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Efficient credit portfolios under IFRS 9

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Abstract

In this paper, we devise a forward-looking methodology to determine efficient credit portfolios under the IFRS 9 framework. We define and implement a credit loss model based on prospective point-in-time probabilities of default. We determine these probabilities of default and the credits' stage allocation through a credit stochastic simulation. This simulation is based on the estimation of transition matrices. Using data from 1981 to 2019, in a non-homogeneous Markov chain setting, we estimate transition matrices conditional on the global real gross domestic product growth. This allows considering the effects of the economic cycle, which are of great importance in bank management. Finally, we develop a robust optimization model that allows the bank manager to analyze the trade-off between the annual average portfolio income and the corresponding portfolio volatility. According to the proposed bi-objective model, we compute the efficient credit portfolios constructed based on 10-year maturity credits. We compare their structure to those generated by the IAS 39 and CECL accounting frameworks. The results indicate that the IFRS 9 and CECL frameworks generate efficient credit portfolios whose structure penalizes riskier-rated credits. In turn, the riskier efficient credit portfolios under the IAS 39 framework concentrate entirely on speculative-grade credits. This pattern is also encountered in efficient credit portfolios constructed based on credits with different maturities, namely 5 and 15 years. Moreover, the longer the maturity of the credits that enter into the composition of the efficient portfolios, the more the speculative-grade credits tend to be penalized.

Keywords: IFRS 9; IAS 39; CECL; credit risk; transition matrices; stochastic simulation

1. Introduction

Before the Global Financial Crisis of 2007–08, the credit losses of a banks' portfolio credit were assessed based on the incurrent loss (IL) approach settled by the International Accounting Standard

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N° 39—IAS 39 (International Accounting Standards Board, 2004). Under the IL approach, institutions should not account for expected credit losses arising from future events. Several authors see this procedure as one of the causes of the depth of the Global Financial Crisis (Financial Stability Forum, 2009) and nicknamed it as "too little, too late" (Basel Comittee on Banking Supervision, 2015).

In response to the criticism of the IL approach, the International Accounting Standards Board issued the International Financial Reporting Standard N° 9 (IFRS 9) (International Financial Reporting Standards, 2017) on 24 July 2014, which become mandatory on 1 January 2018. One of the key aspects of IFRS 9 lies in the shift from the IL approach to an expected credit loss (ECL) approach. Nevertheless, this ECL approach brings a lot of subjectivity about measuring future losses and requires a greater amount of information. This contrasts with the IAS 39 framework, where the losses were measured based on objective facts.

According to IFRS 9, each bank can define its own internal credit loss model. Typically, the ECL computation (KPMG, 2017; Gross et al., 2020) is based on the product of the loss given default by the corresponding point-in-time probability of default (Chawla et al., 2016; Andrija, 2019). Moreover, this computation must contemplate three stages (Taylor, 2017). Stage 1 corresponds to the case where the financial asset has no deterioration in credit quality since acquisition. For a financial asset classified in stage 1, financial institutions should use the one-year ECL and recognize interest income on a gross basis. The occurrence of a significant increase in credit risk (SICR) since acquisition leads to the transfer of the financial asset to stage 2. In stage 2, the lifetime ECL should be recognized and the interest income is computed as in stage 1 (i.e., on a gross basis). Allocation to stage 3 occurs when a financial asset is credit-impaired. In this stage, institutions should compute interest income on a net basis (i.e., gross carrying amount less the ECL).

IFRS 9 generally applies to most countries. However, the United States is a relevant exception since it uses the Generally Accepted Accounting Principles (US GAAP). Under the US GAAP update, issued in June 2016 by the Financial Accounting Standards Board (Financial Accounting Standards Board, 2016), similarly to IFRS 9, businesses should implement the credit loss model based on the prospective credit loss (ECL instead of an IL approach). However, this computation deploys the current expected credit loss (CECL) framework, that is, always applies the lifetime ECL based on point-in-time probabilities of default. Avoiding defining SICR and stage allocation constitutes the main point of divergence from IFRS 9. The CECL model will be mandatory to all institutions in January 2023 (Financial Accounting Standards Board, 2019).

The introduction of the Basel I accord in 1988 (Bank of International Settlements, 1998) had triggered the discussion around the potential procyclical effects of banks' decisions. In periods of economic downturn, there tends to be a depletion of bank capital through the deterioration of credit portfolios. In these periods, banks need more capital to meet the minimum regulatory requirements when access to capital is scarce. Therefore, in economic downturns, banks tend to avoid the investment in high-risk credits precisely when the economy yearns for this kind of investment. More recently, the literature has analyzed the procyclical effects of the different impairment accounting frameworks, namely IFRS 9, IAS 39, and CECL.

Aligned with the opinion of policymakers, Laeven and Majnoni (2003) have found empirical evidence that most banks delay provisioning for deteriorated credits until too late, when cyclical downturns have already occurred, therefore amplifying the impact of the economic cycle on the banks' income and capital. Laeven and Majnoni (2003) agree that provisioning based on a forward-looking approach, as introduced by the IFRS 9 and CECL frameworks, reduces the procyclical effects.

Balla and McKenna (2009) argue that an accounting setting based on the IL model, as the IAS 39 framework, can be suboptimal to avoid bank failures and in the efficiency of credit bank investment/lending. On the one hand, the authors defend that the risk of bank insolvency increases when provisioning occurs after the economic downturn. On the other hand, they defend that the increase of the credit loss reserves, when the economy is in contraction, leads to procyclical effects.

The most recent work suggesting that IFRS 9 has fewer procyclical effects than IAS 39 is the study performed by Buesa et al. (2019). In an empirical exercise based on Italian mortgages from 2006 to 2018, Buesa et al. (2019) have modeled the impact of credit impairments on the Profit and Loss (P&L) statement under the three impairment accounting frameworks: IAS 39, IFRS 9, and CECL. The results suggest that IFRS 9 is less procyclical than IAS 39 and more procyclical than the CECL framework. Nevertheless, the lower procyclicality of the CECL framework comes with a significant increase in provisions.

In the same direction, there are other studies in the literature suggesting that IFRS 9 has fewer procyclical effects than IAS 39 (see, e.g., Basel Comittee on Banking Supervision, 2009; Financial Stability Forum, 2009; Wezel et al., 2012). However, this issue is far from being consensual in the literature.

Recently, Abad and Suarez (2020) have been defending that the estimation of future credit losses under the IFRS 9 and CECL frameworks is challenging. These standards would be countercyclical only if the modelers were able to predict turning points in the cycle or the occurrence of crises (as the case of the recent COVID 19 pandemic) two or three years in advance (Abad and Suarez, 2020). These authors have shown empirical evidence that the IFRS 9 and CECL frameworks will raise the credit loss provisions more suddenly than the IAS 39 approach when the economy switches from expansion to contraction. As a consequence, the P&L and the Common Equity Tier 1 (CET1) ratio will decline more severely (Abad and Suarez, 2018) at the beginning of downturns under IFRS 9 and CECL.

These results are similar to the study performed in Barclays (2017), which concludes that forward-looking approaches, such as IFRS 9 and CECL, may amplify and not reduce the effect of an unexpected increase in risk. Seitz et al. (2018) go further by exposing the high sensitivity of the ECL to the probability of default model used to estimate the corresponding probabilities of default. The point-in-time estimation of the probabilities of default could imply a severe deterioration of the banks' capital when the economy starts to contract, since the banks could not anticipate this adverse shift (European Systematic Risk Board, 2017).

IFRS 9 is, at the time of writing (June 2021), a subject of great debate in light of the current coronavirus crisis. Anticipating the inability of debtors to pay back and the moratoriums given by banks worldwide, government officials have created relief measures to delay the full impact of IFRS 9 so that these moratoriums, for instance, are not immediately recognized as losses. The authorities have thus given flexibility to banks on adopting the assessment of the likeliness to pay (European Central Bank, 2020; European Comission, 2020). These findings highlight the great importance of IFRS 9. However, to our knowledge, there is no current literature that establishes optimal credit portfolios under the IFRS 9 framework. Many specialists and strategists have anticipated stylized impacts in the business model of banks and consequently in the credit portfolios (McKinsey & Co., 2017). For instance, there will be incentives to shorten the maturity of credits and steer away from lower ratings. However, there exists no quantitative methodology to guide a bank manager in selecting a credit portfolio.

Knowing that accounting standards impact banking asset allocation (Argimon et al., 2015) and motivated by the lack of a methodology for credit portfolio allocation under IFRS 9, we design a quantitative method to compute efficient credit portfolios under IFRS 9. The contribution of this paper is twofold: (1) we devise a methodology to determine the efficient credit portfolios under IFRS 9 and compare their structure to those generated by the IAS 39 and CECL frameworks; (2) we identify the accounting frameworks that imply a lower allocation to riskier credit portfolios, which have higher refinancing needs during economic recessions; thus, we add this element to the discussion of the procyclical effects of the different impairment accounting frameworks. Note that if efficient credit portfolios under a particular accounting framework allocate less to riskier credits, this behavior can lead to procyclical effects. In fact, in periods of economic downturn, cash-hungry corporations will find it more difficult to obtain bank financing precisely when they most need it.

Under IFRS 9, we assume that the bank manager aims to select a credit portfolio, constructed with global credits with different ratings (from AAA to B, according to Standard & Poor's Ratings-Direct®, 2014) at the acquisition date. As a standard practice in banking, the credits are classified at amortized cost according to the IFRS 9 framework. Hence, the investment horizon is coincident with the credit maturity. As a baseline case, we assume that each credit has a maturity of 10 years. Furthermore, we posit that the bank manager has a certain amount to invest in the credits, and selects the credit portfolio at the end of 2019, that is, data until the end of 2019 are assumed to be known.

Following the IFRS 9 framework, we begin by defining the credit impairment model by which the bank manager can compute the annual credit income. The definition that we adopt in this paper relies on estimating the credit probability of default to calculate ECL, in line with practice (KPMG, 2017).

Then, to simulate the credit dynamics, we make use of the one-year transition matrices reported annually by Standard & Poor's (S&P) (see, e.g., Standard & Poor's Global Ratings, 2019). We collect annual data from 1981 to 2019. These matrices refer to global corporate bonds that we use as a proxy of global credit data. Since cyclical macroeconomic effects are crucial to bank management, we estimate the transition matrices conditional on the real global gross domestic product (GDP) growth.

The most common approaches to estimate conditional transition matrices are based on the one-factor model suggested in the CreditMetrics™ (J. P. Morgan & Co., 1997) or on numerical adjustments (see, e.g., Jarrow et al., 1997). A comparison between these two approaches can be found in Trück (2008). In this paper, we closely follow the one-factor model approach, constructing a credit cycle measure determined by the real global GDP growth. The construction of the credit cycle measure is based on the historical inverse relationship between the speculative-grade ratings (BB, B, and C/CCC) probabilities of default and the global real GDP growth.

Assuming a non-homogeneous Markov chain model (Brémaud, 2020) for the conditional transition matrices, we simulate the dynamics of 6000 credits (1000 for each rating grade from AAA to B) from acquisition date until maturity. As a result of this stochastic simulation, we determine the stage allocation, according to IFRS 9, for each simulated credit.

To implement the defined credit loss model, we need to compute the probabilities of default for each credit rating grade. IFRS 9 states that these probabilities should be estimated taking into account the economic credit cycle, that is, should be point-in-time (Chawla et al., 2016; Andrija, 2019). Therefore, we evaluate these point-in-time probabilities of default conditional on several different projections for the global real GDP growth. More specifically, for each different prediction of the global real GDP growth, we estimate the corresponding conditional transition matrix, whose last column gives us the point-in-time probabilities of default for each credit rating grade.

Once specified the credit dynamics and estimated the point-in-time probabilities of default, we compute the credit income for each simulated credit. Then, we identify a representative credit, for each rating grade, that corresponds to a weighted average of the 1000 simulated credits. Finally, based on the annual income of each representative credit, we suggest a forward-looking bi-objective optimization model whose solution allows the bank manager to directly analyze the trade-off between the portfolio income and the corresponding portfolio volatility. We show how to tackle such a bi-objective problem by solving different convex optimization quadratic problems.

