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# Influence of plastic wake length on results of 3D numerical modelling of plasticity induced crack closure

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**Abstract**

The numerical analysis of the plasticity induced crack closure requires the development of the plastic wake. Transient behaviour is observed when the crack starts to grow. The plastic wake length has an influence on the crack closure results and a great impact on the computational cost. Previous works have analysed the influence of this parameter considering bi-dimensional specimens in either plane strain or plane stress conditions. Lately, some three-dimensional models have appeared in order to analyse the crack closure phenomenon. The main scope of this study is to quantify and comprehend the minimum length required to stabilise the crack opening and closure values considering a three-dimensional model. On this purpose, a CT aluminium specimen has been modelled three-dimensionally considering a straight crack front and several calculations have been made in order to evaluate the influence of the simulated plastic wake. The numerical analysis is made in terms of crack closure and opening values.

*Keywords:* finite element analysis; plasticity induced crack closure; transient behaviour; plastic wake.

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## 1. Introduction

Crack closure phenomenon was introduced by Elber (Elber (1970), Elber (1971)) in the early 70s. The interaction between the fracture surface of the crack flanks implies a decrease of the stress intensity at the crack tip.

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Consequently, there is an increase in the fatigue life of a mechanical component under cyclic loadings. According to Elber's point of view, a residual plastic wake is created as the crack grows under cyclic loading conditions. The yielded material behaves as a shield to protect the crack tip from the load applied. Therefore, the crack growth rate is influenced by the load applied, the geometry of the mechanical component, but also by the contact of the crack flanks behind the crack tip.

**Nomenclature**

| | |
|---|---|
| $a$ | crack length |
| $b$ | specimen thickness |
| $K_{max}$ | Maximum stress intensity factor |
| $K_{ttop}$ | Crack opening tip tension values |
| $r_{pD}$ | Dugdale's plastic size |
| $s_{me}$ | Minimum element size |
| $R$ | Load ratio |
| $\alpha$ | Constraint factor |

Although there are some sceptical researchers to this phenomenon (Vasudevan et al. (2001), Sadananda and Vasudevan (2003)), there is a great amount of analytical, experimental and numerical work that support the influence of plastic wake on the premature contact between crack flanks and, consequently, influencing fatigue crack propagation.

Numerical models have been previously used to analyse Plasticity Induced Crack Closure (PICC). Most of them were bi-dimensional models considering either plane stress or plane strain conditions (Antunes et al. (2004), Antunes et al. (2015)). Lately, some three-dimensional models have been considered (Chermahini and Blom (1991), Gonzalez-Herrera and Zapatero (2008), Alizadeh et al. (2007)). These analyses allow to obtain crack closure results at the surface, mid-plane and along its evolution through the thickness. Besides, it allows to consider three-dimensional parameters that are disregarded in bi-dimensional analyses such as the crack front curvature or the relationship between the load applied and the specimen thickness. The shape of the crack front has a huge influence on the stress and strain fields around the crack front (Camas et al (2011), Camas et al. (2012)). The yielded area at the surface of the specimen increases when the radius of curvature decreases. Nevertheless, the methodology employed has been inherited from those developed for bi-dimensional cases.

A great number of numerical parameters must be considered when this kind of problems are analysed. The mesh size near the crack front, the elastic-plastic behaviour modelling of the material, the number of loads applied between node releases, the instant during the cycle in which the nodes are released, the methodology considered to measure the crack closure or opening, and the required plastic wake length have a great influence on the results. In the literature, some attempts to optimise these parameters in bi-dimensional models can be easily found (Antunes et al. (2015), Oplt et al. (2019)).

The current computational capabilities allow a comprehensive study of the influence of different modelling parameters on the crack closure results considering three-dimensional models. A previous study of the authors (Camas et al. (2018)) analysed the influence of the mesh size around the crack front considering three-dimensional models and updated the minimum element size classical bi-dimensional recommendation to 60 divisions of the Dugdale's plastic zone size.

