathbb{K}$ arbitrary was studied in [1] where it is proved that besides the matrices permutational similar to tridiagonal matrices, when $\mathbb{K} = \mathbb{Z}_3$ and $n = 5$ there are some matrices with nonzero nondiagonal elements satisfying (1) (see [1] for the details).

Some notation. We use $\mathcal{M}_n(\mathbb{K})$ to denote the set of n-by-n matrices with entries from a field $\mathbb{K}$ and $\mathcal{S}_n(\mathbb{K})$ to denote the subset of $\mathcal{M}_n(\mathbb{K})$ whose elements are the symmetric matrices. When Y denotes an element from $\mathcal{M}_n(\mathbb{K})$, y_{ij} will denote the entry of Y that is in the row i and column j.

Let $A \in \mathcal{M}_n(\mathbb{K})$ and $p \in \mathbb{N}$. The superdiagonal and the subdiagonal whose entries (i,j) such that $|i-j| = p-1$ will be named, jointly, by "diagonals(p)"; the column j and the row j of A will be named, jointly, by "lines(j)".

We denote by $\mathcal{B}_n^p(\mathbb{K})$ be the subset of $\mathcal{S}_n(\mathbb{K})$ whose elements are band matrices of bandwidth $2p-1$, i.e.,

$$\mathcal{B}_n^p(\mathbb{K}) = \{B \in \mathcal{S}_n(\mathbb{K}) : b_{ij} = 0,\ |i-j| \geqslant p\};$$

and denote by $\widehat{\mathcal{B}}_n^p(\mathbb{K})$ the set of matrices $B \in \mathcal{B}_n^p(\mathbb{K})$ such that its diagonals(p) are zero-free, that is,

$$\widehat{\mathcal{B}}_n^p(\mathbb{K}) = \{B \in \mathcal{B}_n^p(\mathbb{K}) : b_{ij} \neq 0,\ |i-j| = p-1\}.$$

We note that $\widehat{\mathcal{B}}_n^1(\mathbb{K})$ is the set of diagonal nonsingular matrices and $\widehat{\mathcal{B}}_n^2(\mathbb{K})$ is the set of tridiagonal irreducible symmetric matrices.

The set of matrices $A \in \mathcal{S}_n(\mathbb{K})$ such that $\operatorname{rank}(A+B) \geqslant n-(p-1)$, for every $B \in \mathcal{B}_n^{p-1}(\mathbb{K})$, will be denoted by $\mathcal{F}_n^p(\mathbb{K})$, i.e.,

$$\mathcal{F}_n^p(\mathbb{K}) = \{A \in \mathcal{S}_n(\mathbb{K}) : \forall\, B \in \mathcal{B}_n^{p-1}(\mathbb{K}),\ \operatorname{rank}(A+B) \geqslant n-(p-1)\}.$$

(although Theorem 2 just say that $\mathcal{F}_n^p(\mathbb{K}) = \widehat{\mathcal{B}}_n^p(\mathbb{K})$ we need some properties of the set $\mathcal{F}_n^p(\mathbb{K})$ to prove that theorem and so it is convenient to have a notation for this set).

Let $\alpha = \{i_1, i_2, ..., i_k\}$ and $\beta = \{j_1, j_2, ..., j_k\}$ be sets of integers such that $1 \leqslant i_1 < i_2 < \cdots < i_k \leqslant n$ and $1 \leqslant j_1 < j_2 < \cdots < j_k \leqslant n$. Let $A \in \mathcal{M}_n(\mathbb{K})$. We use $A[\alpha|\beta]$ to denote the submatrix of A contained in the rows indexed by α and columns indexed by β. Also, we use $A(\alpha|\beta)$ to denote the submatrix of A obtained from A by deleting rows indexed by α and columns indexed by β. When $\beta = \alpha$ we write $A[\alpha]$ and $A(\alpha)$ instead $A[\alpha|\alpha]$ and $A(\alpha|\alpha)$, respectively.

For a matrix $A \in \mathcal{M}_n(\mathbb{K})$, we denote by A' the matrix that results from A by gaussian elimination along the lines(j), for a previous fixed j.