Typically, asset allocation models are very sensitive to input parameters, often leading to corner solutions and unsmooth allocations as a function of the risk parameter. To circumvent this difficulty, Michaud and Michaud (2007, 2008) proposed a resampling methodology. This methodology resamples returns, takes the efficient frontier and portfolios for each sample, and averages efficient frontiers and portfolios across samples. When compared to the optimization without resampling, the resampling methodology is documented to produce smoother allocations as a function of risk, and also avoids corner solutions. Hence, Michaud and Michaud (2007, 2008) qualified the methodology as robust.

In the spirit of Michaud and Michaud (2007, 2008), we apply resampling to our problem, yielding much smoother allocations than one would have obtained if no resampling was used, and refer to it as a robust model. We use the suggested forward-looking bi-objective optimization model over 100 generated scenarios where each scenario corresponds to a global real GDP growth path. With the resulting solutions and following the robust resampling methodology proposed in Michaud and Michaud (2007, 2008), we compute the robust average efficient Pareto frontier not only for the IFRS 9 framework but also for the IAS 39 and CECL frameworks. We finish our analysis by evaluating the composition of the efficient portfolios for varying maturities.

Globally, the results indicate that the efficient credit portfolios constructed under IFRS 9 and CECL have a structure that penalizes riskier-grade rated credits, namely speculative-grade credits. Under IAS 39, we no longer observe such a pattern. On the contrary, we note that the more volatile efficient portfolios under IAS 39 tend to have a structure entirely concentrated in speculative-grade credits. We encounter this pattern in the efficient portfolios with the different maturities considered: 5 years, 10 years, and 15 years. Moreover, based on these three maturities, the results indicate that the longer the maturity, the more the riskier-grade rated credits tend to be penalized. This finding holds under the three accounting frameworks.

The remainder of the paper is organized as follows. Section 2 presents the entire methodology to estimate the credit income under the IFRS 9, IAS 39, and CECL frameworks. In Section 3, we develop the forward-looking robust bi-objective model that allows us to identify, under the different accounting frameworks, efficient portfolios in the correspondent income-risk bi-dimensional space. In Section 4, we present and analyze the results on the robust efficient Pareto frontiers computed under the IFRS 9, IAS 39, and CECL frameworks. Finally, Section 5 concludes the paper.

2. Modeling the bank manager problem

Let us assume that a bank manager wants to construct a credit portfolio from a set of global credits under IFRS 9. According to the S&P Global Rating Classification System (Standard & Poor's RatingsDirect®, 2014), we consider that each global credit can be classified with one of eight rating grades: AAA, AA, AA, A, BBB, BB, B, C/CCC, and D. The rating grades from AAA to BBB correspond to investment-grade ratings, from BB to C/CCC to speculative-grade ratings, and the D corresponds to the default state.

We assume that the bank manager disposes of a funding amount, F, to invest in the set of global credits. Each global credit has a rating grade not inferior to B[1].

Following the IFRS 9 classification system (International Financial Reporting Standards, 2017), we assume that the global credits are classified at amortized cost. We use this particular assumption for two reasons: (1) it is the standard accounting classification for loans, and (2) the vast majority of assets in the bank's balance sheet are classified under this category.

Finally, we assume that all the data until the end of 2019 are known and that the global credits have a 10-year maturity. Motivated by data availability, we use data for global corporate bonds. This allows the reader to reproduce all the results presented in this paper easily. Nevertheless, note that the entire methodology that we develop in the next sections can be applied to any type of credit.

2.1. Credit loss model definition

When the bank manager invests in a global credit, she/he earns a fixed interest, C (annual coupon), until maturity T. The credit income of a global credit acquired with rating grade equal to $G \in \{\text{AAA, AA, A, BBB, BB, B}\}$ and belonging to stage 1 or stage 2 at year $t = 1, \dots, T$, is given by

$$\iota_t^G = C \times x - (IS_t - IS_{t-1}) \times x = [C - (IS_t - IS_{t-1})] \times x, \tag{1}$$

where x is the amount invested in the global credit and IS_τ is the impairment stock in the corresponding year τ. In turn, if at year t the global credit enters in default (belongs to stage 3), the credit income is given by

$$\iota_t^G = C \times RR \times x - (IS_t - IS_{t-1}) \times x = [C \times (1 - LGD) - (IS_t - IS_{t-1})] \times x, \tag{2}$$

where RR is the recovery rate and LGD the loss given default[2]. Note that, as the global credit enters in default in year t, then $\iota_\tau^G = 0,\ \forall \tau = t + 1, \dots, T$.

The fixed interest, C, present in Equations (1) and (2) corresponds to the coupon value of each global corporate bond classified with a rating grade from AAA to B. Accordingly, for a 10-year

[1]The C/CCC rating grade corresponds to substantial risk and therefore we do not consider it as belonging to the investment set.

[2]In this work, we consider a constant $LGD = 62\%$. This value is based on the historical average of the RR for unsecured global corporate bonds reported in Moody's Investors Service (2019).

maturity, these values are[3]: 2.302%, 2.483%, 2.670%, 3.173%, 4.684%, 5.391%, for a AAA, AA, A, BBB, BB, and B rating grade, respectively.

The computation of the impairment stock, IS_τ (i.e., the expected credit loss - ECL), in Equations (1) and (2), is conditional on the stage allocation. Therefore, the impairment stock is given by

$$IS_\tau = \begin{cases} ECL_\tau^{(\text{one-year})} = \dfrac{1}{1+D} \times LGD \times PD_\tau, & \text{if stage 1 at } \tau, \\ ECL_\tau^{(\text{lifetime})} = LGD \times \displaystyle\sum_{n=\tau}^{T} \dfrac{SV_{n-1} \times PD_n}{(1+D)^n}, & \text{if stage 2 at } \tau, \\ LGD \times PD_\tau = LGD, & \text{if stage 3 at } \tau, \end{cases} \tag{3}$$

where D is the discount rate, PD_τ is the one-year probability of default in year $\tau = 1, \ldots, T$, and SV_i is the survival rate until year i. Note that the initial impairment stock is equal to 0, that is, $IS_0 = 0$. The discount rate, D, is set equal to 1.828%, that is, equals the coupon rate of a AAA corporate bond with a five-year maturity[4]. In turn, SV_i is computed as

$$SV_i = \prod_{j=0}^{i} (1 - PD_j), \ \forall i \in \{0, 1, \ldots, T-1\}, \tag{4}$$

with $PD_0 = 0$.

The income under IAS 39 of a non-default global credit at year $t = 1, \ldots, T$ will be simply equal to $C \times x$. In turn, if the credit enters in default at year t, then the credit income will be given by $C \times RR \times x - LGD \times x = [C \times (1 - LGD) - LGD] \times x$, and in the subsequent years, until maturity T, will be equal to 0.

If, instead, we considered the CECL accounting framework, the credit income will be computed similarly as in the IFRS 9 case. Nevertheless, there is an important difference. In the CECL framework, there is no stage allocation. Accordingly, the credit impairment stock will be computed based on the credit lifetime (i.e., it will be equal to the second case of Equation 3).

Within the IFRS 9 framework, the ECL is computed according to three stages (see Equation 3). As IFRS 9 does not explicitly define an SICR, we follow parsimonious criteria: a credit belongs to stage 1 if it shows no downgrade greater than or equal to two grades since acquisition; a credit belongs to stage 2 otherwise.

For the credit loss model implementation, we need to model the credit rating dynamics (to determine the stage allocation) and to estimate the probabilities of default[5] for each global credit rating. We make use of probability transition matrices for both purposes.

[3]The values were obtained in the Bloomberg Terminal (Bloomberg, 2021).

[4]We obtained this value through the Bloomberg Terminal (Bloomberg, 2021).

[5]Note that this estimation should be point-in-time. For further details see, for example, Carruthers et al. (2019).

2.2. Estimation of transition matrices

Every year, S&P publicly discloses the global corporate one-year transition matrices. For example, the reported annual raw transition matrix for 2019 (Standard & Poor's Global Ratings, 2019) is equal to

from/to	AAA	AA	A	BBB	BB	B	CCC/C	D	NR
AAA	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.00	93.25	2.15	0.00	0.00	0.00	0.00	0.00	4.60
A	0.00	0.71	93.71	1.93	0.00	0.00	0.00	0.00	3.64
BBB	0.00	0.00	2.67	91.44	1.23	0.05	0.00	0.11	4.49,
BB	0.00	0.00	0.07	2.61	83.02	4.99	0.30	0.00	9.01
B	0.00	0.00	0.00	0.00	2.21	78.57	5.09	1.49	12.64
CCC/C	0.00	0.00	0.00	0.00	0.49	8.37	45.81	30.05	15.27

(5)

where the probability values are expressed in percentages. Similarly to the transition matrix reported for 2019, we have access to all the transition matrices since 1981 (i.e., all the reports from 1981 to 2019 are available).

The raw transition matrices disclosed by S&P contain a column titled "N.R." (Not Rated). Following the standard procedure in the literature (see, e.g., Wei, 2003), we redistribute the "N.R." portion to the other ratings (leaving the default column unchanged) on a *pro-rata* basis.

As recommended by CreditMetrics^{TM} (J. P. Morgan & Co., 1997) for modeling purposes, we ensure that all transition matrices satisfy some desirable properties: there should not exist nonzero probability events, that is, all the transitions and defaults should have a nonzero probability; all rows should sum exactly one; all the transition matrices should satisfy the so-called monotonicity constraints, that is, within each row and column, the values on each side of the diagonal should monotonically decline (for further details, see J. P. Morgan & Co., 1997).

To ensure that there are nonzero probability events, we consider the existence of a minimum threshold set equal to 1 bp (basis point). Thus, whenever a zero probability arises, we consider that it is equal to the defined threshold, and we subtract this value to the diagonal entry. Moreover, all the rows are modified to sum exactly one by adjusting the corresponding diagonal element.

Regarding the monotonicity constraints, to minimize excessive adjustments, we strictly follow the parsimonious rules described in Wei (2003): (1) whenever the column monotonicity is not satisfied, we swap the element in question with the previous element, and we ensure that the sum of each row is equal to 1 by adjusting the corresponding diagonal elements; (2) whenever the row monotonicity is not satisfied, we set the element in question equal to the previous rating's element and add the difference to the elements between the diagonal and the element in question. We point out that we may need to apply the monotonicity rules more than once to accomplish a smooth transition matrix.

For example, after applying the adjustments described in the previous paragraphs, the smooth version of the 2019 raw transition matrix (see Equation 5) is equal to

from/to	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	99.93	0.01	0.01	0.01	0.01	0.01	0.01	0.01
AA	0.01	97.69	2.25	0.01	0.01	0.01	0.01	0.01
A	0.01	0.74	97.21	2.00	0.01	0.01	0.01	0.01
BBB	0.01	0.01	2.80	95.82	1.29	0.05	0.01	0.01 .
BB	0.01	0.01	0.08	2.87	91.11	5.48	0.33	0.11
B	0.01	0.01	0.01	0.01	2.54	90.09	5.84	1.49
CCC/C	0.01	0.01	0.01	0.01	0.63	10.71	58.57	30.05
D	0	0	0	0	0	0	0	1

(6)

In Equation (6), we added the last row corresponding to the absorbing state (default state). Repeating this procedure to all the historical transition matrices, from 1981 to 2019, we have a time series of one-year smooth transition matrices, P_t, with $t = 1, \ldots, 39$, that can be represented by

$$P_t = \begin{bmatrix} p_t(1,1) & p_t(1,2) & \cdots & p_t(1,8) \\ p_t(2,1) & p_t(2,2) & \cdots & p_t(2,8) \\ \cdots & \cdots & \cdots & \cdots \\ p_t(7,1) & p_t(7,2) & \cdots & p_t(7,8) \\ 0 & 0 & \cdots & 1, \end{bmatrix}, \tag{7}$$

where each number from 1 to 8 corresponds to each possible rating grade: AAA, AA, A, BBB, BB, B, C/CCC, and D, respectively.

