Fertility choices, Demographics and Automation

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Abstract

In this paper we study a theoretical link between the effects of increased automation on labor markets, and the fertility decisions of a representative household that is replaced by robots in the production of tasks. We develop a framework in which children provide utility and impose an opportunity cost to the household due to lost labor income. We show that fertility rate changes are the result of an optimal response to wage variations after the economy is hit by a shock that increases the design quality of robots used in production. Using this model, we characterize an initial equilibrium and simulate the effect of a 10% increase in robot productivity on important endogenous variables, including wages, and find that, in the absence of fixed costs to raising children, the fertility rate increases by approximately 3.4%.

Keywords: Automation, Robots, Tasks, Fertility

JEL codes: I24, J13, J22, J24, J31, O15, O33

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Introduction

Since the beginning of the new millennium, economists have been devoting much of their effort to characterize theoretically and test empirically the effects of robotics and artificial intelligence, on employment, total factor and labor productivity, wages, inequality, as well as the labor share. Apart from some conflicting empirical results, the literature has reached a consensus that ICT and robotics are related to job and wage polarization, a reduction of the labor share while increasing aggregate labor productivity (Acemoglu & Restrepo (2018); Acemoglu & Loebbing (2022); Autor et. Al. (2003); Graetz & Michaels (2018); Feng & Graetz (2020)).

Along with the fast pace of innovations in developed countries, another important trend has consolidated: that of a decreasing fertility rate. Since the 1950's, the average the number of children per woman in the world has consistently dropped from around five to a bit more than two and, in Europe and the United States, from almost three to a little over 1.5 in 2021. Combined with an increase in the average life expectation, this trend points towards an elder society with important implications to labor market outcomes, especially when it interacts with the fast deployment of robotics and artificial intelligence in the economy. In fact, Acemoglu & Restrepo (2021) find that sectors in which the working population is ageing faster, such as Germany, South Korea and Japan, experience rapid growth of robotic automation. This happens due to a shortage of middle-aged worker in certain occupations and the resulting increased equilibrium wages that make automation an interesting option for cost minimizing firms. Focusing on the economic aspect behind family decisions to have children within a context of rapid automation of tasks in commuting regions of the United States, Anelli *et.al.* (2021) presents evidence according to which higher exposure to robots increase substantially the fertility rate of out-of-wedlock, while decreasing the fertility rate of married couples.[1] Using panel data for the Finnish economy, but considering the effects of job displacement due to plant closure or mass layoffs on the fertility rate of women, Huttunen & Kellopkumpu (2016) find that woman's own job losses decrease the fertility rate, and that male job loss has no statistical significance on the fertility rate.

One effort to bridge some gap between empirical and theoretical models in this automation-fertility relationship is Costanzo (2022). The author analyzes empirically the effects of increased robotization on women's fertility decisions using data from 7 European countries for a period of 18 years and develop an optimal stopping model, where children are seen as an irreversible investment in

[1] The authors find that a one standard deviation in robot exposure leads to an increase of 20% in the fertility rate of nonmarried couples, against a decrease of 15% for married couples, showing evidence that agents respond differently to labor-saving technology shocks. They find that the overall effect of higher exposure to robots on the fertility rate is close to zero, because on the aggregate the effects of robot exposure on the fertility behavior of married and unmarried couples counterbalance each other.

an uncertain environment with an opportunity cost associated to the household income, to provide a theoretical framework for the results. The author finds that households in low and high-skilled regional markets experience an anticipation in the decisions of having children while middle-skilled regions experience a delay in this decision. Without explicitly describing labor market dynamics in the presence of robots and other automation technologies, the author uses a stochastic cost-benefit model to find the optimal time for having children: if the current cost of having children is above a certain critical value, it is reasonable for the household to postpone this decision and anticipate otherwise. Since middle-skilled workers face lower demand and wages due to the introduction of robots, they would opt to delay having children.

Although progress has been made in terms of empirically quantifying the effects of labor-saving technologies on the supply side and on empirically assessing household reactions to such labor market shocks, less attention has been giving to the theoretical foundations of the ways in which consumers react to these technological innovations and the changes in the structure of labor demand. One particular decision households can make when facing a dynamic economic environment is related to the number of children they want to have in response to changes in labor market outcomes. This is the main difference of our model when compared to Constanzo (2022). In his model, labor market outcomes coming from robot exposure are only assumed and not modeled directly, whereas in our case all the effects coming from increased use of robots on wages and employment are endogenous responses that lead to changes in the fertility rate. We believe this approach is richer in the sense that it allows for a vast array of labor market outcomes coming from additional automation in the economy and, consequently, allows the household to have different fertility decisions.

The paper is structured in the following way: in Section 2 we present baseline model, starting in Section 2.1 with the decision of households in terms of consumption, labor-supply, and the number of children a dynamic environment, whereas in Section 2.2 we present the supply-side of the model. In Section 2.3 we solve the model for the general equilibrium. In Section 3, we quantify the effects in the equilibrium and present an exercise for comparative statics. In Section 4 we conclude.

2. Baseline model

In this section we describe succinctly the general pieces of the model. The main hypothesis of our model is that households get a certain amount of utility out of having each children, a usual feature in the literature. But together with the biological benefit, having children is followed by direct and indirect costs for the household in the form of childcare (education, health assistance, food, etc.) and opportunity costs. In our model, the total cost of having a child is a function of foregone wage rate, the amount of time devoted to raise her and a fixed cost that encompasses all the direct purchases needed to provide for her basic needs. In our model, the household that makes the decision on having children represents the set of workers that are in risk of being substituted by labor-saving technologies when firms assign tasks to factors of production. The main result is that the optimal number of children in the household will depend directly on fixed costs and the equilibrium wage for the marginal worker and that the number of desired children changes as a response to increased automation, i.e. when machines perform a larger proportion of tasks in the economy.

The supply-side is modeled as in Graetz & Feng (2020), where firms use a technology to combine tasks to produce final goods and tasks are intermediate goods produced by a production function where capital and labor are perfect substitutes. As in the assignment model literature, such as in Sattinger (1993) or in Costinot & Vogel (2010), comparative advantages among heterogenous workers with different levels of training and automation capital determine the equilibrium proportion of tasks produced by workers and machines. Then, due to properties related to task-specific productivity of different workers, there exists a matching function that allocates the labor-intensive tasks with different complexities to workers with different levels of training. The comparative advantage schedule, together with labor market clearing conditions and the matching function determine how wages will respond to increased automation and, ultimately, the optimal number of children in the household.

2.1. Consumer side

As pointed out previously, the consumer represents a set of workers performing the tasks most susceptible to automation technologies and their objective is to choose time paths for consumption, labor supply, number of children and time dedicated to raise children that maximize the present value of utility subject to a law of motion for the capital stock.

$$max_{n(t),c(t),l(t)}\, U = \int_0^\infty [\ln c(t) + \theta \ln(L - l(t) - l_c) + \zeta \ln n(t)]\, e^{-\rho t}\, dt \quad (1)$$

Subject to

$$\dot{k} = r(t)k(t) + w(s(t))l(t) - c(t) - [F + w(s(t))l_c]n(t) - \delta k(t) \quad (2)$$

In this dynamic problem expressed as a Hamiltonian, $c(t)$ is the instantaneous household consumption, L is a fixed total amount of time at the disposal of the household, while $l(t)$ is the desired time spent on productive activities and l_c is the desired time spent raising each children. Finally, $n(t)$ is the desired number of children per household which we approximate to the concept of a fertility rate and F represents an exogenous fixed cost per children that may capture e.g. the quality of education and health-care provided by the household. We treat the quality aspect of raising the children as a fixed cost to avoid complicating the model, while maintaining the consistency of traditional theoretical results in the literature, such as in Becker (1973), where households face a trade-off when deciding the optimal mix of quality and quantity when it comes to having and raising children.

