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Optimal Mapper for OFDM With Index Modulation: A Spectro-Computational Analysis

SAULO QUEIROZ^{1,2}, JOÃO P. VILELA³, AND EDMUNDO MONTEIRO^{1,2}, (Senior Member, IEEE)

¹Academic Department of Informatics, Federal University of Technology (UTFPR), Ponta Grossa 84017-220, Brazil

²CISUC and Department of Informatics Engineering, University of Coimbra, 3030-290 Coimbra, Portugal

³Department of Computer Science, University of Porto, 4169-007 Porto, Portugal

Corresponding author: Saulo Queiroz (sauloqueiroz@utfpr.edu.br)

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ABSTRACT In this work, we present an optimal mapper for OFDM with index modulation (OFDM-IM). By optimal we mean the mapper achieves the lowest possible asymptotic computational complexity (CC) when the spectral efficiency (SE) gain over OFDM maximizes. We propose the spectro-computational (SC) analysis to capture the trade-off between CC and SE and to demonstrate that an N -subcarrier OFDM-IM mapper must run in exact $\Theta(N)$ time complexity. We show that an OFDM-IM mapper running faster than such complexity cannot reach the maximal SE whereas one running slower nullifies the mapping throughput for arbitrarily large N . We demonstrate our theoretical findings by implementing an open-source library that supports all DSP steps to map/demap an N -subcarrier complex frequency-domain OFDM-IM symbol. Our implementation supports different index selector algorithms and is the first to enable the SE maximization while preserving the same time and space asymptotic complexities of the classic OFDM mapper.

INDEX TERMS Computational complexity, index modulation, OFDM, signal mapping, software-defined radio, spectral efficiency.

NOTATION

c_i : Index of the i -th active subcarrier in the symbol
 g : Number of subblocks per symbol
 k : Number of active subcarriers
 m : Total number of bits per symbol
 $m(N)$: Asymptotic number of bits per symbol as function of N
 n : Number of subcarriers per subblock
 p : Total number of bits per subblock
 p_1 : Number of index modulation bits per subblock
 p_2 : Number of bits per active subcarriers in a subblock
 δ : Half-width of the confidence interval
 x : Number of samples of the steady-state mean
 s : List of baseband samples per symbol

s_β : List of baseband samples in the β -th subblock
 $A_{N,k}$: $N \times k$ Johnson association scheme
 I : List of active subcarrier indexes per symbol
 I_β : List of active subcarrier indexes in the β -th subblock
 N : Number of subcarriers per symbol
 M : Constellation size of the modulation diagram
 P_1 : Number of index modulation bits per symbol
 P_2 : Number of bits per active subcarriers in a symbol
 X : Decimal representation of the P_1 -bit mapper input
 $T(N)$: (De)Mapper computational complexity as function of N
 $m(N)/T(N)$: (De)Mapper spectro-computational throughput

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- $\binom{N}{k}$: $N!/(k!(N-k)!)$
- κ : Wall-clock runtime of a computational instruction
- $o(f)$: Order of growth asymptotically smaller than f
- $\omega(f)$: Order of growth asymptotically larger than f
- $O(f)$: Order of growth asymptotically equal or smaller than f
- $\Omega(f)$: Order of growth asymptotically equal or larger than f
- $\Theta(f)$: Order of growth asymptotically equal to f
- Z^T : Transpose of the matrix Z

I. INTRODUCTION

Index Modulation (IM) is a physical layer technique that can improve the spectral efficiency (SE) of OFDM. IM’s basic idea for OFDM [1], [2] consists in activating $k \in [1, N]$ out of N subcarriers of the symbol to enable extra $\binom{N}{k} = N!/(k!(N-k)!)$ waveforms. Of these, OFDM-IM employs $2^{\lfloor \log_2 C(N,k) \rfloor}$ to map $P_1 = \lfloor \log_2 \binom{N}{k} \rfloor$ bits. Besides, modulating the k active subcarriers with an M -ary constellation, the OFDM-IM symbol can transmit more $P_2 = \log_2 M$ bits along with P_1 . Thus, the OFDM-IM mapper takes a total of $m = P_1 + P_2$ bits as input and gives k complex baseband samples as output for the modulation of the k subcarriers. In this process, the index selector (IxS) determines the k -size list of indexes – out of 2^{P_1} possibles – from the P_1 -bit input. The remainder $N-k$ subcarriers are nullified. The other DSP steps follow as usual in OFDM, except for the signal detector at the receiver. In this sense, several research efforts have been done to improve the receiver’s bit error rate at low computational complexity [3]–[7]. Since our focus is on the OFDM-IM mapper, we refer the reader to the survey works [8]–[11] for other aspects of the index modulation technique.

A. PROBLEM

In this work, we concern about whether the OFDM-IM mapper can reach the maximal SE gain over its OFDM counterpart keeping the same computational complexity (CC) asymptotic constraints. The SE maximization of OFDM-IM over OFDM happens when the IM technique is applied on all N subcarriers of the symbol with $k = N/2$ and the active subcarriers are BPSK-modulated, i.e., $M = 2$ [12], [13]. We refer to this setup as the optimal OFDM-IM configuration.

The computational complexity of the OFDM-IM mapper under the optimal SE configuration has been conjectured as an “impossible task” [9], [14]. This belief comes from the fact that the number of mappable OFDM-IM waveforms grows as fast as $O(\binom{N}{k})$, which becomes exponential if the optimal SE configuration is allowed. Indeed, according to the theory of computation, a problem of size N is computationally intractable if its time complexity lower bound is $\Omega(2^N)$. Despite that, as far as we know, the CC lower bound required

to sustain the maximal SE gain of OFDM-IM remains an open question across the literature. Consequently, no prior work can answer whether the OFDM-IM mapper indeed needs more asymptotic computational resources than its OFDM counterpart to sustain the maximal SE gain.

B. RELATED WORK

In this subsection, we review the literature related to the design and computational complexity of the OFDM-IM mapper.

1) EARLY ATTEMPT

The earliest mapper for OFDM-IM we find is due to [1]. The authors suggest a Look-Up Table (LUT) to map P_1 bits into one out of 2^{P_1} unique waveforms for relatively small P_1 . To avoid the exponential increase in storage implied by the optimal SE configuration, the authors employ a Johnson association scheme [15] to map P_1 based on the recursive matrix $A_{N,k} = [[1 \ 0]^T [A_{N-1,k-1} \ A_{N-1,k}]^T]$, in which Z^T is the transpose of a given matrix Z . Those authors remark that the matrix indexes decrease linearly with N towards the base case of recursion. However, we remark that the overall CC to write all rows of $A_{N,k}$ is exponential under the optimal SE configuration. To verify that, consider firstly that $A_{N,k}$ can be lower-bounded by $A_{k,k}$, since $k \leq N$. To build $A_{k,k}$, one needs at least two computational instructions to write the numbers 1 and 0 and two other independent and distinct recursive calls $A_{k-1,k-1}$ and $A_{k-1,k}$. In the worst-case analysis, the number of computational steps T to write all entries of $A_{k,k}$ can be captured by the recurrence $T(k) = 2 + 2T(k-1)$, which is trivially verified as $\Omega(2^k)$. Under the optimal SE setup, the proposed recursive scheme is $\Omega(2^N)$.

2) SUB-BLOCK PARTITIONING

To handle the OFDM-IM mapping overhead, Basar et al. [2], [7] propose the subblock partitioning (SP) approach. According to the survey work of [9], SP and the IxS algorithm presented by [2], [7] were (along with a low complexity detector) the distinctive methods responsible to release the true potential of the IM scheme, thereby shaping the family of index modulation waveforms as we know today. The key idea of SP is to attenuate the mapper CC by restricting the application of the IM technique to smaller portions of the symbol called “subblocks”. The length $n = \lfloor N/g \rfloor$ of each subblock depends on the number g of subblocks, which is a configuration parameter of OFDM-IM. Increasing g , decreases n , which causes the complexity of the IxS algorithm to decrease too. This way, SP introduces a trade-off between SE and CC, since the number of OFDM-IM waveforms increases for lower g [2], [7]. Thus, setting $g = 1$ (i.e., deactivating SP) means maximizing the SE efficiency. SP has represented the state of the art approach to balance SE and CC across the family of IM-based multi-carrier waveforms [8], [9], [12]–[14], [16]–[31].