We could model the transition matrices as a homogeneous Markov chain model as in Jarrow et al. (1997) or in Ludwig (2019). Nevertheless, this will lead to estimation error by not including cyclical effects, which are of great importance in bank management. Instead, in this work, we adopt a non-homogeneous Markov chain model approach (Brémaud, 2020). Inspired by the CreditMetrics^{TM} (J. P. Morgan & Co., 1997) methodology and closely following the one-factor model presented in Belkin et al. (1998) and discussed in Gross et al. (2020), we assume that the rating transitions reflect an underlying continuous credit-change indicator, u_t, such that

$$u_t = (\sqrt{1-\rho}x_t + \sqrt{\rho}h_t) \sim N(0,1), \tag{8}$$

where x_t represents a standard normal distributed idiosyncratic component, h_t represents a standard normal distributed systematic component (shared by all issuers), and ρ corresponds to the correlation between h_t and u_t. It is assumed that these standard normal random variables are independent.

In Equation (8), h_t can be interpreted as a measure of the credit cyclical effects. A positive h_t shifts the distribution of u_t to the right-hand side. In turn, a negative h_t shifts the distribution of u_t to the left-hand side.

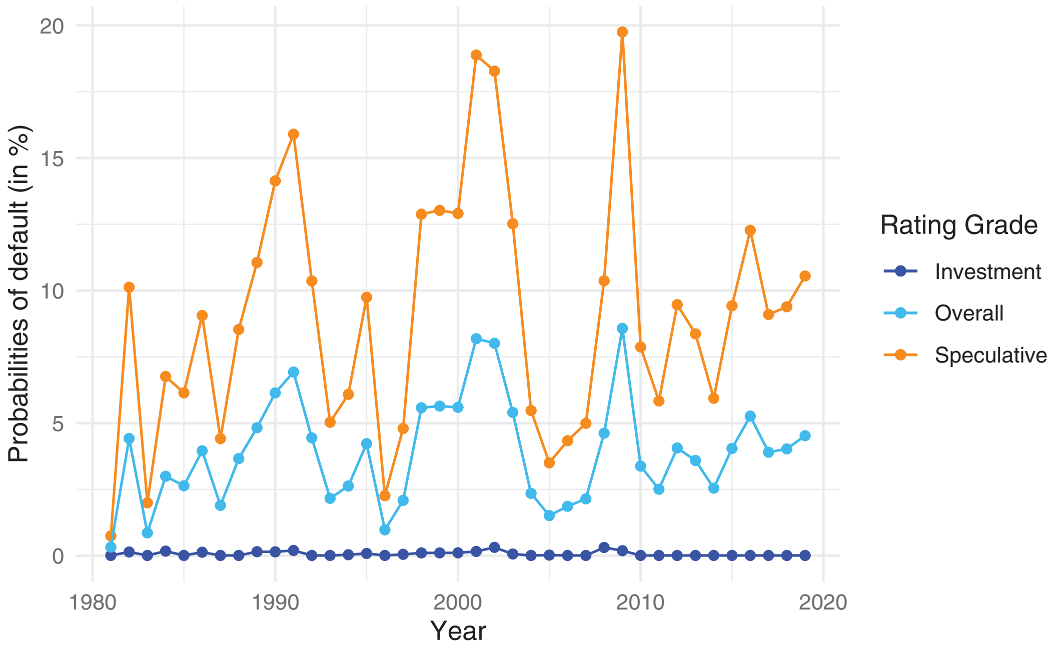


Fig. 1. Historical values of the probabilities of default.

Figure 1 reports, for the period 1981–2019, the aggregate probabilities of default corresponding to investment-grade[6], speculative-grade[7], and overall[8] ratings. As expected, we observe much more volatility of the speculative-grade probabilities of default in contrast with the more steady behavior of the investment-grade defaults.

An interesting fact, that we explore, lies in contrasting the behavior of the speculative-grade probabilities of default (see Fig. 1) with the evolution of the real GDP growth[9] (see Fig. 2).

By contrasting Figs. 1 and 2, we observe a synchronous inverse relationship between the speculative-grade default probabilities and the global real GDP growth. Thus, whenever global real GDP growth goes up, default probabilities go down and vice versa. Moreover, it is well known that nonspeculative probabilities of default are insensitive to the economic cycle (see, e.g., Wilson, 1997; Belkin et al., 1998).

Motivated by these facts, we suggest constructing a credit cycle measure (based on the global real GDP growth) calibrated through BB, B, and CCC/C default probabilities.

Let $PD_t^{(S)}$ be the speculative-grade default probability (i.e., a weighted average of the BB, B and CCC/C probabilities of default) at time t. Inspired in previous works, such as Krüger et al. (2018),

[6]Weighted average of AAA, AA, A, and BBB ratings.

[7]Weighted average of BB, B, and C/CCC ratings.

[8]Weighted average of investment-grade and speculative-grade ratings.

[9]Data for the global real GDP growth are publicly available at: https://data.worldbank.org/ (accessed June 2021).

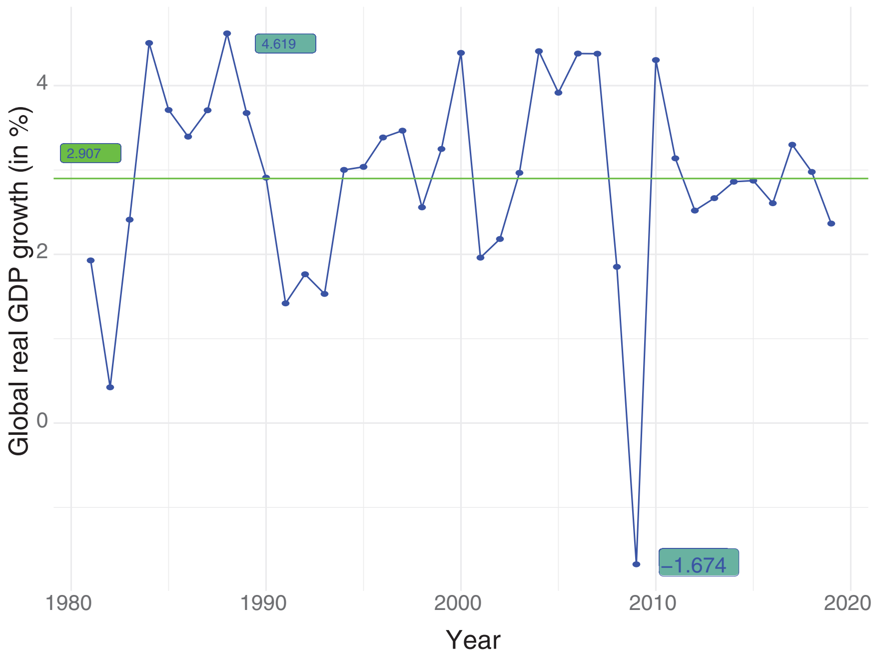


Fig. 2. Historical global real GDP growth.

this probability can be modeled by a probit model

$$PD_t^{(S)} = \Phi(\alpha + \beta GDPG_t + \varepsilon_t), \tag{9}$$

where $GDPG_t$ represents the change in the global real GDP from $t-1$ to t, $\Phi(\cdot)$ is the cumulative distribution function (CDF) of a normal distribution, and ε_t represents an error term. Hence, the relationship

$$\Phi^{-1}(PD_t^{(S)}) = \alpha + \beta GDPG_t + \varepsilon_t, \tag{10}$$

allow us to estimate the inverse CDF of the speculative-grade default probability. Estimating Equation (10) by ordinary least squares (OLS), for the period 1981–2019, leads to the following estimates: $\hat{\alpha} = -1.152$ (std. error $= 0.131$) and $\hat{\beta} = -8.691$ (std. error $= 4.175$). Both estimates are statistically significant at the level of 5%.

We can define the credit cycle measure, h_t, as

$$h_t = -\left(\frac{\Phi^{-1}(PD_t^{(S)}) - \mu_{PD^{(S)}}}{\sigma_{PD^{(S)}}}\right), \tag{11}$$

where $\mu_{PD^{(S)}}$ and $\sigma_{PD^{(S)}}$ denote, respectively, the average and the standard deviation of the inverse normal distribution of $PD_t^{(S)}$. The minus sign in Equation (11) turns the credit cycle measure

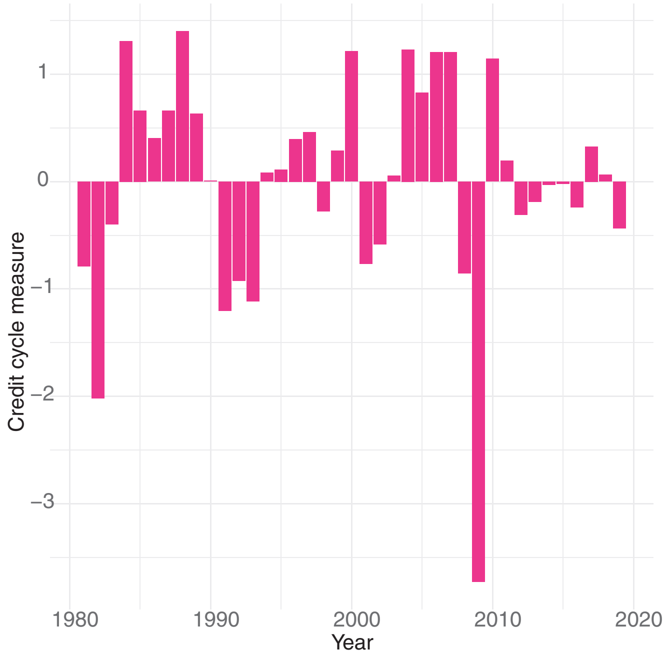


Fig. 3. Historical values of the credit cycle measure.

positive whenever the GDP growth is greater than its historical average and negative whenever the GDP growth is less than its historical average.

Using the estimates of $\Phi^{-1}(PD_t^{(S)})$, computed according to Equation (10), in Fig. 3 we report the evolution of $\hat{h}_t$ during the historical period from 1981 to 2019.

With the time series of the one-year smooth transition matrices (see Equation 7), we can compute an average historical transition matrix, $\bar{P}$, where each entry is given by

$$\bar{p}_{i,j} = \frac{1}{\#TM} \sum_{t=1}^{\#TM} p_t(i, j), \ \forall i, j = 1, \ldots, 8, \tag{12}$$

and $\#TM$ corresponds to the number of historical transition matrices (for the period from 1981 to 2019, $\#TM = 39$). Now, we can map the average transition probabilities into credit scores (Belkin et al., 1998). This correspondence amounts to inverting the cumulative normal distribution function, beginning from the average transition probability matrix column for the default state. Accordingly, the average transition matrix can be transformed to a z-score matrix

$$Z = \begin{bmatrix} +\infty & z_{1,2} & z_{1,3} & \cdots & z_{1,8} & -\infty \\ +\infty & z_{2,2} & z_{2,3} & \cdots & z_{2,8} & -\infty \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ +\infty & z_{7,2} & z_{7,3} & \cdots & z_{7,8} & -\infty \end{bmatrix}, \tag{13}$$

where each entry is given by $z_{i,j} = \Phi^{-1}(\sum_{k=j}^{8} \bar{p}_{i,k}),\ \forall i = 1, \ldots, 7, \forall j = 1, \ldots, 8 \ \wedge \ z_{i,9} = -\infty \ \forall i = 1, \ldots, 7$.