First-Order Conditions for a maximum, assuming the Mangasarian conditions related to the concavity of the objective function hold, are presented below. Maximizing intertemporal utility with respect to the variables above imply equating the derivatives with respect to the control variables $n(t), c(t)\ and\ l(t)$ to zero. So we first start with consumption and the number of desired children:

$$\frac{\partial \mathcal{H}}{\partial c(t)} = \frac{1}{c(t)} e^{-\rho t} - \mu(t) = 0 \ (3)$$

$$\mu(t) = \frac{1}{c(t)} e^{-\rho t} \ (4)$$

$$\frac{\partial \mathcal{H}}{\partial n(t)} = \zeta \frac{1}{n(t)} e^{-\rho t} - \mu(t)\big[F + w\big(s(t)\big) l_c\big] = 0 \ (5)$$

Inserting (4) in (5), we find an intratemporal function relating the fertility rate $n(t)$ to the total cost per children, which is given by a function of fixed costs and the opportunity cost in terms of the fraction of wage lost while raising each child.

$$n(t) = \zeta \frac{c(t)}{\big[F + w\big(s(t)\big) l_c\big]} \ (6)$$

We can observe that expression (6) represents a demand function for children, which is inversely related to its price.

Now we go through the same process to obtain the optimal supply of labor and time spent raising each children. Again, we insert expression (4) in the FOC for these control variables as follows:

$$\frac{\partial \mathcal{H}}{\partial l(t)} = -\theta \frac{1}{L - l(t) - l_c} e^{-\rho t} + \mu(t) w\big(s(t)\big) = 0 \ (7)$$

$$l(t) = (L - l_c) - \theta \frac{c(t)}{w\big(s(t)\big)} \ (8)$$

Expression (8) shows that labor supply varies positively with wages - which highlights the substitution of leisure hours for working hours as wages increase - and negatively with consumption, highlighting the presence of income effects that disincentivize the worker to supply additional labor hours. Therefore, labor supply is going to be weakly increasing in the real wage if the substitution effect is at least equal to the income effect for all levels of the real wage.

Now we turn to the characterization of optimal consumption over time by taking the derivative of the present value Hamiltonian with respect to our state variable (capital stock):

$$\frac{\dot{\mu}(t)}{\mu(t)} = -\frac{\partial \mathcal{H}}{\partial k(t)} \ (9)$$

$$\frac{\dot{\mu}(t)}{\mu(t)} = -r(t) + \delta \ (10)$$

Using the derivation in (4), we have the Euler equation:

$$\mu(t) = \frac{1}{c(t)} e^{-\rho t} \ \ (11)$$

$$\ln \mu(t) = -\ln c(t) - \rho t \ \ (12)$$

$$\frac{\dot{\mu}(t)}{\mu(t)} = -\frac{1}{c(t)} \frac{dc}{dt} - \rho \ \ (13)$$

Defining $\frac{1}{c(t)} \frac{dc}{dt} = \frac{\dot{c}}{c}$, we have:

$$\frac{\dot{c}}{c} = r(t) - \rho - \delta \ (14)$$

2.2. Supply-side

Imagine an economy where we have two types of firms: final goods producing firms and intermediate goods or task producing firms. The final goods producing firms use a constant returns to scale (CRS) Cobb-Douglas technology to combine a continuum of tasks defined in the set T: $[\underline{\sigma}, \bar{\sigma}] \times [0,1]$ and ordered from the least to the most complex tasks in the economy.

$$\ln Y = \frac{1}{\gamma} \int_{\underline{\sigma}}^{\bar{\sigma}} \left(\sum_{\lambda} \beta_\lambda \ln y_\lambda(\sigma) \right) d\sigma \ (15)$$

The set of tasks is divided into two disjoint subsets, those where $\lambda = 0$ and those where $\lambda = 1$. The first subset of tasks is composed by those that require innate abilities to be performed, e.g. those intensive in the use of vision, fine motor skills and human interaction. While the second subset of tasks is composed by those that require education/training to be performed, e.g. those intensive in the use of logic and analytical thinking. The parameter β_λ represents the weight of either innate ability or learned task production in the process of obtaining the final good with the property that $\sum_\lambda \beta_\lambda = 1$, while the parameter $\gamma = \bar{\sigma} - \underline{\sigma}$, which guarantees the property of CRS to the production function.

Intermediate goods or task producing firms (divided by $\lambda = 0$, $\lambda = 1$ types) hire either capital or labor to produce a given $\sigma - complexity$ task using a technology that combines perfectly substitutable labor and capital:

$$\begin{cases} y_0(\sigma) = A_N \int_{\underline{s}}^{\bar{s}} [\alpha_0(s,\sigma)L_0(s,\sigma)]ds + A_K\beta(s_K,\sigma)K_0(\sigma) \; (16) \\ y_1(\sigma) = A_N \int_{\underline{s}}^{\bar{s}} [\alpha_1(s,\sigma)L_1(s,\sigma)]ds + A_K\beta(s_K,\sigma)K_1(\sigma) \; (17) \end{cases}$$

In equations (1), A_N and A_K are factor-augmenting technology parameters while $\alpha_{\lambda=0,1}(s,\sigma)$ is the productivity of $s-type$ worker in a $\sigma-complexity$ task of type λ task and $\beta(s_K,\sigma)$ is the productivity of capital in same $\sigma-complexity$ task. $L_{\lambda=0,1}(s,\sigma)$ is the amount of $s-type$ workers producing the $\sigma-complexity$ task of type λ task and $K(\sigma)$ is the amount of capital producing the $\sigma-complexity$ task.