3) (UN)RANKING ALGORITHMS

The IxS algorithm is a mandatory part for the asymptotic analysis of the OFDM-IM mapper. As observed by authors in [2], [7], the IxS task at the OFDM-IM transmitter (receiver) can be implemented as an unranking (ranking) algorithm. By reviewing the literature in combinatorics, one can find out several different (un)ranking algorithms, running at different time complexities [32]–[40]. At a first glance, building the optimal OFDM-IM mapper may just be a matter of adopting the IxS algorithm that establishes the complexity upper-bound for the (un)ranking problem, i.e., the fastest currently known algorithm. However, in the particular domain of OFDM-IM, k represents a trade-off between SE and CC. Thus, because the literature in pure combinatorics does not concern about SE as a performance indicator, it does not suffice to guide the design of an optimal OFDM-IM mapper. Therefore, to the best of our knowledge, *no prior analysis concerns whether the OFDM-IM mapper complexity minimization under the constraint of SE maximization.*

4) NOVEL SP-FREE OFDM-IM MAPPERS

In [41], the authors propose the concept of sparsely indexing modulation to improve the trade-off between SE and energy efficiency of OFDM-IM. Because this concept imposes k to be much less than N , the authors rely on [37] to perform IxS in $O(k \log N)$ time. With the achieved time complexity reduction, the authors present the first SP-free OFDM-IM mapper. However, the constraint on the value of k prevents the SE maximization. To identify the largest tolerable computational complexity to support the maximal SE, in a prior work [42] we present the spectro-computational efficiency (SCE) analysis. We define the SC throughput of an N -subcarrier mapper as the ratio $m(N)/T(N)$ (in bits per computational instructions¹), where $T(N)$ is the mapper's asymptotic complexity to map $m(N)$ bits into an N -subcarrier complex OFDM symbol. From this, the largest computational complexity $T(N)$ must satisfy $\lim_{N \rightarrow \infty} m(N)/T(N) > 0$, i.e., the SC throughput must not nullify as the system is assigned an arbitrarily large amount of spectrum. Based on that, in [43] we present the first mapper that supports all $2^{\lfloor \log_2 \binom{N}{N/2} \rfloor}$ waveforms of OFDM-IM in the same asymptotic time of the classic OFDM mapper. However, that proposed mapper still requires an extra space of $\Theta(N^2)$ look-up table entries in comparison to the classic OFDM mapper.

C. OUR CONTRIBUTION

In this work, we build upon [42] and [43] to demonstrate the first asymptotically optimal OFDM-IM mapper. By optimal, we mean our mapper enables all $2^{\lfloor \log_2 \binom{N}{N/2} \rfloor}$ waveforms of OFDM-IM under the same asymptotic time and space complexities of the classic OFDM mapper. Thus, we enhance our prior work [43] by reducing the space complexity of

¹or seconds, given the time each instruction takes in a particular computational apparatus e.g. FPGA, ASIC.

the mapper from $\Theta(N^2)$ to $\Theta(N)$. Besides, we enhance the upper-bound analysis of [42] by also showing the corresponding asymptotic lower-bounds that holds for any OFDM-IM implementation. In summary, we achieve the following contributions:

- We derive the general OFDM-IM mapper lower-bound $\Omega(k \log_2 M + \log_2 \binom{N}{k} + k)$ and show it becomes the same of the classic OFDM mapper under the optimal configuration (i.e., $g = 1, k = N/2, M = 2$). *This formally proves that enabling all OFDM-IM waveforms is not computationally intractable, as previously conjectured [9], [14];*
- Based on the upper and lower bound we identify, we show that the optimal OFDM-IM mapper must run in exact $\Theta(N)$ asymptotic complexity. An implementation running above this complexity (i.e. $T(N) = \omega(N)$) nullifies the SC throughput for arbitrarily large N , whereas one running below that (i.e., $T(N) = o(N)$) prevents the SE maximization;
- We present the first worst-case computational complexity analysis of the original OFDM-IM (de)mapper when the maximal SE is allowed. In this context, we show that the OFDM-IM mapper/demapper runs in $O(N^2)$ and becomes more complex than the Inverse Discrete fast Fourier Transform (IDFT)/DFT algorithm;
- We present an OFDM-IM mapper that runs in $\Theta(N)$ time;
- We implement an open-source library that supports all steps to map/demap an N -subcarrier complex frequency-domain OFDM-IM symbol. In our library, the IxS block is implemented with C++ *callbacks* to enable flexible addition of other unranking/ranking algorithms in the mapper. This facilitates the enhancement of currently supported algorithms to consider aspects not studied in this work, e.g. equiprobable IM waveforms [44], Hamming distance minimization [16]. Based on our theoretical findings, our OFDM-IM mapper library is the first implementation that enables the OFDM-IM SE maximization while consuming the same time and space asymptotic complexities of the classic OFDM mapper.

D. ORGANIZATION OF WORK

The remainder of this work is organized as follows. In Section II, we present the system model and the assumptions of our work. In Section III, we present the computational complexity scaling laws of the OFDM-IM mapper, namely, the lower and upper CC bounds under maximal SE. In Section IV, we analyze the throughput of the original OFDM-IM mapper. Because such analysis requires the IxS complexity, in that section we also analyze the CC of the original IxS algorithm and show how to achieve the lowest possible CC under the maximal SE. In Section V, we present a practical case study to validate our theoretical findings. Finally, in Section VI, we conclude our work and point future directions.

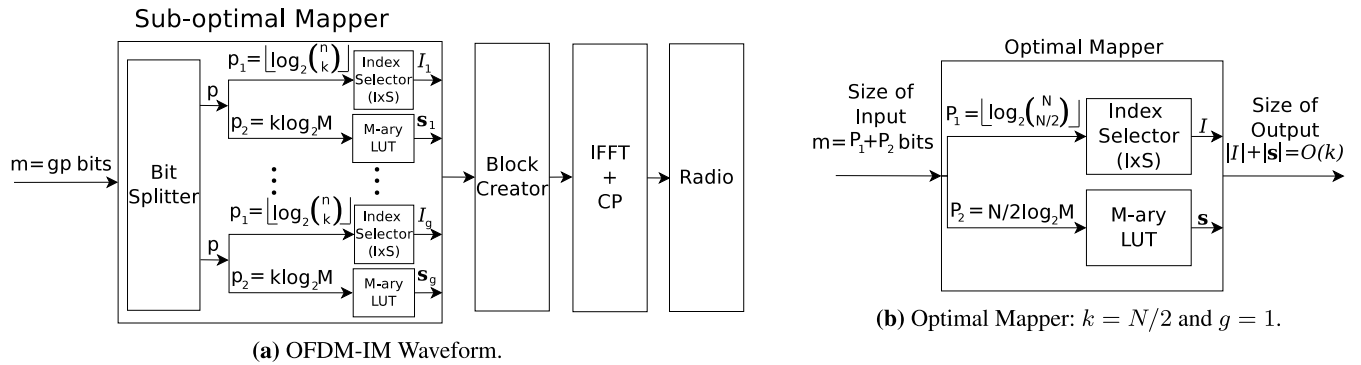


FIGURE 1. The OFDM-IM block diagram (Fig. 1a) mitigates the mapping computational complexity by subdividing the symbol into g small subblocks. To maximize the spectral efficiency (SE) gain over OFDM, the mapper has to set $g = 1$ and $k = N/2$ (Fig. 1b). We prove such optimal mapper can be implemented under the same time and space asymptotic complexities of the classic OFDM mapper.

II. SYSTEM MODEL AND ASSUMPTIONS

In this section, we review the OFDM-IM mapper (subsection II-A) and present its required design for SE maximization (subsection II-B). In subsection II-C, we present the assumptions to determine the lower and upper bound complexities for the OFDM-IM mapper.

A. OFDM-IM BACKGROUND

The SP mapping approach [2], [7] is responsible for the main changes OFDM-IM causes to the classic OFDM transmitter block diagram (as illustrated in Fig. 1a). SP is characterized by the configuration parameter $g \geq 1$, which stands for the number of subblocks within the N -subcarrier OFDM-IM symbol. Each subblock has $n = \lfloor N/g \rfloor$ subcarriers out of which k must be active. Considering an M -point modulator for the active subcarriers, each subblock maps $p = p_1 + p_2 = k \log_2 M + \lfloor \log_2 \binom{n}{k} \rfloor$ bits and the entire symbol has gp bits. The IxS algorithm of the β -th subblock ($\beta = 1, \dots, g$) is fed with $p_1 = \lfloor \log_2 \binom{n}{k} \rfloor$ bits and outputs vector I_β , the k -size vector containing the indexes of the subcarriers that must be active in the β -th subblock. To modulate the k active subcarriers, the “ M -ary modulator” step takes the remainder $p_2 = k \log_2 M$ bits as input and outputs the vector s_β , which consists of k complex baseband signals taken from an M constellation diagram. Then, each subblock forwards $2k$ values (i.e., $|s_\beta| + |I_\beta|$) to the “OFDM block creator”, which refers to s_β and I_β to modulate the k active subcarriers in each subblock and build the full N -subcarrier frequency domain OFDM-IM symbol. The remaining steps proceed as usual in OFDM [45].