For the period from 1981 to 2019, the z-score matrix is equal to

$$Z = \begin{bmatrix} +\infty & -1.28 & -2.56 & -3.20 & -3.35 & -3.43 & -3.54 & -3.72 & -\infty \\ +\infty & 2.54 & -1.40 & -2.46 & -3.00 & -3.27 & -3.42 & -3.52 & -\infty \\ +\infty & 3.22 & 2.07 & -1.56 & -2.48 & -2.84 & -3.21 & -3.31 & -\infty \\ +\infty & 3.51 & 2.78 & 1.68 & -1.61 & -2.28 & -2.69 & -2.88 & -\infty \\ +\infty & 3.72 & 3.22 & 2.49 & 1.51 & -1.32 & -2.15 & -2.40 & -\infty \\ +\infty & 3.72 & 3.54 & 2.85 & 2.36 & 1.49 & -1.38 & -1.76 & -\infty \\ +\infty & 3.72 & 3.54 & 3.37 & 3.09 & 2.27 & 1.00 & -0.76 & -\infty \end{bmatrix}. \tag{14}$$

This matrix can be visualized in terms of bins, as

from/to	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	$(+\infty, -1.28)$	$[-1.28, -2.56)$	$[-2.56, -3.20)$	$[-3.20, -3.35)$	$[-3.35, -3.43)$	$[-3.43, -3.54)$	$[-3.54, -3.72)$	$[-3.72, -\infty)$
AA	$(+\infty, 2.54)$	$[+2.54, -1.40)$	$[-1.40, -2.46)$	$[-2.46, -3.00)$	$[-3.00, -3.27)$	$[-3.27, -3.42)$	$[-3.42, -3.52)$	$[-3.52, -\infty)$
A	$(+\infty, 3.22)$	$[+3.22, +2.07)$	$[+2.07, -1.56)$	$[-1.56, -2.48)$	$[-2.48, -2.84)$	$[-2.84, -3.21)$	$[-3.21, -3.31)$	$[-3.31, -\infty)$
BBB	$(+\infty, 3.51)$	$[+3.51, +2.78)$	$[+2.78, +1.68)$	$[+1.68, -1.61)$	$[-1.61, -2.28)$	$[-2.28, -2.69)$	$[-2.69, -2.88)$	$[-2.88, -\infty)$.
BB	$(+\infty, 3.72)$	$[+3.72, +3.22)$	$[+3.22, +2.49)$	$[+2.49, +1.51)$	$[+1.51, -1.32)$	$[-1.32, -2.15)$	$[-2.15, -2.40)$	$[-2.40, -\infty)$
B	$(+\infty, 3.72)$	$[+3.72, +3.54)$	$[+3.54, +2.85)$	$[+2.85, +2.36)$	$[+2.36, +1.49)$	$[+1.49, -1.38)$	$[-1.38, -1.76)$	$[-1.76, -\infty)$
CCC/C	$(+\infty, 3.72)$	$[+3.72, +3.54)$	$[+3.54. + 3.37)$	$[+3.37, +3.09)$	$[+3.09, +2.27)$	$[+2.27, +1.00)$	$[+1.00, -0.76)$	$[-0.76, -\infty)$

(15)

Now, we can define the conditional (on $\hat{h}_t$) transition probability from the rating grade i to rating j, as

$$\begin{aligned} p_t(i, j|\hat{h}_t) &= Pr\big(z_{i,j} \le u_t \le z_{i,j+1}\big) \\ &= Pr\Big(z_{i,j} \le \sqrt{1-\rho}x_t + \sqrt{\rho}\hat{h}_t \le z_{i,j+1}\Big) \\ &= Pr\left(\frac{z_{i,j} - \sqrt{\rho}\hat{h}_t}{\sqrt{1-\rho}} \le x_t \le \frac{z_{i,j+1} - \sqrt{\rho}\hat{h}_t}{\sqrt{1-\rho}}\right) \\ &= \Phi\left(\frac{z_{i,j} - \sqrt{\rho}\hat{h}_t}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{z_{i,j+1} - \sqrt{\rho}\hat{h}_t}{\sqrt{1-\rho}}\right). \end{aligned} \tag{16}$$

Accordingly, the conditional transition probability matrix is given by

$$P_{t|\hat{h}_t} = \begin{bmatrix} p_t(1,1|\hat{h}_t) & p_t(1,2|\hat{h}_t) & \cdots & p_t(1,8|\hat{h}_t) \\ p_t(2,1|\hat{h}_t) & p_t(2,2|\hat{h}_t) & \cdots & p_t(2,8|\hat{h}_t) \\ \cdots & \cdots & \cdots & \cdots \\ p_t(7,1|\hat{h}_t) & p_t(7,2|\hat{h}_t) & \cdots & p_t(7,8|\hat{h}_t) \\ 0 & 0 & \cdots & 1 \end{bmatrix}. \tag{17}$$

Based on Belkin et al. (1998), we estimate ρ as the solution of the following least-squares problem

$$\min_{\rho \in R} \quad D(P_t(i,j), P_t(i,j|\hat{h}_t)) = \sum_{j=1}^{8} \sum_{i=1}^{7} \frac{(p_t(i,j) - p_t(i,j|\hat{h}_t))^2}{p_t(i,j|\hat{h}_t)(1 - p_t(i,j|\hat{h}_t))}. \tag{18}$$

In Problem (18), the squared deviation is normalized by a factor[10] that represents the sample variance for a transition from rating i to rating j under a binomial sampling approximation[11].

Using the 1981–2019 data, the estimated $\hat{\rho}$ is equal to 4.84%.

We have devised a methodology to model transition probabilities matrices conditional on the global real GDP growth (note that $\hat{h}_t$ is estimated based on Equation 10). Therefore, we can estimate future transition matrices based on future possible global real GDP growth paths.

Now, we assume that the bank manager is interested in modeling the global real GDP growth as a linear process, that is, as an Auto Regressive Integrated Moving Average—ARIMA(p, d, q) model. This choice is in line with several studies in the literature, where GDP growth is modeled as a linear process (see, e.g., Dritsaki, 2015; Arneja et al., 2020). Despite this choice, by no means are we suggesting that this is the model with the best predictive power. To construct a more accurate model in terms of predictive power, possibly one needs to take into account other sources of information (other than the historical time series). Nevertheless, this investigation is out of the scope of the present study. Here, we assume that the bank manager wants to construct the "best" linear model based only on the available historical global real GDP growth time series. Surely, the proposed methodology is ready to accommodate any other global real GDP growth model that the reader finds more suitable.

Based on the Box–Jenkins methodology (Box and Jenkins, 1976), we begin by analyzing the stationarity of the global real GDP growth time series, $GDPG_t$, for the period from 1981 to 2019 (see Fig. 2).

From the augmented Dickey–Fuller test (Dickey and Fuller, 1979), we observe that the time series is nonstationarity at a 5% significant level (Dickey–Fuller = −3.48; p-value = 0.061). However, the first difference of the time series, $\Delta GDPG_t$, is stationary at a 5% level (Dickey–Fuller = −4.03; p-value = 0.019).

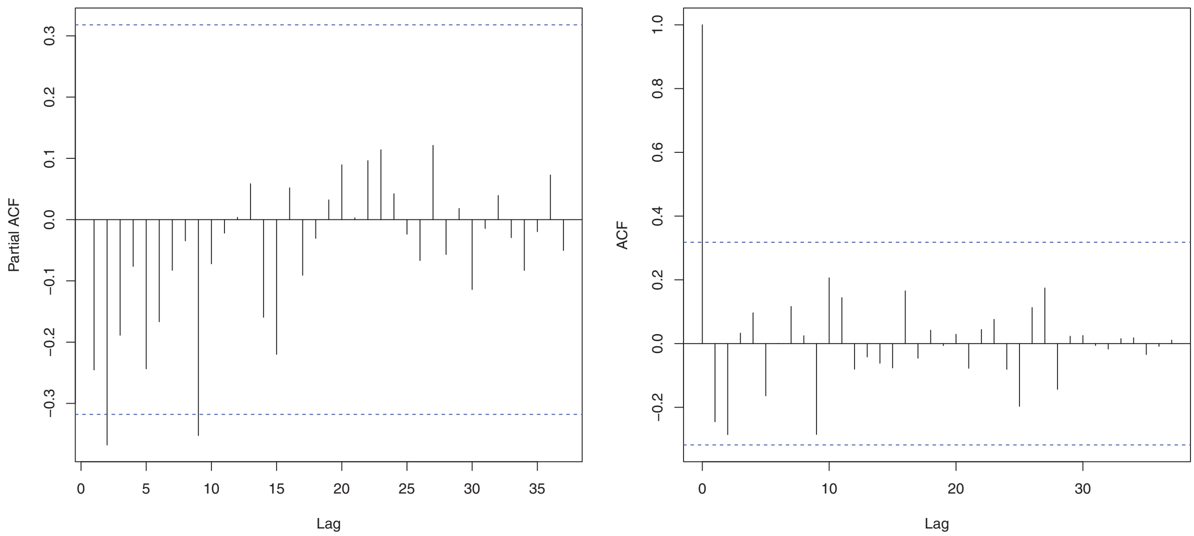
The next step consists of finding the optimal lags, p and q, of the stationary process

$$\left(1 - \sum_{i=1}^{p} \alpha_i L^i\right) \Delta GDPG_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) \varepsilon_i, \tag{19}$$

where L is the lag operator, α_i represent the parameters of the autoregressive part of the model, θ_i are the parameters of the moving average part, and ε_t are the innovations.

[10]This factor does not take into account the number of observations (i.e., credits) for each row because it is not relevant, since it remains constant across columns (Wei, 2003).

[11]Where "success" corresponds to a transition from rating i to rating j and "failure" corresponds to any other transition.



(a) Partial autocorrelogram (b) Autocorrelogram

Fig. 4. Partial autocorrelogram and autocorrelogram for the $\Delta GDPG_t$ time series.

Based on the partial autocorrelogram and autocorrelogram (see Fig. 4), we identify an ARIMA(2,1,0) as a possible model[12]. This model can be estimated through the maximum likelihood (computed via a state-space representation) where the model's innovations and correspondent variances are found by a Kalman filter (a detailed implementation of this procedure can be found in Gardner et al. (1980)). From the model's estimation, we get that both autoregressive coefficients are statistically significant at a 5% level ($\alpha_1 = -0.3289$, std. error $= 0.1508$; $\alpha_2 = -0.3664$, std. error $= 0.1504$). The Ljung–Box test (Ljung and Box, 1978) applied to the innovations gives the test statistic $Q = 3.997$ (p-value $= 0.677$), which allows us to accept the assumption that the model's innovations are independent and identically distributed.

With the estimated ARIMA(2,1,0) model, we can simulate different global real GDP growth paths. Figure 5 displays 100 simulated paths for the next 10 years. In the simulation, we take into account the nonnormal behavior of the model's innovations. The Jarque–Bera test (Jarque and Bera, 1980) applied to the ARIMA(2,1,0) innovations gives a test statistic $JB = 11.45$ (p-value$= 0.003$), which leads to the rejection of the normal distribution assumption of the innovations. Hence, we have done this simulation using the bootstrap innovations (i.e., according to the empirical distribution of the innovations) instead of unrealistically assuming that they follow a normal distribution. This simulation was performed using the R language (R Core Team, 2017).

We observe that the simulated paths (see Fig. 5) exhibit plenty of heterogeneity to be assumed as a good approximation for a set of possible economic scenarios.

[12]We have tested all possible combinations of ARIMA(p, 1, 0) with $p \in \{1, \ldots, 10\}$ and the model with the minimum Akaike's entropy-based Information Criterion—AIC (Akaike, 1974)—is the ARIMA(2,1,0) (AIC$= -212.26$).

Fig. 5. Simulated paths for the global real GDP growth.

2.3. The credit rating dynamics and the estimation of the point-in-time probabilities of default

This section applies the conditional transition matrix methodology to simulate the credit rating dynamics and estimate the point-in-time probabilities of default.

We saw in the previous section how to generate s-scenarios based on the simulation of 100 paths, with length 10 (i.e., 10 years), of the global real GDP growth (see Fig. 5). Fixing a simulated path s ($s = 1, \ldots, 100$), for each year $t = 1, \ldots, T$ (with $T = 10$), we can compute the respective conditional transition matrix, $\hat{P}_{t|\hat{h}_t}$, according to the methodology described in Section 2.2. Thus, for each s-scenario, we have a set, $M^{(s)}$, of conditional transition matrices such that $M^{(s)} = \{\hat{P}^{(s)}_{1|\hat{h}_1}, \ldots, \hat{P}^{(s)}_{T|\hat{h}_T}\}$.