Workers are heterogeneous and differ in terms of their ability or skills (acquired by means of formal education and training) to perform a task with $\sigma-complexity$. The distribution of training/skills in the economy is completely characterized by a Cumulative Distribution Function (CDF) $V(s)$ with support $[\underline{s},\bar{s}]$ and Probability Density Function (PDF) $v(s)$ and market clearing conditions for labor and capital are given respectively by:

$$v(s) = \sum_{\lambda} \int_{\underline{\sigma}}^{\bar{\sigma}} L_{\lambda=0,1}(s,\sigma)]d\sigma \;\; (18)$$

$$K(\sigma) = \int_{\underline{\sigma}}^{\bar{\sigma}} K_{\lambda=0,1}(\sigma)]d\sigma \; (19)$$

Let's now focus on the optimization problem for the intermediate goods or task producing firm that will generate factor demands and the assignment of tasks to factors. For tasks that can be produced with either labor or capital, the factors of production are perfect substitutes, which implies that firms solve the following linear programming (cost minimization) problem:

$$C_{\lambda=0,1}(\sigma) = \min_{L(s,\sigma),K(\sigma)} \int_{\underline{s}}^{\bar{s}} \left[w_{\lambda=0,1}(s)L_{\lambda=0,1}(s,\sigma)\right]ds + rK_{\lambda=0,1}(\sigma) \;\; (20)$$

$$s.t.\, g_{\lambda=0,1}(K,L) = A_N \int_{\underline{s}}^{\bar{s}} \left[\alpha_{\lambda=0,1}(s,\sigma)L_{\lambda=0,1}(s,\sigma)\right]ds + A_K\beta(s_K,\sigma)K_{\lambda=0,1}(\sigma) \geq y_{\lambda=0,1}$$

And the non-negativity constraints:

$$-L_{\lambda=0,1}(s,\sigma) \leq 0$$

$$-K_{\lambda=0,1}(\sigma) \leq 0$$

Setting up the Lagrangian

$$\mathcal{L} = \int_{\underline{s}}^{\bar{s}} [w_{\lambda=0,1}(s) L_{\lambda=0,1}(s,\sigma)] ds + rK_{\lambda=0,1}(\sigma) - \lambda_1 \left(A_N \int_{\underline{s}}^{\bar{s}} [\alpha_{\lambda=0,1}(s,\sigma) L_{\lambda=0,1}(s,\sigma)] ds + A_K \beta(s_K,\sigma) K_{\lambda=0,1}(\sigma) - y_{\lambda=0,1} \right)$$
$$- \lambda_2 L_{\lambda=0,1}(s,\sigma) - \lambda_3 K_{\lambda=0,1}(\sigma)$$

From which we obtain the following Kuhn-Tucker conditions:

$$\lambda_1 \left(\int_{s}^{\bar{s}} [\alpha(s,\sigma) L(s,\sigma)] ds + A_K \beta(s_K,\sigma) K(\sigma) - Y A_N \int_{\underline{s}}^{\bar{s}} [\alpha_{\lambda=0,1}(s,\sigma) L_{\lambda=0,1}(s,\sigma)] ds + A_K \beta(s_K,\sigma) K_{\lambda=0,1}(\sigma) - y_{\lambda=0,1} \right)$$
$$= 0. If\ \lambda_1 > 0, g_{\lambda=0,1}(K,L) = 0\ or\ if\ \lambda_1 = 0, g_{\lambda=0,1}(K,L) - y_{\lambda=0,1} < 0$$

$$\lambda_2 L_{\lambda=0,1}(s,\sigma) = 0. If\ \lambda_2 > 0, h\left(L_{\lambda=0,1}(s,\sigma)\right) = 0\ or\ if\ \lambda_2 = 0, -h\left(L_{\lambda=0,1}(s,\sigma)\right) < 0$$

$$\lambda_3 K_{\lambda=0,1}(\sigma) = 0. If\ \lambda_3 > 0, z\left(K_{\lambda=0,1}(\sigma)\right) = 0\ or\ if\ \lambda_3 = 0, -z\left(K_{\lambda=0,1}(\sigma)\right) < 0$$

The First Order Conditions (FOC) for a $\sigma - complexity$ task give us three possible outcomes, assuming the Production Function binds in equilibrium. First, firms only choose robotic capital in production of tasks if, for that task, $\frac{w_{\lambda=0,1}(s)}{A_N\ \alpha_{\lambda=0,1}(s,\sigma)} = \frac{r}{A_K \beta(s_K,\sigma)} + \frac{\lambda_2}{A_N\ \alpha_{\lambda=0,1}(s,\sigma)}$. That is, whenever the unit cost of labor is strictly higher than the unit cost of robotic capital. Since the second constraint binds, meaning $L = 0$, we are sure that $\lambda_2 > 0$ and that $\frac{w_{\lambda=0,1}(s)}{A_N\ \alpha_{\lambda=0,1}(s,\sigma)} > \frac{r}{A_K \beta(s_K,\sigma)}$. Second, for another given task, firms will only choose labor in production of tasks if $\frac{r}{\beta(s_K,\sigma)} > \frac{w_{\lambda=0,1}(s)}{A_N\ \alpha_{\lambda=0,1}(s,\sigma)}$, which is the case if the third constraint binds, for the same reasons above, i.e. $\lambda_3 > 0$. Third and most interesting result comes when, for a given task, none of the factor constraints bind. In that case, we have an expression that characterizes the marginal or threshold task that defines a unique point that divides the set T: $[\underline{\sigma}, \bar{\sigma}] \times [0,1]$ in two disjoint parts, given by the following no arbitrage condition:

$$\frac{w_{\lambda=0,1}(s^*)}{A_N\ \alpha_{\lambda=0,1}(s^*,\sigma^*)} = \frac{r}{A_K \beta(s_K,\sigma^*)} \quad (21)$$

We illustrate in the below diagram how tasks (ordered by complexity) are allocated to factors according to the intermediate task producing firms' optimal choice algorithm.

Figure 1. Equilibrium assignment of tasks to factors when $\lambda = 0$ ($innate\ ability\ tasks$)

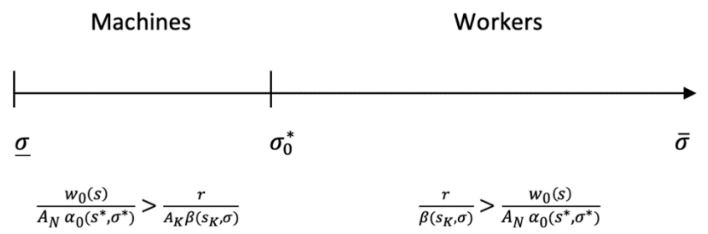
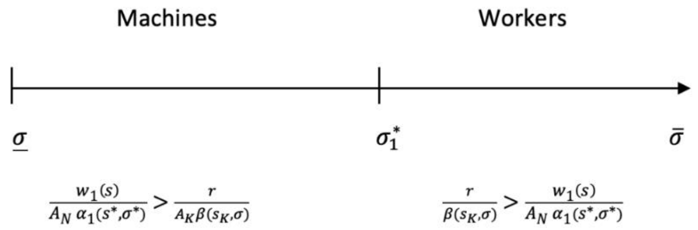


Figure 2. Equilibrium assignment of tasks to factors when $\lambda = 1$ ($learned\ tasks$)



Given this result, let's take a step back now. The term $A_N\alpha(s,\sigma)$ represents a productivity parameter for the worker, where (again) A_N is some labor-augmenting technological parameter. Like Graetz & Feng (2020), we define the task-specific productivity parameter as $\alpha_\lambda = \left(1 - \lambda\frac{\sigma}{s}\right)$, which is a strict log-super modular function[2] that adjusts how much the increase in one factor will translate into increased production of the task. For the innate ability tasks, where $\lambda = 0$, this function is constant and equal to 1, meaning that to perform innate ability tasks worker don't incur in any type of time related cost to acquire education/training. So every additional labor input in the production of these types of tasks is fully converted in task production.