B. OPTIMAL OFDM-IM MAPPER DESIGN

A requisite to maximize the OFDM-IM SE is to deactivate SP (i.e., set g to 1) and k to $N/2$ [7]. In theory, achieving the maximal SE is just a matter of setting OFDM-IM with the proper parameters. Indeed, by setting g to 1 (i.e., deactivating SP) and k to $N/2$, the resulting mapper (Fig. 1b) enables all 2^{P_1} waveforms of OFDM-IM [7]. However, the authors of the original OFDM-IM waveform recommend avoiding the

ideal setup because of the resulting computational complexity (compared with the classic OFDM mapper). In fact, by looking at Fig. 1b, one may observe that the ideal OFDM-IM mapper can be seen as a classic OFDM mapper with the addition of the IxS step. Because of this extra-step, the optimal OFDM-IM mapper requires more computational steps than its OFDM counterpart. However, our rationale is that, *if one can design an OFDM-IM mapper under the same asymptotic computational complexity of the classic OFDM mapper, then the extra computational operations required by the OFDM-IM mapper (compared to OFDM’s) are bounded by a constant even for arbitrarily large N .* Since the IxS complexity is not affected by M , without loss of generality, in this work we adopt $M = 2$ to achieve the largest gain in comparison to the OFDM counterpart [12], [13]. We refer to this as the optimal OFDM-IM setup.

C. ASYMPTOTIC ANALYSIS OF MULTICARRIER MAPPERS

We study the scaling laws of the OFDM-IM mapper as a function of the number N of subcarriers. In particular, for an N -subcarrier OFDM-IM symbol, we study the number $m(N)$ of bits per symbol and the mapper’s computational complexity $T(N)$ to map these bits into N complex baseband samples. We concern about the minimum and maximum asymptotic number of computational instructions required by any OFDM-IM mapper implementation. For this end, we employ the asymptotic notation as usual in the analysis of algorithms [46]. Our asymptotic analysis assumes the classic Random-Access Machine (RAM) model which is shown to be equivalent to the universal Turing machine [47]. The RAM model focus on counting the amount of basic computational instructions (e.g., data reading, data writing, basic arithmetic, data comparison) regardless of the technology of the underlying computational apparatus. For example, based on the RAM model, one verifies that a classic N -subcarrier BPSK-modulated OFDM mapper needs to perform N basic computational instructions of data reading, each as wide as $\log_2 2$ bits. This imposes a minimum of $\Omega(N)$ basic reading operations, regardless of a serial or parallel implementation.

Of course, performing these instructions in parallel yields more efficient runtime than performing them on a single processor. Anyway, the resources consumed by the parallel solution must scale on the derived computational complexity. Besides, for each reading, N independent baseband samples must feed N variables in the input of the IDFT DSP block, demanding a minimum space of $\Omega(N)$ complex variables.

III. INDEX MODULATION MAPPING COMPLEXITY BOUNDS

In this section, we derive the CC lower and upper bounds for an OFDM-IM mapper implementation through asymptotic analysis as a function of the number of subcarriers N .

A. OFDM-IM MAPPING TIME COMPLEXITY LOWER BOUND

To derive the general asymptotic lower bound for any OFDM-IM implementation, we refer to Fig. 1b. Recall we are considering an SP-free mapper design (i.e., $g = 1$) to enable the IM principle on the entire N -subcarrier OFDM-IM symbol. In this case, the lower bound is readily derived by observing that any implementation needs at least m basic computational steps to read the binary input to be mapped. Also, $O(k)$ basic computational steps are required to write the baseband samples in the mapper's output. Based on this, in Lemma 1 we derive the general CC lower bound for any OFDM-IM mapper implementation.

Lemma 1 (OFDM-IM Mapper General CC Lower Bound):

The minimum number of computational steps of any OFDM-IM mapper implementation is $\Omega(k \log_2 M + \lfloor \log_2 \binom{N}{k} \rfloor + k)$.

Proof: In the optimal OFDM-IM mapper, $g = 1$. Thus, the minimum number of computational steps to read the input is $m = P_1 + P_2 = \lfloor \log_2 \binom{N}{k} \rfloor + k \log_2 M$. Further, the OFDM-IM mapper must feed the "OFDM block creator" DSP step with the vectors of the active subcarriers indexes I_β and their corresponding baseband samples s_β ($\beta = 1, \dots, g$). Since the optimal mapper requires $g = 1$, there is only a single k -size vector I_1 and another k -size vector s_1 , yielding to the total output size of $2k = O(k)$. Thus, any OFDM-IM mapper implementation must write at least $O(k)$ units of data in its output. Therefore, because of the computational effort to read (input) and write (output) units of data, any OFDM-IM mapper solution will demand at least $\Omega(k \log_2 M + \lfloor \log_2 \binom{N}{k} \rfloor + k)$ computational steps. \square

When the optimal OFDM-IM setup is allowed, the general asymptotic lower bound of Lemma 1 becomes $\Omega(N)$ (Corollary 1). This stems from the fact that the number of index modulated bits P_1 approaches $N - \log_2 \sqrt{N}$ as $N \rightarrow \infty$ (Lemma 2). Therefore, although the number of waveforms of the optimal OFDM-IM setup grows exponentially on N , the CC of the IM mapping problem is not intractable (i.e., $\Omega(2^N)$) as previously conjectured [9], [14].

Lemma 2 (Maximum Number P_1 of Index Modulation Bits): The maximum number of index modulated bits P_1 approaches $N - \log_2 \sqrt{N}$ for arbitrarily large N .

Proof: By definition, $P_1 = \lfloor \log_2 \binom{N}{k} \rfloor$. If the maximum SE gain of OFDM-IM over OFDM is allowed, $\binom{N}{k}$ becomes the so-called central binomial coefficient $\binom{N}{N/2}$, whose well-known asymptotic growth is $O(2^N / \sqrt{N})$ [48]. From this, it follows that P_1 approaches $\log_2(2^N N^{-0.5}) = N - \log_2 \sqrt{N} = O(N)$ as $N \rightarrow \infty$. \square

Corollary 1 (OFDM-IM Mapper CC Lower Bound under Maximal Spectral Efficiency): Under the optimal spectral efficiency setup, the general mapping CC lower bound of OFDM-IM (Lemma 1) becomes $\Omega(N + P_1)$, which is the same of OFDM, i.e., $\Omega(N)$.

Proof: Since P_1 approaches $N - \log_2 \sqrt{N} = O(N)$ for arbitrarily large N (Lemma 2), the general asymptotic lower-bound $\Omega(N + P_1)$ becomes $\Omega(N)$, which is the minimum asymptotic number of computational steps performed by the classic OFDM mapper. \square

Lemma 1 and Corollary 1 imply that it is not possible to implement an OFDM-IM mapper with less than $\Omega(N)$ computational steps without sacrificing the SE optimality (Corollary 2). The corollary 2 states that any OFDM-IM mapper running in sub-linear complexity, i.e., $k = o(N)$ (which excludes the ideal $k = N/2$), prevents the maximal SE gain over OFDM. However, sub-optimal SE setups can be useful for sparse OFDM-IM systems, in which one gives up the maximal throughput on behalf of energy consumption minimization [41].

Corollary 2 (OFDM-IM Mapper Spectro-Computational Lower-Bound Trade-Off): No OFDM-IM mapper implementation can maximize the spectral efficiency (SE) gain over OFDM while running in $o(N)$ computational steps.

Proof: The asymptotic number of steps of any OFDM-IM mapper is subject to the general lower bound of $\Omega(k \log_2 M + \lfloor \log_2 \binom{N}{k} \rfloor + k)$ (Lemma 1). Thus, the only way to improve that bound consists of changing the OFDM-IM configuration parameters M and k . Out of all possible values of M and k , the *maximum* SE gain of OFDM-IM over OFDM is achieved *only* when $M = 2$ and $k = N/2$ [12], [13]. Also, under such optimal SE configuration, the general CC lower bound becomes $\Omega(N)$ (Corollary 1). Therefore, an OFDM-IM implementation cannot run below this bound (i.e., in sub-linear time) unless a non-optimal SE configuration is adopted for k . \square

B. OFDM-IM MAPPING TIME COMPLEXITY UPPER BOUND

The CC upper bound of a problem is usually defined as the complexity of the fastest currently known algorithm that solves it [49]. This definition does not suffice to our study because our asymptotic analysis is further constrained by the SE maximization. In fact, if the fastest known algorithm does not suffice to avoid an increasing bottleneck in the mapping throughput as N grows, then its complexity cannot be considered suitable to scale the mapper throughput on N . From this, we define the spectro-computational mapper throughput (Def. 1) and, based on its condition of scalability

(Def. 2), we derive the required computational complexity upper bound for any OFDM-IM mapper implementation (Lemma 3).

Definition 1 (The Spectro-Computational (SC) Throughput): Let $T(N)$ be the computational complexity (CC) to map $m(N)$ input bits into an N -subcarrier OFDM-IM symbol. We define $m(N)/T(N)$ in bits per computational steps (or seconds), as the spectro-computational (SC) throughput of the mapper.