To simulate the credit rating dynamics, we consider a universe of 1000 credits within each credit rating grade AAA to B, where the bank manager intends to invest. Therefore, at $t = 0$ we can define, $\forall k = 1, \ldots, 1000$, each initial credit rating grade as

$$\begin{aligned} AAA_0^{(k)} &= \text{AAA}, \\ AA_0^{(k)} &= \text{AA}, \\ A_0^{(k)} &= \text{A}, \\ BBB_0^{(k)} &= \text{BBB}, \\ BB_0^{(k)} &= \text{BB}, \\ B_0^{(k)} &= \text{B}. \end{aligned} \tag{20}$$

Algorithm 1. Simulation of the credit rating dynamics

```
1  Input: The set of conditional transition matrices for a s-scenario M^(s);
2  The set of initial rating grades I.
3  Output: The set of rating grades for each credit J.
4  Set J = {}.
5  for t = 1, ..., T do
6      Obtain the corresponding conditional transition matrix \hat{P}^(s)_{t|\hat{h}_t} ∈ M^(s).
7      Select the row of \hat{P}^(s)_{t|\hat{h}_t} corresponding to the previous rating grade of each credit.
8      for k = 1, ..., 1000 do
9          Generate AAA_t^(K), AA_t^(K), A_t^(K), BBB_t^(K), BB_t^(K), and B_t^(K) from a multinomial
           distribution with probability vector equal to the corresponding row (obtained in line 7).
10     end
11     Update J = J ∪ {AAA_t^(k), AA_t^(k), A_t^(k), BBB_t^(k), BB_t^(k), B_t^(k) : k = 1, ..., 1000}.
12 end
```

Let us define $I = \{AAA_0^{(k)}, AA_0^{(k)}, A_0^{(k)}, BBB_0^{(k)}, BB_0^{(k)}, B_0^{(k)} : k = 1, \ldots, 1000\}$ as the set of the initial credit rating grades where the bank manager wants to invest. I has a total of 6000 elements (i.e., $|I| = 6000$).

In Algorithm 1, we describe the simulation of the credit rating dynamics. Considering the credits' maturity $T = 10$, from the implementation of Algorithm 1 we obtain the set J ($|J| = 6000 \times 10 = 60{,}000$), where each generic element $X_t^{(K)}$ represents the rating grade at time t ($t = 1, \ldots, 10$) of a credit k ($k = 1, \ldots, 1000$) initially rated with rating X ($X =$ AAA, AA, A, BBB, BB, B). Applying Algorithm 1 to each simulated path of the global real GDP growth leads to the identification of 100 J's sets that completely establish the credit rating dynamics for each credit from $t = 0$ to $t = T$. Therefore, the bank investment manager can now define the stage allocation inherent in the implementation of the credit loss model defined under IFRS 9 (see Section 2.1, for further details).

We also use the conditional transition matrices (see Section 2.2) for the estimation of the point-in-time probabilities of default. These marginal point-in-time probabilities of default can be obtained by following the steps presented in Algorithm 2.

Algorithm 2 can be applied to each simulated path $s = 1, \ldots, 100$ for the global real GDP growth (see Fig. 5). This leads to the identification of 100 Y's sets. Considering the global credits' maturity $T = 10$, each Y set (see Algorithm 2) contains ($6 \times \sum_{i=1}^{T} i$) one-year point-in-time probabilities of default. With these probabilities, we can compute the impairment stock (see Equation 3) for any simulated global credit.

We now possess all the data needed to compute, for each s-scenario ($s = 1, \ldots, 100$), the annual credit's income, from year $t = 1$ to year $t = T$ (with $T = 10$), for each simulated credit $k = 1, \ldots, 6,000$. Let us designate the credit's income from a credit k in the s-scenario and at year t, as $I_{s,t}^k$.

Algorithm 2. Estimation of the point-in-time probabilities of default

Input: The historical (1981-2019) transition matrices $\{P_1, \ldots, P_{39}\}$;
The historical (1981-2019) time series of the global real GDP growth $\{GDPG_1, \ldots, GDPG_{39}\}$;
A simulated path for the global real GDP growth $Q^{(s)} = \left\{G\hat{DPG}_1^{(s)}, \ldots, G\hat{DPG}_T^{(s)}\right\}$ and the corresponding set of conditional transition matrices $M^{(s)} = \left\{\hat{P}^{(s)}_{1|\hat{h}_1}, \ldots, \hat{P}^{(s)}_{T|\hat{h}_T}\right\}$.
Output: The set of marginal point-in-time probabilities of default Y.
Set $Y = \{\}$, $N = \{P_1, P_2, \ldots, P_{39}\}$, and $O = \{GDPG_1, \ldots, GDPG_{39}\}$.
for $t = 0, \ldots, T-1$ **do**
  Compute the current average transition matrix (according to Equation (12)), P, using the set of known conditional transition matrices N (data from 1981 to $2019 + t$).
  Compute the corresponding z-score matrix (according to Equation (13)).
  Predict the global real GDP growth, $G\hat{DPG}_\tau$ with $\tau = 1, \ldots, T-t$, for the next $T-t$ years: estimate an ARIMA$(3, 2, 1)$ model (see Equation (19)), using the set of known global real GDP growth rates O (data from 1981 to $2019 + t$), then perform the prediction of the next $T-t$ values according to the estimated model.
  Estimate the credit cycle measure, $\hat{h}_\tau$ with $\tau = 1, \ldots, T-t$, for the next $T-t$ years: estimate the linear model of Equation (10), using the set of known global real GDP growth O, and then estimate the $T-t$ values of $\hat{h}_\tau$ (according to Equation (11)).
  for $\tau = 1, \ldots, T-t$ **do**
    Compute the $\hat{\rho}$ value (according to Problem (18)) and the corresponding conditional transition matrix $\hat{P}_{\tau|\hat{h}_\tau}$ (according to Equation (17)).
    The last column of matrix $\hat{P}_{\tau|\hat{h}_\tau}$ (computed in the previous line), gives the marginal point-in-time probabilities of default of each credit rating grade. Based on the rating grades from AAA to B update $Y = Y \cup \left\{p_t(1,8|\hat{h}_t), p_t(2,8|\hat{h}_t), p_t(3,8|\hat{h}_t), p_t(4,8|\hat{h}_t), p_t(5,8|\hat{h}_t), p_t(6,8|\hat{h}_t)\right\}$.
  end
  Update $O = O \cup \left\{G\hat{DPG}^{(s)}_{t+1}\right\}$ and $N = N \cup \left\{\hat{P}^{(s)}_{t+1|\hat{h}_{t+1}}\right\}$.
end

We are interested in a holistic analysis by looking at each rating grade as a whole instead of looking at each one of the 6000 simulated credits. Thereby, we define the average income of a representative credit, for each rating grade, as

$$\iota^G_{s,t} = \frac{1}{1{,}000}\sum_{k=1}^{1{,}000} I^k_{s,t}, \quad \forall G = \text{AAA, AA, A, BBB, BB, B}. \tag{21}$$

Based on the annual estimated credits' incomes, from year $t = 1$ to year $t = T$, in the next section, we will implement a robust optimization approach that will allow the bank manager to identify efficient credit portfolios according to a determined income-risk measure.

3. Forward-looking robust optimization

In Section 2, we have presented a methodology to estimate the annual income, from year $t = 1$ to year $t = T$ (where T represents the credit's maturity and equals the investment horizon), for each one out of six representative credits: $G \in \{\text{AAA, AA, A, BBB, BB, B}\}$. We have estimated these data for 100 different s-scenarios corresponding to $s = 1, \ldots, 100$ projections of the global real GDP growth.

In each s-scenario, let us define the annual average income of a credit with rating grade equal to G, μ_s^G, as

$$\mu_s^G = \frac{1}{T} \sum_{y=1}^{T} \iota_{s,y}^G. \tag{22}$$

According to the bank manager problem, we assume that a portfolio consists of credits with rating grades from AAA to B. Moreover, it is assumed that the bank manager has a certain funding amount, F, to allocate to credits in different ratings. Let w_s^G represent the proportion of F allocated to each credit rating grade in a specific s-scenario. The portfolio income is assumed linear in $w_s^{\text{AAA}}, \ldots, w_s^{\text{B}}$ and thus for each s-scenario, the portfolio average income, m_s, can be written as

$$m_s(w_s) = \frac{1}{T} \sum_{y=1}^{T} \left(w_s^{\text{AAA}} \iota_{s,y}^{\text{AAA}} + \cdots + w_s^{\text{B}} \iota_{s,y}^{\text{B}} \right) = w_s^{\text{AAA}} \mu_s^{\text{AAA}} + \cdots + w_s^{\text{B}} \mu_s^{\text{B}} = w_s^{\top} \mu_s, \tag{23}$$

with

$$\mu_s = (\mu_s^{\text{AAA}}, \ldots, \mu_s^{\text{B}})^{\top} \text{ and } w_s = (w_s^{\text{AAA}}, \ldots, w_s^{\text{B}})^{\top}. \tag{24}$$

In turn, for each s-scenario the portfolio income variance, v_s, can be computed as

$$v_s(w_s) = \frac{1}{T} \sum_{y=1}^{T} \left[\sum_{G \in \{\text{AAA},\ldots,\text{B}\}} w_s^G \iota_{s,y}^G - \frac{1}{T} \sum_{y=1}^{T} \left(\sum_{G \in \{\text{AAA},\ldots,\text{B}\}} w_s^G \iota_{s,y}^G \right) \right]^2. \tag{25}$$

Therefore,

$$v_s(w_s) = \sum_{i \in \{\text{AAA},\ldots,\text{B}\}} \sum_{j \in \{\text{AAA},\ldots,\text{B}\}} \frac{1}{T} \sum_{y=1}^{T} \left(\iota_{s,y}^i - \mu_s^i \right) \left(\iota_{s,y}^j - \mu_s^j \right) w_s^k w_s^l. \tag{26}$$

Defining each entry i, j of the covariance matrix Q_s by

$$\sigma_{ij} = \frac{1}{T} \sum_{y=1}^{T} \left(\iota_{s,y}^i - \mu_s^i \right) \left(\iota_{s,y}^j - \mu_s^j \right), \tag{27}$$

we have

$$v_s(w_s) = w_s^\top Q_s w_s. \tag{28}$$

According to Equation (28), the corresponding portfolio income standard deviation, $d_s(w_s)$, is simply given by

$$d_s(w_s) = \sqrt{w_s^\top Q_s w_s}. \tag{29}$$

Let us consider, for each s-scenario, the following bi-objective problem

$$\begin{array}{ll} \min\limits_{w_s \in \mathbb{R}^6} & F(w_s) = (-m_s(w_s), v_s(w_s))^\top \\ \text{subject to} & w_s \in \Omega_s, \end{array} \tag{30}$$

where $\Omega_s = \{w_s \in \mathbb{R}^6 : w_s^{\text{AAA}} + \cdots + w_s^{\text{B}} = 1 \wedge w_s^G \geq 0,\ \forall G \in \{\text{AAA}, \ldots, \text{B}\}\}$ denotes the feasible region. The formulation of this problem is motivated by the classical mean-variance optimization problem (Markowitz, 1952), which constitutes the base of the Modern Portfolio Theory.

Definition 1 (Pareto minimizer). *$w_s \in \Omega_s$ is a Pareto minimizer of $F(\cdot)$ if*

$$\nexists_{x_s} \in \Omega_s : -m_s(x_s) < -m_s(w_s) \wedge v_s(x_s) < v_s(w_s). \tag{31}$$

A Pareto minimizer, $w_s \in \Omega_s$, is also known as a nondominated point.

Definition 2 (Set of Pareto minimizers, W_s). *The set of Pareto minimizers, W_s, of $F(\cdot)$, is defined as*

$$W_s = \left\{w_s \in \Omega_S : \nexists_{x_s} \in \Omega_s : -m_s(x_s) < -m_s(w_s) \wedge v_s(x_s) < v_s(w_s)\right\}. \tag{32}$$

The set of Pareto minimizers, W_s, defines the set of nondominated points of Problem (30). The efficient Pareto frontier, solution of Problem (30), corresponds to the mapping of $F(\cdot)$ on the set of nondominated points, W_s.