As we explained before, σ is the complexity level of a particular task and s is a measure of schooling or training of the worker. So it means that as s increases for a given task complexity, the cost in terms of "wasted" production decreases and additional labor is able to generate more additional output. That happens because, for a given task, the higher the skills of the worker, the less time is needed of training to perform it. In the other way, the more complex is a task for a given level of training/education the less an additional unit of labor can generate output. Therefore, the ratio $\frac{\sigma}{s}$ is an opportunity cost related to the balance between task complexity and skills, where the higher it is, the more difficult is for labor to generate additional output.

Much in the same way A_K is a capital-augmenting technological parameter whereas $\beta = \left(1 - \frac{\sigma}{s_K}\right)$. Like schooling for workers, the parameter s_K is fixed for all tasks and embody the state of robotic innovations.

Due to the strict log-supermodularity of task specific productivities, we assume the existence of a continuous and strictly increasing mapping function $M: [\underline{s}, \bar{s}] \to [\underline{\sigma}, \bar{\sigma}]$ that maps training/educational requirements with support $[\underline{s}, \bar{s}]$ to task complexity with support $[\underline{\sigma}, \bar{\sigma}]$. That means that more skilled

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[2] Which means that higher skilled workers have a strict comparative advantage to lower skilled workers in more complex tasks: $\frac{\alpha(s\prime,\sigma\prime)}{\alpha(s\prime,\sigma)} > \frac{\alpha(s,\sigma\prime)}{\alpha(s,\sigma)}$ for any $\sigma' > \sigma$ and $s' > s$.

workers are mapped to more complex tasks and defining the matching function allows us to write the previous equilibrium condition for the marginal task as:

$$\frac{w(s^*)}{A_N \alpha(s^*, M(s^*))} = \frac{r}{A_K \beta(s_K, M(s^*))} \quad (22)$$

Now that we have described the equilibrium allocation of tasks to factors of productions, we can try to visualize what happens to this equilibrium if there is an improvement in the design or quality of the robotic capital reflected by the parameter s_K. In the diagram below, we have a plot with marginal costs for the marginal worker and machines in the y-axis and the complexity of task in the x-axis.

In this diagram, we have the first task allocation equilibrium defined at σ_1^* where the marginal cost of employing robotic capital or $s^* - type$ workers is the same. If we have $ds_k > 0$, i.e. an increase in the quality of robotic capital that will allow it to be more productive in all tasks, independently of complexity, we see that the whole marginal cost schedule for machines shifts downwards, which pushes the equilibrium allocation of tasks to the right for a given $s - type$ worker. After this technological improvement in robotic capital, more complex tasks are now performed by machines in equilibrium which causes job displacement in tasks performed by workers similar to the $s^* - type$.

This diagram is drawn for a particular $s - type$ worker, but as we described earlier, there is a matching function that is responsible for allocating workers in the set $[s^*, \overline{s}\,]$ to tasks in the set $[\sigma_1^*, \overline{\sigma}\,]$. So whenever there is change in the equilibrium allocation of tasks, there will also be a changes in the $s - type$ workers associated to the new equilibrium allocation.

The diagram below then shows the result of the comparative static exercise. So the improvements in robotic capital shifts the matching function upwards, where the new marginal task is now associated with higher $s - type$ workers then before.

From this expression, we can observe that the equilibrium training/schooling s^* increases with advancements in robotic innovation. That happens because as the unit cost of robots reduce due to the increase in productivity, firms assign more (complex) tasks to robots, increasing the educational requirements of workers in the threshold tasks.

So for this setting, automation requires more training and schooling for the workers that remain in the job market. For us to know the effects of this change on the wage schedule across all $s - type$ workers and knowing that market-clearing conditions hold in the factor markets, we are going to assume, for now, that the matching function satisfies the following condition:

$$\frac{dM(s)}{ds} = \frac{\mu}{\beta_1} \frac{w(s)v(s)}{Y} \quad (23)$$

And the wage schedule is given by the following condition:

$$\frac{d \ln w(s)}{ds} = \frac{\partial \ln \alpha(s, M(s))}{\partial s} \quad (24)$$

This system of differential equations together with the boundary conditions $M(\underline{s}) = \underline{\sigma}$ and $M(\overline{s}) = \overline{\sigma}$ provide a unique solution for the matching function and the wage schedule across all $s-type$ workers.

In our case, we are interested in determining the wage schedule for the marginal task worker as a function of the automation threshold. For that matter, it is important to define a no-arbitrage condition for the marginal worker between innate ability and learned tasks:

$$w_0(s^*) = w_1(s^*) \quad (25)$$

This condition tells us that the marginal work should be indifferent, in equilibrium, between performing innate ability or learned tasks. If there were any kind of imbalance in this expression, the assignment of tasks to factors would not be in equilibrium. As a result, if we want to understand how the wage of the marginal task worker changes with automation, we simply need to understand how the wages of innate ability tasks relate to the range of tasks performed by labor in that subset. We start from the market clearing condition in innate ability tasks and integrate with respect s for the range $[\underline{s}, s^*]$:

$$V(s^*) = \int_{\sigma_0^*}^{\overline{\sigma}} \int_{\underline{s}}^{s^*} L_0(s, \sigma) ds d\sigma \quad (26)$$

Since we normalize the labor force, this equation gives us the cumulative supply of workers with education/training $s \leq s^*$ in innate tasks that were not automated, ranging between $[\sigma_0^*, \overline{\sigma}]$.

In this range of tasks, the production function for some task $\sigma \in [\sigma_0^*, \overline{\sigma}]$ takes the form:

$$Y(\sigma) = A_N \int_{\underline{s}}^{s^*} [L(s, \sigma)] ds \quad (27)$$

Because we are talking about innate ability tasks, there is no education/training costs associated to production and, therefore, $\alpha(s, \sigma) = 1 \; for \; all \; s \in [\underline{s}, s^*]$.

If we integrate the production function with respect to task complexity over the range of innate ability tasks performed by workers, we have:

$$\int_{\sigma_0^*}^{\overline{\sigma}} y(\sigma) d\sigma = A_N \int_{\sigma_0^*}^{\overline{\sigma}} \int_{\underline{s}}^{s^*} [L(s, \sigma)] ds d\sigma \quad (28)$$

Integrating the LHS and using the former expression to substitute for the integral in the RHS, we have:

$$(\bar{\sigma} - \sigma_0^*)y_0 = A_N V(s^*) \rightarrow y_0 = \frac{A_N V(s^*)}{(\bar{\sigma} - \sigma_0^*)} \quad (29)$$

Using the optimality condition in the final goods production (Equation A.1 in the appendix) as well as the optimality condition of innate ability range of task producers, we have:

$$P_0(\sigma) = \frac{\beta_0 Y}{\mu y_0(\sigma)} \quad (30)$$

Substituting (29) in (30), we have:

$$P_0(\sigma) = \frac{\beta_0 Y(\bar{\sigma} - \sigma_0^*)}{\mu A_N V(s^*)} \quad (31)$$

Above is the demand for innate ability tasks and below the demand for labor in innate ability tasks:

$$P_0(\sigma) = \frac{w(s)}{A_N} \quad (32)$$

Now, to obtain the wage of the marginal worker as a function of the range of non-automated tasks, we plug (31) in (32).

$$\frac{\beta_0 Y(\bar{\sigma} - \sigma_0^*)}{\mu A_N V(s^*)} = \frac{w(s^*)}{A_N} \quad (33)$$

$$w(s^*) = \frac{\beta_0 Y(\bar{\sigma} - \sigma_0^*)}{\mu V(s^*)} \quad (34)$$

In this framework, σ_0^* represents the threshold task in the range of innate ability tasks. It shows up in this expression because there is also a no-arbitrage condition for workers in the threshold for learned tasks, i.e. workers will be indifferent, at the margin, to perform innate ability tasks in the range $[\sigma_0^*, \bar{\sigma}]$ or the threshold learned tasks σ_1^*. So this no-arbitrage condition is an important connection used to understand the reaction of wages of displaced workers to advancements in automation.