Definition 2 (Spectro-Computational Throughput Scalability): The SC throughput $m(N)/T(N)$ of a mapper is not scalable unless the inequality (1) does hold.

$$\lim_{N \rightarrow \infty} \frac{m(N)}{T(N)} > 0 \quad (1)$$

As a side note about our Def. 2, we call attention to the fact that it consists of the asymptotic analysis. As such, “time complexity” means “amount of computational instructions” which can be translated to (but does not necessarily mean) wall clock runtime. That said, we recognize that a radio implementation that does not meet our Def. 2 can achieve the same wall clock runtime of another one that does. However, in this case, the CC $T(N)$ will translate into other relevant radio’s design performance indicators. For example, suppose that the largest complexity $T(N)$ to satisfy our Def. 2 in a particular DSP study is $O(N)$. A design that violates such a requirement by employing a more complex algorithm, let us say $O(N^2)$, can still reach the same wall clock runtime of a design that does not. However, since the overall number of performed computational instructions depends on the algorithm’s CC rather than the hardware technology, the average wall clock time to run a single computational instruction must be (much) lower in the $O(N^2)$ solution in comparison to the $O(N)$ counterpart. This pushes the algorithm’s CC to the hardware design rather than to the wall clock runtime. Therefore, the SC throughput of a radio design that violates our Def. 2 can scale with N but at the expense of impairing other relevant design performance indicators, such as the number of hardware components (e.g., logic gates), circuit area, energy consumption and manufacturing cost [50].

C. REQUIRED COMPLEXITY FOR MAXIMAL SE

Based on Def. 2, in Lemma 3 we show that the upper bound complexity any OFDM-IM mapper implementation must meet to ensure the optimal SE configuration is $O(N)$.

Lemma 3 (OFDM-IM Mapper Upper Bound under Optimal SE Configuration): Under the optimal SE configuration, the OFDM-IM mapper CC must be upper bounded by $O(N)$.

Proof: To meet the inequality 1 of Def. 2, $T(N)$ must be asymptotically less or equal than $m(N)$, i.e., $T(N) = O(m(N)) = O(P_1 + P_2)$. Under the optimal SE configuration, $k = N/2$ and $P_1 = \log_2 \binom{N}{N/2} = O(N)$ bits (Lemma 2). Therefore, $T(N)$ must be $O(N)$. \square

Based on the fact that the required OFDM-IM mapper upper bound complexity matches its lower bound order of growth in the optimal SE configuration, Theorem 1 tells us

that *the OFDM-IM mapper must run in $\Theta(N)$ time*. A solution requiring more asymptotic steps (i.e., $\omega(N)$) nullifies the mapper throughput as N grows, whereas one requiring fewer steps (i.e., $o(N)$) prevents the SE gain maximization (Corollary 2).

Theorem 1 (Required OFDM-IM Mapping Complexity): If the configuration that maximizes the OFDM-IM spectral efficiency gain over OFDM is allowed (i.e., $g = 1, k = N/2, M = 2$), the OFDM-IM mapper block of [2], [7] must run in $\Theta(N)$ computational steps.

Proof: Corollaries 1 and 2 show that any OFDM-IM mapper implementation running with less than $\Omega(N)$ computational steps cannot achieve the optimal SE gain over OFDM. In turn, Lemma 3 tells us that the mapper throughput nullifies for arbitrarily large N if its complexity requires more than $O(N)$ steps. Therefore, the exact asymptotic number of computational steps for any OFDM-IM mapper implementation under the optimal SE configuration must be $\Theta(N)$. \square

IV. THROUGHPUT ANALYSIS

Our theoretical findings summarized in Theorem 1, disclose the conditions for the computational feasibility of the optimal OFDM-IM mapper. The theorem requires exactly $\Theta(N)$ steps for the mapper. Since the M -ary LUT block of the OFDM-IM mapper (Fig. 1b) already runs in $N/2 = O(N)$ computational steps, to meet the theorem we just need to demonstrate the IxS block can be implemented with $\Theta(N)$ computational steps.

By relying on the literature in combinatorics, one can achieve (un)ranking complexities faster than the $\Theta(N)$ time required by our Theorem 1 e.g., [32], [33]. Such a performance, however, demands $k = o(N)$. Translated to the OFDM-IM domain, this means such algorithms prevent the SE maximization (Corollary 2). We identify that the original OFDM-IM mapper (and its variants) refer to the (un)ranking algorithm named “Combinadic” [34], [40].² In Subsection IV-A, we analyze the OFDM-IM SCE having Combinadic as the IxS block. We show that the Combinadic algorithm not only prevents the mapper to meet our Theorem 1 but also surpasses the $O(N \log_2 N)$ complexity of the IDFT DSP algorithm. In Subsection IV-B, we propose an optimal OFDM-IM mapper by adapting Combinadic to run in linear rather than quadratic complexity.

A. OFDM-IM MAPPER WITH COMBINADIC

We start this subsection by explaining how the Combinadic algorithm works. Then, we analyze its CC when the optimal SE configuration of OFDM-IM is allowed. Based on that, we conduct the spectro-computational analysis of the OFDM-IM mapper.

1) COMBINADIC TERMINOLOGY

The Combinadic algorithm relies on the fact that each decimal number X in the integer range $[0, \binom{N}{k} - 1]$ has a unique representation (c_k, \dots, c_2, c_1) in the combinatorial number

²In [51], the author points a fix to the algorithm of [40].

Algorithm 1 Combinadic Unranking (OFDM-IM IxS Transmitter)

```

1: {Inputs:  $X, N$ , and  $k \in [1, N]$ }
2: {Output: Array  $c_i$  ( $i \in [1, k]$ ) such that  $X = \sum_{i=1}^k \binom{c_i}{i}$  (Eq. 2)}
3:  $cc \leftarrow N$ ; {the current next candidate for  $c_i$ };
4: for  $i$  from  $k$  downto 1 do
5:   repeat
6:      $cc \leftarrow cc - 1$ ; {the first candidate for  $c_k$  is  $N - 1$ };

7:      $ccBinCoef \leftarrow \binom{cc}{i}$ ;
8:     until  $ccBinCoef \leq X$ 
9:      $c_i \leftarrow cc$ ;
10:     $X \leftarrow X - ccBinCoef$ ;
11:  end for
12: return array  $c$ ;

```

Algorithm 2 Combinadic Ranking (OFDM-IM IxS Receiver)

```

1: {Inputs: Array  $c_k > \dots > c_2 > c_1 \geq 0, N > c_k$ , and  $k \in [1, N]$ }
2: {Output:  $X = \sum_{i=1}^k \binom{c_i}{i}$  (Eq. 2)};
3:  $X \leftarrow 0$ ;
4: for  $i$  from 1 to  $k$  do
5:    $X \leftarrow X + \binom{c_i}{i}$ ;
6: end for
7: return  $X$ ;

```

Combinadic unranking and ranking algorithms referred to by the IxS block of original OFDM-IM mapper. In the maximal spectral efficiency OFDM-IM mapper (Fig. 1b), these algorithms run in $O(N^2)$, surpassing the computational complexity of the Fourier transform algorithm.

system [52] (Eq. 2). For OFDM-IM, X represents the P_1 -bit input (in base-10) and the coefficients $c_k > \dots > c_2 > c_1 \geq 0$ represent the indexes of the k subcarriers that must be active in the subblock.

$$X = \binom{c_k}{k} + \dots + \binom{c_2}{2} + \binom{c_1}{1} \quad (2)$$

Combinadic may refer to two distinct tasks, namely, unranking and ranking. The Combinadic unranking (shown in Alg. 1) consists in computing the array of coefficients c_i , $i \in [1, k]$, of Eq. (2) from the input X (along with N and k). The Combinadic unranking takes place in the IxS of the OFDM-IM transmitter. The reverse process, i.e., computing X given all k coefficients c_i , $i \in [1, k]$, is known as ranking and is performed by the IxS of the OFDM-IM receiver (Alg. 2).

2) COMBINADIC UNRANKING FUNCTIONING

The Combinadic unranking is shown in Alg. 1. It takes N , k and X as input parameters and outputs the array c_i , $i \in [1, k]$ such that $X = \sum_{i=1}^k \binom{c_i}{i}$ (Eq. 2). The *candidate* values for the coefficients c_i considered by the algorithm are $0, 1, \dots, N - 1$, which represent the indexes of the N subcarriers. The coefficients are determined from c_k until c_1 and the variable cc (line 3) stores the next candidate value for the current coefficient being computed. The first coefficient to be computed is c_k and its first candidate is $N - 1$. This is the value of cc in the very first execution of line 6. For every candidate value cc , the corresponding binomial coefficient $\binom{cc}{i}$ is computed and stored in the variable $ccBinCoef$ (line 7). If condition $ccBinCoef \leq X$ is satisfied (line 8), then the candidate value cc is confirmed as the value of c_i (line 9) and X is updated accordingly (line 10). This entire process repeats until all the remainder $k - 1$ coefficients are determined.