A key issue is the analysis of the plastic wake. A large plastic wake developed increases the numerical accuracy, but this implies an increase in the computational cost. The numerical effort is especially clear when three-dimensional models are considered. When the crack propagation starts, a transient behaviour can be observed in the numerical results. This behaviour depends on the two-dimensional state of stress (Antunes et al. (2014)). Under plane stress conditions, an increase of crack opening results with crack growth occurs when constant amplitude loads are applied. On the contrary, under plane strain conditions, the same results show a maximum value followed by a steady decrease.

Besides, one of the main issues that these numerical models face is the validation with experimental data. Lately, digital image correlation have been used to validate numerical models (Camas et al. (2016), Camas et al. (2017)). However, with this technique, only information at the surface is available.

The aim of this work is to analyse the influence of the transient behaviour on plasticity induced crack closure results considering three-dimensional models. The numerical accuracy is analysed in terms of crack closure and opening values along the specimen thickness. For this purpose, a CT aluminium specimen has been modelled three-dimensionally and several calculations have been made to evaluate the influence of the simulated plastic wake length.

## 2. Finite element model

Figure 1 shows the geometry of the compact tension (CT) specimen studied in the present work. The main dimensions of this specimen are *W*=50mm, *a*=20mm and *b*=3mm. Although it is well known that the crack front presents some kind of curvature, a straight crack front was considered in the numerical model for all the cases analysed. Because of the symmetry of the geometry and the applied loads, only a quarter of the specimen was modelled considering the appropriates boundary conditions. The commercial finite element software ANSYS was used to build the three-dimensional model and to run the simulations.

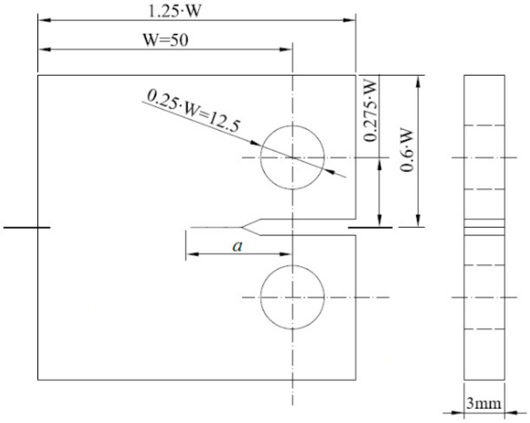

Fig. 1. CT specimen scheme.

In this kind of problems, the most critical region is the area close to the crack front. In this area, there are deep stress and strain gradients. The elements near the crack front must be very small in order to capture the stress and strain variations with enough precision. However, the element sizes of the specimen must increase with the distance from the crack front to avoid an excessive computational cost. For this reason, the specimen was meshed considering two different approaches. The specimen was divided in two different volumes: one, close to the crack front and the other one, the rest of the specimen. Near the crack front, a homogeneous and structured mesh with hexahedral elements was used, while the rest of the specimen was meshed considering an unstructured mesh. In this last region, the material behaviour is always elastic, never reaching any point of this volume, the yield stress. The mesh size and the dimension of the first area was determined by the Dugdale's equation (equation 1).

$$r_{pD} = \frac{\pi}{8}\left(\frac{K_I}{\sigma_y}\right)^2 \tag{1}$$

Previous fracture mechanic analyses (Camas et al. (2011), Camas et al. (2012), Garcia-Manrique et al. (2017), Lopez-Crespo et al. (2008)) showed that an important issue to obtain with accuracy the behaviour along the thickness is to mesh the specimen along the thickness with a fine mesh. Specially near the surface.

The minimum element size in the direction of the crack propagation and through the thickness was determined following the conclusions obtained in a previous study in which the effect of the mesh size in PICC results was analysed (Camas et al. (2018)). Figure 2 shows the different meshes considered in this model.

An aluminium alloy Al-2024-T351 is considered in this study. This material shows weak hardening and its main constants are $E$=73.5GPa, $\sigma_y$=425MPa, $K'$=685MPa and n'=0.073. The cyclic stress strain curve was considered to model the material behaviour. This approach implies savings in the computational cost as the number of loading cycles can be reduced. A three-linear stress-strain curve with an isotropic hardening law was employed to model the cyclic material behaviour.