Graetz & Feng (2020) show that after the new equilibrium allocation occurs, the workers that get displaced by the advances in robotic capital, i.e. workers in the range $[s^*, \hat{s}^*]$ experience a decline in wages as shown in the diagram below. Those that workers at the top of the skill distribution, that are to the far right of $\hat{s}^*$ are those that experience an increase in wages in this context. In this way, we can show a link between the advancement of automation and the fertility rate by means of an increase in the required level of education for workers to have a job. We already developed the dynamic optimization problem and proved that the fertility rate growth is negatively related to the growth rate in the cost of having children (of which education is a large part).

So in this way, we developed an approach that shows that the current decline in the fertility rate might be (in part) determined by the rapid increase in the development and deployment of automation in the economy.

2.3. General Equilibrium

Focusing on condition number (2), which is related to the optimal decision of the consumer in terms of the fertility rate, the marginal worker takes expression (15) to substitute for wages. Therefore, we have:

$$n(t) = \zeta \frac{c(t)}{[F + w(s(t))l_c]} \quad (35)$$

$$n(t) = \zeta \frac{c(t)V(s^*)\gamma}{[F + \beta_0(\bar{\sigma} - \sigma_0^*)Yl_c]} \quad (36)$$

Proposition 1. *(Ceteris Paribus)* Advances in automation , represented by a decrease in $(\bar{\sigma} - \sigma_0^*)$, have a positive effect on the fertility rate of the marginal worker as the opportunity cost of having children declines with smaller wages.

Proof.

$$\frac{\partial n(t)}{\partial \sigma_0^*} = \zeta \frac{c(t)V(s^*)\gamma}{[F+\beta_0(\bar{\sigma}-\sigma_0^*)Yl_c]^2}\beta_0 Y l_c(t) \ > 0 \ \ (37) \ \blacksquare$$

Proposition 2. *(Ceteris Paribus)* The creation of new tasks in the economy, that would represent an increase in $(\bar{\sigma} - \sigma_0^*)$, increases the opportunity cost of having children and reduces the fertility rate for the marginal worker.

Proof.

$$\frac{\partial n(t)}{\partial \bar{\sigma}} = -\zeta \frac{c(t)V(s^*)\gamma}{[F+\beta_0(\bar{\sigma}-\sigma_0^*)Yl_c]^2} \ \beta_0 Y l_c < 0. (38) \ \blacksquare$$

Proposition 3. *(Ceteris Paribus)* Advances in automation, represented by a decrease in $(\bar{\sigma} - \sigma_0^*)$, reduce the marginal worker's supply of hours of work.

Proof.

$$l(t) = (L - l_c) - \theta \frac{c(t)}{w(s(t))} \ (39);$$

$$l(t) = (L - l_c) - \theta \frac{c(t)V(s^*)\gamma}{\beta_0(\bar{\sigma} - \sigma_0^*)Y} \ (40)$$

$$\frac{\partial l(t)}{\partial \sigma_0^*} = -\theta \frac{c(t)V(s^*)\gamma}{\beta_0 Y}(\bar{\sigma} - \sigma_0^*)^{-2} < 0 \quad (41) \blacksquare$$

Given a particular equilibrium allocation of factors to tasks, an increase in the design sophistication of machines allows them to be more productive in tasks with a given complexity, which symbolically can be represented as $\hat{s}_K > s_K$ for a given $\sigma = \sigma^*$.

That increase in machine productivity (reduction in marginal cost of machines in a given task) creates an imbalance in the previous equilibrium condition, which allows machines to execute the previous marginal task σ^* with a lower unit cost (or in other words: have a strict comparative advantage to workers in the task σ^*) and therefore push the economy to a new equilibrium where the marginal task complexity σ^* moves to a higher complexity σ^{**}.

Due to the existence of an increasing matching function $M: [\underline{s}, \bar{s}] \to [\underline{\sigma}, \bar{\sigma}]$ that maps training/educational requirements to task complexity, the new equilibrium task σ^{**} is now related to a higher training/educational requirement $s^{**} > s^*$.

This equilibrium movement between $(s^*, M(s^*))$ to $(s^{**}, M(s^{**}))$ will have effects over wages in general and in particular for the marginal worker, given by:

$$\frac{d \log w(s)}{ds} = \frac{\partial \log \alpha(s, M(s))}{\partial s} \quad (42)$$

The assumptions of the matching function create a condition in which the wage of the marginal worker falls as a response to the increase in the threshold level of training/educational requirements. Either because this worker lost his job and started doing innate ability tasks or because he acquired more training/education to perform the new and more complex threshold tasks for which they have a comparative disadvantage. This result is consistent with task-based model predictions that the demand for labor, wages, and the labor share (which is a function of the proportion of non-automated tasks) decline as automation increases (see appendix for a formal treatment of the labor share).

From the consumer side, we know that the optimal decision regarding the number of children is inversely related to hourly wages, since it also represents an important opportunity cost for the worker that must spend time with their children. The marginal worker's wage is directly affected by the proportion of automated tasks in the economy. The proportion of automated tasks in the innate ability subset determines the lower bound for wages in the market and due to existence of a no-arbitrage condition for the marginal worker, this is also the equilibrium wage for the marginal worker at the 'learned tasks' subset.

From this relationship, we can infer that as automation advances, it reduces the demand and the wage for labor in the marginal task and, therefore, the opportunity cost of having children for a given fixed cost. The creation of new tasks is the opposite: increases wages and negatively influences the fertility rate for a given fixed cost.

3. Quantifying Effects

To solve the model, we must start by assuming an initial equilibrium condition for the threshold level of skills and task complexity in learned tasks (s^*, σ_1^*). This vector represents the level of skills and task complexity that divide the task space into automated and non-automated tasks according to each factors cost advantages to the producer. Assuming this equilibrium initial condition, as we will see, provides an important part of the initial condition necessary to solve the system of Ordinary Differential Equations (ODE) for the matching function and the wage distribution.