3) COMBINADIC UNRANKING COMPLEXITY

In a particular *worst-case* instance of Combinadic unranking (Alg. 1), the logic test of the inner loop (line 8) fails for $cc = N - 1, N - 2, \dots, k$ in the first iteration of the outer loop, i.e. when the first coefficient c_k is being determined. Thus, c_k is assigned to $k - 1$. This narrows the list of candidates (for the remainder $k - 1$ coefficients) to the values $k - 2, k - 3, \dots, 1, 0$. Since the combinatorial number system ensures that all k coefficients are distinct and that c_k is the largest one, a candidate value that fails for c_k can be discarded for c_{k-1} and so on. Thus, after c_k is determined, there must be at least $k - 1$ candidate values for the remainder $k - 1$ coefficients. Because of this, there is only one logic test per candidate value in the inner loop regardless of the number of coefficients. Since there are N candidate values, the inner loop takes $O(N)$ time regardless of the outer loop. In each test of the inner loop, Combinadic relies on the multiplicative identity (Eq. 3) to compute the binomial coefficient value in $O(k)$ time.

$$\binom{N}{k} = \prod_{i=1}^k \frac{N - i + 1}{i} \quad (3)$$

Therefore, the overall CC of the Combinadic unranking algorithm is $O(Nk)$. Considering the optimal SE configuration, $k = N/2$ and the complexity becomes $O(N^2)$, which is asymptotically higher than the $O(N \log N)$ complexity of the IDFT block.

4) COMBINADIC RANKING FUNCTIONING AND COMPLEXITY

The Combinadic ranking is shown in Alg. 2. It takes the array of coefficients c_i , $i \in [1, k]$ from the OFDM-IM detector and performs a summation of the k binomial

coefficients $\binom{c_1}{1} + \binom{c_2}{2} + \dots + \binom{c_k}{k}$ (Eq. 2). Since each binomial coefficient $\binom{c_i}{i}$ can be calculated in $O(i)$ time by the multiplicative formula (Eq. 3), and i ranges from 1 to k , the total number of multiplications performed by the algorithm is $1 + 2 + \dots + k = k(k + 1)/2 = O(k^2)$. Considering the optimal OFDM-IM setup, $k = N/2$, the overall complexity becomes $O(N^2)$ as with Combinadic unranking.

5) OFDM-IM MAPPER THROUGHPUT WITH COMBINADIC

We now analyze the SC throughput of the OFDM-IM mapper assuming the IxS block is implemented by the Combinadic algorithm [34], [40] as in the original OFDM-IM design [7]. Considering the optimal OFDM-IM setup, the total number of bits per symbol is $N/2 + \lfloor \log_2 \binom{N}{N/2} \rfloor$, whereas the IxS complexity is $O(N^2)$, as previously analyzed. Thus, according to Def. 2, the resulting SC throughput must satisfy Ineq. (4) as follows, otherwise it nullifies over N .

$$\lim_{N \rightarrow \infty} \frac{N/2 + \lfloor \log_2 \binom{N}{N/2} \rfloor}{O(N^2)} \stackrel{?}{>} 0 \tag{4}$$

According to the theory of computational complexity, the wall-clock time taken by a particular implementation of a $O(N^2)$ algorithm is bounded by the function κN^2 , in which the constant $\kappa > 0$ captures the wall-clock runtime taken by the asymptotic dominant instruction of the algorithm³ on a real machine. In turn, the number of index modulated bits tends to $N - \log_2 \sqrt{N}$ as N grows (Lemma 2). With basic calculus, one can verify that the limit in Ineq. (4) tends to zero for arbitrarily large N regardless of the value of κ , as follows.

$$\lim_{N \rightarrow \infty} \frac{N/2 + N - \log_2 \sqrt{N}}{\kappa \cdot N^2} = 0 \tag{5}$$

Therefore, referring to the original Combinadic algorithm to implement the IxS block in the optimal SE configuration causes the SC throughput of the OFDM-IM mapper to nullify as N grows.

B. OPTIMAL SPECTRO-COMPUTATIONAL MAPPER

To avoid the asymptotic nullification of the OFDM-IM mapper throughput while assuring the maximal SE, the IxS (un)ranking algorithm must run nor faster nor slower than $\Theta(N)$ (Thm. 1). In [37], the author presents four unranking algorithms, out of which one (called “unranking-comb-D”) can meet that requirement. Therefore, one can consider that algorithm to validate our theoretical findings. However, we remark that the Combinadic algorithm (referred to by the original OFDM-IM design) can benefit from the same properties of unranking-comb-D to run in $\Theta(N)$ rather than $O(N^2)$ under the optimal OFDM-IM setup. Similarly, the ranking algorithm (not proposed in [37]) can also run in $O(N)$ as well. Next, we explain how to adapt Combinadic to enable the minimum possible CC when the maximal SE is allowed.

³The instruction we choose to count in the analysis. Mostly, real or complex arithmetic instructions for DSP algorithms.

1) LINEAR-TIME COMBINADIC UNRANKING

The main bottleneck in the time complexity of Combinadic (un)ranking (Alg. 1) is the inner loop. As previously explained, the inner loop takes k iterations, each of which demands further $O(i)$ iterations to compute the binomial coefficients $\binom{c_i}{i}$. Since i ranges from k to 1 and the optimal OFDM-IM setup imposes $k = O(N)$, this yields $k \cdot O(i) = N/2 \times O(N/2) = O(N^2)$. To improve this complexity, note that only the first candidate binomial coefficient $\binom{c_k}{k} = \binom{N-1}{N/2}$ needs to be computed from scratch (in $O(k)$ time). Thus, such computation can be performed outside both loops of Combinadic (Alg. 1) and stored in a variable we refer to as *ccBinCoef*. The resulting modification is shown in line 4 of the Linear-time Combinadic unranking (Alg. 3). In this algorithm, the variables *cc* and *ccBinCoef* denote the candidate values for c_i and $\binom{c_i}{i}$, respectively. Following *ccBinCoef* = $\binom{c_k}{k}$, the next candidate binomial coefficient, either $\binom{N-1}{N/2-1}$ or $\binom{N-2}{N/2-1}$, can be computed from *ccBinCoef* itself in $O(1)$ time. In general, one can calculate $\binom{c_i-1}{i}$ and $\binom{c_i-1}{i-1}$ from $\binom{c_i}{i}$ by relying on the following respective equations [37]:

$$\binom{c_i - 1}{i} = ((c_i - i) * \binom{c_i}{i}) / c_i \tag{6}$$

$$\binom{c_i - 1}{i - 1} = (i * \binom{c_i}{i}) / c_i \tag{7}$$

The Eqs. (6) and (7) are exploited by lines 9 and 18 of Alg. 3, respectively. Thus, all remainder binomial coefficients within the logic test of the inner loop are computed in $O(1)$ time. Therefore, the complexity of Combinadic unranking improves from $k \cdot O(i) = N/2 \times O(N/2) = O(N^2)$ to $O(k) + k \cdot O(1)$, yielding $N/2 + N/2 \times O(1) = O(N)$ in the optimal OFDM-IM configuration.

2) LINEAR-TIME COMBINADIC RANKING

As with the Combinadic unranking, one can also reduce the time complexity of the Combinadic ranking (Alg. 2) from $O(N^2)$ to $O(N)$ by computing $\binom{c_i+1}{i}$ and $\binom{c_i+1}{i+1}$ from $\binom{c_i}{i}$ in $O(1)$ time rather than from scratch in $O(i)$ time with the multiplicative formula (Eq. 3). However, these $O(1)$ -time properties require the values in the array c to be consecutive, which can not be the case of OFDM-IM because these values depend on the data the user transmits. One can avoid calculating all k binomial coefficients from scratch by relying on the fact that the values $c_k > \dots > c_2 > c_1$ are restricted to the integer range $[0, N - 1]$. Based on this, the linear-time Combinadic ranking (Alg. 4) computes from scratch only one binomial coefficient (we refer to as *ccBinCoef*, line 10) from which at most $N - 1$ other coefficients can be computed sequentially in $O(1)$ time each. Since the value of all other coefficients is computed from *ccBinCoef*, this variable cannot be initialized with null binomial coefficients i.e., $\binom{c_i}{i}$ such that $c_i < i$. Thus, from lines 4 to 9, Alg. 4 looks for the largest i in the range $[0, \dots, i, \dots, N - 1]$ such that $c_i \geq i$. These lines take $O(k)$ iterations. In line 10, *ccBinCoef* is initialized as $\binom{c_i}{i}$ in $O(i)$ time, yielding a cumulative complexity of