Six different plastic wake lengths were considered ranging from 0.05 to 0.8 times the Dugdale's plastic size. For all of them, constant amplitude loading with crack growth was considered. The load applied is said in terms of stress intensity factor, being $K$=25MPa·m$^{1/2}$ and the stress ratios considered was $R$=0.1.

The crack advance in the numerical model was simulated by releasing nodes. A whole load cycle, load and unload, was applied between node releases. The minimum element size considered was 90 times smaller than $r_{pD}$.

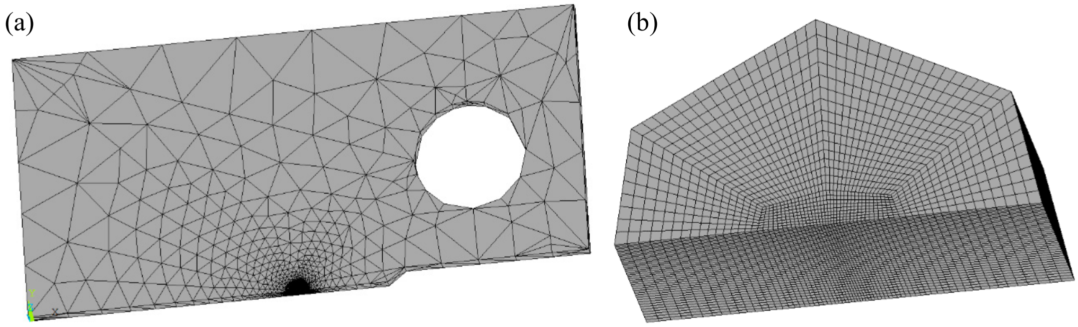

Fig. 2. (a) 3D finite element model; (b) mesh around the crack front.

## 3. Results

In this section, the results for each plastic wake length are presented. It is important to note that the final crack length is always the same and equal to $a$=20mm. The starting distance at which the crack begins to grow changes for each case being the total plastic wake length simulated, the one indicated in each case. This approach allows to reduce the computational cost.

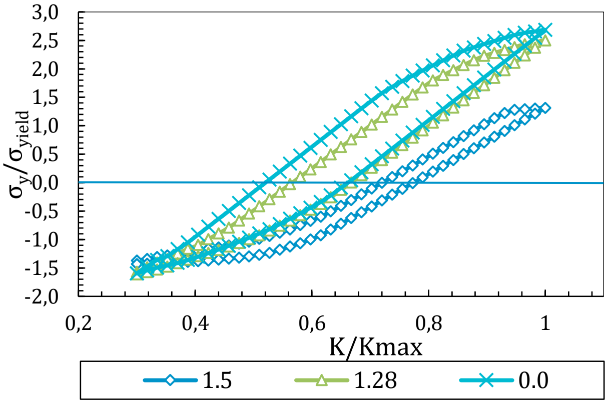

Fig. 3. $y$-stresses for three nodes at different positions along the thickness.

The $y$-stresses at the nodes located along the crack front define the opening and closure values. The crack is open when the stresses go over compression to tension and is closed when go over tension to compression. Figure 3 shows the $y$-stresses of the nodes at the crack front at three different positions along the thickness during the last loading cycle. The values at 1.5 represent the results obtained at the surface, while values at 0.0 refer to the mid-plane. It can be seen that the stresses at the surface are smaller than the ones at the mid-plane. Garcia-Manrique et al. (2013) showed that the stress distribution along the thickness is uneven, supporting more stress the interior than the surface. Besides, the crack closure effect influences the stress distribution, being more prominent at the surface than in the interior. As a consequence of a previous bi-dimensional analysis (Gonzalez-Herrera and Zapatero (2009)), the minimum plastic wake simulated in this three-dimensional study is $0.05{\cdot}r_{pD}$. Plane strain results showed that simulating a plastic wake length of $0.1{\cdot}r_{pD}$ was enough, but for plane stress conditions, a longer plastic wake was required to stabilise the opening and closure results.

Figure 4 shows the crack opening tip tension values along the thickness for the different plastic wake lengths considered. Figure 4(a) shows the evolution along the thickness of the crack opening values for different plastic wake lengths. It can be seen that the results are quite similar at the interior of the specimen for all the plastic wakes considered. Some differences can be observed near the surface, although for plastic wake lengths greater than $0.2{\cdot}r_{pD}$ values collapse in a single curve. As expected, the opening values at the surface are greater than the values at the interior. This implies that the crack opens later at the surface than in the mid-plane.