Assuming a feasible equilibrium vector in the skill-task space $(s^*, \sigma_1^*) = \left(\frac{3}{2}, \frac{1}{2}\right)$ and the automation capital's productivity parameter $s_k = 1$, it is possible for us to solve for the equilibrium innate ability task threshold using equation (43):

$$\beta(s_k, \sigma_1^*) = \beta(s_k, \sigma_0^*)\alpha(s^*, \sigma_1^*)(43)$$

$$(1 - \sigma_0^*) = \frac{\left(1 - \frac{1}{2}\right)}{\left(1 - \frac{1/2}{3/2}\right)}$$

$$\sigma_0^* = \frac{1}{4}$$

This result for the threshold task in innate ability task, lower than in learned tasks, is consistent with automation capital comparative advantages in tasks that demand higher analytical skills. Using the threshold task in innate ability tasks, we can solve for the threshold wage-product ratio. For this exercise, we assume that the distribution of skills follow a uniform distribution, in which case $V(s) = s$, the share of innate ability tasks is $\beta_0 = \frac{1}{3}$, consistent with Graetz & Feng (2020), and the task range is of unit length $\gamma = 1$.

$$\frac{w(s^*)}{Y} = \frac{\beta_0}{\gamma} \frac{(\bar{\sigma} - \sigma_0^*)}{V(s^*)} \quad (44)$$

$$\frac{w(s^*)}{Y} = \frac{\frac{1}{3}\left(1 - \frac{1}{4}\right)}{\frac{3}{2}}$$

$$\frac{w(s^*)}{Y} = \frac{1}{6}$$

This result provides the second necessary equilibrium initial condition for us to solve numerically the system of ODEs both for the matching function and the wage distribution. In order to solve the system of ODEs, we need the equilibrium initial condition established in the beginning $(s^*, \sigma_1^*) = \left(\frac{3}{2}, \frac{1}{2}\right)$ and the $\left(s^*, \frac{w(s^*)}{Y}\right) = \left(\frac{3}{2}, \frac{1}{6}\right)$. Because the matching function reveals the allocation process of skills to tasks starting from the threshold task complexity, we have that $M(s^*) = \sigma_1^*$.

When we substitute for the task-specific productivity function, the system becomes:

$$\frac{dM(s)}{ds} = \frac{\gamma}{\beta_1}\frac{w(s)v(s)}{Y} \quad (45)$$

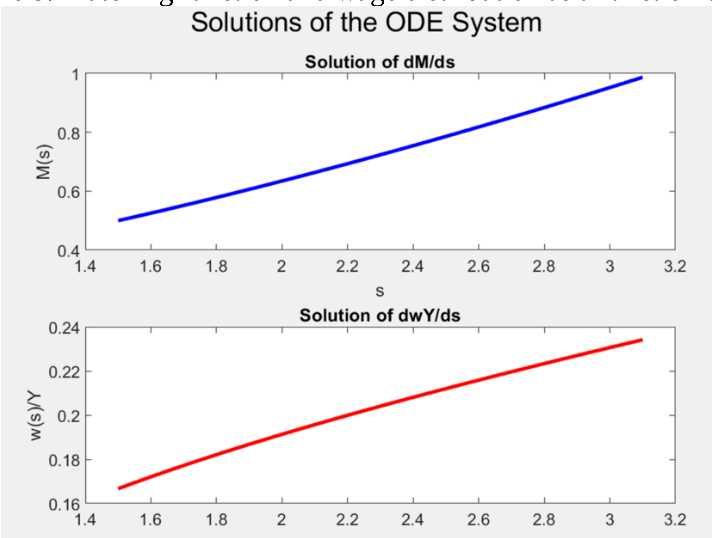
$$\frac{dw(s)}{ds} = w(s)\frac{1}{\left(1 - \frac{M(s)}{s}\right)}\frac{M(s)}{s^2} \quad (46)$$

This system does not have an analytical solution and therefore needs an approximate solution by means of numerical methods. Below we provide graphical representations for the numerical solution to the system of differential equations, along with the boundary conditions, that characterize how the matching and wage function change with worker skill, as well as assumptions about the main parameters of the model:

Table 1. Initial conditions and parameters

Initial Conditions		Endogenous Variables		Parameters				
s^*	$M(s^*)$	$\frac{w(s^*)}{Y}$	s_k	β_1	$v(s)$	γ	A_K	A_N
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	1	$\frac{2}{3}$	1	1	1	1

Figure 3. Matching function and wage distribution as a function of worker skill



Source: Authors. Graphs obtained using the solutions to the system of ODEs in Matlab.

The solutions for the matching function and the wage distribution are exactly as expected, which is that high skill workers are effectively assigned to high complexity tasks, respecting the log-supermodularity of the task productivity schedule or the structure of comparative advantages embedded in the model. At the same time, the solution to the system of ODEs also predicts that workers with higher skills are payed accordingly better than their lower skill counterparts, with the worker with threshold skill s^* receiving the lowest pay among workers assigned to learned tasks.

Using the solutions obtained previously for $\frac{w(s^*)}{Y}$, together with the equilibrium boundary conditions $(s^*, M(s^*))$, we can also use the no-arbitrage condition in factor markets to solve for the interest rate to GDP ratio, as follows:

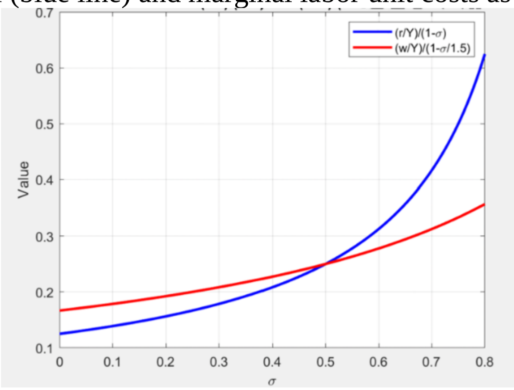
$$\frac{\frac{r}{Y}}{\beta(s_k, M(s^*))} = \frac{\frac{w(s^*)}{Y}}{\alpha(s^*, M(s^*))} \quad (47)$$

$$\frac{\frac{r}{Y}}{\left(1-\frac{1}{2}\right)} = \frac{\frac{1}{6}}{\left(1-\frac{\frac{1}{2}}{\frac{3}{2}}\right)}$$

$$\frac{r}{Y} = \frac{1}{8}$$

To close the internal consistency of the model's workings, we plot the unit costs of automation capital and the threshold worker as a function of task complexity, using the endogenous equilibrium values obtained, and observe that the equilibrium threshold task matches the initial equilibrium condition, which means that the unit cost of automation capital is below that of labor for tasks $\Psi = \{\sigma \epsilon \mathbb{R} \mid 0 < \sigma \leq 0.5\}$.

Figure 4. Automation capital (blue line) and marginal labor unit costs as functions of task complexity



Sources: Authors. Graphs obtained using the solutions to the system of ODEs in Matlab.

In the appendix, we provide a full description of solutions for all the relevant endogenous variables of the model.

3.1. Comparative statics

Now we proceed to understand how a positive exogenous shock in robot's task productivity (or analogously a negative shock in the design cost of robots given by its inverse $\frac{1}{s_k}$) affect the amount of tasks allocated to robots and, consequently, the wage variation experienced by the new threshold workers.