Algorithm 3 Linear-Time Combinadic Unranking (OFDM-IM Index Selector Transmitter)

```

1: {Inputs:  $X, N$ , and  $k \in [1, N]$ }
2: {Output: Array  $c_i$  ( $i \in [1, k]$ ) such that  $X = \sum_{i=1}^k \binom{c_i}{i}$  (Eq. 2)}
3:  $cc \leftarrow N - 1$ ; {largest candidate for  $c_i$ };
4:  $ccBinCoef \leftarrow \binom{cc}{k}$ ; {candidate value for  $\binom{c_k}{k}$ };
5: for  $i$  from  $k$  downto 1 do
6:    $c_i \leftarrow cc$ ;
7:   while  $ccBinCoef > X$  do
8:     {Below,  $\binom{c_i-1}{i}$  is computed from  $\binom{c_i}{i}$  in  $O(1)$ };
9:      $ccBinCoef \leftarrow ((c_i-i) * ccBinCoef) / c_i$ ;
10:     $c_i \leftarrow c_i - 1$ ;
11:   end while
12:    $X \leftarrow X - ccBinCoef$ ;
13:   {Below,  $\binom{c_i-1}{i-1}$  is computed from  $\binom{c_i}{i}$  in  $O(1)$ };
14:    $cc \leftarrow c_i - 1$ ;
15:   if  $cc = 0$  then
16:     return array  $c$ 
17:   end if
18:    $ccBinCoef \leftarrow (i * ccBinCoef) / c_i$ ;
19: end for
20: return array  $c$ 

```

Algorithm 4 Linear-Time Combinadic Ranking (OFDM-IM Index Selector Receiver)

```

1: {Inputs: Array  $c_k > \dots > c_2 > c_1 \geq 0, N > c_k$ , and  $k \in [1, N]$ }
2: {Output:  $X = \sum_{i=1}^k \binom{c_i}{i}$  (Eq. 2)};
3:  $i \leftarrow 1$ ;
4: while  $i \leq k$  and  $c_i < i$  do
5:    $i \leftarrow i + 1$ ;
6: end while
7: if  $i > k$  then
8:   return 0;
9: end if
10:  $ccBinCoef \leftarrow \binom{c_i}{i}$ ;  $X \leftarrow 0$ ;
11: for  $cc$  from  $c_i$  to  $N - 1$  do
12:   if  $c_i = cc$  then
13:      $X \leftarrow X + ccBinCoef$ ;
14:      $ccBinCoef \leftarrow (ccBinCoef * (c_i + 1)) / (i + 1)$ ;
15:      $i \leftarrow i + 1$ ;
16:   else
17:      $ccBinCoef \leftarrow (ccBinCoef * (cc + 1)) / (cc + 1 - i)$ ;
18:   end if
19: end for
20: return  $X$ ;

```

Adaptation of the Combinadic algorithms (unranking 1 and ranking 2) referred to by the original OFDM-IM mapper to run in $O(N)$ time. We prove these adaptations enable the overall OFDM-IM mapper to maximize the spectral efficiency gain over OFDM while consuming the same time and space computational complexities of the classic OFDM mapper.

$O(k) + O(k) = O(k)$. From this, any consecutive binomial coefficient (either $\binom{c_i+1}{i}$ or $\binom{c_i+1}{i+1}$) can be computed in $O(1)$ time from $ccBinCoef = \binom{c_i}{i}$ as in the linear-time unranking algorithm. Since the total number of remainder binomial coefficients ranges from i to $N - 1$, the loop in line 11 computes all of them in $O(N - i) = O(N)$ time. Therefore, the overall complexity is $O(k) + O(k) + O(N)$ which becomes $O(N)$ under the optimal OFDM-IM setup (i.e., $k = N/2$).

3) SCALABLE OFDM-IM MAPPER THROUGHPUT

We now proceed with the SC analysis of the optimal OFDM-IM mapper (Fig. 1b) considering an IxS implementation that meets our Theorem 1. The analysis is as in subsection IV-A5, except for the fact that the IxS algorithm runs in $\Theta(N)$ time complexity. Thus, the SC throughput is given by

$$\lim_{N \rightarrow \infty} \frac{N/2 + N - \log_2 \sqrt{N}}{\kappa \cdot N} \quad (8)$$

As N grows, the time complexity is bounded by κN for some constant $\kappa > 0$. Similarly, the SC throughput of the mapper results in a non-null constant $\kappa > 0$, meeting the Def. 2. As explained in the subsection IV-A5, $\kappa > 0$ is constant that depends on the computational apparatus running the algorithm. Under the linear-time IxS complexity, the throughput of the OFDM-IM mapper does not nullify for

arbitrarily large N ,

$$\lim_{N \rightarrow \infty} \frac{N/2 + N - \log_2 \sqrt{N}}{\kappa \cdot N} = \frac{3}{2\kappa} > 0 \quad (9)$$

Note also that the throughput can increase with N if one achieves a $o(N)$ mapper. However, as demonstrated in Corollary 2, this conflicts with the optimal SE setup, thereby preventing the SE maximization.

V. IMPLEMENTATION AND EVALUATION

In this section, we present a practical case study to validate our theoretical findings. In subsection V-A, we introduce the open-source library we develop for the case study. In subsection V-B, we describe the methodology to assess and reproduce the empirical values of our experiments. Finally, in subsection V-C, we present the results of our practical case study that validate our theoretical findings.

A. OPEN-SOURCE OFDM-IM MAPPER LIBRARY

We wrote a C++ library that implements all OFDM-IM steps to map/demap an N -subcarrier complex frequency-domain symbol. We implement the IxS block with C++ *callbacks* to enable flexible addition of novel (un)ranking algorithms. In the released version, we implement the original IxS algorithm [7] and all the algorithms presented in this work (Algs. 3 and 4). We do not implement (un)ranking algorithms

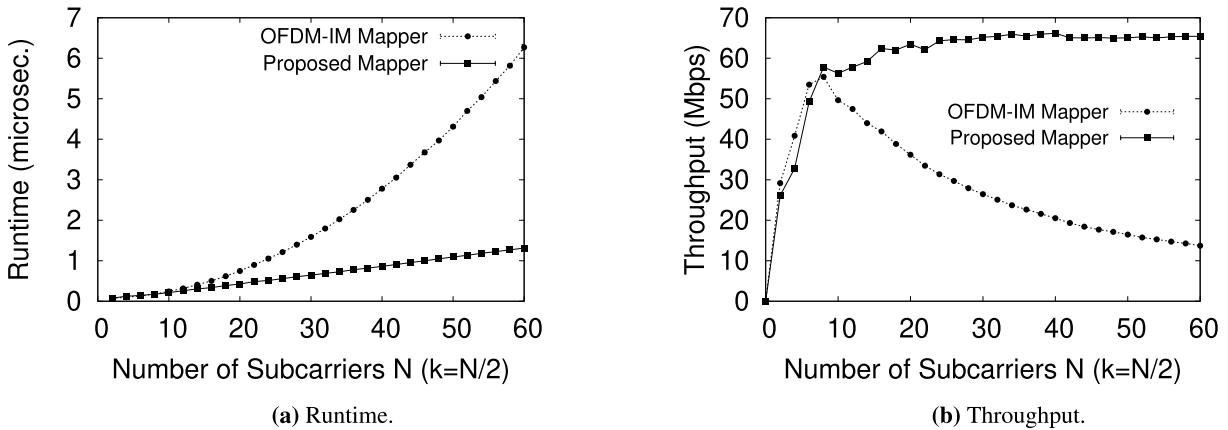


FIGURE 2. Mapper performance: Proposed vs. OFDM-IM.

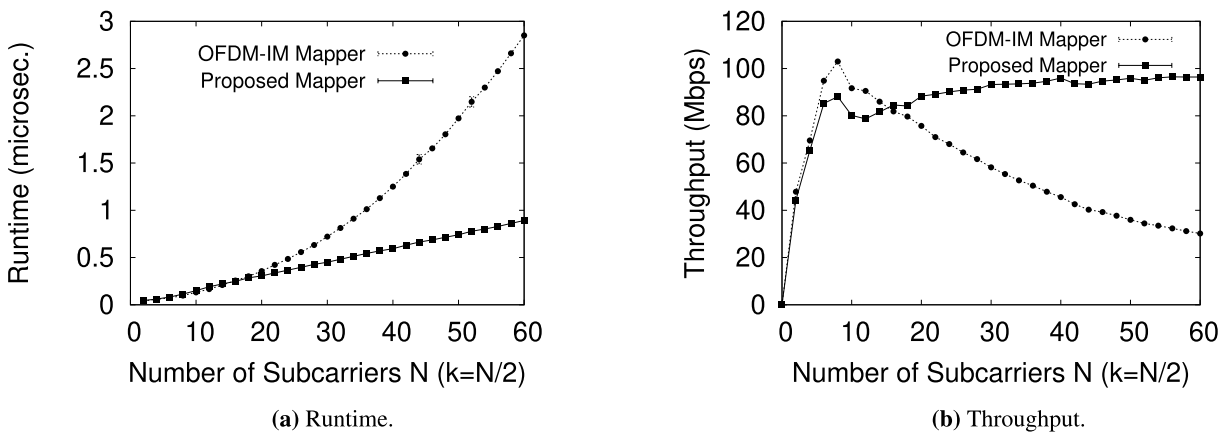


FIGURE 3. Demapper performance: Proposed vs. OFDM-IM.

that can reach a complexity that is asymptotically faster than required by our Theorem 1 e.g. [32], [33]. As previously explained (Corollary 2), performing (un)ranking faster than $\Theta(N)$ would require $k \neq N/2$, thereby preventing the SE maximization (Corollary 2). However, future works may implement IxS algorithms that improve the original OFDM-IM using other criteria (e.g. BER [16], [44].) than CC and SE. These and other IxS algorithms can also be included/evaluated in our library. The entire source code of our library, as well as detailed instructions on how to enhance it with novel IxS algorithms, are publicly available under the GPLv2 license in [53].