Now, consider the following linear scalarization problem

$$\begin{array}{ll} \min\limits_{w_s \in \mathbb{R}^6} & \alpha w_s^\top Q_s w_s - (1-\alpha) w_s^\top \mu_s \\ \text{subject to} & w_s \in \Omega_s, \end{array} \tag{33}$$

where $\alpha \in [0, 1]$ represents a tuning parameter. Note that Problem (33) corresponds to a convex quadratic programming (QP) problem.

It is easy to prove that a solution of Problem (33) is a Pareto minimizer for Problem (30). In fact, let $w_s^* \in \Omega_s$ be a solution of Problem (33). Now, suppose that w_s^* is not a Pareto minimizer of Problem (30). Then, according to Definition 1, we have

$$\exists z_s \in \Omega_s : -z_s^\top \mu_s < -w_s^{*\top} \mu_s \ \wedge \ z_s^\top Q_s z_s < w_s^{*\top} Q_s w_s^*. \tag{34}$$

By the fact that $\alpha \in (0, 1)$, we have

$$-(1-\alpha) z_s^\top \mu_s < -(1-\alpha) w_s^{*\top} \mu_s \ \wedge \ \alpha z_s^\top Q_s z_s < \alpha w_s^{*\top} Q_s w_s^*, \quad \forall \alpha \in (0, 1) \tag{35}$$

Finally, summing these two inequalities, we have

$$\alpha z_s^\top Q_s z_s - (1-\alpha) z_s^\top \mu_s < \alpha w_s^{*\top} Q_s w_s^* - (1-\alpha) w_s^{*\top} \mu_s, \quad \forall \alpha \in [0, 1], \tag{36}$$

from which follows that w_s^* is not an optimal solution of Problem (33). This concludes the proof by *reductio ad absurdum*.

Accordingly, for each s-scenario, we can find the corresponding set of Pareto minimizers, W_s, by solving Problem (33) for different values of α. Setting $\alpha = 0$, we obtain the maximum income portfolio and setting $\alpha = 1$ give us the global minimum variance portfolio. We solve Problem (33), varying α between 0 and 1, in order to achieve $|W_s| = 10,002$ (besides the efficient allocations corresponding to $\alpha = 0$ and $\alpha = 1$, we find 10,000 more different efficient allocations).

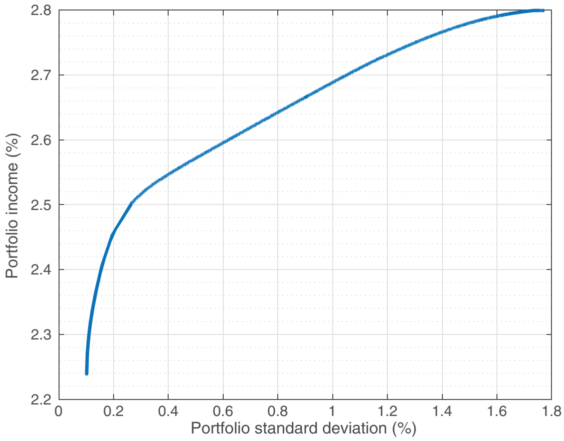
The value of α defines an efficient allocation for the corresponding s-scenario. Motivated by the robust resampling methodology proposed in Michaud and Michaud (2007, 2008), we define the optimal efficient allocations by averaging over all s-scenarios. As a result, we obtain 10,002 optimal efficient portfolios, where each portfolio is determined by the α value and corresponds to the average (over the 100 considered scenarios) efficient allocations.

4. Computational results

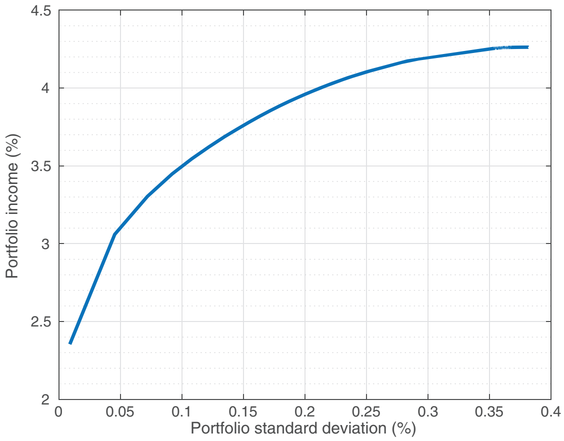
We have implemented the forward-looking robust optimization approach, described in the previous section, using MATLAB® (Mathworks™, 2021). Considering the baseline case, where each credit has a 10-year maturity, we obtain the robust efficient frontiers under the IFRS 9 framework (see Fig. 6a). This frontier allows the bank manager to directly analyze the tradeoff between the portfolio income and the portfolio income volatility. Therefore, the bank manager can select an efficient portfolio according to a desirable income-risk rule.

Instead of focusing on the selection of a specific income-risk rule or efficient portfolio, we report the weight variation with the risk level for all the computed efficient portfolios (see Fig. 7a). We can observe that a less risky portfolio has a larger AAA-grade allocation. In turn, a more risky portfolio has a larger BB-grade allocation. Under the IFRS 9 accounting framework, the efficient riskiest credit portfolio has the following structure: 12%-AAA, 22%-AA, 15%-AA, 5%-BBB, 46%-BB, and 0%-B. This portfolio has an average annual income equal to 2.80% with a standard deviation equal to 1.77%, which, for example, dominates the equally weighted portfolio that achieves an average annual income equal to 1.78% with a standard deviation of 2.49%.

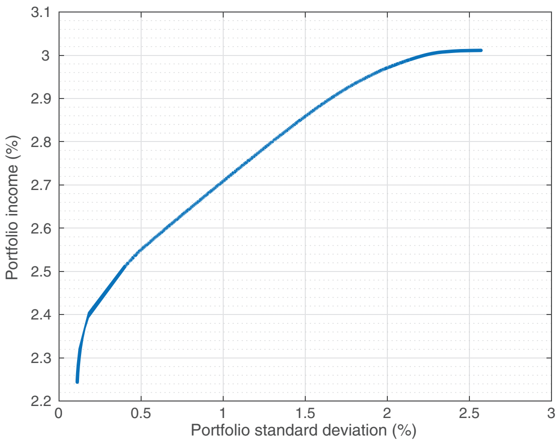
In Fig. 6, we also report the robust efficient frontiers under the IAS 9 and CECL accounting frameworks. We observe that under the IAS 39 framework, in opposition to the IFRS 9 and CECL frameworks, the efficient credit portfolios are less risky and achieve a higher average annual income. Moreover, looking at how the allocations vary by risk level (see Fig. 7), we observe that the structure of the efficient credit portfolios under IFRS 9 and CECL is very different from the one under IAS 39. The riskier credit portfolio under IAS 39 (0%-AAA, 0%-AA, 0%-A, 0%-BBB, 74%-BB, and 26%-B) has a portfolio income standard deviation equal to 0.38% and an average annual income equal to 4.26%. These numbers correspond to approximately 4.66 times less risk and 1.93 times more average annual income than the riskier efficient portfolio under IFRS 9. The riskier efficient portfolio under CECL (8%-AAA, 19%-AA, 2%-A, 3%-BBB, 68%-BB, and 0%-B) achieves a higher



(a) IFRS 9 framework

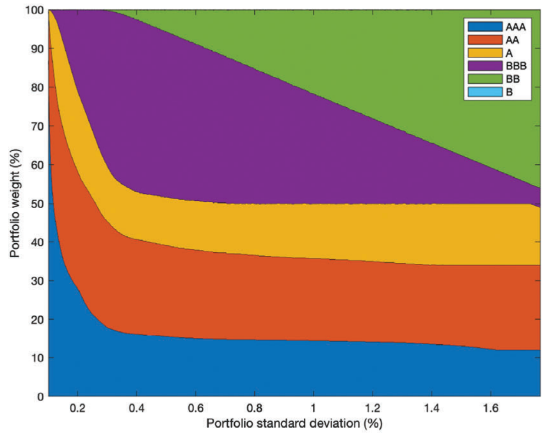


(b) IAS 39 framework

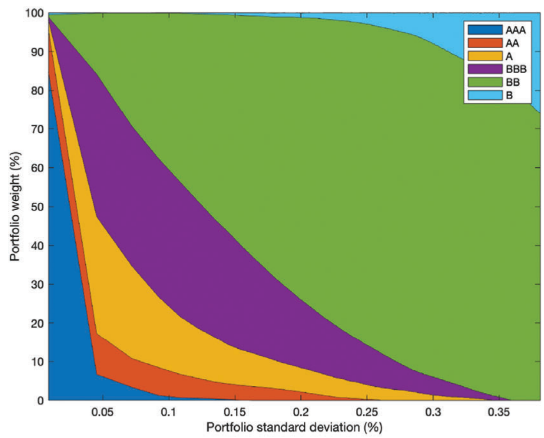


(c) ECL framework

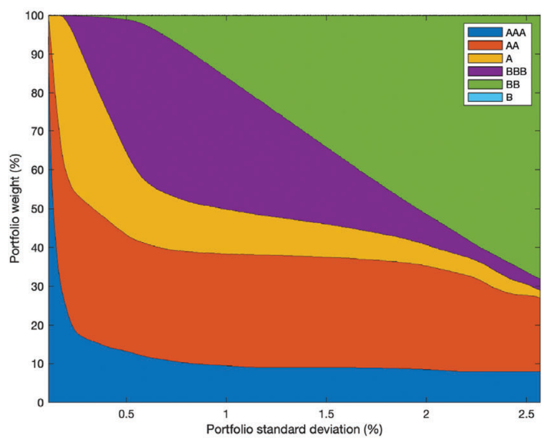
Fig. 6. Robust efficient frontiers for a 10-year maturity time frame.



(a) IFRS 9 framework



(b) IAS 39 framework



(c) ECL framework

Fig. 7. Risk maps for a 10-year maturity time frame.

income in comparison with the riskier efficient portfolio under IFRS 9 (3.01% > 2.80%) but is riskier (2.57% > 1.77%).

In Fig. 7, we observe that, under the IFRS 9 and CECL frameworks, riskier rating allocations are penalized. Under the IAS 39 framework, the allocation of the riskier portfolios (see Fig. 7b) concentrates 100% around speculative-grade rated credits (BB and B). In turn, under IFRS 9 more than 50% of the allocation (see Fig. 7a) of the riskier portfolios is concentrated in investment-grade ratings (AAA, AA, A, and BBB). Under the CECL framework (see Fig. 7b), this percentage is slightly less. Even so, riskier credit ratings are also very penalized. These results suggest that the IFRS 9 and CECL frameworks can lead to procyclical effects when compared to the IAS 39 framework. This is in line with the results presented in previous studies (Barclays, 2017; Abad and Suarez, 2018; Seitz et al., 2018).

To give some insight into the sensitivity of these results to the maturity of the credits, we apply the methodology described in Section 2 and the forward-looking robust optimization model developed in Section 3 to credits with a 5-year and a 15-year maturity. We assume that the investment horizon coincides with the credit maturities.

The credit income computation uses as fixed interest, C, the coupon value of global corporate bonds. In the case of five-year maturity credits, these values are equal to: 1.828%, 1.999%, 2.146%, 2.531%, 3.536%, and 4.156% for a AAA, AA, A, BBB, BB, and B ratings, respectively. In turn, for the case of 15-year maturity credits, these values are equal to 2.631%, 2.777%, 3.075%, 3.684%, 5.412%, and 6.065% or a AAA, AA, A, BBB, BB, and B ratings, respectively.

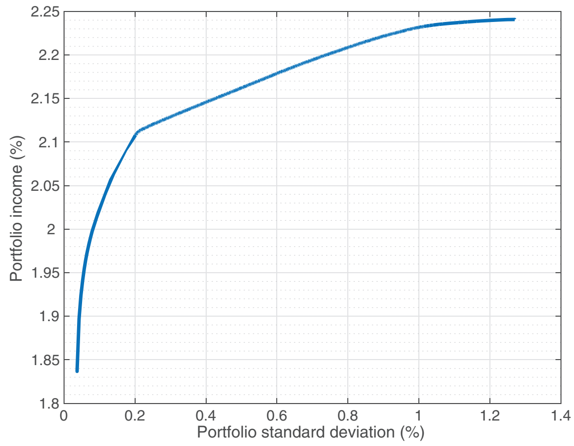
For the five-year horizon, the discount rate D for the credit impairment stock, under the IFRS 9 and CECL frameworks, is set to 1.659%, equaling the two-year AAA global corporate bond rate[13]. In the 15-year horizon, D is set equal to the seven-year AAA corporate bond rate, 2.030%. All these values were obtained through the Bloomberg Terminal (Bloomberg, 2021). Using these data, the procedure presented in Sections 2 and 3, which was applied initially to 10-year maturities, is replicated to 5-year and 15-year horizons.