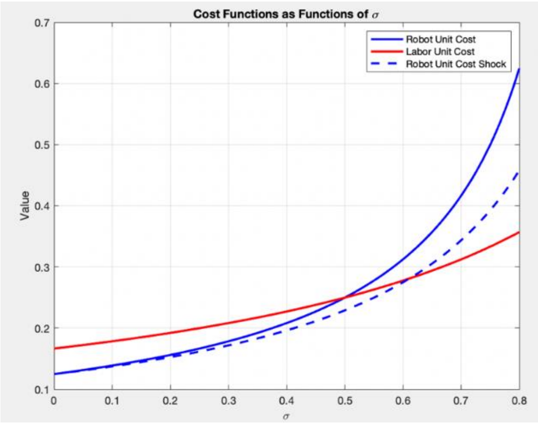
Assuming a 10% increase in robot's productivity, from our baseline of 1 to 1.1, that may arise e.g. due to innovations that come from the expansion of research on automation tools, our simulation suggests that the level of learned tasks automated in the economy increases as much as 22%, from 0.5 to 0.61 as illustrated by figure 3. We use equation (47) and update for the new level of robot productivity (s_k) as follows:

$$\frac{\frac{r}{Y}}{\beta(s_k, M(s^*))} = \frac{\frac{w(s^*)}{Y}}{\alpha(s^*, M(s^*))} \quad (47)$$

$$\frac{\frac{1}{8}}{(1 - M'(s^*))} = \frac{\frac{1}{6}}{\left(1 - \frac{M'(s^*)}{\frac{3}{2}}\right)}$$

$$M'^{(s^*)} = 0.611$$

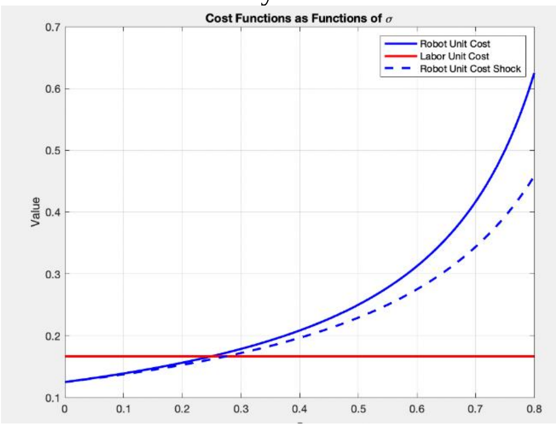
Figure 5. The effect of an increase of 10% of robot's task productivity parameter of the number of learned tasks automated



Source: Authors using Matlab.

The same pattern is followed in the subset of innate ability tasks, where an increase in robot's task-specific productivity also increases the number of tasks performed by automation machinery, as can be seen in figure 4. But, as the patterns of robotic comparative advantage suggest, this increase is less intense than in the case of learned tasks. Robots have a comparative advantage on the production of learned tasks and the so the increase in the automated innate ability tasks generated by a 10% increase in productivity is of 10%, from 0.25 to 0.275.

Figure 6. The effect of an increase of 10% of robot's task productivity parameter of the number of innate ability tasks automated



Source: Authors using Matlab.

Given the increase in the level of automated tasks in both subsets of tasks, i.e. innate ability and learned tasks, we are able to calculate the effects on the wage to GDP ratio using equation (44) – along with the parameters already used for the share of innate ability tasks in the economy (β_0), the distribution of skills $(V(s))$and the total range of tasks (γ) in the last exercise.

$$\frac{w(s^*)}{Y} = \frac{\beta_0}{\gamma}\frac{(\bar{\sigma} - \sigma_0^*)}{V(s^*)} \quad (44)$$

$$\frac{w'(s^*)}{Y} = \frac{\frac{1}{3}(1 - 0.275)}{1.5} = 0.161$$

Therefore, the effect of additional automation on the threshold wage to GDP ratio is about -3.4%, which means that lower labor demand in automated tasks, caused by direct substitution of capital for labor, is linked to a relative wage loss for those workers displaced from production.

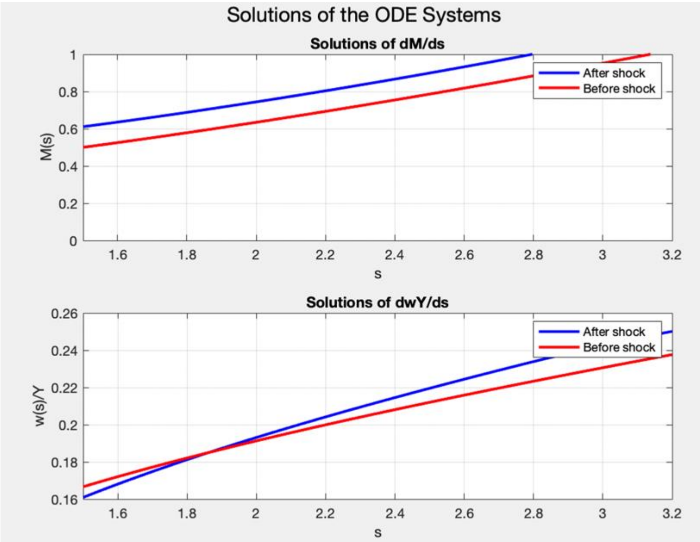
Solutions to equations (47) and (48) provide new initial conditions for the system of differential equations that characterize the behavior of the matching function and wage distribution. Now we can update Table 1 for the new values of s'_k, $M'(s^*)$ and $\frac{w\prime(s^*)}{Y}$.

Table 2. New Initial Conditions and Endogenous Variables

s^*	$M'(s^*)$	$\frac{w'(s^*)}{Y}$	s'_k
$\frac{3}{2}$	0.611	0.161	1.1

Using again the system of differential equations given by equations (45) and (46), we obtain the matching function and wage distribution after the productivity shock has hit the economy. The numerical solutions obtained are shown in figure 5.

Figure 7. Matching function and wage distribution as a function of worker skills



Source: Authors. Graphs obtained using the solutions to the system of ODEs in Matlab.

The graph compares both the matching function and the wage distribution before and after the increase in robot's productivity parameter. The simulation is consistent with the theoretical results presented in Graetz & Feng (2020) in which the matching function moves upward for every level of skill. As in Costinot & Vogel (2010) as well as Graetz & Feng (2020), this shock leads to skill downgrading in the perspective of the firm and task upgrading from the perspective of the worker.

In the case of the wage to GDP ratio, the shock leads to a decrease in the relative wage of displaced workers, from skill levels between 1.5 and 1.9, and an increase for workers with skills higher than that. Again the result is consistent with the literature, where automation shocks display patterns of wage polarization.