We may see our Theorem 2 (as well as the Theorem 1) as results about *Completion Problems* (see e.g. [3], [4]): by a *partial matrix* we mean a matrix in which some of the entries are specified (or prescribed) elements of a certain set S, while others are independent indeterminate variables over S (the unspecified elements). A *completion* of a partial matrix is the matrix with elements in S obtained from the partial matrix when we specify for each of these variables a value of S. So if $A = [a_{ij}]$ is a partial matrix, a completion of A will be any matrix $B = [b_{ij}]$ with elements in S, the same dimensions of A, and such that if a_{ij} is specified in A, $b_{ij} = a_{ij}$. A matrix completion problem asks whether any (in some problems, at least one) completion of a partial matrix has a completion with certain properties.

In our case, the prescribed entries of A are the entries in positions (i, j) with $|i - j| \geqslant p - 1$, while we may see the remaining ones as free variables. We want that for any completion C of A, C is a symmetric n-by-n matrix with rank$C \geqslant n - (p - 1)$. What Theorem 2 says is that this is only possible if and only A is a band matrix of bandwidth $2p - 1$ with the entries (i, j) such that $|i - j| = p - 1$ different from zero. In sequel we sometimes use these kind of ideas and we think of an $A \in \mathcal{M}_n(\mathbb{K})$ as a partial matrix viewing the entries in positions (i, j) with $|i - j| < p - 1$ as free variables.

2. Basic properties of $\mathcal{F}_n^p(\mathbb{K})$

From the characterization of $\mathcal{F}_n^p(\mathbb{K})$ follows:

Proposition 3. *Let $A \in \mathcal{F}_n^p(\mathbb{K})$. If B is a symmetric band matrix of bandwidth not greater than $2p-3$ then $A+B \in \mathcal{F}_n^p(\mathbb{K})$.* ∎

The following result, which is a slight generalization of a result in [5], allows us to use induction in the proof of Theorem 2.

Proposition 4. *Let $i \in \{1, n\}$. If $A \in \mathcal{F}_n^p(\mathbb{K})$ and $a_{ii} \neq 0$ then $A'(i) \in \mathcal{F}_{n-1}^p(\mathbb{K})$.*

Proof. We assume that $i = n$. We take A partitioned as

$$A = \begin{bmatrix} A(n) & b \\ b^T & a_{nn} \end{bmatrix}.$$

The gaussian elimination along the lines(n) is equivalent to the product of

$$E = \begin{bmatrix} I_{n-1} & -a_{nn}^{-1} b \\ 0 & 1 \end{bmatrix}$$

by A and the resulting matrix, EA, by E^T, i.e., it is equivalent to computation of the matrix EAE^T. In fact:

$$EAE^T = \begin{bmatrix} A(n) - a_{nn}^{-1} bb^T & 0 \\ 0 & a_{nn} \end{bmatrix} = \begin{bmatrix} A'(n) & 0 \\ 0 & a_{nn} \end{bmatrix}.$$

Let $B_1 \in \mathcal{B}_{n-1}^{p-1}(\mathbb{K})$ and we take $B = B_1 \oplus 0$. The matrix E is nonsingular; so: $\operatorname{rank}\left(E(A+B)E^T\right) = \operatorname{rank}(A+B)$. On the other hand,

$$E(A+B)E^T = \begin{bmatrix} A'(n) + B_1 & 0 \\ 0 & a_{nn} \end{bmatrix}.$$

From these two facts, we have $\operatorname{rank}(A+B) = \operatorname{rank}(A'(n) + B_1) + 1$; from this and by the hypothesis, we have: $\operatorname{rank}(A'(n) + B_1) \geqslant n - p$. Therefore, $A'(n) \in \mathcal{F}_{n-1}^p(\mathbb{K})$.