For a five-year horizon, Figs. 8 and 9, report, under the three different accounting frameworks, the robust efficient frontiers and the portfolio weights for different risk levels, respectively. In Fig. 8, we observe that the efficient portfolios have low income and low risk, regardless of the accounting framework, when compared to 10-year efficient portfolios (Fig. 6).

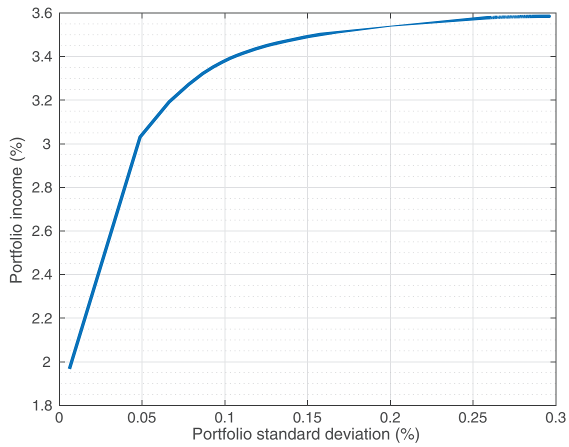
For example, we observe that the riskiest efficient portfolio under the IFRS 9 accounting framework, with the structure 1%-AAA, 17.98%-AA, 13.02%-A, 28%-BBB, 40%-BB, and 0%-B, achieves an annual average income equal to 2.24% with an income standard deviation of 1.27%. This allocation earns a lower average yearly income than the riskiest 10-year efficient portfolio (2.24% < 2.80%), but also entails less risk (1.27% < 1.77%). Besides the difference between the maturities of the credits, this pattern is expected due to the positive slope of the bond coupon curves.

Under the IFRS 9 and CECL frameworks, the structure of the five-year efficient portfolios (Fig. 9) and 10-year portfolios (Fig. 7) is similar. Despite the similarities, we highlight that, for both frameworks, shorter maturities lead to lower concentration on the AAA-rated credits.

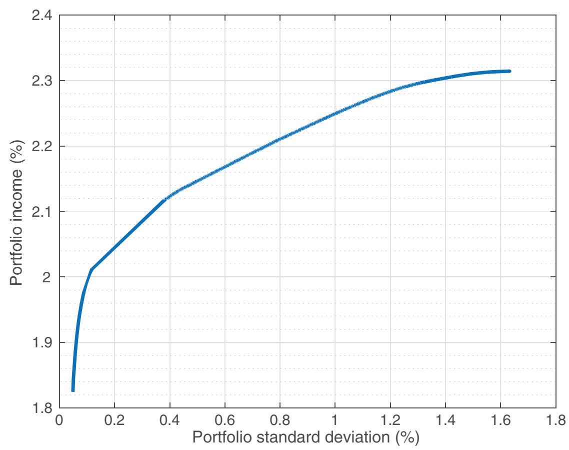
[13]Similarly to the baseline case of credits with a 10-year maturity, for credits with 5-year and 15-year maturity, we use as a discount rate, D, the coupon rate of an AAA global corporate bond with roughly half the maturity of the credits.



(a) IFRS 9 framework

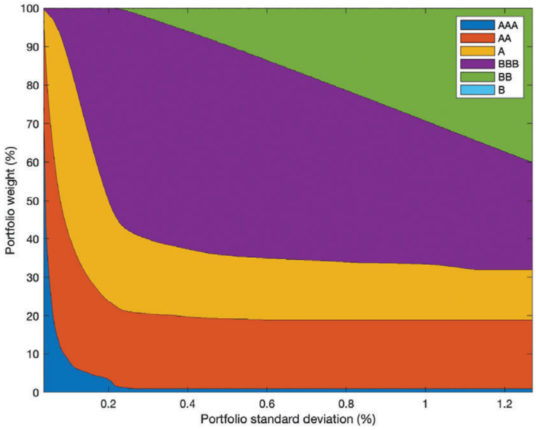


(b) IAS 39 framework

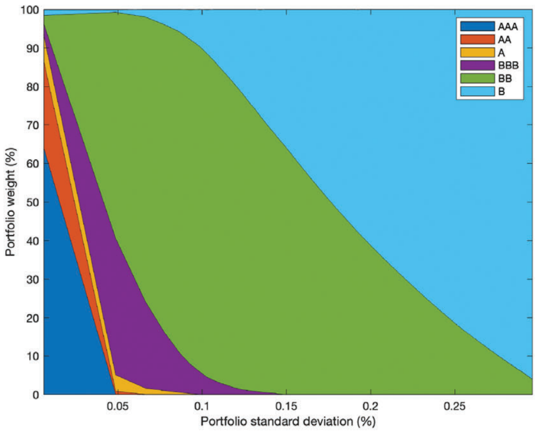


(c) ECL framework

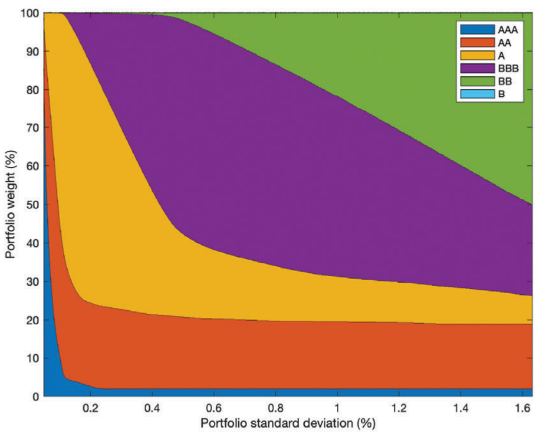
Fig. 8. Robust efficient frontiers for a five-year maturity time frame.



(a) IFRS 9 framework



(b) IAS 39 framework



(c) ECL framework

Fig. 9. Risk maps for a five-year maturity time frame.

As in the case of 10-year maturity credits, we also observe that the five-year efficient portfolios under the IFRS 9 and CECL tend to penalize riskier ratings, namely speculative-grade classes (BB and B). In turn, under the IAS 39 framework, the short-dated riskiest efficient portfolios tend to have a higher concentration in speculative-grade credit ratings (Figs. 9b). For example, under IAS-39, the riskiest efficient credit portfolio has the following structure: 0%-AAA, 0%-AA, 0%-A, 0%-BBB, 4%-BB, 96%-B.

Finally, we test efficient frontiers and allocations for a 15-year horizon under the IFRS 9, IAS 39, and CECL frameworks, in Figs. 10 and 11, respectively. These efficient credit portfolios exhibit a higher annual average income and volatility (Fig. 10) when compared to the 10-year efficient credit portfolios (Fig. 6). The higher risk stems from the higher investment horizon, while the higher income is explained again by the positive slope of the coupon curve.

The structure of the 15-year efficient portfolios shows identical features to 10-year and 5-year portfolios (see Figs. 11, 7, and 9). Nevertheless, under the IFRS 9 and CECL frameworks, we observe that the 15-year efficient portfolios tend to allocate a larger proportion to AAA ratings, avoiding the volatility caused by lower credit ratings.

We continue to observe that, under IAS 39, riskier ratings are less penalized. For example, the riskiest 15-year efficient portfolio shows the following allocations: 0%-AAA, 0%-AA, 0%-A, 0%-BBB, 95%-BB, and 5%-B. However, for the same level of risk, 15-year portfolios display a higher concentration in investment-grade ratings, and thus a lower concentration in speculative-grade assets, when compared to the 10-year horizons (see Figs. 11b and 7b).

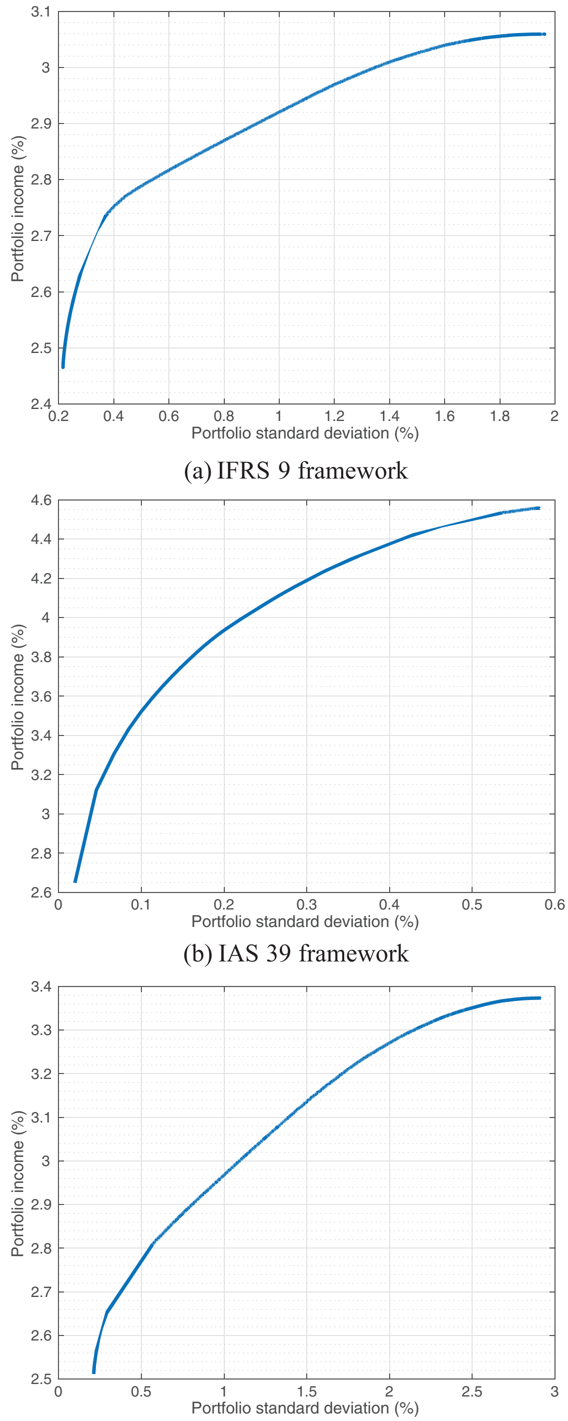
5. Conclusion

In this paper, we have developed a methodology to construct credit portfolios under the IFRS 9 framework. Based on a point-in-time probability model, we have defined the credit loss model to compute the credit income under IFRS 9. We have also explained the differences between the credit loss model under the IFRS 9 framework and the credit loss models under the IAS 39 and CECL accounting frameworks.

We have devised a nonhomogeneous Markov chain setting, based on CreditMetrics^{TM}, to estimate transition matrices conditional on global real GDP growth. We have used these conditional transition matrices to simulate the credit dynamics and estimate the point-in-time probabilities of default, both needed to compute credit income under the IFRS 9 framework. The credit income is thus estimated taking into account the cyclical economic effects that are of great importance to bank management.

For each rating class, we estimated the forward-looking credit income based on a portfolio of representative credits. We subsequently implemented a robust optimization model to analyze the trade-off between expected revenue and volatility, measured as the annual standard deviation of portfolio income. As future work, this model can be easily adapted to use other risk measures such as VaR or CVaR.

According to the proposed model, we have obtained the efficient Pareto frontiers under the IFRS 9, IAS 39, and CECL accounting frameworks. We began our tests by evaluating 10-year efficient allocations and subsequently examined the 5-year and 15-year Pareto frontiers, the impact of investment horizons on income, risk, and efficient allocations.