3.2. Effects on fertility

Given the framework that we have constructed so far, the important element connecting the supply side to consumer's decisions is the wage to GDP ratio schedule. Because we are analyzing decisions made by workers that are displaced from the production process by means of increased automation, we can use equation (6) to deduce the effects of additional robotization on optimal fertility decisions.

$$n(t) = \zeta \frac{c(t)}{\left[F + w\big(s(t)\big)l_c\right]} \quad (6)$$

If we divide both numerator and denominator by output in period t, we obtain the following equation, which connects better the variables from supply and demand sides:

$$n(t) = \zeta \frac{\frac{c(t)}{Y(t)}}{\frac{\left[F + w\big(s(t)\big)l_c\right]}{Y(t)}}$$

Taking the natural log of both sides, we have:

$$\ln n(t) = \ln \zeta + \ln\left(\frac{c(t)}{Y(t)}\right) - \ln\left(\frac{\left[F + w\big(s(t)\big)l_c\right]}{Y(t)}\right)$$

Taking the derivative with respect to time to obtain growth rates:

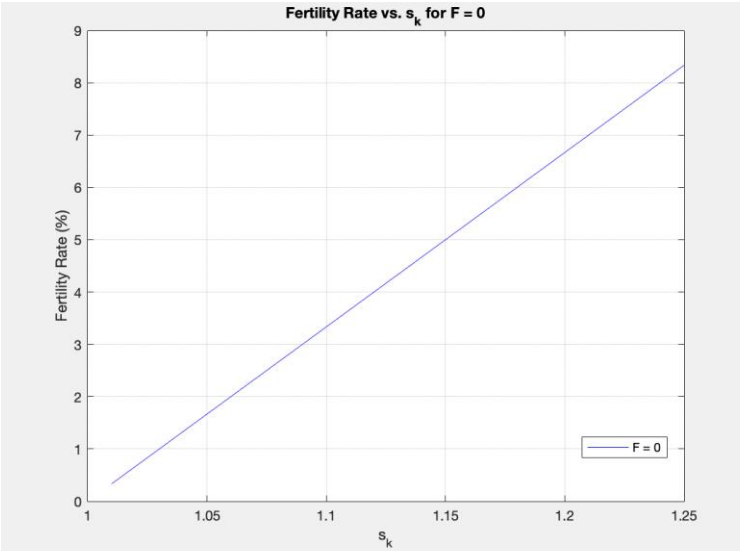
$$\frac{\mathrm{d}\ln n(t)}{dt} = \dot{\zeta} + \left(\dot{\frac{c(t)}{Y(t)}}\right) - \frac{1}{\frac{\left[F + w\big(s(t)\big)l_c\right]}{Y(t)}}\left(\frac{d\left(\frac{F}{Y(t)}\right)}{dt} + \frac{d\left(\frac{w(s(t))}{Y(t)}\right)}{dt}l_c\right)$$

If we assume for now that the fixed costs of having children are zero and that we have stable utility parameter ζ associated to having children and stable ratio between consumption and production (which is reasonable in an economy that is on a steady state path), we have that:

$$\frac{\mathrm{d}\ln n(t)}{dt} = -\frac{1}{\frac{\left[w\big(s(t)\big)l_c\right]}{Y(t)}}\left(\frac{d\left(\frac{w(s(t))}{Y(t)}\right)}{dt}l_c\right) = -\left(\dot{\frac{w\big(s(t)\big)}{Y(t)}}\right) \quad (48)$$

Substituting the variation of the wage to GDP ratio obtained from the simulation, we arrive at a growth rate of 3.4% for the fertility rate of the marginal worker following a 10% productivity shock that automates a larger fraction of tasks in the economy. The increase in the fertility rate in the absence of fixed costs follows completely from the assumption that raising children is understood as an opportunity cost. So the logical conclusion is that, if wages decrease for the marginal worker, the opportunity cost of raising a child decreases and the optimizing consumer opts to increase the number of children in the household. Figure 8 shows the behavior of the growth rate of fertility in the marginal household when faced with different magnitudes for the productivity shock for robots (ranging from 1% to 25%) in the absence of fixed costs.

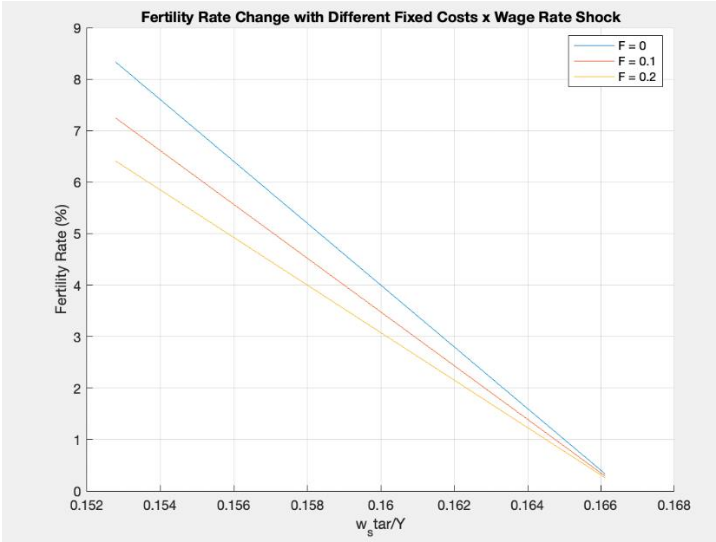
Figure 8. Fertility Rate Change for different



Source: Authors using Matlab.

It is important to note however that the assumption about the existence of a fixed cost associated to having children does entail different results for the consumer's decision. In fact, if we assume that fixed costs vary at least at the same rate as GDP, the fertility rate will increase by less than the fall in the wage to GDP ratio – and the increase will be lower the larger the fixed cost level faced by the household, no matter the magnitude of the robot productivity shock - as figure 9 suggests.

Figure 9. Fertility rate change after a productivity shock to robots productivity with different hypothesis for the fixed costs as a proportion of GDP



Source: Author using Matlab.

For this exercise, we considered constant fixed costs throughout the comparative statics that range between zero and 20% of GDP which is a reasonable range, considering that fixed costs per children could represent general expenditures in education, health, and housing. The graph shows how the fertility rate changes according to how wages change for a given fixed cost following a technological shock. As we showed, the larger the technological shock, the more wages will fall for the marginal worker. The slope is now reversed because now we are in the fertility rate-wage space where larger technological shocks are associated with lower levels of the wage rate and, therefore, to higher fertility growth.

5. Conclusions

In this paper we model the fertility decisions of a representative worker that is displaced by an automation technology following an increase in capacity of robots to perform tasks. We use a task-based framework developed by Graetz & Feng (2020) and Costinot & Vogel (2010) to model how tasks are assigned to robots and workers and how factor incomes are determined in factor markets. Upon an initial characterization for the equilibrium, we simulate a shock in this economy by means a 10% increase in the productivity of robots or a 10% reduction in the cost of robot design to analyze the effects over all relevant endogenous variables, with a particular interest in the behavior of the wage distribution and the wage of the marginal or displaced worker. Our simulation suggests that, in the absence of fixed costs related to having children, a 10% increase in the productivity of robots results in a 3.4% increase in the fertility rate of the representative household. This positive effect is due to the assumption that raising children imposes an opportunity cost on the household by preventing the worker from working the hours it needs to take care of their offspring. In this way, by facing a reduction in their wage, the opportunity cost of having children decrease and pushes utility optimizing agents to increase the number of children. It is also important to notice that this is a limiting situation in which we only consider the behavior of the wage function to determine the sign and magnitude of the variation in fertility. In our model, there is another element that may define the sign and magnitude of the variation in fertility, which is the fixed cost of having children. Depending on the level and the possible pattern of variation in time of this cost, the positive effect of the reduction of wages on the fertility rate can attenuated and even reverse completely this effect.

This paper open prospects of future work also. First, several extensions are planned to improve the quantitative exercise. Second and most important, automation can be endogenized through an improving-quality innovation process. This should highlight new results because the population ageing (low fertility) can have a detrimental effect on innovation, creating a bidirectional link between fertility and automation.

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