The case $i = 1$ is proved similarly. ∎

Lemma 5. *Let $n \in \mathbb{N}$ such that $3 \leqslant n$. If $A \in \mathcal{F}_n^n(\mathbb{K})$ then $A \in \widehat{\mathcal{B}}_n^n(\mathbb{K})$.*

Proof. Let $A \in \mathcal{F}_n^n(\mathbb{K})$. We may consider A in the form

$$A = \begin{bmatrix} x_{11} & \cdots & x_{1\,n-1} & a_{1n} \\ \vdots & \ddots & \ddots & x_{2n} \\ x_{1\,n-1} & \ddots & \ddots & \cdots \\ a_{1n} & x_{2n} & \cdots & x_{nn} \end{bmatrix},$$

where x_{ij} denotes the non prescribed entries. The unique prescribed entries in A are the $(1,n)$ and $(n,1)$ entries. These are the entries of the diagonals(n). We claim that these entries are different from zero. In fact, if they were zero, we take $B \in \mathcal{B}_n^{n-1}(\mathbb{K})$ such that

$$B = -\begin{bmatrix} x_{11} & \cdots & x_{1\,n-1} & 0 \\ \vdots & \ddots & \ddots & x_{2n} \\ x_{1\,n-1} & \ddots & \ddots & \cdots \\ 0 & x_{2n} & \cdots & x_{nn} \end{bmatrix}$$

and, then, the matrix $A+B$ has all its entries equal to zero. Therefore, we have $\operatorname{rank}(A+B) = 0$ and we get a contradiction. So, $a_{1n} \neq 0$ and $A \in \widehat{\mathcal{B}}_n^n(\mathbb{K})$. ∎

3. The main result

We restate Theorem 2, using the notation we introduce in Section 2.

Theorem 6. *Let $\mathbb{K}$ be a field and p, n integers such that $3 \leqslant p \leqslant n$. Then, we have: $A \in \mathcal{F}_n^p(\mathbb{K})$ if and only if $A \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$.*

Proof. We start by proving that the condition is sufficient for $A \in \mathcal{F}_n^p(\mathbb{K})$. Let $A \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$. We take any matrix $B \in \mathcal{B}_n^{p-1}(\mathbb{K})$. The matrix $A+B$ is an element from $\widehat{\mathcal{B}}_n^p(\mathbb{K})$ and has diagonals(p) equal to the diagonals(p) of A. So, the submatrix $(A+B)[1,...,n-p+1|p,...,n]$ is lower triangular and has diagonals elements different from zero and then has rank $n-p+1$. Therefore, as $\operatorname{rank}(A+B) \geqslant \operatorname{rank}((A+B)[1,...,n-p+1|p,...,n])$, we have:

$$\operatorname{rank}(A+B) \geqslant n-p+1.$$

Hence, as B is any matrix from $\mathcal{B}_n^{p-1}(\mathbb{K})$, we have $A \in \mathcal{F}_n^p(\mathbb{K})$.

We show, now, that the condition is necessary for $A \in \mathcal{F}_n^p(\mathbb{K})$.

We will use induction on n. By Lemma 5, the first step $(n=p)$ is done.

We assume that the theorem is satisfied for all matrices of order n, with $n \geqslant p$. We show that it remains valid for matrices of order $n+1$. Let

$A \in \mathcal{F}_{n+1}^{p}(\mathbb{K})$ such that

$$A=\begin{bmatrix} x_{11} & \cdots & x_{1\,p-1} & a_{1\,p} & a_{1\,p+1} & \cdots & a_{1\,n+1} \\ \vdots & \ddots & \ddots & x_{2\,p} & a_{2\,p+1} & \cdots & \vdots \\ x_{1\,p-1} & \ddots & \ddots & \ddots & x_{3\,p+1} & \ddots & \vdots \\ a_{1\,p} & x_{2\,p} & \ddots & \ddots & \vdots & \ddots & a_{n-p+1\,n+1} \\ a_{1\,p+1} & a_{2\,p+1} & x_{3\,p+1} & \cdots & \ddots & \ddots & x_{n-p+2\,n+1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ a_{1\,n+1} & \cdots & \cdots & a_{n-p+1\,n+1} & x_{n-p+2\,n+1} & \cdots & x_{n+1\,n+1} \end{bmatrix}.$$

We start proving that $a_{1\,n+1}=0$. For that we suppose not, i. e., we suppose that we have $a_{1\,n+1}\neq 0$.