B. PERFORMANCE ASSESSMENT METHODOLOGY

We assess the runtime $T(N)$ (in secs.) and the throughput $m(N)/T(N)$ (in megabits per seconds, Def. 1) for both the original OFDM-IM mapper and our proposed mapper under the optimal SE configuration (i.e., $g = 1, k = N/2$ and $M = 2$). For each mapper, we assess the performance indicators at both the transmitter (mapper) and the receiver (demapper) on a 3.5-GHz Intel i7-3770K processor.

We sampled the wall-clock runtime $T(N)$ of each mapper with the standard C++ `timespace` library [54]

under the profile `CLOCK_MONOTONIC`. In each execution, we assigned our process with the largest real-time priority and employed the `isolcpus` Linux kernel directive to allocate one physical CPU core exclusively for each process. We generate the input for the mappers with the standard C++ 64-bit version of the Mersenne Twister (MT) 19937 pseudo-random number generator [55]. We set up three independent instances of MT19937_64 with seeds 1973272912, 1822174485 and 1998078925 [56]. Every iteration, three sampled $T(N)$ are forwarded to the Akaroa-2 tool [57] for statistical treatment. Akaroa-2 determines the minimum number of samples required to reach the steady-state mean estimation of a given precision. In our experiments, this precision corresponds to a relative error below 5% and a confidence interval of 95%. Besides, in all experiments the highest observed variance was below 10^{-3} and the average number of samples in the transient state was about 300.

Table 1 reports all assessed results for both the original OFDM-IM mapper and the proposed mapper at the transmitter (mapper). The table 2 reports the analogous results assessed at the receiver (demapper). From left to right, the tables present the following columns: the number N of symbol’s subcarriers, the number $m(N)$ of bits per

TABLE 1. Mapper performance: Proposed ("Prop.") vs. original OFDM-IM ("Orig").

N	$m(N)$ bits	IM Gain	IM Mapper	Runtime (μs)	$\pm\delta$ (μs)	Through put (Mbps)	x
2	2	1.00	Prop.	0.08	0.004	26.08	3792
			Orig.	0.07	0.002	29.15	2358
			Prop.	0.12	0.001	32.81	1854
4	4	1.00	Orig.	0.10	0.002	40.86	1710
			Prop.	0.14	0.003	49.23	1704
			Orig.	0.13	0.003	53.48	1656
8	10	1.25	Prop.	0.17	0.001	57.84	1686
			Orig.	0.18	0.001	55.40	1602
			Prop.	0.21	0.001	56.29	1536
10	12	1.20	Orig.	0.24	0.002	49.65	2208
			Prop.	0.26	0.002	57.78	1614
			Orig.	0.32	0.003	47.47	1872
14	18	1.28	Prop.	0.30	0.002	59.21	1524
			Orig.	0.41	0.003	43.99	1542
			Prop.	0.34	0.002	62.39	1728
16	21	1.31	Orig.	0.50	0.004	41.92	1476
			Prop.	0.39	0.002	62.03	1596
			Orig.	0.62	0.005	38.85	1494
20	27	1.35	Prop.	0.43	0.002	63.45	1524
			Orig.	0.75	0.007	36.18	1554
			Prop.	0.48	0.002	62.10	1884
22	30	1.36	Orig.	0.90	0.008	33.46	1518
			Prop.	0.51	0.002	64.31	1554
			Orig.	1.05	0.043	31.35	1512
26	36	1.38	Prop.	0.56	0.001	64.61	1560
			Orig.	1.21	0.007	29.70	1470
			Prop.	0.60	0.002	64.55	1536
28	39	1.39	Orig.	1.40	0.012	27.92	1512
			Prop.	0.64	0.003	65.19	1518
			Orig.	1.59	0.016	26.43	1476
30	42	1.40	Prop.	0.69	0.010	65.43	1524
			Orig.	1.79	0.012	25.07	1548
			Prop.	0.73	0.003	65.93	1560
34	48	1.41	Orig.	2.03	0.018	23.70	1518
			Prop.	0.78	0.008	65.47	1500
			Orig.	2.25	0.015	22.63	1482
38	54	1.42	Prop.	0.82	0.002	65.96	1608
			Orig.	2.50	0.017	21.57	1776
			Prop.	0.86	0.002	66.16	1524
40	57	1.42	Orig.	2.78	0.027	20.51	1530
			Prop.	0.91	0.003	65.06	1620
			Orig.	3.05	0.019	19.33	1458
44	62	1.41	Prop.	0.95	0.003	65.02	1686
			Orig.	3.37	0.027	18.41	1518
			Prop.	1.00	0.002	65.10	2118
46	65	1.41	Orig.	3.68	0.055	17.68	1548
			Prop.	1.05	0.002	64.98	1536
			Orig.	3.97	0.022	17.13	1476
50	71	1.42	Prop.	1.09	0.010	65.04	1530
			Orig.	4.31	0.035	16.47	1494
			Prop.	1.13	0.002	65.31	1578
52	74	1.42	Orig.	4.70	0.022	15.75	1494
			Prop.	1.18	0.002	65.03	1470
			Orig.	5.04	0.025	15.28	1500
56	80	1.43	Prop.	1.23	0.002	65.31	1440
			Orig.	5.44	0.026	14.71	1536
			Prop.	1.27	0.004	65.42	2064
58	83	1.43	Orig.	5.82	0.035	14.27	1512
			Prop.	1.31	0.003	65.42	1614
			Orig.	6.27	0.073	13.72	1476
60	86	1.43	Prop.	1.36	0.003	65.45	1548
			Orig.	6.70	0.027	13.28	1500
			Prop.	1.36	0.003	65.45	1548
62	89	1.43	Orig.	6.70	0.027	13.28	1500
			Prop.	$\Theta(N)$	0	$3/(2\kappa)$	∞
			Orig.	$\Theta(N^2)$	0	0	∞

symbol, the SE gain of the original OFDM-IM waveform against the classic OFDM mapper,⁴ the assessed (de)mapper,

⁴The maximum SE gain is $m(N)/N$ [13].

TABLE 2. Demapper performance: Proposed ("Prop.") vs. original OFDM-IM ("Orig").

N	$m(N)$ bits	IM Gain	IM Mapper	Runtime (μs)	$\pm\delta$ (μs)	Through put (Mbps)	x
2	2	1.00	Prop.	0.05	0.001	44.35	1644
			Orig.	0.04	0.001	47.85	1680
			Prop.	0.06	0.001	65.25	1674
4	4	1.00	Orig.	0.06	0.000	69.57	1620
			Prop.	0.08	0.001	85.26	1758
			Orig.	0.07	0.001	94.85	1746
8	10	1.25	Prop.	0.11	0.001	88.11	2208
			Orig.	0.10	0.001	102.99	1626
			Prop.	0.15	0.002	80.16	1518
10	12	1.20	Orig.	0.13	0.001	91.67	1704
			Prop.	0.19	0.002	78.70	1524
			Orig.	0.17	0.002	90.53	1536
14	18	1.28	Prop.	0.22	0.001	81.63	1536
			Orig.	0.21	0.001	86.00	1614
			Prop.	0.25	0.002	84.58	1512
16	21	1.31	Orig.	0.26	0.001	81.87	1758
			Prop.	0.29	0.003	84.18	1536
			Orig.	0.30	0.002	79.68	1524
20	27	1.35	Prop.	0.31	0.002	88.32	1542
			Orig.	0.36	0.001	75.74	1614
			Prop.	0.34	0.004	89.13	1596
22	30	1.36	Orig.	0.42	0.002	70.99	1542
			Prop.	0.37	0.003	90.11	1488
			Orig.	0.49	0.007	68.03	1704
26	36	1.38	Prop.	0.40	0.003	90.86	1500
			Orig.	0.56	0.001	64.49	1530
			Prop.	0.43	0.008	91.23	1482
28	39	1.39	Orig.	0.63	0.003	61.69	1626
			Prop.	0.45	0.002	93.23	1566
			Orig.	0.72	0.001	58.20	1548
30	42	1.40	Prop.	0.48	0.007	93.38	1464
			Orig.	0.81	0.002	55.35	1476
			Prop.	0.51	0.003	93.51	1602
34	48	1.41	Orig.	0.91	0.001	52.67	1560
			Prop.	0.54	0.011	93.82	1548
			Orig.	1.01	0.002	50.42	1878
38	54	1.42	Prop.	0.57	0.002	94.62	1500
			Orig.	1.13	0.002	47.85	1512
			Prop.	0.59	0.003	95.99	1464
40	57	1.42	Orig.	1.25	0.003	45.58	1548
			Prop.	0.63	0.003	93.58	1548
			Orig.	1.39	0.014	42.58	1722
44	62	1.41	Prop.	0.66	0.002	93.37	1464
			Orig.	1.54	0.051	40.27	2259
			Prop.	0.69	0.001	94.68	1512
46	65	1.41	Orig.	1.66	0.002	39.27	1656
			Prop.	0.71	0.006	95.18	1554
			Orig.	1.80	0.006	37.68	1548
50	71	1.42	Prop.	0.74	0.002	95.88	1458
			Orig.	1.97	0.013	35.98	1488
			Prop.	0.78	0.009	95.05	1530
52	74	1.42	Orig.	2.15	0.055	34.45	1506
			Prop.	0.80	0.002	96.08	1542
			Orig.	2.30	0.002	33.49	1548
56	80	1.43	Prop.	0.83	0.002	96.50	1572
			Orig.	2.47	0.003	32.35	1512
			Prop.	0.86	0.003	96.42	1566
58	83	1.43	Orig.	2.66	0.002	31.19	1524
			Prop.	0.89	0.003	96.41	1482
			Orig.	2.85	0.002	30.16	1506
60	86	1.43	Prop.	0.92	0.003	96.33	1614
			Orig.	3.09	0.064	28.76	1488
			Prop.	$\Theta(N)$	0	$3/(2\kappa)$	∞
62	89	1.43	Orig.	$\Theta(N^2)$	0	0	∞