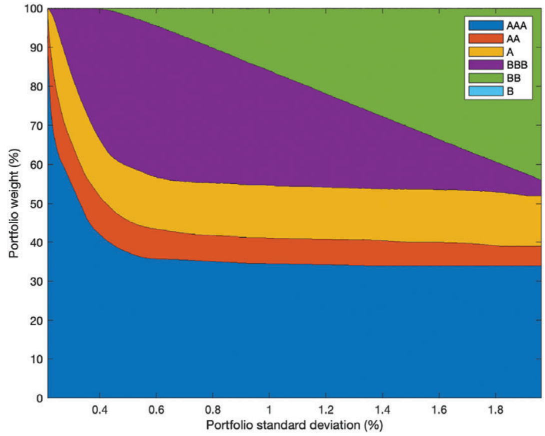


(a) IFRS 9 framework

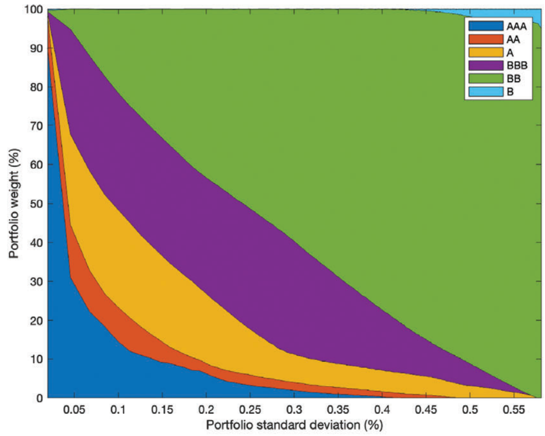
(b) IAS 39 framework

(c) ECL framework

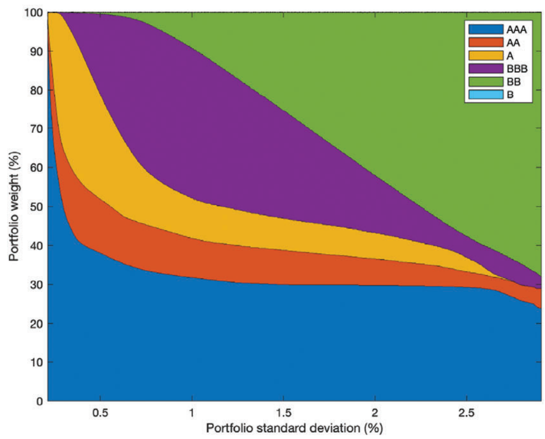
Fig. 10. Robust efficient frontiers for a 15-year maturity time frame.



(a) IFRS 9 framework



(b) IAS 39 framework



(c) ECL framework

Fig. 11. Risk maps for a 15-year maturity time frame.

The results indicate that efficient credit portfolios constructed under IFRS 9 and CECL severely penalize riskier ratings more than IAS 39. Moreover, when changing the investment horizon, we observed that the longer the maturity, the lower the allocation to riskier ratings.

These results highlight the potential procyclical effects of the IFRS 9 and CECL frameworks. If the bank manager anticipates an economic downturn, under the IFRS 9 and CECL, she/he will tend to penalize riskier credits. The problem is that it is precisely in downturn periods that several credits suffer downgrades as the result of the economic conditions. Therefore, the investment decisions made by the bank manager could lead to procyclical effects by contributing to intensifying the economic downturn.

Furthermore, the several subjective decisions associated with implementing IFRS 9 and CECL may cause a lack of comparability and transparency among institutions, compared to the more objective procedure to account for losses under IAS 39. This feature of more recent accounting regimes demonstrates the importance of conduction more research on their practical implications.

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References

Abad, J., Suarez, J., 2018. The procyclicality of expected credit loss provisions. Working paper, Centre for Economic Policy Research. Available at https://ssrn.com/ (accessed November 2021).

Abad, J., Suarez, J., 2020. IFRS 9 and COVID-19: delay and freeze the transitional arrangements clock. Available at https://voxeu.org/ (accessed November 2021).

Akaike, H., 1974. A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19, 6, 716–723.

Andrija, D., 2019. Macroeconomic approach to point in time probability of defaul modeling - IFRS 9 challenges. *Journal of Central Banking Theory and Practice* 8, 1, 209–223.

Argimon, I., Düllmann, K., Foos, D., Francis, B., Gaul, L., Georgescu, O., Nordal, K., Pramor, M., 2015. The interplay of accounting and regulation and its impact on bank behaviour: literature review. Working paper, Bank for International Settlements. Available at https://www.bis.org/ (accessed November 2021).

Arneja, R.S., kaur, N., Saihjpal, A.K., 2020. Analyses and forecasting evaluation of GDP of India using ARIMA model. *International Journal of Advanced Science and Technology* 29, 11, 1102–1107.

Balla, E., McKenna, A., 2009. Dynamic provisioning: a countercyclical tool for loan loss reserves. *Economic Quarterly* 95, 4, 383–418.

Bank of International Settlements, 1998. International convergence of capital measurement and capital standards. Available at https://www.bis.org/ (accessed November 2021).

Barclays, 2017. IFRS 9 - Bigger than Basel IV. Available at http://www.eifr.eu/ (accessed November 2021).

Basel Comittee on Banking Supervision, 2009. Strengthening the resilience of the banking sector. Available at https://www.bis.org/ (accessed:November 2021).

Basel Comittee on Banking Supervision, 2015. Guidance on accounting for expected credit losses. Available at https://www.bis.org/ (accessed November 2021).

Belkin, B., Suchower, S., Forest Jr., L., 1998. A one-parameter representation of credit risk and transition matrices. CreditMetrics™ Monitor (Third Quarter). Available at https://www.z-riskengine.com/ (accessed November 2021).

Bloomberg, 2021. Bloomberg Terminal [online]. Available at Subscription Service (accessed December 2020).

Box, G.E., Jenkins, G.M., 1976. *Time series analysis: forecasting and control*. Holden-Day, Inc.

Brémaud, P., 2020. *Non-homogeneous Markov Chains*. Springer, Cham.

Buesa, A., García, F.J.P., Tarancón, J., 2019. Measuring the procyclicality of impairment accounting regimes: a comparison between IFRS 9 and US GAAP. Working paper, European Central Bank, Available at https://www.ecb.europa.eu/ (accessed November 2021).

Carruthers, D., Makova, B., Siddiqui, S., 2019. Point-in-Time PD Curves: IFRS 9/CECL Applications. Working paper, Credit Benchmark, Available at https://www.creditbenchmark.com/ (accessed November 2021).

Chawla, G., Forest Jr, L.R., Aguais, S.D., 2016. Point-in-time loss-given default rates and exposures at default models for IFRS 9/CECL and stress testing. *Journal of Risk Management in Financial Institutions* 9, 3, 249–263.

Dickey, D.A., Fuller, W.A., 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association* 74, 366, 427–431.

Dritsaki, C., 2015. Forecasting real GDP rate through econometric models: an empirical study from Greece. *Journal of International Business and Economics* 3, 1, 13–19.

European Central Bank, 2020. Press Release: ECB Banking Supervision provides further flexibility to banks in reaction to coronavirus. Available at https://www.bankingsupervision.europa.eu/ (accessed November 2021).

European Comission, 2020. Remarks by Executive Vice-President Dombrovskis at the press conference on the Comission banking package to facilitate lending to households and businesses in the EU. Available at https://ec.europa.eu/ (accessed November 2021).

European Systematic Risk Board, 2017. Financial stability implications of IFRS 9. Available at https://www.esrb.europa.eu/ (accessed November 2021).

Financial Accounting Standards Board, 2016. Financial instruments - credit losses (topic 326): measurement of credit losses on financial instruments. Available at https://www.fasb.org/ (accessed November 2021).

Financial Accounting Standards Board, 2019. Accounting standards update 2019-10–financial Instruments–credit losses (topic 326), derivatives and hedging (topic 815), and leases (topic 842): effective dates. Available at https://www.fasb.org/ (Last accessed November 2021).

Financial Stability Forum, 2009. Report of the Financial Stability Forum on addressing procyclicality in the Financial System. Available at https://www.fsb.org/ (Last accessed November 2021).

Gardner, G., Harvey, A.C., Philips, G.D.A., 1980. Algorithm as 154: an algorithm for exact maximum likelihood estimation of autoregressive-moving average models by means of Kalman filtering. *Applied Statistics* 29, 3, 311–322.

Gross, M., Laliotis, D., Leika, M., Lukyantsau, P., 2020. Expected credit loss modeling from a top-down stress testing perspective. Working paper, International Monetary Fund. Available at https://www.imf.org/ (accessed February 2022).

International Accounting Standards Board, 2004. IAS 39 - Financial instruments: recognition and measurement. Available at https://www.iasplus.com/ (accessed November 2021).

International Financial Reporting Standards, 2017. IFRS Standard 9, Financial Instruments. Available at https://www.ifrs.org/ (accessed November 2021).

J. P. Morgan & Co., 1997. Creditmetrics™—technical document. Available at https://www.msci.com/ (accessed November 2021).

Jarque, C.M., Bera, A.K., 1980. Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics Letters* 6, 3, 255–259.

Jarrow, R., Lando, D., Turnbull, S., 1997. A Markov model for the term structure of credit risk spreads. *Review of Financial Studies* 10, 2, 481–523.

KPMG, 2017. Demystifying expected credit loss. Available at https://assets.kpmg/ (accessed November 2021).

Krüger, S., Rösch, D., Scheule, H., 2018. The impact of loan loss provisioning on bank capital requirements. *Journal of Financial Stability* 36, 114–129.

Laeven, L., Majnoni, G., 2003. Loan loss provisioning and economic slowdowns: too much, too late? *Journal of Financial Intermediation* 12, 2, 178–197.

Ljung, G.M., Box, G.E.P., 1978. On a measure of a lack of fit in time series models. *Biometrika* 65, 2, 297–303.

Ludwig, T., 2019. Markov Chain based PD term structure modelling in an IFRS 9 framework. Master's thesis, University of Innsbruck.

Markowitz, H.M., 1952. Portfolio selection. *The Journal of Finance* 7, 1, 77–91.
Mathworks^{TM}, 2021. MATLAB®: The language of technical computing. Available at https://www.mathworks.com/ (accessed November 2021).
McKinsey & Co., 2017. IFRS 9: A silent revolution in banks' business models. Available at https://www.mckinsey.com/ (accessed November 2021).
Michaud, R.O., Michaud, R., 2007. Estimation Error and Portfolio Optimization: a resampling solution. Working paper, New Frontier Advisors, LLC. Available at https://ssrn.com/ (accessed November 2021).
Michaud, R.O., Michaud, R.O., 2008. *Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation*. Oxford University Press, Oxford.
Moody's Investors Service, 2019. Corporates - global: Annual default study: defaults will rise modestly in 2019 amid higher volatility. Available at https://www.moodys.com/ (accessed November 2021).
R Core Team, 2017. *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. Available at https://www.r-project.org/ (accessed November 2021).
Seitz, B., Dinh, T., Rathgeber, A., 2018. Understanding loan loss reserves under IFRS 9: a simulated-based approach. *Advances in Quantitative Analysis of Finance and Accounting* 16, 311–357.
Standard & Poor's Global Ratings, 2019. Default, transitions, and recovery: 2019 Annual Global Corporate Default And Rating Transition Study. Available at https://www.spglobal.com/ (accessed November 2021).
Standard & Poor's RatingsDirect®, 2014. Standard & Poor's Ratings Definitions. Available at https://www.spratings.com/ (accessed November 2021).
Taylor, D., 2017. IFRS 9 explained - the new expected credit loss model. Available at https://www.bdo.co.uk/ (accessed November 2021).
Trück, S., 2008. Forecasting credit migration matrices with business cycle effects – a model comparison. *The European Journal of Finance* 14, 5, 359–379.
Wei, J.Z., 2003. A multi-factor, credit migration model for sovereign and corporate debts. *Journal of International Money and Finance* 22, 709–735.
Wezel, T., Chan-Lau, J.A., Columba, F., 2012. Dynamic loan loss provisioning: Simulations in effectiveness and guide to implementation. Working paper, International Monetary Fund, Available at https://www.imf.org/ (accessed November 2021).
Wilson, T., 1997. Portfolio credit risk i/ii.