If $n+1=p+1$ we apply the gaussian elimination along the lines(1) choosing the entries x_{11} and x_{12} in a way that we get

$$a'_{2\,n+1}=a'_{2\,p+1}=a_{2\,p+1}+\frac{-x_{12}}{x_{11}}a_{1\,p+1}=0.$$

So, the matrix $A'(1)$ has zero diagonals(p) and has order p. On the other hand, by Proposition 4, we have $A'(1)\in\mathcal{F}_{(n+1)-1}^{p}(\mathbb{K})$ and then, by the inductive hypothesis (or by Lemma 5), $A'(1)$ has diagonals(p) zeros-free (here, the order of $A'(1)$ is equal to p). Therefore, we get a contradiction. Then, there must be $a_{1\,n+1}=0$.

If $n+1>p+1$ ($n\geqslant p+1$) we apply the gaussian elimination along the lines(1) choosing the entries x_{11} and x_{12} in a way that we get

$$a'_{2\,n+1}=a_{2\,n+1}+\frac{-x_{12}}{x_{11}}a_{1\,n+1}\neq 0.$$

Therefore, the matrix $A'(1)$ is not a band matrix of bandwidth $(2p-1)$, whence the entry $(1,n)$ of $A'(1)$ is different from zero and is outside of the band because $|1-n|=n-1\geqslant(p+1)-1=p$. But, by Proposition 4, $A'(1)\in\mathcal{F}_{n}^{p}(\mathbb{K})$ and then $A'(1)$ is a band matrix of bandwidth $(2p-1)$ by inductive hypothesis. A contradiction. So, there must be $a_{1\,n+1}=0$.

Now, we show that

$$a_{1\,p}\neq 0,\quad a_{1\,p+1}=\cdots=a_{1\,n}=0.$$

We choose $x_{n+1\,n+1}\neq 0$ and apply the gaussian elimination along the lines($n+1$). As $a_{1n+1}=0$, the entries of the lines(1) remains unchanged.

From this and by inductive hypothesis — which guarantees that $A'(n+1) \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$ — we can conclude that $a_{1p} \neq 0$ and, likewise, $a_{1\,j} = 0$, $j = p+1, ..., n$.

To finish the "only if" part of the proof, we need to show that other entries of the diagonals(p) are different from zero and that the entries of the diagonals(j), $j = p+1, ..., n+1$, are all equal to zero. We choose $x_{11} \neq 0$ and apply the gaussian elimination along the lines(1). This operation only interfere with the entries of the submatrix $A[2, ..., p]$ and no one entry of this submatrix is out of the band because

$$\max_{i,j=2,\ldots,p} |i-j| = p-2 < p-1.$$

So, we have (†) $A'(1) = A(1)$ up to the entries of the submatrix $A[2, ..., p]$ and, therefore, all entries in diagonals(p) and out of the band remains unchanged. On the other hand, by Proposition 4, $A'(1) \in \mathcal{F}_n^p(\mathbb{K})$. Therefore, by inductive hypothesis, (‡) $A'(1) \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$. From these two facts, (†) and (‡), we have $A(1) \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$, i.e., $a_{ij} \neq 0$ if $|i-j| = p-1$ and $a_{ij} = 0$ if $|i-j| > p-1$, for $i, j \geqslant 2$.

Finally, we may conclude that $A \in \widehat{\mathcal{B}}_{n+1}^p(\mathbb{K})$, which ends the proof relative to the necessity of the condition. ∎

The next theorem shows that, for a suitable choice of its free elements, the rank of a matrix in $\widehat{\mathcal{B}}_n^p(\mathbb{K})$ can achieve all its possible values from $n-p+1$ up to n.