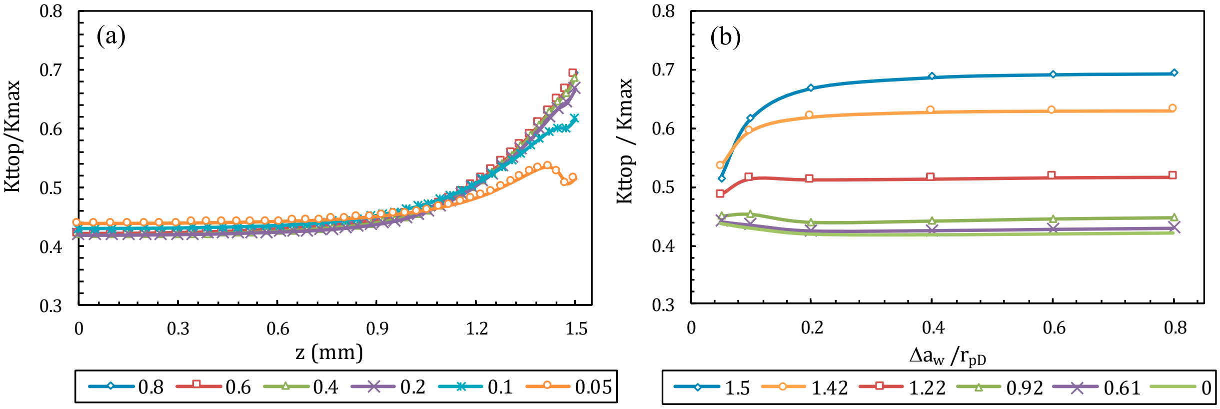

Fig. 4. Crack opening in terms of plastic wake (a) along the thickness (b) and for different planes along the thickness when considering a straight crack front.

To clarify the evolution along the thickness, Figure 4(b) shows the opening results and its evolution with the plastic wake length at different planes along the thickness. As the main variations in the results happen in the first 0.5mm close to the surface, three different planes were considered within this distance, while in the rest of the thickness, other three were considered. The surface is represented at 1.50 while 0.00 represent the values in the mid-plane. As occurs in the bi-dimensional analysis at plane stress state, the opening values at the surface increase gradually with the simulated plastic wake length. This behaviour remains at the planes near the surface, but the increase becomes smaller as we move into the specimen until a certain plane is reached and the trend is inverted.

In the mid-plane, the opening values slightly decrease with the plastic wake length. This trend is different at what was observed in the previous bi-dimensional analysis at plane strain. However, the fact that the values at plane strain stabilize with less plastic wake than at plane stress state, remains the same for the three-dimensional analysis.

Same behaviour can be observed when the displacements are considered instead of the stresses. There is a little difference. It is necessary to simulate at least 0.6 times the Dugdale's plastic zone size to stabilize the results.

These results correspond to a straight crack front. As said above, it is well known that the crack front presents some kind of curvature. This curvature is affecting to the stress and strain distribution along the thickness. In this way, the yielded area on the surface increases when the radius of curvature decreases, while the yielded area in the mid-plane stays constant (Camas et al. (2012)). Therefore, an increase in the opening values in the surface is expected, although further investigation is needed. It is important to note the time consuming of each simulation which varies, for the same computer configuration (i7 with 8Gb RAM), from 55 hours for the $0.8{\cdot}r_{pD}$ case to the 5 hours of the $0.05{\cdot}r_{pD}$ one.

## 4. Conclusions

In this study, the influence of the plastic wake length previously simulated on plasticity induced crack closure results has been analysed considering a three-dimensional model with a straight crack front. The huge relevance that this parameter has on the crack opening results has been shown, analysing their evolution along the thickness. These results indicate that there is an initial transient behaviour, which is different depending on the plane considered. At least, a plastic wake length of 0.4 times Dugdale's plastic size is necessary to develop in order to stabilize the main results when considering the tip tension opening values.

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