the assessed runtime $T(N)$, the half-width of the confidence interval δ for $T(N)$, the achieved (de)mapping throughput, and the number x of samples needed to achieve the required

precision. The source-code of all our experiments is publicly available under GPLv2 license in [53].

C. RESULTS

In Fig. 2a and Fig. 2b, we respectively plot the runtime and the throughput performances of the compared mappers for $N = 2, 4, \dots, 62$. Although only particular values of N verify in industry standards (e.g. $N = 48$ [58], $N = 52$ [59]), we range it from small to large values to illustrate the asymptotic shape predicted by our throughput analysis. Detailed information about these plots are reported on the Table 1. As predicted by our theoretical analysis (Subsections IV-A5 and IV-B), in the ideal setup, the runtime order of growth of the original OFDM-IM mapper is asymptotically larger than our proposed mapper (Fig. 2a). From the theoretical analysis, we know these complexities are $O(N^2)$ and $O(N)$, respectively. Naturally, the runtime curves of both mappers increase monotonically towards infinite as the number N of subcarriers grows. However, because the runtime order of growth of the original OFDM-IM mapper is larger than the number $m(N) = N/2 + \log_2 \binom{N}{N/2} = O(N)$ of bits per symbol, the throughput $m(N)/T(N)$ of this mapper nullifies as N grows (Fig. 2b). This validates the theoretical analysis we show in subsection IV-A5.

By contrast, when our proposed mapper takes place, both the resulting computational complexity $T(N)$ and the total of bits $m(N)$ per symbol increases in the same order of growth. Thus, the throughput $m(N)/T(N)$ tends to a non-null constant. In particular, according to our theoretical analysis in subsection IV-B3, this is $m(N)/T(N) = 3/(2\kappa)$. Recall that the constant $\kappa > 0$ captures the wall-clock runtime taken by the asymptotic dominant instruction of the algorithm on a real machine. However, in our practical case study, the assessed runtime $T(N)$ encompasses all computational instructions performed by each (de)mapper. Thus, κ represents an average of the runtime taken by each kind of instruction on the machine of our testbed i.e., the Intel i7-3770K processor. From the assessed throughput $m(N)/T(N)$, the average value of κ can be computed based on Eq. (9), which is $\kappa = 3/2 \cdot 1/(m(N)/T(N))$. In our testbed, the average runtime per computational instruction was $0.02 \mu s$.

In Fig. 3a and Fig. 3b, we respectively plot the runtime and the throughput performances of the compared demappers for different values of N . Detailed informations of these plots are reported on the Table 2. As in the mapper analysis, the throughput of the original OFDM-IM demapper tends to zero as N grows whereas the throughput of our proposed demapper tends to a non-null constant under the same conditions. If compared against its corresponding mapper, we verify that our proposed demapper presents larger throughput. This means that, although both our mapper and demapper have the same $O(N)$ asymptotic complexity, the demapper implementation is less complex concerning the constant κ . Indeed, we verify an average $\kappa = 0.015 \mu s$ for the demapper in contrast with the $0.02 \mu s$ for the mapper.

VI. CONCLUSION AND FUTURE DIRECTIONS

In this work, we studied the trade-off between spectral efficiency (SE) and computational complexity (CC) $T(N)$ of an N -subcarrier OFDM with Index Modulation (OFDM-IM) mapper. We identified that the CC lower bound to map any of all $2^{\lfloor \log_2 \binom{N}{N/2} \rfloor}$ OFDM-IM waveforms is $\Omega(N)$. With this, we formally proved that enabling all OFDM-IM waveforms is not computationally intractable, as previously conjectured [9], [14]. Besides, we showed that any algorithm running faster than this lower bound prevents the OFDM-IM SE maximization. We also presented the spectro-computational efficiency (SCE) metric both to analyze the mapper's throughput and identify an upper bound for the mapper's complexity $T(N)$ under the maximal SE. In this context, we proved that the worst tolerable CC for the mapper is $O(N)$ otherwise, the mapper's throughput nullifies as the system is assigned more and more subcarriers. We showed that this is the case of the original OFDM-IM mapper [7], in which the $O(N^2)$ CC surpasses the $O(N \log_2 N)$ CC of the IDFT/DFT algorithm. Then, we presented an OFDM-IM mapper that enables the largest SE under the minimum possible CC.

We demonstrate our theoretical findings by implementing an open-source library that supports all DSP steps to map/demap an N -subcarrier complex frequency-domain OFDM-IM symbol. Our implementation supports different index selector algorithms and is the first to enable the SE maximization while preserving the same time and space asymptotic complexities of the classic OFDM mapper. With our library, we showed that the OFDM-IM mapper does not need compromise approaches that prevail in the OFDM-IM literature such as subblock partitioning (SP) [7]–[9], [13], [25], adoption of few active subcarriers [41] or extra space complexity [43].

Future works may consider extra performance indicators in the analysis (in addition to CC and SE) such as bit-error rate [16], [44]. Moreover, our mapper can be directly applied to other IM systems that rely on the same index selector of the original OFDM-IM mapper such as spatial modulation systems [60] and dual mode OFDM-IM [25]. Besides, our methodology can guide the activation of all waveforms in other variants of OFDM-IM such as multiple mode OFDM-IM [26].

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SAULO QUEIROZ received the B.Sc. and M.Sc. degrees from the Federal University of Amazonas, Brazil, in 2006 and 2009, respectively, with the focus on the efficiency of networking algorithms. He is currently pursuing the Ph.D. degree with the University of Coimbra, Portugal. He is also a Permanent Lecturer at the Department of Computer Science, Federal University of Technology (UTFPR), Brazil. Over the last decade, he has lectured disciplines on computer science such as design and analysis of algorithms and signal communication processing. With his research on networking, he has contributed with open source initiatives such as Google Summer of Code. His current research interests comprise the design and analysis of signal communication algorithms.



JOÃO P. VILELA received the Ph.D. degree in computer science from the University of Porto, Portugal, in 2011. He was a Professor with the Department of Informatics Engineering, University of Coimbra. He was a Visiting Researcher at the Coding, Communications and Information Theory Group, Georgia Tech, working on physical-layer security, and the Network Coding and Reliable Communications Group, MIT, working on security for network coding. He is currently a Professor with the Department of Computer Science, University of Porto. In recent years, he has been the coordinator and team member of several national, bilateral, and European-funded projects in security and privacy of computer and communication systems, with the focus on wireless networks, the Internet of Things, and mobile devices.



EDMUNDO MONTEIRO (Senior Member, IEEE) is currently a Full Professor with the University of Coimbra, Portugal. He has more than 30 years of research experience in the field of computer communications, wireless networks, quality of service and experience, network and service management, and computer and network security. He has participated in many Portuguese, European, and international research projects and initiatives. His publication list includes over 200 publications in journals, books, and international refereed conferences. He has coauthored nine international patents. He is a member of the Editorial Board of *Wireless Networks* (Springer) journal and is involved in the organization of many national and international conferences and workshops. He is a Senior Member of the IEEE Communications Society and the ACM Special Interest Group on Communications. He is also a Portuguese Representative in IFIP TC6 (Communication Systems).

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