Theorem 7. *Let $\mathbb{K}$ be a field and p, n integers such that $1 \leqslant p \leqslant n$. Then, for any matrix $A \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$ and any integer k such that $n-p+1 \leqslant k \leqslant n$, there exists a matrix $B \in \mathcal{B}_n^{p-1}(\mathbb{K})$ such that $A+B$ has rank k (we make the convention that $\mathcal{B}_n^0(\mathbb{K})$ consists only of the null matrix of order n).*

Proof. The theorem is clearly true for $p = 1$. For $p = 2$, $\widehat{\mathcal{B}}_n^2(\mathbb{K})$ is just the set of irreducible tridiagonal matrices with free diagonal: let us prove, by induction over n, that for a suitable choice of the diagonal elements, such a matrix has rank n. For $n = 1$ the result is clearly true. Suppose that this is true for $n-1$ and let us prove that it remains valid for n. Let $A \in \widehat{\mathcal{B}}_n^2(\mathbb{K})$; denote the diagonal element of A in position (i, i) by x_i and the elements in position (i, j), $i \neq j$, by a_{ij}; it is well known that

$$\det A = x_i \det A(1) - a_{12}^2 \det A(1, 2).$$

Now, $A(1) \in \widehat{\mathcal{B}}_n^2(\mathbb{K})$ and so, by induction hypothesis, we may choose the diagonal elements of $A(1)$ in such a way that $\det A(1) \neq 0$. Now we just choose x_i in such a way that $\det A \neq 0$ and A will have rank n. A similar argument, choosing again the diagonal elements of $A(1)$ so that $\det A(1) \neq 0$, allows us to choose x_1 such that $\det A = 0$ and in this case A will have rank $n-1$.

Suppose now $p \geqslant 1$. Let $A \in \widehat{\mathcal{B}}_n^p(\mathbb{K})$.

As we said in the introduction we may see the entries in A in positions (i,j), $|i-j| < p-1$, as free variables: now take all these entries in the non-diagonal positions as zero and keep the diagonal elements free. This special matrix A looks like as follows:

$$A = \begin{bmatrix} x_1 & 0 & \cdots & 0 & \star & 0 & \cdots & 0 \\ 0 & x_2 & \ddots & \ddots & \ddots & \star & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \star \\ \star & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \star & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \star & 0 & \cdots & 0 & x_n \end{bmatrix} \qquad (2)$$

We are going to prove, by induction on n, that, for a suitable choice of its diagonal elements, the above matrix verify the thesis of the theorem.

For $n = 1$ there is nothing to prove. Suppose the theorem true for all positive integers less than n and let us prove that it remains true for n. Let A be a n-by-n matrix of the above type. Let r be the greatest integer such that $1 + r(p-1) \leqslant n$. It is not difficult to see (think for example of the graph of A) that $A[1, 1+(p-1), ..., 1+r(p-1)]$ is an irreducible tridiagonal matrix, while $A(1, 1+(p-1), ..., 1+r(p-1))$ is a band matrix, in fact an element of $\widehat{\mathcal{B}}_{n-(r+1)}^{p-1}(\mathbb{K})$. Moreover, for $i = 1, ..., p-2$, the (i)diagonals have all its entries equal to 0, that is, $A(1, 1+(p-1), ..., 1+r(p-1))$ is a matrix of type (2) and so, by induction hypothesis, its rank can, for a suitable choice of its diagonal elements, achieve all the values from $n-(r+1)-(p-2)$ up to $n-(r+1)$. Let $\{j_1, j_2, ..., j_{n-(r+1)}\}$ be the complement set of $\{1, 1+(p-1), ..., 1+r(p-1)\}$ in $\{1, 2, ..., n\}$. It is not difficult to see that the both matrices $A[1, 1+(p-1), ..., 1+r(p-1)|j_1, j_2, ..., j_{n-(r+1)}]$ and $A[j_1, j_2, ..., j_{n-(r+1)}|1, 1+(p-1), ..., 1+r(p-1)]$ are null matrices. This means that, up to a permutation similarity, the matrix A is the direct sum of a $(r+1)$-by-$(r+1)$ irreducible tridiagonal matrix (whose rank can achieve all the values $r+1$ and r) with a matrix of type (2) (whose rank can achieve all the values from $n-(r+1)-(p-2)$ up to $n-(r+1)$). So, our result follows.■

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Américo Bento
Departmento de Matemática, Univ. de Trás-os-Montes e Alto Douro, 5000-911 Vila Real, Portugal

E-mail address: abento@utad.pt

António Leal Duarte
Dep. de Matemática, Univ. de Coimbra, Apartado 3008, 3001-454 Coimbra, Portugal

E-mail address: leal@mat.uc.pt